Computer algebra independent integration tests

4-Trig-functions/4.5-Secant/4.5.0-a-sec-^m-b-trg-^n

Nasser M. Abbasi

May 23, 2020 Compiled on May 23, 2020 at 1:09am

Contents

Chapter 1

Introduction

This report gives the result of running the computer algebra independent integration problems. The listing of the problems are maintained by and can be downloaded from `https://rulebasedintegration.org`

The number of integrals in this report is [299]. This is test number [115].

1.1 Listing of CAS systems tested

The following systems were tested at this time.

1. Mathematica 12.1 (64 bit) on windows 10.
2. Rubi 4.16.1 in Mathematica 12 on windows 10.
3. Maple 2020 (64 bit) on windows 10.
4. Maxima 5.43 on Linux. (via sagemath 8.9)
5. Fricas 1.3.6 on Linux (via sagemath 9.0)
6. Sympy 1.5 under Python 3.7.3 using Anaconda distribution.
7. Giac/Xcas 1.5 on Linux. (via sagemath 8.9)

Maxima, Fricas and Giac/Xcas were called from inside SageMath. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	solved	Failed
Rubi	% 100. (299)	% 0. (0)
Mathematica	% 100. (299)	% 0. (0)
Maple	% 75.25 (225)	% 24.75 (74)
Maxima	% 31.1 (93)	% 68.9 (206)
Fricas	% 35.45 (106)	% 64.55 (193)
Sympy	% 5.35 (16)	% 94.65 (283)
Giac	% 12.04 (36)	% 87.96 (263)

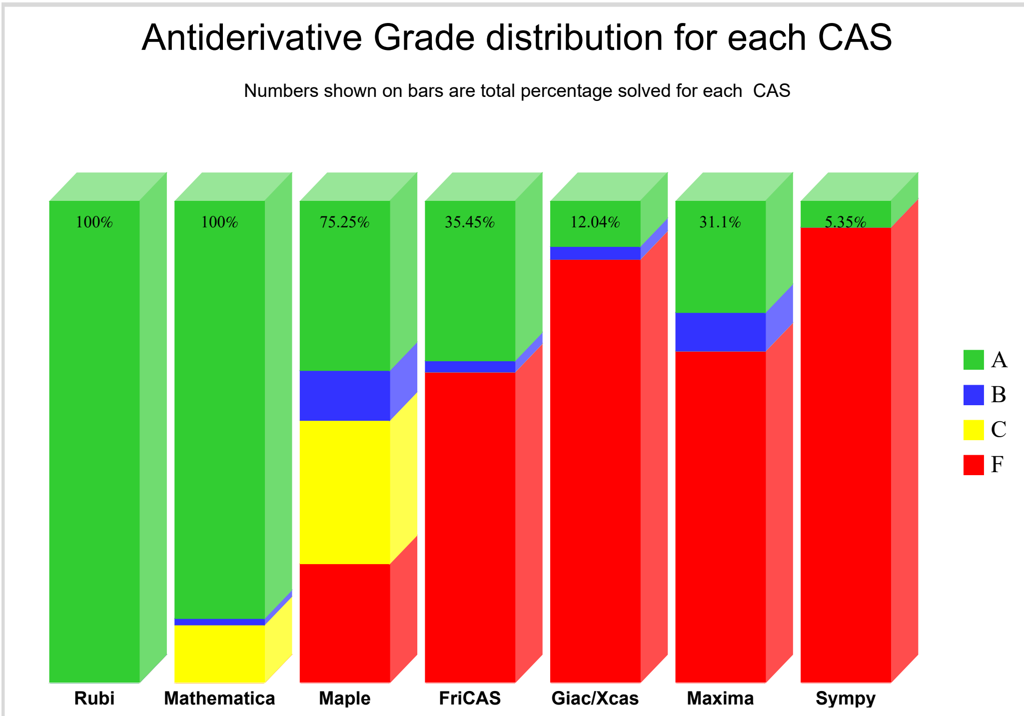
The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

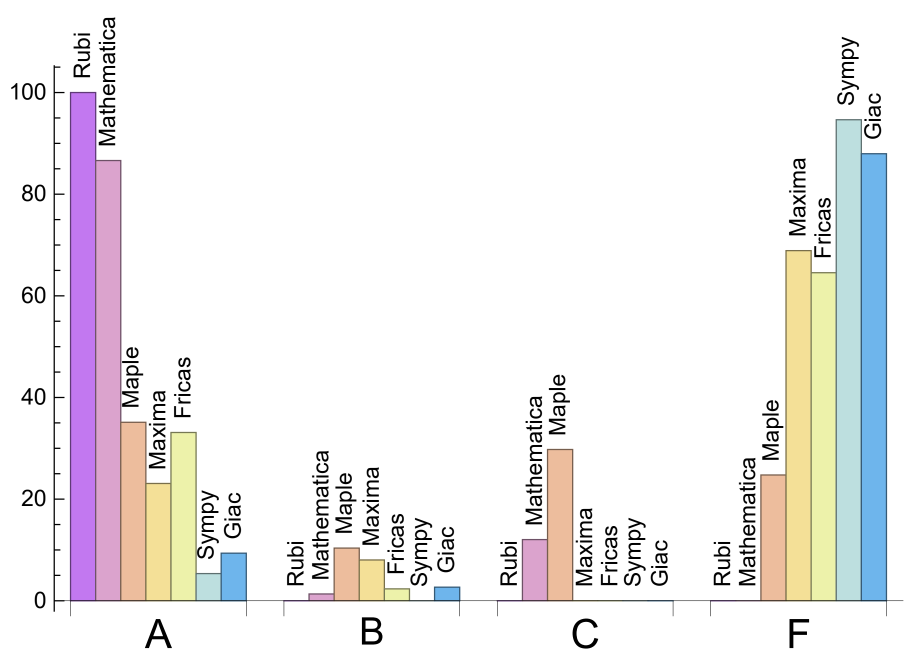
Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

System	% A grade	% B grade	% C grade	% F grade
Rubi	100.	0.	0.	0.
Mathematica	86.62	1.34	12.04	0.
Maple	35.12	10.37	29.77	24.75
Maxima	23.08	8.03	0.	68.9
Fricas	33.11	2.34	0.	64.55
Sympy	5.35	0.	0.	94.65
Giac	9.36	2.68	0.	87.96

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



1.3 Performance

The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.06	79.	1.	70.	1.
Mathematica	0.21	63.33	0.95	56.	0.87
Maple	0.19	193.93	2.33	122.	1.57
Maxima	1.94	227.58	3.3	57.	1.3
Fricas	1.68	201.01	4.18	135.	2.7
Sympy	24.87	32.56	1.	36.	1.03
Giac	1.38	57.11	1.95	55.	1.5

1.4 list of integrals that has no closed form antiderivative

{}

1.5 list of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {}

Mathematica {31, 32, 33, 34, 35, 36, 38, 68, 69, 186, 187, 191, 192, 193, 196, 197, 198, 199, 201, 202, 203, 213, 214, 215, 216, 280, 281, 282, 283, 298}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

1.7 Timing

The command `AboluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since these integrals are run in a batch mode, using an automated script, and by using sagemath (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 30 such integrals out of total 705, or about 4 percent. This pecrentage can be higher or lower depending on the specific input test file.

Such integrals can be indentified by looking at the output of the integration in each section for Maxima. If the output was an exception `ValueError` then this is most likely due to this reason.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima `abs_integrate` was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/ for reference.

1.9.2 Important note about FriCAS and Giac/XCAS results

There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error `Exception raised: NotImplementedError`

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi and Maple, the buildin system function `LeafSize` is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special buildin function for this purpose at this time. Therefore the leaf size is determined as follows.

For Fricas, Giac and Maxima (all called via sagemath) the following code is used

```
#see https://stackoverflow.com/questions/25202346/how-to-obtain-leaf-count-expression-size-in-s

def tree(expr):
    if expr.operator() is None:
       return expr
    else:
       return [expr.operator()]+map(tree, expr.operands())

try:
    # 1.35 is a fudge factor since this estimate of leaf count is bit lower than
    #what it should be compared to Mathematica's
    leafCount = round(1.35*len(flatten(tree(anti))))
except Exception as ee:
    leafCount =1
```

For Sympy, called directly from Python, the following code is used

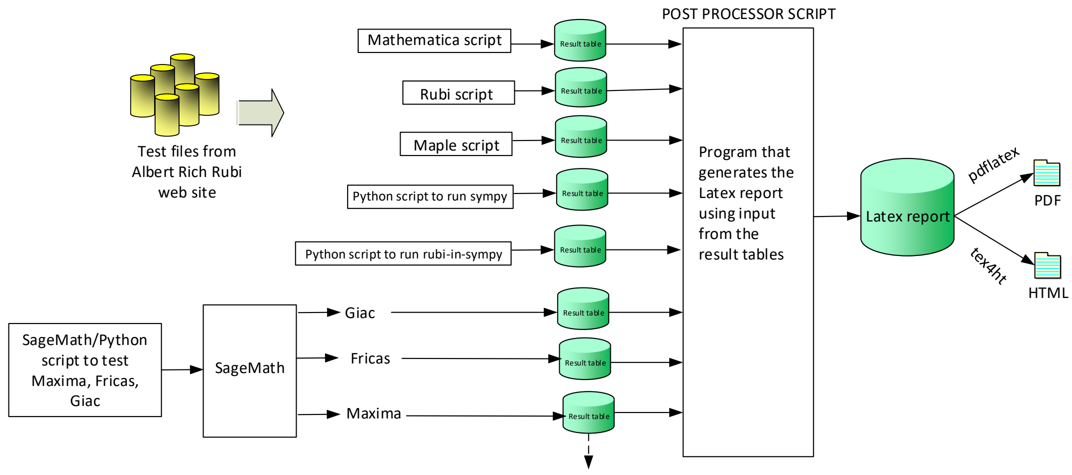
```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
         leafCount =1
```

When these cas systems have a buildin function to find the leaf size of expressions, it will be used instead, and these tests run again.

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



One record (line) per one integral result. The line is CSV comma separated. It contains 13 fields. This is description of each record (line)

1. integer, the problem number.
2. integer. 0 or 1 for failed or passed. (this is not the grade field)
3. integer. Leaf size of result.
4. integer. Leaf size of the optimal antiderivative.
5. number. CPU time used to solve this integral. 0 if failed.
6. string. The integral in Latex format
7. string. The input used in CAS own syntx.
8. string. The result (antiderivative) produced by CAS in Latex format
9. string. The optimal antiderivative in Latex format.
10. integer. 0 or 1. Indicates if problem has known antiderivative or not
11. String. The result (antiderivative) in CAS own syntax.
12. String. The grade of the antiderivative. Can be “A”, “B”, “C”, or “F”
13. String. The optimal antiderivative in CAS own syntax.

High level overview of the CAS independent integration test build system

Nasser M. Abbasi
June 22, 2018

Chapter 2

detailed summary tables of results

2.1 List of integrals sorted by grade for each CAS

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299 }

B grade: { }

C grade: { }

F grade: { }

2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 231, 233, 235, 237, 238, 240, 242, 247, 249, 251, 253, 254, 256, 262, 264, 266, 268, 270, 271, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 296, 297, 299 }

B grade: { 41, 42, 294, 295 }

C grade: { 230, 232, 234, 236, 239, 241, 243, 244, 245, 246, 248, 250, 252, 255, 257, 258, 259, 260, 261, 263, 265, 267, 269, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 298 }

F grade: { }

2.1.3 Maple

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 11, 39, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 61, 62, 63, 64, 65, 66, 67, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 223, 224, 225, 226, 227, 229, 231, 233, 236, 238, 240, 242, 247, 249, 250, 251, 252, 254, 256, 262, 264, 267, 269, 271 }

B grade: { 9, 10, 12, 13, 14, 15, 16, 40, 41, 42, 228, 230, 232, 234, 239, 241, 243, 245, 246, 248, 255, 257, 259, 261, 263, 265, 272, 274, 276, 278, 287 }

C grade: { 17, 18, 19, 20, 21, 22, 23, 24, 55, 56, 57, 58, 59, 60, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 235, 237, 244, 253, 258, 260, 266, 268, 270, 273, 275, 277, 279 }

F grade: { 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 68, 69, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 280, 281, 282, 283, 284, 285, 286, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299 }

2.1.4 Maxima

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 39, 40, 41, 42, 43, 44, 45, 46, 50, 51, 52, 53, 54, 61, 62, 63, 64, 65, 66, 67, 135, 137, 138, 139, 140, 141, 145, 147, 148, 149, 150, 151, 155, 157, 158, 159, 160, 164, 165, 166, 167, 171, 172, 173, 174, 175, 179, 180, 181, 182, 183, 223, 224, 225, 226, 227, 228, 229, 287, 288, 289

B grade: { 47, 48, 49, 132, 133, 134, 136, 142, 143, 144, 146, 152, 153, 154, 156, 161, 162, 163, 168, 169, 170, 176, 177, 178 }

C grade: { }

F grade: { 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 55, 56, 57, 58, 59, 60, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299 }

2.1.5 FriCAS

A grade: { 2, 4, 5, 6, 7, 8, 39, 40, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 61, 62, 63, 64, 65, 66, 67, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 223, 224, 225, 226, 227, 228, 229, 231, 233, 238, 240, 242, 247, 249, 254, 256, 262, 287, 288, 289 }

B grade: { 1, 3, 41, 42, 251, 264, 271 }

C grade: { }

F grade: { 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 55, 56, 57, 58, 59, 60, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 230, 232, 234, 235, 236, 237, 239, 241, 243, 244, 245, 246, 248, 250, 252, 253, 255, 257, 258, 259, 260, 261, 263, 265, 266, 267, 268, 269, 270, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299 }

2.1.6 Sympy

A grade: { 1, 43, 44, 45, 51, 52, 53, 137, 138, 147, 164, 165, 166, 171, 172, 173 }

B grade: { }

C grade: { }

F grade: { 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 46, 47, 48, 49, 50, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 139, 140, 141, 142, 143, 144, 145, 146, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 167, 168, 169, 170, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299 }

2.1.7 Giac

A grade: { 2, 3, 4, 5, 6, 7, 8, 39, 43, 44, 45, 46, 47, 48, 49, 50, 51, 53, 54, 61, 62, 63, 64, 65, 225, 226, 228, 229 }

B grade: { 1, 40, 41, 42, 52, 223, 224, 227 }

C grade: { }

F grade: { 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 55, 56, 57, 58, 59, 60, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$

Problem 1	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	11	19	24	76	36	59
normalized size	1	1.	1.	1.73	2.18	6.91	3.27	5.36
time (sec)	N/A	0.004	0.002	0.003	1.104	1.526	5.915	1.312

Problem 2	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	10	10	11	14	42	0	14
normalized size	1	1.	1.	1.1	1.4	4.2	0.	1.4
time (sec)	N/A	0.009	0.004	0.056	1.266	1.403	0.	1.25

Problem 3	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	34	38	62	162	0	65
normalized size	1	1.	1.	1.12	1.82	4.76	0.	1.91
time (sec)	N/A	0.014	0.01	0.133	1.001	1.412	0.	1.353

Problem 4	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	23	24	30	81	0	30
normalized size	1	1.	0.88	0.92	1.15	3.12	0.	1.15
time (sec)	N/A	0.011	0.041	0.043	1.166	1.346	0.	1.324

Problem 5	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	42	57	96	200	0	85
normalized size	1	1.	0.76	1.04	1.75	3.64	0.	1.55
time (sec)	N/A	0.025	0.071	0.044	1.03	1.423	0.	1.32

Problem 6	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	35	34	46	108	0	46
normalized size	1	1.	0.85	0.83	1.12	2.63	0.	1.12
time (sec)	N/A	0.015	0.101	0.043	1.201	1.362	0.	1.137

Problem 7	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	52	76	123	231	0	99
normalized size	1	1.	0.68	1.	1.62	3.04	0.	1.3
time (sec)	N/A	0.039	0.166	0.046	1.07	1.476	0.	1.276

Problem 8	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	43	44	59	135	0	59
normalized size	1	1.	0.81	0.83	1.11	2.55	0.	1.11
time (sec)	N/A	0.017	0.219	0.043	1.075	1.393	0.	1.274

Problem 9	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	59	358	0	0	0	0
normalized size	1	1.	0.69	4.21	0.	0.	0.	0.
time (sec)	N/A	0.037	0.175	2.199	0.	0.	0.	0.

Problem 10	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	46	213	0	0	0	0
normalized size	1	1.	0.74	3.44	0.	0.	0.	0.
time (sec)	N/A	0.027	0.067	1.266	0.	0.	0.	0.

Problem 11	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	45	101	0	0	0	0
normalized size	1	1.	0.78	1.74	0.	0.	0.	0.
time (sec)	N/A	0.026	0.048	1.062	0.	0.	0.	0.

Problem 12	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	36	133	0	0	0	0
normalized size	1	1.	1.	3.69	0.	0.	0.	0.
time (sec)	N/A	0.017	0.029	0.883	0.	0.	0.	0.

Problem 13	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	36	133	0	0	0	0
normalized size	1	1.	1.	3.69	0.	0.	0.	0.
time (sec)	N/A	0.017	0.034	0.936	0.	0.	0.	0.

Problem 14	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	49	179	0	0	0	0
normalized size	1	1.	0.79	2.89	0.	0.	0.	0.
time (sec)	N/A	0.028	0.044	1.485	0.	0.	0.	0.

Problem 15	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	55	202	0	0	0	0
normalized size	1	1.	0.89	3.26	0.	0.	0.	0.
time (sec)	N/A	0.029	0.069	1.212	0.	0.	0.	0.

Problem 16	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	61	199	0	0	0	0
normalized size	1	1.	0.72	2.34	0.	0.	0.	0.
time (sec)	N/A	0.04	0.099	1.247	0.	0.	0.	0.

Problem 17	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	62	354	0	0	0	0
normalized size	1	1.	0.63	3.61	0.	0.	0.	0.
time (sec)	N/A	0.056	0.166	0.32	0.	0.	0.	0.

Problem 18	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	51	128	0	0	0	0
normalized size	1	1.	0.73	1.83	0.	0.	0.	0.
time (sec)	N/A	0.034	0.066	0.198	0.	0.	0.	0.

Problem 19	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	48	322	0	0	0	0
normalized size	1	1.	0.73	4.88	0.	0.	0.	0.
time (sec)	N/A	0.04	0.039	0.225	0.	0.	0.	0.

Problem 20	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	38	98	0	0	0	0
normalized size	1	1.	1.	2.58	0.	0.	0.	0.
time (sec)	N/A	0.019	0.02	0.153	0.	0.	0.	0.

Problem 21	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	38	306	0	0	0	0
normalized size	1	1.	1.	8.05	0.	0.	0.	0.
time (sec)	N/A	0.027	0.029	0.164	0.	0.	0.	0.

Problem 22	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	59	131	0	0	0	0
normalized size	1	1.	0.82	1.82	0.	0.	0.	0.
time (sec)	N/A	0.046	0.058	0.155	0.	0.	0.	0.

Problem 23	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	60	323	0	0	0	0
normalized size	1	1.	0.83	4.49	0.	0.	0.	0.
time (sec)	N/A	0.038	0.068	0.2	0.	0.	0.	0.

Problem 24	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	66	153	0	0	0	0
normalized size	1	1.	0.66	1.53	0.	0.	0.	0.
time (sec)	N/A	0.059	0.096	0.19	0.	0.	0.	0.

Problem 25	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	55	0	0	0	0	0
normalized size	1	1.	1.08	0.	0.	0.	0.	0.
time (sec)	N/A	0.023	0.051	0.068	0.	0.	0.	0.

Problem 26	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	55	0	0	0	0	0
normalized size	1	1.	1.08	0.	0.	0.	0.	0.
time (sec)	N/A	0.028	0.036	0.094	0.	0.	0.	0.

Problem 27	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	53	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.033	0.034	0.1	0.	0.	0.	0.

Problem 28	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	53	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.025	0.082	0.093	0.	0.	0.	0.

Problem 29	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	55	0	0	0	0	0
normalized size	1	1.	1.04	0.	0.	0.	0.	0.
time (sec)	N/A	0.025	0.083	0.093	0.	0.	0.	0.

Problem 30	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	55	0	0	0	0	0
normalized size	1	1.	1.04	0.	0.	0.	0.	0.
time (sec)	N/A	0.025	0.066	0.055	0.	0.	0.	0.

Problem 31	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	54	54	57	0	0	0	0	0
normalized size	1	1.	1.06	0.	0.	0.	0.	0.
time (sec)	N/A	0.034	0.049	0.059	0.	0.	0.	0.

Problem 32	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	54	54	57	0	0	0	0	0
normalized size	1	1.	1.06	0.	0.	0.	0.	0.
time (sec)	N/A	0.03	0.041	0.084	0.	0.	0.	0.

Problem 33	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	56	56	55	0	0	0	0	0
normalized size	1	1.	0.98	0.	0.	0.	0.	0.
time (sec)	N/A	0.035	0.04	0.105	0.	0.	0.	0.

Problem 34	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	56	56	55	0	0	0	0	0
normalized size	1	1.	0.98	0.	0.	0.	0.	0.
time (sec)	N/A	0.032	0.052	0.094	0.	0.	0.	0.

Problem 35	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	56	56	57	0	0	0	0	0
normalized size	1	1.	1.02	0.	0.	0.	0.	0.
time (sec)	N/A	0.028	0.053	0.097	0.	0.	0.	0.

Problem 36	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	56	56	57	0	0	0	0	0
normalized size	1	1.	1.02	0.	0.	0.	0.	0.
time (sec)	N/A	0.029	0.069	0.054	0.	0.	0.	0.

Problem 37	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	61	0	0	0	0	0
normalized size	1	1.	0.87	0.	0.	0.	0.	0.
time (sec)	N/A	0.033	0.05	0.291	0.	0.	0.	0.

Problem 38	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	73	73	61	0	0	0	0	0
normalized size	1	1.	0.84	0.	0.	0.	0.	0.
time (sec)	N/A	0.031	0.047	0.265	0.	0.	0.	0.

Problem 39	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	74	72	57	162	0	80
normalized size	1	1.	1.48	1.44	1.14	3.24	0.	1.6
time (sec)	N/A	0.017	0.29	0.159	1.741	1.367	0.	1.286

Problem 40	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	68	64	41	139	0	72
normalized size	1	1.	1.89	1.78	1.14	3.86	0.	2.
time (sec)	N/A	0.012	0.14	0.091	1.648	1.409	0.	1.278

Problem 41	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	52	53	24	109	0	59
normalized size	1	1.	2.36	2.41	1.09	4.95	0.	2.68
time (sec)	N/A	0.009	0.057	0.065	1.719	1.331	0.	1.367

Problem 42	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	3	3	44	21	4	61	0	47
normalized size	1	1.	14.67	7.	1.33	20.33	0.	15.67
time (sec)	N/A	0.006	0.009	0.071	1.732	1.421	0.	1.339

Problem 43	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	11	14	15	12	10	8
normalized size	1	1.	1.	1.27	1.36	1.09	0.91	0.73
time (sec)	N/A	0.007	0.006	0.074	1.18	1.24	0.483	1.272

Problem 44	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	23	21	34	38	27	22
normalized size	1	1.	0.79	0.72	1.17	1.31	0.93	0.76
time (sec)	N/A	0.011	0.015	0.059	1.089	1.312	1.506	1.35

Problem 45	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	31	29	50	59	44	34
normalized size	1	1.	0.72	0.67	1.16	1.37	1.02	0.79
time (sec)	N/A	0.015	0.025	0.072	1.165	1.444	17.704	1.257

Problem 46	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	37	35	66	78	0	46
normalized size	1	1.	0.65	0.61	1.16	1.37	0.	0.81
time (sec)	N/A	0.019	0.038	0.088	1.069	1.421	0.	1.405

Problem 47	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	84	78	74	2936	186	0	107
normalized size	1	1.	0.93	0.88	34.95	2.21	0.	1.27
time (sec)	N/A	0.041	0.124	0.145	16.739	1.478	0.	1.301

Problem 48	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	72	66	1500	159	0	90
normalized size	1	1.	1.11	1.02	23.08	2.45	0.	1.38
time (sec)	N/A	0.031	0.121	0.087	2.673	1.57	0.	1.409

Problem 49	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	55	55	437	119	0	57
normalized size	1	1.	1.2	1.2	9.5	2.59	0.	1.24
time (sec)	N/A	0.022	0.051	0.061	1.946	1.471	0.	1.306

Problem 50	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	46	23	51	171	0	42
normalized size	1	1.	1.84	0.92	2.04	6.84	0.	1.68
time (sec)	N/A	0.015	0.008	0.056	1.936	1.464	0.	1.343

Problem 51	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	13	16	8	46	15	15
normalized size	1	1.	1.	1.23	0.62	3.54	1.15	1.15
time (sec)	N/A	0.029	0.005	0.069	1.881	1.445	0.538	1.229

Problem 52	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	27	23	19	74	37	78
normalized size	1	1.	0.75	0.64	0.53	2.06	1.03	2.17
time (sec)	N/A	0.018	0.018	0.057	1.945	1.432	1.487	1.379

Problem 53	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	36	31	30	96	60	113
normalized size	1	1.	0.65	0.56	0.55	1.75	1.09	2.05
time (sec)	N/A	0.027	0.028	0.064	1.834	1.445	17.485	1.441

Problem 54	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	42	37	38	115	0	149
normalized size	1	1.	0.57	0.5	0.51	1.55	0.	2.01
time (sec)	N/A	0.035	0.035	0.081	1.784	1.451	0.	1.608

Problem 55	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	117	59	223	0	0	0	0
normalized size	1	1.	0.5	1.91	0.	0.	0.	0.
time (sec)	N/A	0.052	0.093	0.368	0.	0.	0.	0.

Problem 56	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	43	87	0	0	0	0
normalized size	1	1.	0.66	1.34	0.	0.	0.	0.
time (sec)	N/A	0.035	0.035	0.182	0.	0.	0.	0.

Problem 57	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	32	191	0	0	0	0
normalized size	1	1.	0.76	4.55	0.	0.	0.	0.
time (sec)	N/A	0.026	0.017	0.205	0.	0.	0.	0.

Problem 58	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	31	76	0	0	0	0
normalized size	1	1.	0.7	1.73	0.	0.	0.	0.
time (sec)	N/A	0.027	0.04	0.14	0.	0.	0.	0.

Problem 59	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	43	198	0	0	0	0
normalized size	1	1.	0.59	2.71	0.	0.	0.	0.
time (sec)	N/A	0.037	0.094	0.156	0.	0.	0.	0.

Problem 60	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	117	59	114	0	0	0	0
normalized size	1	1.	0.5	0.97	0.	0.	0.	0.
time (sec)	N/A	0.056	0.097	0.206	0.	0.	0.	0.

Problem 61	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	163	163	54	53	82	228	0	90
normalized size	1	1.	0.33	0.33	0.5	1.4	0.	0.55
time (sec)	N/A	0.039	0.168	0.233	1.643	1.587	0.	1.327

Problem 62	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	117	42	41	58	165	0	66
normalized size	1	1.	0.36	0.35	0.5	1.41	0.	0.56
time (sec)	N/A	0.03	0.091	0.113	1.792	1.407	0.	1.244

Problem 63	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	30	29	34	101	0	30
normalized size	1	1.	0.49	0.48	0.56	1.66	0.	0.49
time (sec)	N/A	0.022	0.057	0.067	1.692	1.449	0.	1.305

Problem 64	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	8	43	0	8
normalized size	1	1.	1.	0.93	0.53	2.87	0.	0.53
time (sec)	N/A	0.016	0.005	0.063	1.739	1.364	0.	1.258

Problem 65	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	23	22	34	74	0	53
normalized size	1	1.	0.64	0.61	0.94	2.06	0.	1.47
time (sec)	N/A	0.015	0.023	0.074	1.664	1.42	0.	1.311

Problem 66	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	38	41	78	126	0	0
normalized size	1	1.	0.44	0.48	0.91	1.47	0.	0.
time (sec)	N/A	0.032	0.039	0.104	1.646	1.461	0.	0.

Problem 67	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	132	132	55	57	119	177	0	0
normalized size	1	1.	0.42	0.43	0.9	1.34	0.	0.
time (sec)	N/A	0.051	0.082	0.221	1.711	1.452	0.	0.

Problem 68	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	81	81	69	0	0	0	0	0
normalized size	1	1.	0.85	0.	0.	0.	0.	0.
time (sec)	N/A	0.053	0.094	0.444	0.	0.	0.	0.

Problem 69	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	83	83	71	0	0	0	0	0
normalized size	1	1.	0.86	0.	0.	0.	0.	0.
time (sec)	N/A	0.05	0.073	0.197	0.	0.	0.	0.

Problem 70	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	69	152	0	0	0	0
normalized size	1	1.	0.71	1.57	0.	0.	0.	0.
time (sec)	N/A	0.06	0.212	0.224	0.	0.	0.	0.

Problem 71	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	69	356	0	0	0	0
normalized size	1	1.	0.73	3.75	0.	0.	0.	0.
time (sec)	N/A	0.061	0.165	0.208	0.	0.	0.	0.

Problem 72	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	51	130	0	0	0	0
normalized size	1	1.	0.74	1.88	0.	0.	0.	0.
time (sec)	N/A	0.042	0.084	0.155	0.	0.	0.	0.

Problem 73	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	47	316	0	0	0	0
normalized size	1	1.	0.75	5.02	0.	0.	0.	0.
time (sec)	N/A	0.041	0.051	0.214	0.	0.	0.	0.

Problem 74	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	38	98	0	0	0	0
normalized size	1	1.	1.	2.58	0.	0.	0.	0.
time (sec)	N/A	0.02	0.02	0.141	0.	0.	0.	0.

Problem 75	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	39	303	0	0	0	0
normalized size	1	1.	1.	7.77	0.	0.	0.	0.
time (sec)	N/A	0.03	0.035	0.149	0.	0.	0.	0.

Problem 76	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	51	123	0	0	0	0
normalized size	1	1.	0.76	1.84	0.	0.	0.	0.
time (sec)	N/A	0.052	0.051	0.164	0.	0.	0.	0.

Problem 77	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	57	315	0	0	0	0
normalized size	1	1.	0.81	4.5	0.	0.	0.	0.
time (sec)	N/A	0.051	0.059	0.208	0.	0.	0.	0.

Problem 78	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	63	145	0	0	0	0
normalized size	1	1.	0.66	1.53	0.	0.	0.	0.
time (sec)	N/A	0.07	0.09	0.188	0.	0.	0.	0.

Problem 79	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	71	325	0	0	0	0
normalized size	1	1.	0.72	3.32	0.	0.	0.	0.
time (sec)	N/A	0.069	0.244	0.181	0.	0.	0.	0.

Problem 80	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	64	152	0	0	0	0
normalized size	1	1.	0.67	1.6	0.	0.	0.	0.
time (sec)	N/A	0.059	0.183	0.195	0.	0.	0.	0.

Problem 81	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	64	356	0	0	0	0
normalized size	1	1.	0.65	3.63	0.	0.	0.	0.
time (sec)	N/A	0.059	0.175	0.19	0.	0.	0.	0.

Problem 82	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	49	122	0	0	0	0
normalized size	1	1.	0.73	1.82	0.	0.	0.	0.
time (sec)	N/A	0.041	0.072	0.146	0.	0.	0.	0.

Problem 83	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	48	320	0	0	0	0
normalized size	1	1.	0.73	4.85	0.	0.	0.	0.
time (sec)	N/A	0.034	0.037	0.199	0.	0.	0.	0.

Problem 84	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	39	98	0	0	0	0
normalized size	1	1.	1.	2.51	0.	0.	0.	0.
time (sec)	N/A	0.03	0.019	0.136	0.	0.	0.	0.

Problem 85	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	41	309	0	0	0	0
normalized size	1	1.	1.	7.54	0.	0.	0.	0.
time (sec)	N/A	0.038	0.024	0.147	0.	0.	0.	0.

Problem 86	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	52	129	0	0	0	0
normalized size	1	1.	0.74	1.84	0.	0.	0.	0.
time (sec)	N/A	0.054	0.049	0.16	0.	0.	0.	0.

Problem 87	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	58	321	0	0	0	0
normalized size	1	1.	0.81	4.46	0.	0.	0.	0.
time (sec)	N/A	0.054	0.049	0.196	0.	0.	0.	0.

Problem 88	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	64	151	0	0	0	0
normalized size	1	1.	0.65	1.54	0.	0.	0.	0.
time (sec)	N/A	0.073	0.078	0.184	0.	0.	0.	0.

Problem 89	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	72	331	0	0	0	0
normalized size	1	1.	0.72	3.31	0.	0.	0.	0.
time (sec)	N/A	0.075	0.16	0.182	0.	0.	0.	0.

Problem 90	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	61	152	0	0	0	0
normalized size	1	1.	0.62	1.55	0.	0.	0.	0.
time (sec)	N/A	0.059	0.18	0.191	0.	0.	0.	0.

Problem 91	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	61	348	0	0	0	0
normalized size	1	1.	0.63	3.59	0.	0.	0.	0.
time (sec)	N/A	0.057	0.164	0.204	0.	0.	0.	0.

Problem 92	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	51	128	0	0	0	0
normalized size	1	1.	0.73	1.83	0.	0.	0.	0.
time (sec)	N/A	0.032	0.017	0.159	0.	0.	0.	0.

Problem 93	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	50	324	0	0	0	0
normalized size	1	1.	0.74	4.76	0.	0.	0.	0.
time (sec)	N/A	0.045	0.031	0.218	0.	0.	0.	0.

Problem 94	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	41	98	0	0	0	0
normalized size	1	1.	1.	2.39	0.	0.	0.	0.
time (sec)	N/A	0.038	0.016	0.153	0.	0.	0.	0.

Problem 95	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	38	311	0	0	0	0
normalized size	1	1.	0.93	7.59	0.	0.	0.	0.
time (sec)	N/A	0.037	0.043	0.158	0.	0.	0.	0.

Problem 96	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	54	131	0	0	0	0
normalized size	1	1.	0.75	1.82	0.	0.	0.	0.
time (sec)	N/A	0.055	0.049	0.167	0.	0.	0.	0.

Problem 97	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	60	321	0	0	0	0
normalized size	1	1.	0.83	4.46	0.	0.	0.	0.
time (sec)	N/A	0.054	0.051	0.201	0.	0.	0.	0.

Problem 98	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	66	153	0	0	0	0
normalized size	1	1.	0.66	1.53	0.	0.	0.	0.
time (sec)	N/A	0.071	0.073	0.188	0.	0.	0.	0.

Problem 99	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	74	333	0	0	0	0
normalized size	1	1.	0.74	3.33	0.	0.	0.	0.
time (sec)	N/A	0.074	0.166	0.186	0.	0.	0.	0.

Problem 100	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	62	354	0	0	0	0
normalized size	1	1.	0.63	3.61	0.	0.	0.	0.
time (sec)	N/A	0.048	0.085	0.222	0.	0.	0.	0.

Problem 101	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	69	152	0	0	0	0
normalized size	1	1.	0.69	1.52	0.	0.	0.	0.
time (sec)	N/A	0.057	0.083	0.23	0.	0.	0.	0.

Problem 102	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	61	356	0	0	0	0
normalized size	1	1.	0.63	3.67	0.	0.	0.	0.
time (sec)	N/A	0.059	0.226	0.236	0.	0.	0.	0.

Problem 103	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	51	130	0	0	0	0
normalized size	1	1.	0.71	1.81	0.	0.	0.	0.
time (sec)	N/A	0.039	0.074	0.181	0.	0.	0.	0.

Problem 104	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	48	319	0	0	0	0
normalized size	1	1.	0.74	4.91	0.	0.	0.	0.
time (sec)	N/A	0.04	0.078	0.218	0.	0.	0.	0.

Problem 105	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	41	98	0	0	0	0
normalized size	1	1.	1.	2.39	0.	0.	0.	0.
time (sec)	N/A	0.021	0.016	0.158	0.	0.	0.	0.

Problem 106	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	38	306	0	0	0	0
normalized size	1	1.	1.	8.05	0.	0.	0.	0.
time (sec)	N/A	0.019	0.015	0.157	0.	0.	0.	0.

Problem 107	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	60	126	0	0	0	0
normalized size	1	1.	0.87	1.83	0.	0.	0.	0.
time (sec)	N/A	0.043	0.068	0.16	0.	0.	0.	0.

Problem 108	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	60	316	0	0	0	0
normalized size	1	1.	0.9	4.72	0.	0.	0.	0.
time (sec)	N/A	0.051	0.082	0.215	0.	0.	0.	0.

Problem 109	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	66	148	0	0	0	0
normalized size	1	1.	0.68	1.53	0.	0.	0.	0.
time (sec)	N/A	0.069	0.075	0.204	0.	0.	0.	0.

Problem 110	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	70	328	0	0	0	0
normalized size	1	1.	0.74	3.45	0.	0.	0.	0.
time (sec)	N/A	0.069	0.189	0.202	0.	0.	0.	0.

Problem 111	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	69	152	0	0	0	0
normalized size	1	1.	0.69	1.52	0.	0.	0.	0.
time (sec)	N/A	0.059	0.166	0.219	0.	0.	0.	0.

Problem 112	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	64	356	0	0	0	0
normalized size	1	1.	0.64	3.56	0.	0.	0.	0.
time (sec)	N/A	0.058	0.053	0.223	0.	0.	0.	0.

Problem 113	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	56	125	0	0	0	0
normalized size	1	1.	0.78	1.74	0.	0.	0.	0.
time (sec)	N/A	0.038	0.097	0.17	0.	0.	0.	0.

Problem 114	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	51	322	0	0	0	0
normalized size	1	1.	0.75	4.74	0.	0.	0.	0.
time (sec)	N/A	0.04	0.057	0.224	0.	0.	0.	0.

Problem 115	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	41	98	0	0	0	0
normalized size	1	1.	1.	2.39	0.	0.	0.	0.
time (sec)	N/A	0.023	0.022	0.135	0.	0.	0.	0.

Problem 116	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	41	311	0	0	0	0
normalized size	1	1.	1.	7.59	0.	0.	0.	0.
time (sec)	N/A	0.022	0.031	0.144	0.	0.	0.	0.

Problem 117	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	59	131	0	0	0	0
normalized size	1	1.	0.82	1.82	0.	0.	0.	0.
time (sec)	N/A	0.033	0.049	0.155	0.	0.	0.	0.

Problem 118	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	60	321	0	0	0	0
normalized size	1	1.	0.87	4.65	0.	0.	0.	0.
time (sec)	N/A	0.045	0.032	0.197	0.	0.	0.	0.

Problem 119	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	66	153	0	0	0	0
normalized size	1	1.	0.67	1.56	0.	0.	0.	0.
time (sec)	N/A	0.072	0.091	0.184	0.	0.	0.	0.

Problem 120	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	73	333	0	0	0	0
normalized size	1	1.	0.75	3.43	0.	0.	0.	0.
time (sec)	N/A	0.071	0.077	0.19	0.	0.	0.	0.

Problem 121	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	64	152	0	0	0	0
normalized size	1	1.	0.64	1.52	0.	0.	0.	0.
time (sec)	N/A	0.057	0.132	0.217	0.	0.	0.	0.

Problem 122	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	64	351	0	0	0	0
normalized size	1	1.	0.64	3.51	0.	0.	0.	0.
time (sec)	N/A	0.058	0.113	0.222	0.	0.	0.	0.

Problem 123	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	51	130	0	0	0	0
normalized size	1	1.	0.71	1.81	0.	0.	0.	0.
time (sec)	N/A	0.038	0.066	0.174	0.	0.	0.	0.

Problem 124	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	51	322	0	0	0	0
normalized size	1	1.	0.75	4.74	0.	0.	0.	0.
time (sec)	N/A	0.038	0.053	0.234	0.	0.	0.	0.

Problem 125	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	41	98	0	0	0	0
normalized size	1	1.	1.	2.39	0.	0.	0.	0.
time (sec)	N/A	0.022	0.016	0.14	0.	0.	0.	0.

Problem 126	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	38	311	0	0	0	0
normalized size	1	1.	0.93	7.59	0.	0.	0.	0.
time (sec)	N/A	0.023	0.036	0.143	0.	0.	0.	0.

Problem 127	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	62	131	0	0	0	0
normalized size	1	1.	0.86	1.82	0.	0.	0.	0.
time (sec)	N/A	0.04	0.031	0.147	0.	0.	0.	0.

Problem 128	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	60	321	0	0	0	0
normalized size	1	1.	0.83	4.46	0.	0.	0.	0.
time (sec)	N/A	0.034	0.018	0.199	0.	0.	0.	0.

Problem 129	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	66	153	0	0	0	0
normalized size	1	1.	0.68	1.58	0.	0.	0.	0.
time (sec)	N/A	0.063	0.069	0.178	0.	0.	0.	0.

Problem 130	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	73	333	0	0	0	0
normalized size	1	1.	0.74	3.4	0.	0.	0.	0.
time (sec)	N/A	0.072	0.121	0.183	0.	0.	0.	0.

Problem 131	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	66	153	0	0	0	0
normalized size	1	1.	0.66	1.53	0.	0.	0.	0.
time (sec)	N/A	0.051	0.02	0.187	0.	0.	0.	0.

Problem 132	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	107	64	131	2236	608	0	0
normalized size	1	1.	0.6	1.22	20.9	5.68	0.	0.
time (sec)	N/A	0.033	0.141	0.178	2.489	1.608	0.	0.

Problem 133	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	45	52	397	115	0	0
normalized size	1	1.	0.64	0.74	5.67	1.64	0.	0.
time (sec)	N/A	0.017	0.087	0.129	2.08	1.412	0.	0.

Problem 134	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	50	112	892	528	0	0
normalized size	1	1.	0.69	1.56	12.39	7.33	0.	0.
time (sec)	N/A	0.019	0.052	0.135	2.113	1.548	0.	0.

Problem 135	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	32	39	73	78	0	0
normalized size	1	1.	1.	1.22	2.28	2.44	0.	0.
time (sec)	N/A	0.012	0.018	0.13	2.147	1.367	0.	0.

Problem 136	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	33	52	88	290	0	0
normalized size	1	1.	1.	1.58	2.67	8.79	0.	0.
time (sec)	N/A	0.007	0.011	0.116	2.064	1.496	0.	0.

Problem 137	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	24	32	35	266	5	0
normalized size	1	1.	1.	1.33	1.46	11.08	0.21	0.
time (sec)	N/A	0.002	0.014	0.108	1.786	1.638	1.55	0.

Problem 138	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	32	41	18	76	36	0
normalized size	1	1.	1.	1.28	0.56	2.38	1.12	0.
time (sec)	N/A	0.007	0.036	0.127	2.057	1.402	20.9	0.

Problem 139	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	45	54	34	428	0	0
normalized size	1	1.	0.71	0.86	0.54	6.79	0.	0.
time (sec)	N/A	0.014	0.064	0.14	2.019	1.649	0.	0.

Problem 140	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	45	52	57	130	0	0
normalized size	1	1.	0.64	0.74	0.81	1.86	0.	0.
time (sec)	N/A	0.016	0.109	0.135	2.072	1.383	0.	0.

Problem 141	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	55	74	66	537	0	0
normalized size	1	1.	0.56	0.76	0.67	5.48	0.	0.
time (sec)	N/A	0.025	0.116	0.157	2.132	1.79	0.	0.

Problem 142	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	110	64	131	2352	621	0	0
normalized size	1	1.	0.58	1.19	21.38	5.65	0.	0.
time (sec)	N/A	0.035	0.111	0.139	2.631	1.585	0.	0.

Problem 143	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	45	52	404	117	0	0
normalized size	1	1.	0.62	0.72	5.61	1.62	0.	0.
time (sec)	N/A	0.017	0.088	0.118	2.103	1.354	0.	0.

Problem 144	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	50	112	933	536	0	0
normalized size	1	1.	0.68	1.51	12.61	7.24	0.	0.
time (sec)	N/A	0.021	0.058	0.117	2.191	1.648	0.	0.

Problem 145	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	32	39	73	81	0	0
normalized size	1	1.	0.97	1.18	2.21	2.45	0.	0.
time (sec)	N/A	0.011	0.02	0.113	2.036	1.416	0.	0.

Problem 146	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	33	52	92	293	0	0
normalized size	1	1.	0.97	1.53	2.71	8.62	0.	0.
time (sec)	N/A	0.007	0.021	0.117	2.061	1.783	0.	0.

Problem 147	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	24	32	35	269	5	0
normalized size	1	1.	0.96	1.28	1.4	10.76	0.2	0.
time (sec)	N/A	0.003	0.025	0.089	1.787	1.857	38.559	0.

Problem 148	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	32	41	18	78	0	0
normalized size	1	1.	0.97	1.24	0.55	2.36	0.	0.
time (sec)	N/A	0.007	0.044	0.114	2.002	1.578	0.	0.

Problem 149	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	45	54	38	436	0	0
normalized size	1	1.	0.69	0.83	0.58	6.71	0.	0.
time (sec)	N/A	0.015	0.079	0.124	2.038	1.967	0.	0.

Problem 150	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	45	52	61	135	0	0
normalized size	1	1.	0.62	0.72	0.85	1.88	0.	0.
time (sec)	N/A	0.017	0.138	0.109	2.083	1.668	0.	0.

Problem 151	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	55	74	72	551	0	0
normalized size	1	1.	0.54	0.73	0.71	5.46	0.	0.
time (sec)	N/A	0.028	0.154	0.123	2.153	2.016	0.	0.

Problem 152	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	116	57	62	952	158	0	0
normalized size	1	1.	0.49	0.53	8.21	1.36	0.	0.
time (sec)	N/A	0.024	0.217	0.119	2.537	1.728	0.	0.

Problem 153	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	45	52	420	123	0	0
normalized size	1	1.	0.59	0.68	5.53	1.62	0.	0.
time (sec)	N/A	0.017	0.079	0.106	1.978	1.693	0.	0.

Problem 154	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	50	112	1008	544	0	0
normalized size	1	1.	0.64	1.44	12.92	6.97	0.	0.
time (sec)	N/A	0.02	0.077	0.132	2.191	1.98	0.	0.

Problem 155	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	32	39	73	84	0	0
normalized size	1	1.	0.91	1.11	2.09	2.4	0.	0.
time (sec)	N/A	0.012	0.031	0.127	2.06	1.636	0.	0.

Problem 156	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	33	52	97	296	0	0
normalized size	1	1.	0.92	1.44	2.69	8.22	0.	0.
time (sec)	N/A	0.008	0.034	0.097	1.916	1.845	0.	0.

Problem 157	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	24	32	35	271	0	0
normalized size	1	1.	0.89	1.19	1.3	10.04	0.	0.
time (sec)	N/A	0.003	0.019	0.091	1.825	1.973	0.	0.

Problem 158	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	32	41	18	81	0	0
normalized size	1	1.	0.91	1.17	0.51	2.31	0.	0.
time (sec)	N/A	0.007	0.065	0.105	1.879	1.662	0.	0.

Problem 159	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	45	54	43	444	0	0
normalized size	1	1.	0.65	0.78	0.62	6.43	0.	0.
time (sec)	N/A	0.015	0.113	0.105	2.128	2.04	0.	0.

Problem 160	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	45	52	66	140	0	0
normalized size	1	1.	0.59	0.68	0.87	1.84	0.	0.
time (sec)	N/A	0.017	0.163	0.114	2.	1.643	0.	0.

Problem 161	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	50	112	892	533	0	0
normalized size	1	1.	0.69	1.56	12.39	7.4	0.	0.
time (sec)	N/A	0.019	0.057	0.137	2.025	1.964	0.	0.

Problem 162	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	32	39	80	81	0	0
normalized size	1	1.	1.	1.22	2.5	2.53	0.	0.
time (sec)	N/A	0.011	0.026	0.119	2.039	1.642	0.	0.

Problem 163	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	33	52	88	298	0	0
normalized size	1	1.	1.	1.58	2.67	9.03	0.	0.
time (sec)	N/A	0.007	0.016	0.12	1.982	1.9	0.	0.

Problem 164	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	24	32	35	274	5	0
normalized size	1	1.	1.	1.33	1.46	11.42	0.21	0.
time (sec)	N/A	0.002	0.015	0.092	1.58	1.972	19.077	0.

Problem 165	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	32	41	18	81	36	0
normalized size	1	1.	1.	1.28	0.56	2.53	1.12	0.
time (sec)	N/A	0.007	0.036	0.154	2.093	1.748	25.96	0.

Problem 166	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	45	54	34	437	82	0
normalized size	1	1.	0.71	0.86	0.54	6.94	1.3	0.
time (sec)	N/A	0.014	0.077	0.135	2.054	2.005	44.261	0.

Problem 167	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	45	52	57	132	0	0
normalized size	1	1.	0.64	0.74	0.81	1.89	0.	0.
time (sec)	N/A	0.016	0.094	0.124	2.122	1.685	0.	0.

Problem 168	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	50	112	905	539	0	0
normalized size	1	1.	0.64	1.44	11.6	6.91	0.	0.
time (sec)	N/A	0.02	0.057	0.117	2.15	1.955	0.	0.

Problem 169	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	32	39	90	84	0	0
normalized size	1	1.	0.91	1.11	2.57	2.4	0.	0.
time (sec)	N/A	0.012	0.035	0.101	2.174	1.672	0.	0.

Problem 170	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	33	52	88	301	0	0
normalized size	1	1.	0.92	1.44	2.44	8.36	0.	0.
time (sec)	N/A	0.007	0.026	0.092	2.053	1.886	0.	0.

Problem 171	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	24	32	35	277	5	0
normalized size	1	1.	0.89	1.19	1.3	10.26	0.19	0.
time (sec)	N/A	0.003	0.025	0.083	1.643	1.937	133.827	0.

Problem 172	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	32	41	18	84	36	0
normalized size	1	1.	0.91	1.17	0.51	2.4	1.03	0.
time (sec)	N/A	0.008	0.041	0.105	1.93	1.658	19.552	0.

Problem 173	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	45	54	34	443	82	0
normalized size	1	1.	0.65	0.78	0.49	6.42	1.19	0.
time (sec)	N/A	0.014	0.066	0.123	2.044	1.977	49.173	0.

Problem 174	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	45	52	57	135	0	0
normalized size	1	1.	0.59	0.68	0.75	1.78	0.	0.
time (sec)	N/A	0.017	0.085	0.107	2.049	1.718	0.	0.

Problem 175	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	107	55	74	66	552	0	0
normalized size	1	1.	0.51	0.69	0.62	5.16	0.	0.
time (sec)	N/A	0.027	0.121	0.139	1.871	2.154	0.	0.

Problem 176	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	53	112	929	539	0	0
normalized size	1	1.	0.68	1.44	11.91	6.91	0.	0.
time (sec)	N/A	0.02	0.051	0.117	2.12	1.957	0.	0.

Problem 177	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	32	39	90	84	0	0
normalized size	1	1.	0.91	1.11	2.57	2.4	0.	0.
time (sec)	N/A	0.012	0.033	0.106	1.856	1.638	0.	0.

Problem 178	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	33	52	88	301	0	0
normalized size	1	1.	0.92	1.44	2.44	8.36	0.	0.
time (sec)	N/A	0.008	0.022	0.088	1.949	1.904	0.	0.

Problem 179	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	24	32	35	277	0	0
normalized size	1	1.	0.89	1.19	1.3	10.26	0.	0.
time (sec)	N/A	0.003	0.026	0.077	1.772	1.956	0.	0.

Problem 180	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	35	41	18	84	0	0
normalized size	1	1.	1.	1.17	0.51	2.4	0.	0.
time (sec)	N/A	0.007	0.025	0.094	1.942	1.646	0.	0.

Problem 181	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	48	54	34	443	0	0
normalized size	1	1.	0.7	0.78	0.49	6.42	0.	0.
time (sec)	N/A	0.015	0.052	0.127	2.174	1.984	0.	0.

Problem 182	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	48	52	57	135	0	0
normalized size	1	1.	0.63	0.68	0.75	1.78	0.	0.
time (sec)	N/A	0.017	0.072	0.134	2.161	1.698	0.	0.

Problem 183	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	107	58	74	66	552	0	0
normalized size	1	1.	0.54	0.69	0.62	5.16	0.	0.
time (sec)	N/A	0.028	0.058	0.127	1.976	2.463	0.	0.

Problem 184	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	60	0	0	0	0	0
normalized size	1	1.	1.03	0.	0.	0.	0.	0.
time (sec)	N/A	0.033	0.075	0.067	0.	0.	0.	0.

Problem 185	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	60	0	0	0	0	0
normalized size	1	1.	1.13	0.	0.	0.	0.	0.
time (sec)	N/A	0.031	0.047	0.063	0.	0.	0.	0.

Problem 186	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	56	56	55	0	0	0	0	0
normalized size	1	1.	0.98	0.	0.	0.	0.	0.
time (sec)	N/A	0.025	0.042	0.131	0.	0.	0.	0.

Problem 187	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	58	58	58	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.036	0.067	0.11	0.	0.	0.	0.

Problem 188	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	59	0	0	0	0	0
normalized size	1	1.	1.02	0.	0.	0.	0.	0.
time (sec)	N/A	0.043	0.143	0.166	0.	0.	0.	0.

Problem 189	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	60	0	0	0	0	0
normalized size	1	1.	1.03	0.	0.	0.	0.	0.
time (sec)	N/A	0.031	0.08	0.068	0.	0.	0.	0.

Problem 190	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	60	0	0	0	0	0
normalized size	1	1.	1.09	0.	0.	0.	0.	0.
time (sec)	N/A	0.03	0.049	0.064	0.	0.	0.	0.

Problem 191	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	54	54	57	0	0	0	0	0
normalized size	1	1.	1.06	0.	0.	0.	0.	0.
time (sec)	N/A	0.027	0.008	0.055	0.	0.	0.	0.

Problem 192	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	58	58	56	0	0	0	0	0
normalized size	1	1.	0.97	0.	0.	0.	0.	0.
time (sec)	N/A	0.038	0.009	0.106	0.	0.	0.	0.

Problem 193	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	58	58	60	0	0	0	0	0
normalized size	1	1.	1.03	0.	0.	0.	0.	0.
time (sec)	N/A	0.046	0.054	0.194	0.	0.	0.	0.

Problem 194	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	60	0	0	0	0	0
normalized size	1	1.	1.03	0.	0.	0.	0.	0.
time (sec)	N/A	0.031	0.06	0.065	0.	0.	0.	0.

Problem 195	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	60	0	0	0	0	0
normalized size	1	1.	1.13	0.	0.	0.	0.	0.
time (sec)	N/A	0.031	0.047	0.092	0.	0.	0.	0.

Problem 196	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	56	56	55	0	0	0	0	0
normalized size	1	1.	0.98	0.	0.	0.	0.	0.
time (sec)	N/A	0.025	0.054	0.091	0.	0.	0.	0.

Problem 197	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	58	58	58	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.037	0.115	0.068	0.	0.	0.	0.

Problem 198	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	58	58	60	0	0	0	0	0
normalized size	1	1.	1.03	0.	0.	0.	0.	0.
time (sec)	N/A	0.045	0.07	0.142	0.	0.	0.	0.

Problem 199	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	56	56	60	0	0	0	0	0
normalized size	1	1.	1.07	0.	0.	0.	0.	0.
time (sec)	N/A	0.031	0.043	0.087	0.	0.	0.	0.

Problem 200	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	58	0	0	0	0	0
normalized size	1	1.	1.05	0.	0.	0.	0.	0.
time (sec)	N/A	0.03	0.015	0.085	0.	0.	0.	0.

Problem 201	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	56	56	57	0	0	0	0	0
normalized size	1	1.	1.02	0.	0.	0.	0.	0.
time (sec)	N/A	0.026	0.004	0.059	0.	0.	0.	0.

Problem 202	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	58	58	58	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.038	0.017	0.069	0.	0.	0.	0.

Problem 203	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	58	58	60	0	0	0	0	0
normalized size	1	1.	1.03	0.	0.	0.	0.	0.
time (sec)	N/A	0.047	0.099	0.138	0.	0.	0.	0.

Problem 204	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	83	0	0	0	0	0
normalized size	1	1.	1.02	0.	0.	0.	0.	0.
time (sec)	N/A	0.04	0.092	0.102	0.	0.	0.	0.

Problem 205	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	83	0	0	0	0	0
normalized size	1	1.	1.01	0.	0.	0.	0.	0.
time (sec)	N/A	0.039	0.138	0.094	0.	0.	0.	0.

Problem 206	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	83	0	0	0	0	0
normalized size	1	1.	1.01	0.	0.	0.	0.	0.
time (sec)	N/A	0.041	0.141	0.086	0.	0.	0.	0.

Problem 207	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	83	0	0	0	0	0
normalized size	1	1.	1.01	0.	0.	0.	0.	0.
time (sec)	N/A	0.039	0.141	0.082	0.	0.	0.	0.

Problem 208	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	83	0	0	0	0	0
normalized size	1	1.	1.01	0.	0.	0.	0.	0.
time (sec)	N/A	0.041	0.13	0.091	0.	0.	0.	0.

Problem 209	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	83	0	0	0	0	0
normalized size	1	1.	0.98	0.	0.	0.	0.	0.
time (sec)	N/A	0.043	0.183	0.082	0.	0.	0.	0.

Problem 210	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	76	0	0	0	0	0
normalized size	1	1.	0.85	0.	0.	0.	0.	0.
time (sec)	N/A	0.039	0.065	0.594	0.	0.	0.	0.

Problem 211	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	71	0	0	0	0	0
normalized size	1	1.	0.99	0.	0.	0.	0.	0.
time (sec)	N/A	0.044	0.047	0.328	0.	0.	0.	0.

Problem 212	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	65	0	0	0	0	0
normalized size	1	1.	1.07	0.	0.	0.	0.	0.
time (sec)	N/A	0.037	0.04	0.396	0.	0.	0.	0.

Problem 213	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	73	73	61	0	0	0	0	0
normalized size	1	1.	0.84	0.	0.	0.	0.	0.
time (sec)	N/A	0.034	0.042	0.243	0.	0.	0.	0.

Problem 214	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	75	75	68	0	0	0	0	0
normalized size	1	1.	0.91	0.	0.	0.	0.	0.
time (sec)	N/A	0.049	0.094	0.859	0.	0.	0.	0.

Problem 215	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	75	75	71	0	0	0	0	0
normalized size	1	1.	0.95	0.	0.	0.	0.	0.
time (sec)	N/A	0.055	0.095	0.968	0.	0.	0.	0.

Problem 216	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	75	75	73	0	0	0	0	0
normalized size	1	1.	0.97	0.	0.	0.	0.	0.
time (sec)	N/A	0.056	0.108	1.35	0.	0.	0.	0.

Problem 217	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	81	0	0	0	0	0
normalized size	1	1.	1.01	0.	0.	0.	0.	0.
time (sec)	N/A	0.04	0.104	0.12	0.	0.	0.	0.

Problem 218	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	81	0	0	0	0	0
normalized size	1	1.	1.01	0.	0.	0.	0.	0.
time (sec)	N/A	0.038	0.092	0.117	0.	0.	0.	0.

Problem 219	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	81	0	0	0	0	0
normalized size	1	1.	1.01	0.	0.	0.	0.	0.
time (sec)	N/A	0.038	0.101	0.128	0.	0.	0.	0.

Problem 220	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	81	0	0	0	0	0
normalized size	1	1.	1.01	0.	0.	0.	0.	0.
time (sec)	N/A	0.039	0.117	0.128	0.	0.	0.	0.

Problem 221	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	81	0	0	0	0	0
normalized size	1	1.	1.01	0.	0.	0.	0.	0.
time (sec)	N/A	0.04	0.139	0.12	0.	0.	0.	0.

Problem 222	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	81	0	0	0	0	0
normalized size	1	1.	1.01	0.	0.	0.	0.	0.
time (sec)	N/A	0.039	0.18	0.123	0.	0.	0.	0.

Problem 223	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	20	17	31	66	0	45
normalized size	1	1.	1.	0.85	1.55	3.3	0.	2.25
time (sec)	N/A	0.035	0.055	0.021	1.127	1.633	0.	1.274

Problem 224	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	20	17	31	63	0	45
normalized size	1	1.	1.	0.85	1.55	3.15	0.	2.25
time (sec)	N/A	0.036	0.041	0.017	1.145	1.622	0.	1.279

Problem 225	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	18	17	31	38	0	34
normalized size	1	1.	1.	0.94	1.72	2.11	0.	1.89
time (sec)	N/A	0.036	0.033	0.018	1.158	1.663	0.	1.339

Problem 226	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	18	17	31	54	0	30
normalized size	1	1.	1.	0.94	1.72	3.	0.	1.67
time (sec)	N/A	0.033	0.038	0.025	1.058	1.658	0.	1.325

Problem 227	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	20	17	31	65	0	47
normalized size	1	1.	1.	0.85	1.55	3.25	0.	2.35
time (sec)	N/A	0.033	0.053	0.025	1.185	1.709	0.	3.622

Problem 228	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	32	357	49	97	0	66
normalized size	1	1.	0.78	8.71	1.2	2.37	0.	1.61
time (sec)	N/A	0.049	0.22	0.199	1.097	1.658	0.	1.189

Problem 229	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	42	36	51	105	0	66
normalized size	1	1.	0.98	0.84	1.19	2.44	0.	1.53
time (sec)	N/A	0.049	0.103	0.149	1.205	1.695	0.	1.343

Problem 230	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	128	128	122	542	0	0	0	0
normalized size	1	1.	0.95	4.23	0.	0.	0.	0.
time (sec)	N/A	0.204	1.487	0.253	0.	0.	0.	0.

Problem 231	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	56	54	0	155	0	0
normalized size	1	1.	0.81	0.78	0.	2.25	0.	0.
time (sec)	N/A	0.097	0.116	0.191	0.	1.826	0.	0.

Problem 232	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	93	109	285	0	0	0	0
normalized size	1	1.	1.17	3.06	0.	0.	0.	0.
time (sec)	N/A	0.143	0.858	0.2	0.	0.	0.	0.

Problem 233	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	31	42	0	85	0	0
normalized size	1	1.	1.	1.35	0.	2.74	0.	0.
time (sec)	N/A	0.049	0.063	0.164	0.	1.709	0.	0.

Problem 234	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	68	157	0	0	0	0
normalized size	1	1.	1.28	2.96	0.	0.	0.	0.
time (sec)	N/A	0.093	0.671	0.196	0.	0.	0.	0.

Problem 235	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	270	270	171	276	0	0	0	0
normalized size	1	1.	0.63	1.02	0.	0.	0.	0.
time (sec)	N/A	0.14	1.326	0.148	0.	0.	0.	0.

Problem 236	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	93	80	188	0	0	0	0
normalized size	1	1.	0.86	2.02	0.	0.	0.	0.
time (sec)	N/A	0.147	0.673	0.186	0.	0.	0.	0.

Problem 237	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	322	322	222	522	0	0	0	0
normalized size	1	1.	0.69	1.62	0.	0.	0.	0.
time (sec)	N/A	0.212	1.789	0.225	0.	0.	0.	0.

Problem 238	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	104	57	64	0	203	0	0
normalized size	1	1.	0.55	0.62	0.	1.95	0.	0.
time (sec)	N/A	0.162	0.298	0.196	0.	2.447	0.	0.

Problem 239	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	166	166	114	996	0	0	0	0
normalized size	1	1.	0.69	6.	0.	0.	0.	0.
time (sec)	N/A	0.268	1.12	0.207	0.	0.	0.	0.

Problem 240	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	45	54	0	135	0	0
normalized size	1	1.	0.65	0.78	0.	1.96	0.	0.
time (sec)	N/A	0.103	0.135	0.162	0.	2.062	0.	0.

Problem 241	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	125	125	99	503	0	0	0	0
normalized size	1	1.	0.79	4.02	0.	0.	0.	0.
time (sec)	N/A	0.205	0.512	0.187	0.	0.	0.	0.

Problem 242	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	31	42	0	84	0	0
normalized size	1	1.	1.	1.35	0.	2.71	0.	0.
time (sec)	N/A	0.048	0.061	0.169	0.	1.658	0.	0.

Problem 243	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	66	505	0	0	0	0
normalized size	1	1.	0.74	5.67	0.	0.	0.	0.
time (sec)	N/A	0.143	0.367	0.191	0.	0.	0.	0.

Problem 244	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	327	327	64	656	0	0	0	0
normalized size	1	1.	0.2	2.01	0.	0.	0.	0.
time (sec)	N/A	0.219	0.242	0.171	0.	0.	0.	0.

Problem 245	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	94	69	520	0	0	0	0
normalized size	1	1.	0.73	5.53	0.	0.	0.	0.
time (sec)	N/A	0.153	0.407	0.17	0.	0.	0.	0.

Problem 246	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	166	166	92	563	0	0	0	0
normalized size	1	1.	0.55	3.39	0.	0.	0.	0.
time (sec)	N/A	0.265	1.423	0.232	0.	0.	0.	0.

Problem 247	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	106	106	57	64	0	207	0	0
normalized size	1	1.	0.54	0.6	0.	1.95	0.	0.
time (sec)	N/A	0.162	0.193	0.165	0.	1.871	0.	0.

Problem 248	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	131	131	87	304	0	0	0	0
normalized size	1	1.	0.66	2.32	0.	0.	0.	0.
time (sec)	N/A	0.204	0.652	0.195	0.	0.	0.	0.

Problem 249	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	45	54	0	132	0	0
normalized size	1	1.	0.65	0.78	0.	1.91	0.	0.
time (sec)	N/A	0.102	0.207	0.161	0.	1.55	0.	0.

Problem 250	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	93	68	190	0	0	0	0
normalized size	1	1.	0.73	2.04	0.	0.	0.	0.
time (sec)	N/A	0.141	0.579	0.206	0.	0.	0.	0.

Problem 251	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	33	42	0	127	0	0
normalized size	1	1.	1.	1.27	0.	3.85	0.	0.
time (sec)	N/A	0.049	0.112	0.149	0.	1.363	0.	0.

Problem 252	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	70	192	0	0	0	0
normalized size	1	1.	0.71	1.96	0.	0.	0.	0.
time (sec)	N/A	0.15	0.528	0.176	0.	0.	0.	0.

Problem 253	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	329	329	280	544	0	0	0	0
normalized size	1	1.	0.85	1.65	0.	0.	0.	0.
time (sec)	N/A	0.217	1.73	0.172	0.	0.	0.	0.

Problem 254	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	45	54	0	185	0	0
normalized size	1	1.	0.65	0.78	0.	2.68	0.	0.
time (sec)	N/A	0.1	0.258	0.164	0.	2.328	0.	0.

Problem 255	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	128	128	104	992	0	0	0	0
normalized size	1	1.	0.81	7.75	0.	0.	0.	0.
time (sec)	N/A	0.193	1.048	0.207	0.	0.	0.	0.

Problem 256	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	33	42	0	116	0	0
normalized size	1	1.	1.	1.27	0.	3.52	0.	0.
time (sec)	N/A	0.048	0.112	0.153	0.	2.066	0.	0.

Problem 257	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	80	502	0	0	0	0
normalized size	1	1.	0.9	5.64	0.	0.	0.	0.
time (sec)	N/A	0.139	0.466	0.185	0.	0.	0.	0.

Problem 258	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	270	270	55	322	0	0	0	0
normalized size	1	1.	0.2	1.19	0.	0.	0.	0.
time (sec)	N/A	0.131	0.125	0.184	0.	0.	0.	0.

Problem 259	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	66	517	0	0	0	0
normalized size	1	1.	1.25	9.75	0.	0.	0.	0.
time (sec)	N/A	0.088	0.218	0.172	0.	0.	0.	0.

Problem 260	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	322	322	66	668	0	0	0	0
normalized size	1	1.	0.2	2.07	0.	0.	0.	0.
time (sec)	N/A	0.204	0.256	0.17	0.	0.	0.	0.

Problem 261	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	84	531	0	0	0	0
normalized size	1	1.	0.88	5.59	0.	0.	0.	0.
time (sec)	N/A	0.141	0.354	0.213	0.	0.	0.	0.

Problem 262	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	110	57	54	0	203	0	0
normalized size	1	1.	0.52	0.49	0.	1.85	0.	0.
time (sec)	N/A	0.154	0.281	0.158	0.	2.212	0.	0.

Problem 263	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	135	135	119	550	0	0	0	0
normalized size	1	1.	0.88	4.07	0.	0.	0.	0.
time (sec)	N/A	0.2	1.422	0.193	0.	0.	0.	0.

Problem 264	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	45	42	0	131	0	0
normalized size	1	1.	1.36	1.27	0.	3.97	0.	0.
time (sec)	N/A	0.053	0.133	0.142	0.	1.818	0.	0.

Problem 265	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	105	290	0	0	0	0
normalized size	1	1.	1.07	2.96	0.	0.	0.	0.
time (sec)	N/A	0.148	0.798	0.173	0.	0.	0.	0.

Problem 266	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	327	327	199	975	0	0	0	0
normalized size	1	1.	0.61	2.98	0.	0.	0.	0.
time (sec)	N/A	0.215	0.635	0.178	0.	0.	0.	0.

Problem 267	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	92	84	195	0	0	0	0
normalized size	1	1.	0.91	2.12	0.	0.	0.	0.
time (sec)	N/A	0.143	0.641	0.19	0.	0.	0.	0.

Problem 268	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	322	322	223	526	0	0	0	0
normalized size	1	1.	0.69	1.63	0.	0.	0.	0.
time (sec)	N/A	0.213	1.973	0.19	0.	0.	0.	0.

Problem 269	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	135	135	89	222	0	0	0	0
normalized size	1	1.	0.66	1.64	0.	0.	0.	0.
time (sec)	N/A	0.207	0.555	0.181	0.	0.	0.	0.

Problem 270	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	371	371	246	556	0	0	0	0
normalized size	1	1.	0.66	1.5	0.	0.	0.	0.
time (sec)	N/A	0.297	2.304	0.174	0.	0.	0.	0.

Problem 271	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	45	42	0	151	0	0
normalized size	1	1.	1.36	1.27	0.	4.58	0.	0.
time (sec)	N/A	0.054	0.158	0.138	0.	2.309	0.	0.

Problem 272	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	135	135	101	993	0	0	0	0
normalized size	1	1.	0.75	7.36	0.	0.	0.	0.
time (sec)	N/A	0.202	1.802	0.196	0.	0.	0.	0.

Problem 273	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	329	329	55	1259	0	0	0	0
normalized size	1	1.	0.17	3.83	0.	0.	0.	0.
time (sec)	N/A	0.214	0.2	0.179	0.	0.	0.	0.

Problem 274	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	94	80	515	0	0	0	0
normalized size	1	1.	0.85	5.48	0.	0.	0.	0.
time (sec)	N/A	0.147	0.661	0.169	0.	0.	0.	0.

Problem 275	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	322	322	70	668	0	0	0	0
normalized size	1	1.	0.22	2.07	0.	0.	0.	0.
time (sec)	N/A	0.203	0.221	0.175	0.	0.	0.	0.

Problem 276	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	79	530	0	0	0	0
normalized size	1	1.	0.83	5.58	0.	0.	0.	0.
time (sec)	N/A	0.142	0.363	0.192	0.	0.	0.	0.

Problem 277	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	371	371	81	696	0	0	0	0
normalized size	1	1.	0.22	1.88	0.	0.	0.	0.
time (sec)	N/A	0.291	0.379	0.183	0.	0.	0.	0.

Problem 278	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	135	135	90	544	0	0	0	0
normalized size	1	1.	0.67	4.03	0.	0.	0.	0.
time (sec)	N/A	0.206	0.658	0.165	0.	0.	0.	0.

Problem 279	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	406	406	95	722	0	0	0	0
normalized size	1	1.	0.23	1.78	0.	0.	0.	0.
time (sec)	N/A	0.39	0.411	0.221	0.	0.	0.	0.

Problem 280	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	81	81	278	0	0	0	0	0
normalized size	1	1.	3.43	0.	0.	0.	0.	0.
time (sec)	N/A	0.074	2.249	0.565	0.	0.	0.	0.

Problem 281	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	86	86	280	0	0	0	0	0
normalized size	1	1.	3.26	0.	0.	0.	0.	0.
time (sec)	N/A	0.086	0.549	0.509	0.	0.	0.	0.

Problem 282	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	84	84	281	0	0	0	0	0
normalized size	1	1.	3.35	0.	0.	0.	0.	0.
time (sec)	N/A	0.087	0.312	0.529	0.	0.	0.	0.

Problem 283	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	89	89	283	0	0	0	0	0
normalized size	1	1.	3.18	0.	0.	0.	0.	0.
time (sec)	N/A	0.094	0.188	0.558	0.	0.	0.	0.

Problem 284	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	51	0	0	0	0	0
normalized size	1	1.	1.06	0.	0.	0.	0.	0.
time (sec)	N/A	0.051	0.051	0.338	0.	0.	0.	0.

Problem 285	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	51	0	0	0	0	0
normalized size	1	1.	1.04	0.	0.	0.	0.	0.
time (sec)	N/A	0.051	0.043	0.323	0.	0.	0.	0.

Problem 286	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	51	0	0	0	0	0
normalized size	1	1.	1.06	0.	0.	0.	0.	0.
time (sec)	N/A	0.036	0.033	0.31	0.	0.	0.	0.

Problem 287	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	23	66	39	59	0	0
normalized size	1	1.	0.96	2.75	1.62	2.46	0.	0.
time (sec)	N/A	0.034	0.025	0.07	1.149	1.699	0.	0.

Problem 288	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	52	45	0	78	115	0	0
normalized size	1	1.	0.87	0.	1.5	2.21	0.	0.
time (sec)	N/A	0.052	0.126	1.153	1.155	1.747	0.	0.

Problem 289	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	81	0	116	178	0	0
normalized size	1	1.	1.04	0.	1.49	2.28	0.	0.
time (sec)	N/A	0.064	0.554	1.199	1.143	1.763	0.	0.

Problem 290	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	77	0	0	0	0	0
normalized size	1	1.	1.07	0.	0.	0.	0.	0.
time (sec)	N/A	0.08	0.577	0.352	0.	0.	0.	0.

Problem 291	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	77	0	0	0	0	0
normalized size	1	1.	1.07	0.	0.	0.	0.	0.
time (sec)	N/A	0.079	0.509	0.351	0.	0.	0.	0.

Problem 292	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	75	0	0	0	0	0
normalized size	1	1.	1.04	0.	0.	0.	0.	0.
time (sec)	N/A	0.078	0.443	0.323	0.	0.	0.	0.

Problem 293	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	65	0	0	0	0	0
normalized size	1	1.	0.9	0.	0.	0.	0.	0.
time (sec)	N/A	0.03	0.1	0.471	0.	0.	0.	0.

Problem 294	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	165	0	0	0	0	0
normalized size	1	1.	2.29	0.	0.	0.	0.	0.
time (sec)	N/A	0.08	0.457	0.946	0.	0.	0.	0.

Problem 295	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	246	0	0	0	0	0
normalized size	1	1.	3.42	0.	0.	0.	0.	0.
time (sec)	N/A	0.076	0.62	1.065	0.	0.	0.	0.

Problem 296	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	92	0	0	0	0	0
normalized size	1	1.	1.14	0.	0.	0.	0.	0.
time (sec)	N/A	0.115	1.827	0.159	0.	0.	0.	0.

Problem 297	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	90	0	0	0	0	0
normalized size	1	1.	1.11	0.	0.	0.	0.	0.
time (sec)	N/A	0.097	1.61	0.152	0.	0.	0.	0.

Problem 298	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	81	81	326	0	0	0	0	0
normalized size	1	1.	4.02	0.	0.	0.	0.	0.
time (sec)	N/A	0.096	3.101	0.144	0.	0.	0.	0.

Problem 299	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	115	0	0	0	0	0
normalized size	1	1.	1.42	0.	0.	0.	0.	0.
time (sec)	N/A	0.107	1.006	0.136	0.	0.	0.	0.

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [270] had the largest ratio of [0.44]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	1	1	1.	6	0.167
2	A	2	2	1.	8	0.25
3	A	2	2	1.	8	0.25
4	A	2	1	1.	8	0.125
5	A	3	2	1.	8	0.25
6	A	2	1	1.	8	0.125
7	A	4	2	1.	8	0.25
8	A	2	1	1.	8	0.125
9	A	4	3	1.	10	0.3
10	A	3	3	1.	10	0.3
11	A	3	3	1.	10	0.3
12	A	2	2	1.	10	0.2
13	A	2	2	1.	10	0.2
14	A	3	3	1.	10	0.3
15	A	3	3	1.	10	0.3
16	A	4	3	1.	10	0.3
17	A	4	3	1.	12	0.25
18	A	3	3	1.	12	0.25
19	A	3	3	1.	12	0.25
20	A	2	2	1.	12	0.167
21	A	2	2	1.	12	0.167
22	A	3	3	1.	12	0.25
23	A	3	3	1.	12	0.25
24	A	4	3	1.	12	0.25
25	A	2	2	1.	10	0.2
26	A	2	2	1.	10	0.2
27	A	2	2	1.	10	0.2
28	A	2	2	1.	10	0.2
29	A	2	2	1.	10	0.2
30	A	2	2	1.	10	0.2
31	A	2	2	1.	12	0.167
32	A	2	2	1.	12	0.167
33	A	2	2	1.	12	0.167
34	A	2	2	1.	12	0.167
35	A	2	2	1.	12	0.167
36	A	2	2	1.	12	0.167
37	A	2	2	1.	8	0.25
38	A	2	2	1.	10	0.2
39	A	5	3	1.	8	0.375
40	A	4	3	1.	8	0.375
41	A	3	3	1.	8	0.375
42	A	2	2	1.	8	0.25
43	A	2	2	1.	8	0.25

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
44	A	3	3	1.	8	0.375
45	A	4	3	1.	8	0.375
46	A	5	3	1.	8	0.375
47	A	6	4	1.	10	0.4
48	A	5	4	1.	10	0.4
49	A	4	4	1.	10	0.4
50	A	3	3	1.	10	0.3
51	A	2	2	1.	10	0.2
52	A	3	3	1.	10	0.3
53	A	4	3	1.	10	0.3
54	A	5	3	1.	10	0.3
55	A	7	4	1.	10	0.4
56	A	5	4	1.	10	0.4
57	A	4	4	1.	10	0.4
58	A	4	4	1.	10	0.4
59	A	5	4	1.	10	0.4
60	A	7	4	1.	10	0.4
61	A	3	2	1.	10	0.2
62	A	3	2	1.	10	0.2
63	A	3	2	1.	10	0.2
64	A	3	3	1.	10	0.3
65	A	3	3	1.	10	0.3
66	A	5	3	1.	10	0.3
67	A	7	3	1.	10	0.3
68	A	3	3	1.	12	0.25
69	A	3	3	1.	14	0.214
70	A	5	4	1.	21	0.19
71	A	5	4	1.	21	0.19
72	A	4	4	1.	21	0.19
73	A	4	4	1.	19	0.21
74	A	2	2	1.	12	0.167
75	A	3	3	1.	19	0.158
76	A	4	4	1.	21	0.19
77	A	4	4	1.	21	0.19
78	A	5	4	1.	21	0.19
79	A	5	4	1.	21	0.19
80	A	5	4	1.	21	0.19
81	A	5	4	1.	21	0.19
82	A	4	4	1.	19	0.21
83	A	3	3	1.	12	0.25
84	A	3	3	1.	19	0.158
85	A	3	3	1.	21	0.143
86	A	4	4	1.	21	0.19
87	A	4	4	1.	21	0.19

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
88	A	5	4	1.	21	0.19
89	A	5	4	1.	21	0.19
90	A	5	4	1.	21	0.19
91	A	5	4	1.	19	0.21
92	A	3	3	1.	12	0.25
93	A	4	4	1.	19	0.21
94	A	3	3	1.	21	0.143
95	A	3	3	1.	21	0.143
96	A	4	4	1.	21	0.19
97	A	4	4	1.	21	0.19
98	A	5	4	1.	21	0.19
99	A	5	4	1.	21	0.19
100	A	4	3	1.	12	0.25
101	A	5	4	1.	21	0.19
102	A	5	4	1.	21	0.19
103	A	4	4	1.	21	0.19
104	A	4	4	1.	21	0.19
105	A	3	3	1.	19	0.158
106	A	2	2	1.	12	0.167
107	A	4	4	1.	19	0.21
108	A	4	4	1.	21	0.19
109	A	5	4	1.	21	0.19
110	A	5	4	1.	21	0.19
111	A	5	4	1.	21	0.19
112	A	5	4	1.	21	0.19
113	A	4	4	1.	21	0.19
114	A	4	4	1.	21	0.19
115	A	3	3	1.	21	0.143
116	A	3	3	1.	19	0.158
117	A	3	3	1.	12	0.25
118	A	4	4	1.	19	0.21
119	A	5	4	1.	21	0.19
120	A	5	4	1.	21	0.19
121	A	5	4	1.	21	0.19
122	A	5	4	1.	21	0.19
123	A	4	4	1.	21	0.19
124	A	4	4	1.	21	0.19
125	A	3	3	1.	21	0.143
126	A	3	3	1.	21	0.143
127	A	4	4	1.	19	0.21
128	A	3	3	1.	12	0.25
129	A	5	4	1.	19	0.21
130	A	5	4	1.	21	0.19
131	A	4	3	1.	12	0.25

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
132	A	4	3	1.	23	0.13
133	A	3	2	1.	23	0.087
134	A	3	3	1.	23	0.13
135	A	3	3	1.	23	0.13
136	A	2	2	1.	23	0.087
137	A	2	2	1.	23	0.087
138	A	2	2	1.	23	0.087
139	A	3	3	1.	23	0.13
140	A	3	2	1.	23	0.087
141	A	4	3	1.	23	0.13
142	A	4	3	1.	23	0.13
143	A	3	2	1.	23	0.087
144	A	3	3	1.	23	0.13
145	A	3	3	1.	23	0.13
146	A	2	2	1.	23	0.087
147	A	2	2	1.	23	0.087
148	A	2	2	1.	23	0.087
149	A	3	3	1.	23	0.13
150	A	3	2	1.	23	0.087
151	A	4	3	1.	23	0.13
152	A	3	2	1.	23	0.087
153	A	3	2	1.	23	0.087
154	A	3	3	1.	23	0.13
155	A	3	3	1.	23	0.13
156	A	2	2	1.	23	0.087
157	A	2	2	1.	23	0.087
158	A	2	2	1.	23	0.087
159	A	3	3	1.	23	0.13
160	A	3	2	1.	23	0.087
161	A	3	3	1.	23	0.13
162	A	3	3	1.	23	0.13
163	A	2	2	1.	23	0.087
164	A	2	2	1.	23	0.087
165	A	2	2	1.	23	0.087
166	A	3	3	1.	23	0.13
167	A	3	2	1.	23	0.087
168	A	3	3	1.	23	0.13
169	A	3	3	1.	23	0.13
170	A	2	2	1.	23	0.087
171	A	2	2	1.	23	0.087
172	A	2	2	1.	23	0.087
173	A	3	3	1.	23	0.13
174	A	3	2	1.	23	0.087
175	A	4	3	1.	23	0.13

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
176	A	3	3	1.	23	0.13
177	A	3	3	1.	23	0.13
178	A	2	2	1.	23	0.087
179	A	2	2	1.	23	0.087
180	A	2	2	1.	23	0.087
181	A	3	3	1.	23	0.13
182	A	3	2	1.	23	0.087
183	A	4	3	1.	23	0.13
184	A	3	3	1.	21	0.143
185	A	3	3	1.	19	0.158
186	A	2	2	1.	12	0.167
187	A	3	3	1.	19	0.158
188	A	3	3	1.	21	0.143
189	A	3	3	1.	21	0.143
190	A	3	3	1.	19	0.158
191	A	2	2	1.	12	0.167
192	A	3	3	1.	19	0.158
193	A	3	3	1.	21	0.143
194	A	3	3	1.	21	0.143
195	A	3	3	1.	19	0.158
196	A	2	2	1.	12	0.167
197	A	3	3	1.	19	0.158
198	A	3	3	1.	21	0.143
199	A	3	3	1.	21	0.143
200	A	3	3	1.	19	0.158
201	A	2	2	1.	12	0.167
202	A	3	3	1.	19	0.158
203	A	3	3	1.	21	0.143
204	A	3	3	1.	21	0.143
205	A	3	3	1.	21	0.143
206	A	3	3	1.	21	0.143
207	A	3	3	1.	21	0.143
208	A	3	3	1.	21	0.143
209	A	3	3	1.	21	0.143
210	A	3	3	1.	19	0.158
211	A	3	3	1.	19	0.158
212	A	3	3	1.	17	0.176
213	A	2	2	1.	10	0.2
214	A	3	3	1.	17	0.176
215	A	3	3	1.	19	0.158
216	A	3	3	1.	19	0.158
217	A	3	3	1.	21	0.143
218	A	3	3	1.	21	0.143
219	A	3	3	1.	21	0.143

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
220	A	3	3	1.	21	0.143
221	A	3	3	1.	21	0.143
222	A	3	3	1.	21	0.143
223	A	2	2	1.	19	0.105
224	A	2	2	1.	19	0.105
225	A	2	2	1.	19	0.105
226	A	2	2	1.	19	0.105
227	A	2	2	1.	19	0.105
228	A	3	2	1.	21	0.095
229	A	3	2	1.	21	0.095
230	A	5	4	1.	25	0.16
231	A	2	2	1.	25	0.08
232	A	4	4	1.	25	0.16
233	A	1	1	1.	25	0.04
234	A	3	3	1.	25	0.12
235	A	12	9	1.	25	0.36
236	A	4	4	1.	25	0.16
237	A	13	10	1.	25	0.4
238	A	3	2	1.	25	0.08
239	A	6	5	1.	25	0.2
240	A	2	2	1.	25	0.08
241	A	5	5	1.	25	0.2
242	A	1	1	1.	25	0.04
243	A	4	4	1.	25	0.16
244	A	13	10	1.	25	0.4
245	A	4	4	1.	25	0.16
246	A	6	5	1.	25	0.2
247	A	3	3	1.	25	0.12
248	A	5	5	1.	25	0.2
249	A	2	2	1.	25	0.08
250	A	4	4	1.	25	0.16
251	A	1	1	1.	25	0.04
252	A	4	4	1.	25	0.16
253	A	13	10	1.	25	0.4
254	A	2	2	1.	25	0.08
255	A	5	4	1.	25	0.16
256	A	1	1	1.	25	0.04
257	A	4	4	1.	25	0.16
258	A	12	9	1.	25	0.36
259	A	3	3	1.	25	0.12
260	A	13	10	1.	25	0.4
261	A	4	4	1.	25	0.16
262	A	3	3	1.	25	0.12
263	A	5	5	1.	25	0.2

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
264	A	1	1	1.	25	0.04
265	A	4	4	1.	25	0.16
266	A	13	10	1.	25	0.4
267	A	4	4	1.	25	0.16
268	A	13	10	1.	25	0.4
269	A	5	5	1.	25	0.2
270	A	14	11	1.	25	0.44
271	A	1	1	1.	25	0.04
272	A	5	5	1.	25	0.2
273	A	13	10	1.	25	0.4
274	A	4	4	1.	25	0.16
275	A	13	10	1.	25	0.4
276	A	4	4	1.	25	0.16
277	A	14	11	1.	25	0.44
278	A	5	5	1.	25	0.2
279	A	15	11	1.	25	0.44
280	A	2	2	1.	17	0.118
281	A	2	2	1.	19	0.105
282	A	2	2	1.	19	0.105
283	A	2	2	1.	21	0.095
284	A	2	2	1.	19	0.105
285	A	2	2	1.	19	0.105
286	A	2	2	1.	17	0.118
287	A	2	2	1.	17	0.118
288	A	3	2	1.	19	0.105
289	A	3	2	1.	19	0.105
290	A	2	2	1.	19	0.105
291	A	2	2	1.	19	0.105
292	A	2	2	1.	19	0.105
293	A	2	2	1.	10	0.2
294	A	2	2	1.	19	0.105
295	A	2	2	1.	19	0.105
296	A	2	2	1.	23	0.087
297	A	2	2	1.	23	0.087
298	A	2	2	1.	23	0.087
299	A	2	2	1.	23	0.087

Chapter 3

Listing of integrals

3.1 $\int \sec(a+bx)\,dx$

Optimal. Leaf size=11

$$\frac{\tanh^{-1}(\sin(a+bx))}{b}$$

```
[Out] ArcTanh[Sin[a + b*x]]/b
```

Rubi [A] time = 0.0042149, antiderivative size = 11, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3770}

$$\frac{\tanh^{-1}(\sin(a+bx))}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[a + b*x],x]
```

```
[Out] ArcTanh[Sin[a + b*x]]/b
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sec(a+bx)\,dx = \frac{\tanh^{-1}(\sin(a+bx))}{b}$$

Mathematica [A] time = 0.0022212, size = 11, normalized size = 1.

$$\frac{\tanh^{-1}(\sin(a+bx))}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[a + b*x],x]
```

```
[Out] ArcTanh[Sin[a + b*x]]/b
```

Maple [A] time = 0.003, size = 19, normalized size = 1.7

$$\frac{\ln\left(\sec\left(bx+a\right)+\tan\left(bx+a\right)\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a),x)
```

```
[Out] 1/b*ln(sec(b*x+a)+tan(b*x+a))
```

Maxima [A] time = 1.10381, size = 24, normalized size = 2.18

$$\frac{\log\left(\sec\left(bx+a\right)+\tan\left(bx+a\right)\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a),x, algorithm="maxima")
```

```
[Out] log(sec(b*x + a) + tan(b*x + a))/b
```

Fricas [B] time = 1.52565, size = 76, normalized size = 6.91

$$\frac{\log\left(\sin\left(bx+a\right)+1\right)-\log\left(-\sin\left(bx+a\right)+1\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/2*(log(sin(b*x + a) + 1) - log(-sin(b*x + a) + 1))/b
```

Sympy [A] time = 5.91471, size = 36, normalized size = 3.27

$$\begin{cases} \frac{\log\left(\tan\left(a+bx\right)+\sec\left(a+bx\right)\right)}{b} & \text{for}\: b \neq 0 \\ \frac{x\left(\tan\left(a\right)\sec\left(a\right)+\sec^{2}\left(a\right)\right)}{\tan\left(a\right)+\sec\left(a\right)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a),x)
```

```
[Out] Piecewise((log(tan(a + b*x) + sec(a + b*x))/b, Ne(b, 0)), (x*(tan(a)*sec(a)
 + sec(a)**2)/(tan(a) + sec(a)), True))
```

Giac [B] time = 1.31224, size = 59, normalized size = 5.36

$$\frac{\log\left(\left|\frac{1}{\sin(bx+a)} + \sin\left(bx+a\right) + 2\right|\right) - \log\left(\left|\frac{1}{\sin(bx+a)} + \sin\left(bx+a\right) - 2\right|\right)}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a),x, algorithm="giac")
```

```
[Out] 1/4*(log(abs(1/sin(b*x + a) + sin(b*x + a) + 2)) - log(abs(1/sin(b*x + a) +
 sin(b*x + a) - 2)))/b
```

3.2 $\int \sec^2(a+bx)\,dx$

Optimal. Leaf size=10

$$\frac{\tan(a+bx)}{b}$$

```
[Out] Tan[a + b*x]/b
```

Rubi [A] time = 0.0087791, antiderivative size = 10, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {3767, 8}

$$\frac{\tan(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[a + b*x]^2,x]
```

```
[Out] Tan[a + b*x]/b
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \sec^2(a+bx)\,dx &= -\frac{\text{Subst}\left(\int 1\,dx, x, -\tan(a+bx)\right)}{b}\\ &= \frac{\tan(a+bx)}{b}\end{aligned}$$

Mathematica [A] time = 0.0035802, size = 10, normalized size = 1.

$$\frac{\tan(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[a + b*x]^2,x]
```

```
[Out] Tan[a + b*x]/b
```

Maple [A] time = 0.056, size = 11, normalized size = 1.1

$$\frac{\tan(bx+a)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)^2,x)
```

```
[Out] tan(b*x+a)/b
```

Maxima [A] time = 1.26571, size = 14, normalized size = 1.4

$$\frac{\tan(bx+a)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] tan(b*x + a)/b
```

Fricas [A] time = 1.40265, size = 42, normalized size = 4.2

$$\frac{\sin(bx+a)}{b\cos(bx+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] sin(b*x + a)/(b*cos(b*x + a))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sec^2(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)**2,x)
```

```
[Out] Integral(sec(a + b*x)**2, x)
```

Giac [A] time = 1.24984, size = 14, normalized size = 1.4

$$\frac{\tan(bx+a)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^2,x, algorithm="giac")
```

```
[Out] tan(b*x + a)/b
```

3.3 $\int \sec^3(a+bx)\,dx$

Optimal. Leaf size=34

$$\frac{\tanh^{-1}(\sin(a+bx))}{2b}+\frac{\tan(a+bx)\sec(a+bx)}{2b}$$

```
[Out] ArcTanh[Sin[a + b*x]]/(2*b) + (Sec[a + b*x]*Tan[a + b*x])/(2*b)
```

Rubi [A] time = 0.0143485, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {3768, 3770}

$$\frac{\tanh^{-1}(\sin(a+bx))}{2b}+\frac{\tan(a+bx)\sec(a+bx)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[a + b*x]^3,x]
```

```
[Out] ArcTanh[Sin[a + b*x]]/(2*b) + (Sec[a + b*x]*Tan[a + b*x])/(2*b)
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \sec^3(a+bx)\,dx &= \frac{\sec(a+bx)\tan(a+bx)}{2b}+\frac{1}{2}\int \sec(a+bx)\,dx\\ &= \frac{\tanh^{-1}(\sin(a+bx))}{2b}+\frac{\sec(a+bx)\tan(a+bx)}{2b}\end{aligned}$$

Mathematica [A] time = 0.0098694, size = 34, normalized size = 1.

$$\frac{\tanh^{-1}(\sin(a+bx))}{2b}+\frac{\tan(a+bx)\sec(a+bx)}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[a + b*x]^3,x]
```

```
[Out] ArcTanh[Sin[a + b*x]]/(2*b) + (Sec[a + b*x]*Tan[a + b*x])/(2*b)
```

Maple [A] time = 0.133, size = 38, normalized size = 1.1

$$\frac{\sec(bx+a)\tan(bx+a)}{2b}+\frac{\ln(\sec(bx+a)+\tan(bx+a))}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)^3,x)
```

```
[Out] 1/2*sec(b*x+a)*tan(b*x+a)/b+1/2/b*ln(sec(b*x+a)+tan(b*x+a))
```

Maxima [A] time = 1.00103, size = 62, normalized size = 1.82

$$-\frac{\frac{2\sin(bx+a)}{\sin(bx+a)^2-1}-\log(\sin(bx+a)+1)+\log(\sin(bx+a)-1)}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -1/4*(2*sin(b*x + a)/(sin(b*x + a)^2 - 1) - log(sin(b*x + a) + 1) + log(sin
(b*x + a) - 1))/b
```

Fricas [B] time = 1.41226, size = 162, normalized size = 4.76

$$\frac{\cos(bx+a)^2\log(\sin(bx+a)+1)-\cos(bx+a)^2\log(-\sin(bx+a)+1)+2\sin(bx+a)}{4b\cos(bx+a)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/4*(cos(b*x + a)^2*log(sin(b*x + a) + 1) - cos(b*x + a)^2*log(-sin(b*x + a
) + 1) + 2*sin(b*x + a))/(b*cos(b*x + a)^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\sec^3(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)**3,x)
```

```
[Out] Integral(sec(a + b*x)**3, x)
```

Giac [A] time = 1.35288, size = 65, normalized size = 1.91

$$-\frac{\frac{2\sin(bx+a)}{\sin(bx+a)^2-1}-\log(|\sin(bx+a)+1|)+\log(|\sin(bx+a)-1|)}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^3,x, algorithm="giac")
```

```
[Out] -1/4*(2*sin(b*x + a)/(sin(b*x + a)^2 - 1) - log(abs(sin(b*x + a) + 1)) + lo
g(abs(sin(b*x + a) - 1)))/b
```

3.4 $\int \sec^4(a+bx)\,dx$

Optimal. Leaf size=26

$$\frac{\tan^3(a+bx)}{3b}+\frac{\tan(a+bx)}{b}$$

```
[Out] Tan[a + b*x]/b + Tan[a + b*x]^3/(3*b)
```

Rubi [A] time = 0.0112607, antiderivative size = 26, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3767}

$$\frac{\tan^3(a+bx)}{3b}+\frac{\tan(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[a + b*x]^4,x]
```

```
[Out] Tan[a + b*x]/b + Tan[a + b*x]^3/(3*b)
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}\int \sec^4(a+bx)\,dx &= -\frac{\operatorname{Subst}\left(\int \left(1+x^2\right)dx, x, -\tan(a+bx)\right)}{b}\\ &= \frac{\tan(a+bx)}{b}+\frac{\tan^3(a+bx)}{3b}\end{aligned}$$

Mathematica [A] time = 0.0409817, size = 23, normalized size = 0.88

$$\frac{\frac{1}{3}\tan^3(a+bx)+\tan(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[a + b*x]^4,x]
```

```
[Out] (Tan[a + b*x] + Tan[a + b*x]^3/3)/b
```

Maple [A] time = 0.043, size = 24, normalized size = 0.9

$$-\frac{\tan(bx+a)}{b}\left(-\frac{2}{3}-\frac{(\sec(bx+a))^2}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)^4,x)
```

```
[Out] -1/b*(-2/3-1/3*sec(b*x+a)^2)*tan(b*x+a)
```

Maxima [A] time = 1.16608, size = 30, normalized size = 1.15

$$\frac{\tan(bx+a)^3 + 3\tan(bx+a)}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^4,x, algorithm="maxima")
```

```
[Out] 1/3*(tan(b*x + a)^3 + 3*tan(b*x + a))/b
```

Fricas [A] time = 1.34596, size = 81, normalized size = 3.12

$$\frac{\left(2\cos(bx+a)^2 + 1\right)\sin(bx+a)}{3b\cos(bx+a)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^4,x, algorithm="fricas")
```

```
[Out] 1/3*(2*cos(b*x + a)^2 + 1)*sin(b*x + a)/(b*cos(b*x + a)^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sec^4(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)**4,x)
```

```
[Out] Integral(sec(a + b*x)**4, x)
```

Giac [A] time = 1.32413, size = 30, normalized size = 1.15

$$\frac{\tan(bx+a)^3 + 3\tan(bx+a)}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^4,x, algorithm="giac")
```

```
[Out] 1/3*(tan(b*x + a)^3 + 3*tan(b*x + a))/b
```

3.5 $\int \sec^5(a+bx)\,dx$

Optimal. Leaf size=55

$$\frac{3\tanh^{-1}(\sin(a+bx))}{8b}+\frac{\tan(a+bx)\sec^3(a+bx)}{4b}+\frac{3\tan(a+bx)\sec(a+bx)}{8b}$$

```
[Out] (3*ArcTanh[Sin[a + b*x]])/(8*b) + (3*Sec[a + b*x]*Tan[a + b*x])/(8*b) + (Se
c[a + b*x]^3*Tan[a + b*x])/(4*b)
```

Rubi [A] time = 0.0250803, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {3768, 3770}

$$\frac{3\tanh^{-1}(\sin(a+bx))}{8b}+\frac{\tan(a+bx)\sec^3(a+bx)}{4b}+\frac{3\tan(a+bx)\sec(a+bx)}{8b}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[a + b*x]^5,x]
```

```
[Out] (3*ArcTanh[Sin[a + b*x]])/(8*b) + (3*Sec[a + b*x]*Tan[a + b*x])/(8*b) + (Se
c[a + b*x]^3*Tan[a + b*x])/(4*b)
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \sec^5(a+bx)\,dx &= \frac{\sec^3(a+bx)\tan(a+bx)}{4b}+\frac{3}{4}\int \sec^3(a+bx)\,dx\\ &= \frac{3\sec(a+bx)\tan(a+bx)}{8b}+\frac{\sec^3(a+bx)\tan(a+bx)}{4b}+\frac{3}{8}\int \sec(a+bx)\,dx\\ &= \frac{3\tanh^{-1}(\sin(a+bx))}{8b}+\frac{3\sec(a+bx)\tan(a+bx)}{8b}+\frac{\sec^3(a+bx)\tan(a+bx)}{4b}\end{aligned}$$

Mathematica [A] time = 0.0714815, size = 42, normalized size = 0.76

$$\frac{3\tanh^{-1}(\sin(a+bx))+\tan(a+bx)\sec(a+bx)\left(2\sec^2(a+bx)+3\right)}{8b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[a + b*x]^5,x]
```

```
[Out] (3*ArcTanh[Sin[a + b*x]] + Sec[a + b*x]*(3 + 2*Sec[a + b*x]^2)*Tan[a + b*x]
)/(8*b)
```

Maple [A] time = 0.044, size = 57, normalized size = 1.

$$\frac{(\sec (bx+a))^3 \tan (bx+a)}{4b} + \frac{3 \sec (bx+a) \tan (bx+a)}{8b} + \frac{3 \ln (\sec (bx+a) + \tan (bx+a))}{8b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)^5,x)
```

```
[Out] 1/4*sec(b*x+a)^3*tan(b*x+a)/b+3/8*sec(b*x+a)*tan(b*x+a)/b+3/8/b*ln(sec(b*x+
a)+tan(b*x+a))
```

Maxima [A] time = 1.03009, size = 96, normalized size = 1.75

$$-\frac{\frac{2\left(3 \sin(bx+a)^3 - 5 \sin(bx+a)\right)}{\sin(bx+a)^4 - 2 \sin(bx+a)^2 + 1} - 3 \log(\sin(bx+a)+1) + 3 \log(\sin(bx+a)-1)}{16b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^5,x, algorithm="maxima")
```

```
[Out] -1/16*(2*(3*sin(b*x + a)^3 - 5*sin(b*x + a))/(sin(b*x + a)^4 - 2*sin(b*x +
a)^2 + 1) - 3*log(sin(b*x + a) + 1) + 3*log(sin(b*x + a) - 1))/b
```

Fricas [A] time = 1.42324, size = 200, normalized size = 3.64

$$\frac{3 \cos(bx+a)^4 \log(\sin(bx+a)+1) - 3 \cos(bx+a)^4 \log(-\sin(bx+a)+1) + 2\left(3 \cos(bx+a)^2 + 2\right) \sin(bx+a)}{16 b \cos(bx+a)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^5,x, algorithm="fricas")
```

```
[Out] 1/16*(3*cos(b*x + a)^4*log(sin(b*x + a) + 1) - 3*cos(b*x + a)^4*log(-sin(b*
x + a) + 1) + 2*(3*cos(b*x + a)^2 + 2)*sin(b*x + a))/(b*cos(b*x + a)^4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sec^5(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)**5,x)
```

```
[Out] Integral(sec(a + b*x)**5, x)
```

Giac [A] time = 1.31957, size = 85, normalized size = 1.55

$$-\frac{\frac{2\left(3\sin(bx+a)^3-5\sin(bx+a)\right)}{\left(\sin(bx+a)^2-1\right)^2}-3\log\left(\left|\sin\left(bx+a\right)+1\right|\right)+3\log\left(\left|\sin\left(bx+a\right)-1\right|\right)}{16\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^5,x, algorithm="giac")
```

```
[Out] -1/16*(2*(3*sin(b*x + a)^3 - 5*sin(b*x + a))/(sin(b*x + a)^2 - 1)^2 - 3*log
(abs(sin(b*x + a) + 1)) + 3*log(abs(sin(b*x + a) - 1)))/b
```

3.6 $\int \sec^6(a+bx)\,dx$

Optimal. Leaf size=41

$$\frac{\tan^5(a+bx)}{5b}+\frac{2\tan^3(a+bx)}{3b}+\frac{\tan(a+bx)}{b}$$

```
[Out] Tan[a + b*x]/b + (2*Tan[a + b*x]^3)/(3*b) + Tan[a + b*x]^5/(5*b)
```

Rubi [A] time = 0.0148022, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3767}

$$\frac{\tan^5(a+bx)}{5b}+\frac{2\tan^3(a+bx)}{3b}+\frac{\tan(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[a + b*x]^6,x]
```

```
[Out] Tan[a + b*x]/b + (2*Tan[a + b*x]^3)/(3*b) + Tan[a + b*x]^5/(5*b)
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}\int \sec^6(a+bx)\,dx &= -\frac{\text{Subst}\left(\int\left(1+2x^2+x^4\right)dx,x,-\tan(a+bx)\right)}{b}\\ &= \frac{\tan(a+bx)}{b}+\frac{2\tan^3(a+bx)}{3b}+\frac{\tan^5(a+bx)}{5b}\end{aligned}$$

Mathematica [A] time = 0.100569, size = 35, normalized size = 0.85

$$\frac{\frac{1}{5}\tan^5(a+bx)+\frac{2}{3}\tan^3(a+bx)+\tan(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[a + b*x]^6,x]
```

```
[Out] (Tan[a + b*x] + (2*Tan[a + b*x]^3)/3 + Tan[a + b*x]^5/5)/b
```

Maple [A] time = 0.043, size = 34, normalized size = 0.8

$$-\frac{\tan(bx+a)}{b}\left(-\frac{8}{15}-\frac{(\sec(bx+a))^4}{5}-\frac{4(\sec(bx+a))^2}{15}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)^6,x)
```

```
[Out] -1/b*(-8/15-1/5*sec(b*x+a)^4-4/15*sec(b*x+a)^2)*tan(b*x+a)
```

Maxima [A] time = 1.20084, size = 46, normalized size = 1.12

$$\frac{3\,\tan\left(bx+a\right)^5+10\,\tan\left(bx+a\right)^3+15\,\tan\left(bx+a\right)}{15\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^6,x, algorithm="maxima")
```

```
[Out] 1/15*(3*tan(b*x + a)^5 + 10*tan(b*x + a)^3 + 15*tan(b*x + a))/b
```

Fricas [A] time = 1.36232, size = 108, normalized size = 2.63

$$\frac{\left(8\,\cos\left(bx+a\right)^4+4\,\cos\left(bx+a\right)^2+3\right)\sin\left(bx+a\right)}{15\,b\cos\left(bx+a\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^6,x, algorithm="fricas")
```

```
[Out] 1/15*(8*cos(b*x + a)^4 + 4*cos(b*x + a)^2 + 3)*sin(b*x + a)/(b*cos(b*x + a)
^5)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sec^6\left(a+bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)**6,x)
```

```
[Out] Integral(sec(a + b*x)**6, x)
```

Giac [A] time = 1.13672, size = 46, normalized size = 1.12

$$\frac{3\,\tan\left(bx+a\right)^5+10\,\tan\left(bx+a\right)^3+15\,\tan\left(bx+a\right)}{15\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^6,x, algorithm="giac")
```

```
[Out] 1/15*(3*tan(b*x + a)^5 + 10*tan(b*x + a)^3 + 15*tan(b*x + a))/b
```

3.7 $\int \sec^7(a+bx)\,dx$

Optimal. Leaf size=76

$$\frac{5\tanh^{-1}(\sin(a+bx))}{16b}+\frac{\tan(a+bx)\sec^5(a+bx)}{6b}+\frac{5\tan(a+bx)\sec^3(a+bx)}{24b}+\frac{5\tan(a+bx)\sec(a+bx)}{16b}$$

```
[Out] (5*ArcTanh[Sin[a + b*x]])/(16*b) + (5*Sec[a + b*x]*Tan[a + b*x])/(16*b) + (
5*Sec[a + b*x]^3*Tan[a + b*x])/(24*b) + (Sec[a + b*x]^5*Tan[a + b*x])/(6*b)
```

Rubi [A] time = 0.0386378, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {3768, 3770}

$$\frac{5\tanh^{-1}(\sin(a+bx))}{16b}+\frac{\tan(a+bx)\sec^5(a+bx)}{6b}+\frac{5\tan(a+bx)\sec^3(a+bx)}{24b}+\frac{5\tan(a+bx)\sec(a+bx)}{16b}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[a + b*x]^7,x]
```

```
[Out] (5*ArcTanh[Sin[a + b*x]])/(16*b) + (5*Sec[a + b*x]*Tan[a + b*x])/(16*b) + (
5*Sec[a + b*x]^3*Tan[a + b*x])/(24*b) + (Sec[a + b*x]^5*Tan[a + b*x])/(6*b)
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \sec^7(a+bx)\,dx &= \frac{\sec^5(a+bx)\tan(a+bx)}{6b}+\frac{5}{6}\int \sec^5(a+bx)\,dx\\ &= \frac{5\sec^3(a+bx)\tan(a+bx)}{24b}+\frac{\sec^5(a+bx)\tan(a+bx)}{6b}+\frac{5}{8}\int \sec^3(a+bx)\,dx\\ &= \frac{5\sec(a+bx)\tan(a+bx)}{16b}+\frac{5\sec^3(a+bx)\tan(a+bx)}{24b}+\frac{\sec^5(a+bx)\tan(a+bx)}{6b}+\frac{5}{16}\int \sec(a+\\ &= \frac{5\tanh^{-1}(\sin(a+bx))}{16b}+\frac{5\sec(a+bx)\tan(a+bx)}{16b}+\frac{5\sec^3(a+bx)\tan(a+bx)}{24b}+\frac{\sec^5(a+bx)\tan}{6b}\end{aligned}$$

Mathematica [A] time = 0.166078, size = 52, normalized size = 0.68

$$\frac{15\tanh^{-1}(\sin(a+bx))+\tan(a+bx)\sec(a+bx)\left(8\sec^4(a+bx)+10\sec^2(a+bx)+15\right)}{48b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[a + b*x]^7,x]
```

```
[Out] (15*ArcTanh[Sin[a + b*x]] + Sec[a + b*x]*(15 + 10*Sec[a + b*x]^2 + 8*Sec[a
+ b*x]^4)*Tan[a + b*x])/(48*b)
```

Maple [A] time = 0.046, size = 76, normalized size = 1.

$$\frac{(\sec(bx+a))^5\tan(bx+a)}{6b}+\frac{5(\sec(bx+a))^3\tan(bx+a)}{24b}+\frac{5\sec(bx+a)\tan(bx+a)}{16b}+\frac{5\ln(\sec(bx+a)+\tan(}{16b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)^7,x)
```

```
[Out] 1/6*sec(b*x+a)^5*tan(b*x+a)/b+5/24*sec(b*x+a)^3*tan(b*x+a)/b+5/16*sec(b*x+a
)*tan(b*x+a)/b+5/16/b*ln(sec(b*x+a)+tan(b*x+a))
```

Maxima [A] time = 1.07036, size = 123, normalized size = 1.62

$$-\frac{\frac{2\left(15\sin(bx+a)^5-40\sin(bx+a)^3+33\sin(bx+a)\right)}{\sin(bx+a)^6-3\sin(bx+a)^4+3\sin(bx+a)^2-1}-15\log(\sin(bx+a)+1)+15\log(\sin(bx+a)-1)}{96b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^7,x, algorithm="maxima")
```

```
[Out] -1/96*(2*(15*sin(b*x + a)^5 - 40*sin(b*x + a)^3 + 33*sin(b*x + a))/(sin(b*x
 + a)^6 - 3*sin(b*x + a)^4 + 3*sin(b*x + a)^2 - 1) - 15*log(sin(b*x + a) +
1) + 15*log(sin(b*x + a) - 1))/b
```

Fricas [A] time = 1.47555, size = 231, normalized size = 3.04

$$\frac{15\cos(bx+a)^6\log(\sin(bx+a)+1)-15\cos(bx+a)^6\log(-\sin(bx+a)+1)+2\left(15\cos(bx+a)^4+10\cos(bx+\right.}{96b\cos(bx+a)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^7,x, algorithm="fricas")
```

```
[Out] 1/96*(15*cos(b*x + a)^6*log(sin(b*x + a) + 1) - 15*cos(b*x + a)^6*log(-sin(
b*x + a) + 1) + 2*(15*cos(b*x + a)^4 + 10*cos(b*x + a)^2 + 8)*sin(b*x + a))
/(b*cos(b*x + a)^6)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sec^7(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)**7,x)
```

```
[Out] Integral(sec(a + b*x)**7, x)
```

<u>Giac</u> [A] time = 1.27607, size = 99, normalized size = 1.3

$$-\frac{\frac{2\left(15\sin(bx+a)^5-40\sin(bx+a)^3+33\sin(bx+a)\right)}{\left(\sin(bx+a)^2-1\right)^3}-15\log\left(|\sin\left(bx+a\right)+1|\right)+15\log\left(|\sin\left(bx+a\right)-1|\right)}{96\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^7,x, algorithm="giac")
```

```
[Out] -1/96*(2*(15*sin(b*x + a)^5 - 40*sin(b*x + a)^3 + 33*sin(b*x + a))/(sin(b*x
 + a)^2 - 1)^3 - 15*log(abs(sin(b*x + a) + 1)) + 15*log(abs(sin(b*x + a) -
1)))/b
```

3.8 $\int \sec^8(a+bx)\,dx$

Optimal. Leaf size=53

$$\frac{\tan^7(a+bx)}{7b}+\frac{3\tan^5(a+bx)}{5b}+\frac{\tan^3(a+bx)}{b}+\frac{\tan(a+bx)}{b}$$

```
[Out] Tan[a + b*x]/b + Tan[a + b*x]^3/b + (3*Tan[a + b*x]^5)/(5*b) + Tan[a + b*x]
^7/(7*b)
```

Rubi [A] time = 0.0167995, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3767}

$$\frac{\tan^7(a+bx)}{7b}+\frac{3\tan^5(a+bx)}{5b}+\frac{\tan^3(a+bx)}{b}+\frac{\tan(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[a + b*x]^8,x]
```

```
[Out] Tan[a + b*x]/b + Tan[a + b*x]^3/b + (3*Tan[a + b*x]^5)/(5*b) + Tan[a + b*x]
^7/(7*b)
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}\int \sec^8(a+bx)\,dx&=-\frac{\text{Subst}\left(\int \left(1+3x^2+3x^4+x^6\right)\,dx,x,-\tan(a+bx)\right)}{b}\\&=\frac{\tan(a+bx)}{b}+\frac{\tan^3(a+bx)}{b}+\frac{3\tan^5(a+bx)}{5b}+\frac{\tan^7(a+bx)}{7b}\end{aligned}$$

Mathematica [A] time = 0.218945, size = 43, normalized size = 0.81

$$\frac{\frac{1}{7}\tan^7(a+bx)+\frac{3}{5}\tan^5(a+bx)+\tan^3(a+bx)+\tan(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[a + b*x]^8,x]
```

```
[Out] (Tan[a + b*x] + Tan[a + b*x]^3 + (3*Tan[a + b*x]^5)/5 + Tan[a + b*x]^7/7)/b
```

Maple [A] time = 0.043, size = 44, normalized size = 0.8

$$-\frac{\tan(bx+a)}{b}\left(-\frac{16}{35}-\frac{(\sec(bx+a))^6}{7}-\frac{6\,(\sec(bx+a))^4}{35}-\frac{8\,(\sec(bx+a))^2}{35}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)^8,x)
```

```
[Out] -1/b*(-16/35-1/7*sec(b*x+a)^6-6/35*sec(b*x+a)^4-8/35*sec(b*x+a)^2)*tan(b*x+
a)
```

Maxima [A] time = 1.07473, size = 59, normalized size = 1.11

$$\frac{5\,\tan\left(bx+a\right)^{7}+21\,\tan\left(bx+a\right)^{5}+35\,\tan\left(bx+a\right)^{3}+35\,\tan\left(bx+a\right)}{35\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^8,x, algorithm="maxima")
```

```
[Out] 1/35*(5*tan(b*x + a)^7 + 21*tan(b*x + a)^5 + 35*tan(b*x + a)^3 + 35*tan(b*x
 + a))/b
```

Fricas [A] time = 1.39288, size = 135, normalized size = 2.55

$$\frac{\left(16\,\cos\left(bx+a\right)^{6}+8\,\cos\left(bx+a\right)^{4}+6\,\cos\left(bx+a\right)^{2}+5\right)\sin\left(bx+a\right)}{35\,b\cos\left(bx+a\right)^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^8,x, algorithm="fricas")
```

```
[Out] 1/35*(16*cos(b*x + a)^6 + 8*cos(b*x + a)^4 + 6*cos(b*x + a)^2 + 5)*sin(b*x
+ a)/(b*cos(b*x + a)^7)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sec^{8}\left(a+bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)**8,x)
```

```
[Out] Integral(sec(a + b*x)**8, x)
```

Giac [A] time = 1.27431, size = 59, normalized size = 1.11

$$\frac{5\,\tan\left(bx+a\right)^{7}+21\,\tan\left(bx+a\right)^{5}+35\,\tan\left(bx+a\right)^{3}+35\,\tan\left(bx+a\right)}{35\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^8,x, algorithm="giac")
```

```
[Out] 1/35*(5*tan(b*x + a)^7 + 21*tan(b*x + a)^5 + 35*tan(b*x + a)^3 + 35*tan(b*x
 + a))/b
```

3.9 $\int \sec^{\frac{7}{2}}(a + bx)\, dx$

Optimal. Leaf size=85

$$\frac{2\sin(a+bx)\sec^{\frac{5}{2}}(a+bx)}{5b}+\frac{6\sin(a+bx)\sqrt{\sec(a+bx)}}{5b}-\frac{6\sqrt{\cos(a+bx)}\sqrt{\sec(a+bx)}E\left(\frac{1}{2}(a+bx)\middle|2\right)}{5b}$$

```
[Out] (-6*Sqrt[Cos[a + b*x]]*EllipticE[(a + b*x)/2, 2]*Sqrt[Sec[a + b*x]])/(5*b)
+ (6*Sqrt[Sec[a + b*x]]*Sin[a + b*x])/(5*b) + (2*Sec[a + b*x]^(5/2)*Sin[a +
 b*x])/(5*b)
```

Rubi [A] time = 0.0370816, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {3768, 3771, 2639}

$$\frac{2\sin(a+bx)\sec^{\frac{5}{2}}(a+bx)}{5b}+\frac{6\sin(a+bx)\sqrt{\sec(a+bx)}}{5b}-\frac{6\sqrt{\cos(a+bx)}\sqrt{\sec(a+bx)}E\left(\frac{1}{2}(a+bx)\middle|2\right)}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[a + b*x]^(7/2),x]
```

```
[Out] (-6*Sqrt[Cos[a + b*x]]*EllipticE[(a + b*x)/2, 2]*Sqrt[Sec[a + b*x]])/(5*b)
+ (6*Sqrt[Sec[a + b*x]]*Sin[a + b*x])/(5*b) + (2*Sec[a + b*x]^(5/2)*Sin[a +
 b*x])/(5*b)
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sec^{\frac{7}{2}}(a+bx)\,dx = \frac{2\sec^{\frac{5}{2}}(a+bx)\sin(a+bx)}{5b} + \frac{3}{5}\int \sec^{\frac{3}{2}}(a+bx)\,dx$$
$$= \frac{6\sqrt{\sec(a+bx)}\sin(a+bx)}{5b} + \frac{2\sec^{\frac{5}{2}}(a+bx)\sin(a+bx)}{5b} - \frac{3}{5}\int \frac{1}{\sqrt{\sec(a+bx)}}\,dx$$
$$= \frac{6\sqrt{\sec(a+bx)}\sin(a+bx)}{5b} + \frac{2\sec^{\frac{5}{2}}(a+bx)\sin(a+bx)}{5b} - \frac{1}{5}\left(3\sqrt{\cos(a+bx)}\sqrt{\sec(a+bx)}\right)\int \sqrt{}$$
$$= -\frac{6\sqrt{\cos(a+bx)}E\left(\frac{1}{2}(a+bx)\middle|2\right)\sqrt{\sec(a+bx)}}{5b} + \frac{6\sqrt{\sec(a+bx)}\sin(a+bx)}{5b} + \frac{2\sec^{\frac{5}{2}}(a+bx)\sin}{5b}$$

Mathematica [A] time = 0.174773, size = 59, normalized size = 0.69

$$\frac{\sec^{\frac{5}{2}}(a+bx)\left(7\sin(a+bx)+3\sin(3(a+bx))-12\cos^{\frac{5}{2}}(a+bx)E\left(\frac{1}{2}(a+bx)\middle|2\right)\right)}{10b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[a + b*x]^(7/2),x]
```

```
[Out] (Sec[a + b*x]^(5/2)*(-12*Cos[a + b*x]^(5/2)*EllipticE[(a + b*x)/2, 2] + 7*S
in[a + b*x] + 3*Sin[3*(a + b*x)]))/(10*b)
```

Maple [B] time = 2.199, size = 358, normalized size = 4.2

$$\frac{2}{5b}\sqrt{-\left(-2\left(\cos\left(1/2\,bx+a/2\right)\right)^{2}+1\right)\left(\sin\left(\frac{bx}{2}+\frac{a}{2}\right)\right)^{2}}\left(12\sqrt{2\left(\sin\left(1/2\,bx+a/2\right)\right)^{2}-1}\sqrt{\left(\sin\left(1/2\,bx+a/2\right)\right)^{2}}\mathrm{EllipticE}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)^(7/2),x)
```

```
[Out] 2/5*(-(-2*cos(1/2*b*x+1/2*a)^2+1)*sin(1/2*b*x+1/2*a)^2)^(1/2)/(8*sin(1/2*b*
x+1/2*a)^6-12*sin(1/2*b*x+1/2*a)^4+6*sin(1/2*b*x+1/2*a)^2-1)/sin(1/2*b*x+1/
2*a)^3*(12*(2*sin(1/2*b*x+1/2*a)^2-1)^(1/2)*(sin(1/2*b*x+1/2*a)^2)^(1/2)*El
lipticE(cos(1/2*b*x+1/2*a),2^(1/2))*sin(1/2*b*x+1/2*a)^4-24*sin(1/2*b*x+1/2
*a)^6*cos(1/2*b*x+1/2*a)-12*(2*sin(1/2*b*x+1/2*a)^2-1)^(1/2)*(sin(1/2*b*x+1
/2*a)^2)^(1/2)*EllipticE(cos(1/2*b*x+1/2*a),2^(1/2))*sin(1/2*b*x+1/2*a)^2+2
4*sin(1/2*b*x+1/2*a)^4*cos(1/2*b*x+1/2*a)+3*(2*sin(1/2*b*x+1/2*a)^2-1)^(1/2
)*(sin(1/2*b*x+1/2*a)^2)^(1/2)*EllipticE(cos(1/2*b*x+1/2*a),2^(1/2))-8*sin(
1/2*b*x+1/2*a)^2*cos(1/2*b*x+1/2*a))*(-2*sin(1/2*b*x+1/2*a)^4+sin(1/2*b*x+1
/2*a)^2)^(1/2)/(2*cos(1/2*b*x+1/2*a)^2-1)^(1/2)/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sec\left(bx+a\right)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(b*x + a)^(7/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sec\left(bx+a\right)^{\frac{7}{2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^(7/2),x, algorithm="fricas")
```

```
[Out] integral(sec(b*x + a)^(7/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sec\left(bx+a\right)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^(7/2),x, algorithm="giac")
```

```
[Out] integrate(sec(b*x + a)^(7/2), x)
```

3.10 $\int \sec^{\frac{5}{2}}(a+bx)\,dx$

Optimal. Leaf size=62

$$\frac{2\sqrt{\cos(a+bx)}\sqrt{\sec(a+bx)}\text{EllipticF}\left(\frac{1}{2}(a+bx),2\right)}{3b}+\frac{2\sin(a+bx)\sec^{\frac{3}{2}}(a+bx)}{3b}$$

```
[Out] (2*Sqrt[Cos[a + b*x]]*EllipticF[(a + b*x)/2, 2]*Sqrt[Sec[a + b*x]])/(3*b) +
 (2*Sec[a + b*x]^(3/2)*Sin[a + b*x])/(3*b)
```

Rubi [A] time = 0.0269803, antiderivative size = 62, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {3768, 3771, 2641}

$$\frac{2\sin(a+bx)\sec^{\frac{3}{2}}(a+bx)}{3b}+\frac{2\sqrt{\cos(a+bx)}\sqrt{\sec(a+bx)}F\left(\frac{1}{2}(a+bx)\middle|2\right)}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[a + b*x]^(5/2),x]
```

```
[Out] (2*Sqrt[Cos[a + b*x]]*EllipticF[(a + b*x)/2, 2]*Sqrt[Sec[a + b*x]])/(3*b) +
 (2*Sec[a + b*x]^(3/2)*Sin[a + b*x])/(3*b)
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \sec^{\frac{5}{2}}(a+bx)\,dx &= \frac{2\sec^{\frac{3}{2}}(a+bx)\sin(a+bx)}{3b}+\frac{1}{3}\int \sqrt{\sec(a+bx)}\,dx\\
&= \frac{2\sec^{\frac{3}{2}}(a+bx)\sin(a+bx)}{3b}+\frac{1}{3}\left(\sqrt{\cos(a+bx)}\sqrt{\sec(a+bx)}\right)\int \frac{1}{\sqrt{\cos(a+bx)}}\,dx\\
&= \frac{2\sqrt{\cos(a+bx)}F\left(\frac{1}{2}(a+bx)\middle|2\right)\sqrt{\sec(a+bx)}}{3b}+\frac{2\sec^{\frac{3}{2}}(a+bx)\sin(a+bx)}{3b}
\end{aligned}$$

Mathematica [A] time = 0.0674676, size = 46, normalized size = 0.74

$$\frac{2\sec^{\frac{3}{2}}(a+bx)\left(\cos^{\frac{3}{2}}(a+bx)\text{EllipticF}\left(\frac{1}{2}(a+bx),2\right)+\sin(a+bx)\right)}{3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[a + b*x]^(5/2),x]
```

```
[Out] (2*Sec[a + b*x]^(3/2)*(Cos[a + b*x]^(3/2)*EllipticF[(a + b*x)/2, 2] + Sin[a
 + b*x]))/(3*b)
```

Maple [B] time = 1.266, size = 213, normalized size = 3.4

$$-\frac{2}{3\,b}\left(-2\,\sqrt{(\sin(1/2\,bx+a/2))^2}\sqrt{2\,(\sin(1/2\,bx+a/2))^2-1}\text{EllipticF}\left(\cos(1/2\,bx+a/2),\sqrt{2}\right)(\sin(1/2\,bx+a/2))^2+\sqrt{(\text{s}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)^(5/2),x)
```

```
[Out] -2/3*(-2*(sin(1/2*b*x+1/2*a)^2)^(1/2)*(2*sin(1/2*b*x+1/2*a)^2-1)^(1/2)*Elli
pticF(cos(1/2*b*x+1/2*a),2^(1/2))*sin(1/2*b*x+1/2*a)^2+(sin(1/2*b*x+1/2*a)^
2)^(1/2)*(2*sin(1/2*b*x+1/2*a)^2-1)^(1/2)*EllipticF(cos(1/2*b*x+1/2*a),2^(1
/2))-2*sin(1/2*b*x+1/2*a)^2*cos(1/2*b*x+1/2*a))*((2*cos(1/2*b*x+1/2*a)^2-1)
*sin(1/2*b*x+1/2*a)^2)^(1/2)/(-2*sin(1/2*b*x+1/2*a)^4+sin(1/2*b*x+1/2*a)^2)
^(1/2)/(2*cos(1/2*b*x+1/2*a)^2-1)^(3/2)/sin(1/2*b*x+1/2*a)/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sec(bx+a)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(b*x + a)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sec(bx+a)^{\frac{5}{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sec(b*x + a)^(5/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sec(bx+a)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sec(b*x + a)^(5/2), x)
```

3.11 $\int \sec^{\frac{3}{2}}(a+bx)\,dx$

Optimal. Leaf size=58

$$\frac{2\sin(a+bx)\sqrt{\sec(a+bx)}}{b}-\frac{2\sqrt{\cos(a+bx)}\sqrt{\sec(a+bx)}E\left(\frac{1}{2}(a+bx)\middle|2\right)}{b}$$

```
[Out] (-2*Sqrt[Cos[a + b*x]]*EllipticE[(a + b*x)/2, 2]*Sqrt[Sec[a + b*x]])/b + (2
*Sqrt[Sec[a + b*x]]*Sin[a + b*x])/b
```

Rubi [A] time = 0.0255371, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {3768, 3771, 2639}

$$\frac{2\sin(a+bx)\sqrt{\sec(a+bx)}}{b}-\frac{2\sqrt{\cos(a+bx)}\sqrt{\sec(a+bx)}E\left(\frac{1}{2}(a+bx)\middle|2\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[a + b*x]^(3/2),x]
```

```
[Out] (-2*Sqrt[Cos[a + b*x]]*EllipticE[(a + b*x)/2, 2]*Sqrt[Sec[a + b*x]])/b + (2
*Sqrt[Sec[a + b*x]]*Sin[a + b*x])/b
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \sec^{\frac{3}{2}}(a+bx)\,dx &= \frac{2\sqrt{\sec(a+bx)}\sin(a+bx)}{b}-\int \frac{1}{\sqrt{\sec(a+bx)}}\,dx\\
&= \frac{2\sqrt{\sec(a+bx)}\sin(a+bx)}{b}-\left(\sqrt{\cos(a+bx)}\sqrt{\sec(a+bx)}\right)\int \sqrt{\cos(a+bx)}\,dx\\
&= -\frac{2\sqrt{\cos(a+bx)}E\left(\frac{1}{2}(a+bx)\middle|2\right)\sqrt{\sec(a+bx)}}{b}+\frac{2\sqrt{\sec(a+bx)}\sin(a+bx)}{b}
\end{aligned}$$

Mathematica [A] time = 0.0475868, size = 45, normalized size = 0.78

$$\frac{2\sqrt{\sec(a+bx)}\left(\sin(a+bx)-\sqrt{\cos(a+bx)}E\left(\frac{1}{2}(a+bx)\middle|2\right)\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[a + b*x]^(3/2),x]
```

```
[Out] (2*Sqrt[Sec[a + b*x]]*(-(Sqrt[Cos[a + b*x]]*EllipticE[(a + b*x)/2, 2]) + Si
n[a + b*x]))/b
```

Maple [A] time = 1.062, size = 101, normalized size = 1.7

$$-2\,\frac{\sqrt{2\,(\sin(1/2\,bx+a/2))^2-1}\sqrt{(\sin(1/2\,bx+a/2))^2}\text{EllipticE}\left(\cos(1/2\,bx+a/2),\sqrt{2}\right)-2\,(\sin(1/2\,bx+a/2))^2\cos(1}{\sin(1/2\,bx+a/2)\sqrt{2\,(\cos(1/2\,bx+a/2))^2-1}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)^(3/2),x)
```

```
[Out] -2*((2*sin(1/2*b*x+1/2*a)^2-1)^(1/2)*(sin(1/2*b*x+1/2*a)^2)^(1/2)*EllipticE
(cos(1/2*b*x+1/2*a),2^(1/2))-2*sin(1/2*b*x+1/2*a)^2*cos(1/2*b*x+1/2*a))/sin
(1/2*b*x+1/2*a)/(2*cos(1/2*b*x+1/2*a)^2-1)^(1/2)/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sec(bx+a)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(b*x + a)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sec(bx+a)^{\frac{3}{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sec(b*x + a)^(3/2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sec^{\frac{3}{2}}(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)**(3/2),x)
```

```
[Out] Integral(sec(a + b*x)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sec(bx+a)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sec(b*x + a)^(3/2), x)
```

3.12 $\int \sqrt{\sec(a+bx)}\,dx$

Optimal. Leaf size=36

$$\frac{2\sqrt{\cos(a+bx)}\sqrt{\sec(a+bx)}\text{EllipticF}\left(\frac{1}{2}(a+bx),2\right)}{b}$$

```
[Out] (2*Sqrt[Cos[a + b*x]]*EllipticF[(a + b*x)/2, 2]*Sqrt[Sec[a + b*x]])/b
```

Rubi [A] time = 0.0167242, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {3771, 2641}

$$\frac{2\sqrt{\cos(a+bx)}\sqrt{\sec(a+bx)}F\left(\frac{1}{2}(a+bx)\middle|2\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Sec[a + b*x]],x]
```

```
[Out] (2*Sqrt[Cos[a + b*x]]*EllipticF[(a + b*x)/2, 2]*Sqrt[Sec[a + b*x]])/b
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \sqrt{\sec(a+bx)}\,dx &= \left(\sqrt{\cos(a+bx)}\sqrt{\sec(a+bx)}\right)\int \frac{1}{\sqrt{\cos(a+bx)}}\,dx\\ &= \frac{2\sqrt{\cos(a+bx)}F\left(\frac{1}{2}(a+bx)\middle|2\right)\sqrt{\sec(a+bx)}}{b}\end{aligned}$$

Mathematica [A] time = 0.028913, size = 36, normalized size = 1.

$$\frac{2\sqrt{\cos(a+bx)}\sqrt{\sec(a+bx)}\text{EllipticF}\left(\frac{1}{2}(a+bx),2\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Sec[a + b*x]],x]
```

```
[Out] (2*Sqrt[Cos[a + b*x]]*EllipticF[(a + b*x)/2, 2]*Sqrt[Sec[a + b*x]])/b
```

Maple [B] time = 0.883, size = 133, normalized size = 3.7

$$-2\,\frac{\sqrt{\left(2\,(\cos(1/2\,bx+a/2))^2-1\right)(\sin(1/2\,bx+a/2))^2}\sqrt{(\sin(1/2\,bx+a/2))^2}\sqrt{-2\,(\cos(1/2\,bx+a/2))^2+1}\mathrm{EllipticF}\left(\cos\right.}{\sqrt{-2\,(\sin(1/2\,bx+a/2))^4+(\sin(1/2\,bx+a/2))^2}\sin(1/2\,bx+a/2)\sqrt{2\,(\cos(1/2\,bx+a/2))^2-1}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)^(1/2),x)
```

```
[Out] -2*((2*cos(1/2*b*x+1/2*a)^2-1)*sin(1/2*b*x+1/2*a)^2)^(1/2)*(sin(1/2*b*x+1/2
*a)^2)^(1/2)*(-2*cos(1/2*b*x+1/2*a)^2+1)^(1/2)/(-2*sin(1/2*b*x+1/2*a)^4+sin
(1/2*b*x+1/2*a)^2)^(1/2)*EllipticF(cos(1/2*b*x+1/2*a),2^(1/2))/sin(1/2*b*x+
1/2*a)/(2*cos(1/2*b*x+1/2*a)^2-1)^(1/2)/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{\sec(bx+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(sec(b*x + a)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\sqrt{\sec(bx+a)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(sec(b*x + a)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{\sec(a+bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)**(1/2),x)
```

```
[Out] Integral(sqrt(sec(a + b*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{\sec(bx+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(sec(b*x + a)), x)
```

3.13 $\int \frac{1}{\sqrt{\sec(a+bx)}}\,dx$

Optimal. Leaf size=36

$$\frac{2\sqrt{\cos(a+bx)}\sqrt{\sec(a+bx)}E\left(\frac{1}{2}(a+bx)\middle|2\right)}{b}$$

```
[Out] (2*Sqrt[Cos[a + b*x]]*EllipticE[(a + b*x)/2, 2]*Sqrt[Sec[a + b*x]])/b
```

Rubi [A] time = 0.0171722, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {3771, 2639}

$$\frac{2\sqrt{\cos(a+bx)}\sqrt{\sec(a+bx)}E\left(\frac{1}{2}(a+bx)\middle|2\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[Sec[a + b*x]],x]
```

```
[Out] (2*Sqrt[Cos[a + b*x]]*EllipticE[(a + b*x)/2, 2]*Sqrt[Sec[a + b*x]])/b
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{\sec(a+bx)}}\,dx &= \left(\sqrt{\cos(a+bx)}\sqrt{\sec(a+bx)}\right)\int \sqrt{\cos(a+bx)}\,dx\\ &= \frac{2\sqrt{\cos(a+bx)}E\left(\frac{1}{2}(a+bx)\middle|2\right)\sqrt{\sec(a+bx)}}{b}\end{aligned}$$

Mathematica [A] time = 0.0344761, size = 36, normalized size = 1.

$$\frac{2E\left(\frac{1}{2}(a+bx)\middle|2\right)}{b\sqrt{\cos(a+bx)}\sqrt{\sec(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[Sec[a + b*x]],x]
```

```
[Out] (2*EllipticE[(a + b*x)/2, 2])/(b*Sqrt[Cos[a + b*x]]*Sqrt[Sec[a + b*x]])
```

Maple [B] time = 0.936, size = 133, normalized size = 3.7

$$2\,\frac{\sqrt{\left(2\,(\cos(1/2\,bx+a/2))^2-1\right)(\sin(1/2\,bx+a/2))^2}\sqrt{(\sin(1/2\,bx+a/2))^2}\sqrt{-2\,(\cos(1/2\,bx+a/2))^2+1}\mathrm{EllipticE}\left(\mathrm{co}\right.}{\sqrt{-2\,(\sin(1/2\,bx+a/2))^4+(\sin(1/2\,bx+a/2))^2}\sin(1/2\,bx+a/2)\sqrt{2\,(\cos(1/2\,bx+a/2))^2-1}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sec(b*x+a)^(1/2),x)
```

```
[Out] 2*((2*cos(1/2*b*x+1/2*a)^2-1)*sin(1/2*b*x+1/2*a)^2)^(1/2)*(sin(1/2*b*x+1/2*
a)^2)^(1/2)*(-2*cos(1/2*b*x+1/2*a)^2+1)^(1/2)*EllipticE(cos(1/2*b*x+1/2*a),
2^(1/2))/(-2*sin(1/2*b*x+1/2*a)^4+sin(1/2*b*x+1/2*a)^2)^(1/2)/sin(1/2*b*x+1
/2*a)/(2*cos(1/2*b*x+1/2*a)^2-1)^(1/2)/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{\sec(bx+a)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(sec(b*x + a)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{1}{\sqrt{\sec(bx+a)}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(1/sqrt(sec(b*x + a)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{\sec(a+bx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(b*x+a)**(1/2),x)
```

```
[Out] Integral(1/sqrt(sec(a + b*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{\sec(bx+a)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(sec(b*x + a)), x)
```

3.14 $\int \frac{1}{\sec^{\frac{3}{2}}(a+bx)}\,dx$

Optimal. Leaf size=62

$$\frac{2\sqrt{\cos(a+bx)}\sqrt{\sec(a+bx)}\text{EllipticF}\left(\frac{1}{2}(a+bx),2\right)}{3b}+\frac{2\sin(a+bx)}{3b\sqrt{\sec(a+bx)}}$$

```
[Out] (2*Sqrt[Cos[a + b*x]]*EllipticF[(a + b*x)/2, 2]*Sqrt[Sec[a + b*x]])/(3*b) +
 (2*Sin[a + b*x])/(3*b*Sqrt[Sec[a + b*x]])
```

Rubi [A] time = 0.0280829, antiderivative size = 62, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {3769, 3771, 2641}

$$\frac{2\sin(a+bx)}{3b\sqrt{\sec(a+bx)}}+\frac{2\sqrt{\cos(a+bx)}\sqrt{\sec(a+bx)}F\left(\frac{1}{2}(a+bx)\middle|2\right)}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[a + b*x]^(-3/2),x]
```

```
[Out] (2*Sqrt[Cos[a + b*x]]*EllipticF[(a + b*x)/2, 2]*Sqrt[Sec[a + b*x]])/(3*b) +
 (2*Sin[a + b*x])/(3*b*Sqrt[Sec[a + b*x]])
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sec^{\frac{3}{2}}(a+bx)}\,dx &= \frac{2\sin(a+bx)}{3b\sqrt{\sec(a+bx)}}+\frac{1}{3}\int \sqrt{\sec(a+bx)}\,dx\\
&= \frac{2\sin(a+bx)}{3b\sqrt{\sec(a+bx)}}+\frac{1}{3}\left(\sqrt{\cos(a+bx)}\sqrt{\sec(a+bx)}\right)\int \frac{1}{\sqrt{\cos(a+bx)}}\,dx\\
&= \frac{2\sqrt{\cos(a+bx)}F\left(\frac{1}{2}(a+bx)\middle|2\right)\sqrt{\sec(a+bx)}}{3b}+\frac{2\sin(a+bx)}{3b\sqrt{\sec(a+bx)}}
\end{aligned}$$

Mathematica [A] time = 0.0442057, size = 49, normalized size = 0.79

$$\frac{\sqrt{\sec(a+bx)}\left(2\sqrt{\cos(a+bx)}\text{EllipticF}\left(\frac{1}{2}(a+bx),2\right)+\sin(2(a+bx))\right)}{3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[a + b*x]^(-3/2),x]
```

```
[Out] (Sqrt[Sec[a + b*x]]*(2*Sqrt[Cos[a + b*x]]*EllipticF[(a + b*x)/2, 2] + Sin[2
*(a + b*x)]))/(3*b)
```

Maple [B] time = 1.485, size = 179, normalized size = 2.9

$$-\frac{2}{3\,b}\sqrt{\left(2\,\left(\cos\left(1/2\,bx+a/2\right)\right)^{2}-1\right)\left(\sin\left(\frac{bx}{2}+\frac{a}{2}\right)\right)^{2}}\left(4\,\left(\sin\left(1/2\,bx+a/2\right)\right)^{4}\cos\left(1/2\,bx+a/2\right)+\sqrt{\left(\sin\left(\frac{bx}{2}+\frac{a}{2}\right)\right)^{2}}\sqrt{2\,(s}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sec(b*x+a)^(3/2),x)
```

```
[Out] -2/3*((2*cos(1/2*b*x+1/2*a)^2-1)*sin(1/2*b*x+1/2*a)^2)^(1/2)*(4*sin(1/2*b*x
+1/2*a)^4*cos(1/2*b*x+1/2*a)+(sin(1/2*b*x+1/2*a)^2)^(1/2)*(2*sin(1/2*b*x+1/
2*a)^2-1)^(1/2)*EllipticF(cos(1/2*b*x+1/2*a),2^(1/2))-2*sin(1/2*b*x+1/2*a)^
2*cos(1/2*b*x+1/2*a))/(-2*sin(1/2*b*x+1/2*a)^4+sin(1/2*b*x+1/2*a)^2)^(1/2)/
sin(1/2*b*x+1/2*a)/(2*cos(1/2*b*x+1/2*a)^2-1)^(1/2)/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sec\left(bx+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(b*x + a)^(-3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$${\rm integral}\left(\frac{1}{\sec\left(bx+a\right)^{\frac{3}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sec(b*x + a)^(-3/2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sec^{\frac{3}{2}}(a+bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(b*x+a)**(3/2),x)
```

```
[Out] Integral(sec(a + b*x)**(-3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sec(bx+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sec(b*x + a)^(-3/2), x)
```

3.15 $\int \frac{1}{\sec^{\frac{5}{2}}(a+bx)}\,dx$

Optimal. Leaf size=62

$$\frac{2\sin(a+bx)}{5b\sec^{\frac{3}{2}}(a+bx)}+\frac{6\sqrt{\cos(a+bx)}\sqrt{\sec(a+bx)}E\left(\frac{1}{2}(a+bx)\middle|2\right)}{5b}$$

```
[Out] (6*Sqrt[Cos[a + b*x]]*EllipticE[(a + b*x)/2, 2]*Sqrt[Sec[a + b*x]])/(5*b) +
 (2*Sin[a + b*x])/(5*b*Sec[a + b*x]^(3/2))
```

Rubi [A] time = 0.0286673, antiderivative size = 62, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {3769, 3771, 2639}

$$\frac{2\sin(a+bx)}{5b\sec^{\frac{3}{2}}(a+bx)}+\frac{6\sqrt{\cos(a+bx)}\sqrt{\sec(a+bx)}E\left(\frac{1}{2}(a+bx)\middle|2\right)}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[a + b*x]^(-5/2),x]
```

```
[Out] (6*Sqrt[Cos[a + b*x]]*EllipticE[(a + b*x)/2, 2]*Sqrt[Sec[a + b*x]])/(5*b) +
 (2*Sin[a + b*x])/(5*b*Sec[a + b*x]^(3/2))
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sec^{\frac{5}{2}}(a+bx)}\,dx &= \frac{2\sin(a+bx)}{5b\sec^{\frac{3}{2}}(a+bx)}+\frac{3}{5}\int \frac{1}{\sqrt{\sec(a+bx)}}\,dx\\ &= \frac{2\sin(a+bx)}{5b\sec^{\frac{3}{2}}(a+bx)}+\frac{1}{5}\left(3\sqrt{\cos(a+bx)}\sqrt{\sec(a+bx)}\right)\int \sqrt{\cos(a+bx)}\,dx\\ &= \frac{6\sqrt{\cos(a+bx)}E\left(\frac{1}{2}(a+bx)\middle|2\right)\sqrt{\sec(a+bx)}}{5b}+\frac{2\sin(a+bx)}{5b\sec^{\frac{3}{2}}(a+bx)}\end{aligned}$$

Mathematica [A] time = 0.0693045, size = 55, normalized size = 0.89

$$\frac{\sqrt{\sec(a+bx)}\left(\sin(a+bx)+\sin(3(a+bx))+12\sqrt{\cos(a+bx)}E\left(\frac{1}{2}(a+bx)\middle|2\right)\right)}{10b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[a + b*x]^(-5/2),x]
```

```
[Out] (Sqrt[Sec[a + b*x]]*(12*Sqrt[Cos[a + b*x]]*EllipticE[(a + b*x)/2, 2] + Sin[
a + b*x] + Sin[3*(a + b*x)]))/(10*b)
```

Maple [B] time = 1.212, size = 202, normalized size = 3.3

$$-\frac{2}{5b}\sqrt{\left(2\left(\cos(1/2\,bx+a/2)\right)^2-1\right)\left(\sin\left(\frac{bx}{2}+\frac{a}{2}\right)\right)^2}\left(-8\left(\sin(1/2\,bx+a/2)\right)^6\cos(1/2\,bx+a/2)+8\left(\sin(1/2\,bx+a/2)\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sec(b*x+a)^(5/2),x)
```

```
[Out] -2/5*((2*cos(1/2*b*x+1/2*a)^2-1)*sin(1/2*b*x+1/2*a)^2)^(1/2)*(-8*sin(1/2*b*
x+1/2*a)^6*cos(1/2*b*x+1/2*a)+8*sin(1/2*b*x+1/2*a)^4*cos(1/2*b*x+1/2*a)-3*(
2*sin(1/2*b*x+1/2*a)^2-1)^(1/2)*(sin(1/2*b*x+1/2*a)^2)^(1/2)*EllipticE(cos(
1/2*b*x+1/2*a),2^(1/2))-2*sin(1/2*b*x+1/2*a)^2*cos(1/2*b*x+1/2*a))/(-2*sin(
1/2*b*x+1/2*a)^4+sin(1/2*b*x+1/2*a)^2)^(1/2)/sin(1/2*b*x+1/2*a)/(2*cos(1/2*
b*x+1/2*a)^2-1)^(1/2)/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sec(bx+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(b*x+a)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(b*x + a)^(-5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{1}{\sec(bx+a)^{\frac{5}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(b*x+a)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sec(b*x + a)^(-5/2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sec^{\frac{5}{2}}(a+bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(b*x+a)**(5/2),x)
```

```
[Out] Integral(sec(a + b*x)**(-5/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sec(bx+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(b*x+a)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sec(b*x + a)^(-5/2), x)
```

3.16 $\int \frac{1}{\sec^{\frac{7}{2}}(a+bx)} dx$

Optimal. Leaf size=85

$$\frac{10\sqrt{\cos(a+bx)}\sqrt{\sec(a+bx)}\text{EllipticF}\left(\frac{1}{2}(a+bx),2\right)}{21b}+\frac{2\sin(a+bx)}{7b\sec^{\frac{5}{2}}(a+bx)}+\frac{10\sin(a+bx)}{21b\sqrt{\sec(a+bx)}}$$

```
[Out] (10*Sqrt[Cos[a + b*x]]*EllipticF[(a + b*x)/2, 2]*Sqrt[Sec[a + b*x]])/(21*b)
 + (2*Sin[a + b*x])/(7*b*Sec[a + b*x]^(5/2)) + (10*Sin[a + b*x])/(21*b*Sqrt
[Sec[a + b*x]])
```

Rubi [A] time = 0.0403677, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {3769, 3771, 2641}

$$\frac{2\sin(a+bx)}{7b\sec^{\frac{5}{2}}(a+bx)}+\frac{10\sin(a+bx)}{21b\sqrt{\sec(a+bx)}}+\frac{10\sqrt{\cos(a+bx)}\sqrt{\sec(a+bx)}F\left(\frac{1}{2}(a+bx)\middle|2\right)}{21b}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[a + b*x]^(-7/2),x]
```

```
[Out] (10*Sqrt[Cos[a + b*x]]*EllipticF[(a + b*x)/2, 2]*Sqrt[Sec[a + b*x]])/(21*b)
 + (2*Sin[a + b*x])/(7*b*Sec[a + b*x]^(5/2)) + (10*Sin[a + b*x])/(21*b*Sqrt
[Sec[a + b*x]])
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{\sec^{\frac{7}{2}}(a+bx)}\,dx &= \frac{2\sin(a+bx)}{7b\sec^{\frac{5}{2}}(a+bx)} + \frac{5}{7}\int \frac{1}{\sec^{\frac{3}{2}}(a+bx)}\,dx \\
&= \frac{2\sin(a+bx)}{7b\sec^{\frac{5}{2}}(a+bx)} + \frac{10\sin(a+bx)}{21b\sqrt{\sec(a+bx)}} + \frac{5}{21}\int \sqrt{\sec(a+bx)}\,dx \\
&= \frac{2\sin(a+bx)}{7b\sec^{\frac{5}{2}}(a+bx)} + \frac{10\sin(a+bx)}{21b\sqrt{\sec(a+bx)}} + \frac{1}{21}\left(5\sqrt{\cos(a+bx)}\sqrt{\sec(a+bx)}\right)\int \frac{1}{\sqrt{\cos(a+bx)}}\,dx \\
&= \frac{10\sqrt{\cos(a+bx)}F\left(\frac{1}{2}(a+bx)\middle|2\right)\sqrt{\sec(a+bx)}}{21b} + \frac{2\sin(a+bx)}{7b\sec^{\frac{5}{2}}(a+bx)} + \frac{10\sin(a+bx)}{21b\sqrt{\sec(a+bx)}}
\end{aligned}
$$

Mathematica [A] time = 0.0986761, size = 61, normalized size = 0.72

$$
\frac{\sqrt{\sec(a+bx)}\left(40\sqrt{\cos(a+bx)}\text{EllipticF}\left(\frac{1}{2}(a+bx),2\right)+26\sin(2(a+bx))+3\sin(4(a+bx))\right)}{84b}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[a + b*x]^(-7/2),x]
```

```
[Out] (Sqrt[Sec[a + b*x]]*(40*Sqrt[Cos[a + b*x]]*EllipticF[(a + b*x)/2, 2] + 26*S
in[2*(a + b*x)] + 3*Sin[4*(a + b*x)]))/(84*b)
```

Maple [B] time = 1.247, size = 199, normalized size = 2.3

$$
-\frac{2}{21\,b}\sqrt{\left(2\,\left(\cos\left(1/2\,bx+a/2\right)\right)^{2}-1\right)\left(\sin\left(\frac{bx}{2}+\frac{a}{2}\right)\right)^{2}}\left(48\,\left(\cos\left(1/2\,bx+a/2\right)\right)^{9}-120\,\left(\cos\left(1/2\,bx+a/2\right)\right)^{7}+128\,\left(\cos(1/2\right.\right.
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sec(b*x+a)^(7/2),x)
```

```
[Out] -2/21*((2*cos(1/2*b*x+1/2*a)^2-1)*sin(1/2*b*x+1/2*a)^2)^(1/2)*(48*cos(1/2*b
*x+1/2*a)^9-120*cos(1/2*b*x+1/2*a)^7+128*cos(1/2*b*x+1/2*a)^5-72*cos(1/2*b*
x+1/2*a)^3+5*(sin(1/2*b*x+1/2*a)^2)^(1/2)*(-2*cos(1/2*b*x+1/2*a)^2+1)^(1/2)
*EllipticF(cos(1/2*b*x+1/2*a),2^(1/2))+16*cos(1/2*b*x+1/2*a))/(-2*sin(1/2*b
*x+1/2*a)^4+sin(1/2*b*x+1/2*a)^2)^(1/2)/sin(1/2*b*x+1/2*a)/(2*cos(1/2*b*x+1
/2*a)^2-1)^(1/2)/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$
\int \frac{1}{\sec\left(bx+a\right)^{\frac{7}{2}}}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(b*x+a)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(b*x + a)^(-7/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\sec\left(bx+a\right)^{\frac{7}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(b*x+a)^(7/2),x, algorithm="fricas")
```

```
[Out] integral(sec(b*x + a)^(-7/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(b*x+a)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sec\left(bx+a\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(b*x+a)^(7/2),x, algorithm="giac")
```

```
[Out] integrate(sec(b*x + a)^(-7/2), x)
```

3.17 $\int (c\sec(a+bx))^{7/2}\,dx$

Optimal. Leaf size=98

$$\frac{6c^3\sin(a+bx)\sqrt{c\sec(a+bx)}}{5b}-\frac{6c^4E\left(\frac{1}{2}(a+bx)\middle|2\right)}{5b\sqrt{\cos(a+bx)}\sqrt{c\sec(a+bx)}}+\frac{2c\sin(a+bx)(c\sec(a+bx))^{5/2}}{5b}$$

```
[Out] (-6*c^4*EllipticE[(a + b*x)/2, 2])/(5*b*Sqrt[Cos[a + b*x]]*Sqrt[c*Sec[a + b
*x]]) + (6*c^3*Sqrt[c*Sec[a + b*x]]*Sin[a + b*x])/(5*b) + (2*c*(c*Sec[a + b
*x])^(5/2)*Sin[a + b*x])/(5*b)
```

Rubi [A] time = 0.0558185, antiderivative size = 98, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {3768, 3771, 2639}

$$\frac{6c^3\sin(a+bx)\sqrt{c\sec(a+bx)}}{5b}-\frac{6c^4E\left(\frac{1}{2}(a+bx)\middle|2\right)}{5b\sqrt{\cos(a+bx)}\sqrt{c\sec(a+bx)}}+\frac{2c\sin(a+bx)(c\sec(a+bx))^{5/2}}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[(c*Sec[a + b*x])^(7/2),x]
```

```
[Out] (-6*c^4*EllipticE[(a + b*x)/2, 2])/(5*b*Sqrt[Cos[a + b*x]]*Sqrt[c*Sec[a + b
*x]]) + (6*c^3*Sqrt[c*Sec[a + b*x]]*Sin[a + b*x])/(5*b) + (2*c*(c*Sec[a + b
*x])^(5/2)*Sin[a + b*x])/(5*b)
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (c\sec(a+bx))^{7/2}\,dx = \frac{2c(c\sec(a+bx))^{5/2}\sin(a+bx)}{5b} + \frac{1}{5}\left(3c^2\right)\int (c\sec(a+bx))^{3/2}\,dx$$
$$= \frac{6c^3\sqrt{c\sec(a+bx)}\sin(a+bx)}{5b} + \frac{2c(c\sec(a+bx))^{5/2}\sin(a+bx)}{5b} - \frac{1}{5}\left(3c^4\right)\int \frac{1}{\sqrt{c\sec(a+bx)}}$$
$$= \frac{6c^3\sqrt{c\sec(a+bx)}\sin(a+bx)}{5b} + \frac{2c(c\sec(a+bx))^{5/2}\sin(a+bx)}{5b} - \frac{\left(3c^4\right)\int \sqrt{\cos(a+bx)}\,d}{5\sqrt{\cos(a+bx)}\sqrt{c\sec(a+}}$$
$$= -\frac{6c^4E\left(\frac{1}{2}(a+bx)\middle|2\right)}{5b\sqrt{\cos(a+bx)}\sqrt{c\sec(a+bx)}} + \frac{6c^3\sqrt{c\sec(a+bx)}\sin(a+bx)}{5b} + \frac{2c(c\sec(a+bx))^{5/2}\sin(}{5b}$$

Mathematica [A] time = 0.165577, size = 62, normalized size = 0.63

$$\frac{c(c\sec(a+bx))^{5/2}\left(7\sin(a+bx)+3\sin(3(a+bx))-12\cos^{\frac{5}{2}}(a+bx)E\left(\frac{1}{2}(a+bx)\middle|2\right)\right)}{10b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*Sec[a + b*x])^(7/2),x]
```

```
[Out] (c*(c*Sec[a + b*x])^(5/2)*(-12*Cos[a + b*x]^(5/2)*EllipticE[(a + b*x)/2, 2]
 + 7*Sin[a + b*x] + 3*Sin[3*(a + b*x)]))/(10*b)
```

Maple [C] time = 0.32, size = 354, normalized size = 3.6

$$\frac{2\left(-1+\cos(bx+a)\right)^2\cos(bx+a)\left(\cos(bx+a)+1\right)^2}{5b\left(\sin(bx+a)\right)^5}\left(3i\left(\cos(bx+a)\right)^3\text{EllipticE}\left(\frac{i\left(-1+\cos(bx+a)\right)}{\sin(bx+a)},i\right)\sqrt{(\cos(b}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*sec(b*x+a))^(7/2),x)
```

```
[Out] 2/5/b*(-1+cos(b*x+a))^2*(3*I*cos(b*x+a)^3*EllipticE(I*(-1+cos(b*x+a))/sin(b
*x+a),I)*(1/(cos(b*x+a)+1))^(1/2)*(cos(b*x+a)/(cos(b*x+a)+1))^(1/2)*sin(b*x
+a)-3*I*cos(b*x+a)^3*EllipticF(I*(-1+cos(b*x+a))/sin(b*x+a),I)*(1/(cos(b*x+
a)+1))^(1/2)*(cos(b*x+a)/(cos(b*x+a)+1))^(1/2)*sin(b*x+a)+3*I*(1/(cos(b*x+a
)+1))^(1/2)*(cos(b*x+a)/(cos(b*x+a)+1))^(1/2)*cos(b*x+a)^2*EllipticE(I*(-1+
cos(b*x+a))/sin(b*x+a),I)*sin(b*x+a)-3*I*(1/(cos(b*x+a)+1))^(1/2)*(cos(b*x+
a)/(cos(b*x+a)+1))^(1/2)*EllipticF(I*(-1+cos(b*x+a))/sin(b*x+a),I)*cos(b*x+
a)^2*sin(b*x+a)-3*cos(b*x+a)^3+2*cos(b*x+a)^2+1)*cos(b*x+a)*(cos(b*x+a)+1)^
2*(c/cos(b*x+a))^(7/2)/sin(b*x+a)^5
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (c\sec(bx+a))^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sec(b*x+a))^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((c*sec(b*x + a))^(7/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{c\sec\left(bx+a\right)}c^3\sec\left(bx+a\right)^3,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sec(b*x+a))^(7/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*sec(b*x + a))*c^3*sec(b*x + a)^3, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sec(b*x+a))**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(c\sec\left(bx+a\right)\right)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sec(b*x+a))^(7/2),x, algorithm="giac")
```

```
[Out] integrate((c*sec(b*x + a))^(7/2), x)
```

3.18 $\int (c\sec(a+bx))^{5/2}\,dx$

Optimal. Leaf size=70

$$\frac{2c^2\sqrt{\cos(a+bx)}\text{EllipticF}\left(\frac{1}{2}(a+bx),2\right)\sqrt{c\sec(a+bx)}}{3b}+\frac{2c\sin(a+bx)(c\sec(a+bx))^{3/2}}{3b}$$

```
[Out] (2*c^2*Sqrt[Cos[a + b*x]]*EllipticF[(a + b*x)/2, 2]*Sqrt[c*Sec[a + b*x]])/(
3*b) + (2*c*(c*Sec[a + b*x])^(3/2)*Sin[a + b*x])/(3*b)
```

Rubi [A] time = 0.0336493, antiderivative size = 70, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {3768, 3771, 2641}

$$\frac{2c^2\sqrt{\cos(a+bx)}F\left(\frac{1}{2}(a+bx)\middle|2\right)\sqrt{c\sec(a+bx)}}{3b}+\frac{2c\sin(a+bx)(c\sec(a+bx))^{3/2}}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[(c*Sec[a + b*x])^(5/2),x]
```

```
[Out] (2*c^2*Sqrt[Cos[a + b*x]]*EllipticF[(a + b*x)/2, 2]*Sqrt[c*Sec[a + b*x]])/(
3*b) + (2*c*(c*Sec[a + b*x])^(3/2)*Sin[a + b*x])/(3*b)
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int (c\sec(a+bx))^{5/2}\,dx &= \frac{2c(c\sec(a+bx))^{3/2}\sin(a+bx)}{3b}+\frac{1}{3}c^2\int\sqrt{c\sec(a+bx)}\,dx\\ &= \frac{2c(c\sec(a+bx))^{3/2}\sin(a+bx)}{3b}+\frac{1}{3}\left(c^2\sqrt{\cos(a+bx)}\sqrt{c\sec(a+bx)}\right)\int\frac{1}{\sqrt{\cos(a+bx)}}\,dx\\ &= \frac{2c^2\sqrt{\cos(a+bx)}F\left(\frac{1}{2}(a+bx)\middle|2\right)\sqrt{c\sec(a+bx)}}{3b}+\frac{2c(c\sec(a+bx))^{3/2}\sin(a+bx)}{3b}\end{aligned}$$

Mathematica [A] time = 0.0658061, size = 51, normalized size = 0.73

$$\frac{2c^2\sqrt{c\sec(a+bx)}\left(\sqrt{\cos(a+bx)}\text{EllipticF}\left(\frac{1}{2}(a+bx),2\right)+\tan(a+bx)\right)}{3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*Sec[a + b*x])^(5/2),x]
```

```
[Out] (2*c^2*Sqrt[c*Sec[a + b*x]]*(Sqrt[Cos[a + b*x]]*EllipticF[(a + b*x)/2, 2] +
 Tan[a + b*x]))/(3*b)
```

Maple [C] time = 0.198, size = 128, normalized size = 1.8

$$-\frac{(-2+2\cos(bx+a))\cos(bx+a)(\cos(bx+a)+1)^2}{3b(\sin(bx+a))^3}\left(i\sqrt{(\cos(bx+a)+1)^{-1}}\sqrt{\frac{\cos(bx+a)}{\cos(bx+a)+1}}\text{EllipticF}\left(\frac{i(-1+\cos(}{\sin(bx}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*sec(b*x+a))^(5/2),x)
```

```
[Out] -2/3/b*(-1+cos(b*x+a))*(I*(1/(cos(b*x+a)+1))^(1/2)*(cos(b*x+a)/(cos(b*x+a)+
1))^(1/2)*EllipticF(I*(-1+cos(b*x+a))/sin(b*x+a),I)*cos(b*x+a)*sin(b*x+a)-c
os(b*x+a)+1)*cos(b*x+a)*(cos(b*x+a)+1)^2*(c/cos(b*x+a))^(5/2)/sin(b*x+a)^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (c\sec(bx+a))^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sec(b*x+a))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((c*sec(b*x + a))^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{c\sec(bx+a)}c^2\sec(bx+a)^2,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sec(b*x+a))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*sec(b*x + a))*c^2*sec(b*x + a)^2, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sec(b*x+a))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (c\sec (bx+a))^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sec(b*x+a))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((c*sec(b*x + a))^(5/2), x)
```

3.19 $\int (c\sec(a+bx))^{3/2}\,dx$

Optimal. Leaf size=66

$$\frac{2c\sin(a+bx)\sqrt{c\sec(a+bx)}}{b}-\frac{2c^2E\left(\frac{1}{2}(a+bx)\middle|2\right)}{b\sqrt{\cos(a+bx)}\sqrt{c\sec(a+bx)}}$$

```
[Out] (-2*c^2*EllipticE[(a + b*x)/2, 2])/(b*Sqrt[Cos[a + b*x]]*Sqrt[c*Sec[a + b*x
]]) + (2*c*Sqrt[c*Sec[a + b*x]]*Sin[a + b*x])/b
```

Rubi [A] time = 0.0395353, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {3768, 3771, 2639}

$$\frac{2c\sin(a+bx)\sqrt{c\sec(a+bx)}}{b}-\frac{2c^2E\left(\frac{1}{2}(a+bx)\middle|2\right)}{b\sqrt{\cos(a+bx)}\sqrt{c\sec(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*Sec[a + b*x])^(3/2),x]
```

```
[Out] (-2*c^2*EllipticE[(a + b*x)/2, 2])/(b*Sqrt[Cos[a + b*x]]*Sqrt[c*Sec[a + b*x
]]) + (2*c*Sqrt[c*Sec[a + b*x]]*Sin[a + b*x])/b
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int (c\sec(a+bx))^{3/2}\,dx &= \frac{2c\sqrt{c\sec(a+bx)}\sin(a+bx)}{b}-c^2\int \frac{1}{\sqrt{c\sec(a+bx)}}\,dx\\
&= \frac{2c\sqrt{c\sec(a+bx)}\sin(a+bx)}{b}-\frac{c^2\int \sqrt{\cos(a+bx)}\,dx}{\sqrt{\cos(a+bx)}\sqrt{c\sec(a+bx)}}\\
&= -\frac{2c^2E\left(\frac{1}{2}(a+bx)\middle|2\right)}{b\sqrt{\cos(a+bx)}\sqrt{c\sec(a+bx)}}+\frac{2c\sqrt{c\sec(a+bx)}\sin(a+bx)}{b}
\end{aligned}$$

Mathematica [A] time = 0.0391133, size = 48, normalized size = 0.73

$$\frac{2c\sqrt{c\sec(a+bx)}\left(\sin(a+bx)-\sqrt{\cos(a+bx)}E\left(\frac{1}{2}(a+bx)\middle|2\right)\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*Sec[a + b*x])^(3/2),x]
```

```
[Out] (2*c*Sqrt[c*Sec[a + b*x]]*(-(Sqrt[Cos[a + b*x]]*EllipticE[(a + b*x)/2, 2])
+ Sin[a + b*x]))/b
```

Maple [C] time = 0.225, size = 322, normalized size = 4.9

$$2\frac{(\cos(bx+a)+1)^2(-1+\cos(bx+a))^2\cos(bx+a)}{b(\sin(bx+a))^5}\left(i\mathrm{EllipticE}\left(\frac{i(-1+\cos(bx+a))}{\sin(bx+a)},i\right)\cos(bx+a)\sin(bx+a)\sqrt{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*sec(b*x+a))^(3/2),x)
```

```
[Out] 2/b*(cos(b*x+a)+1)^2*(-1+cos(b*x+a))^2*(I*EllipticE(I*(-1+cos(b*x+a))/sin(b
*x+a),I)*cos(b*x+a)*sin(b*x+a)*(1/(cos(b*x+a)+1))^(1/2)*(cos(b*x+a)/(cos(b*
x+a)+1))^(1/2)-I*(1/(cos(b*x+a)+1))^(1/2)*(cos(b*x+a)/(cos(b*x+a)+1))^(1/2)
*EllipticF(I*(-1+cos(b*x+a))/sin(b*x+a),I)*cos(b*x+a)*sin(b*x+a)+I*Elliptic
E(I*(-1+cos(b*x+a))/sin(b*x+a),I)*sin(b*x+a)*(1/(cos(b*x+a)+1))^(1/2)*(cos(
b*x+a)/(cos(b*x+a)+1))^(1/2)-I*EllipticF(I*(-1+cos(b*x+a))/sin(b*x+a),I)*si
n(b*x+a)*(1/(cos(b*x+a)+1))^(1/2)*(cos(b*x+a)/(cos(b*x+a)+1))^(1/2)-cos(b*x
+a)+1)*cos(b*x+a)*(c/cos(b*x+a))^(3/2)/sin(b*x+a)^5
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (c\sec(bx+a))^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sec(b*x+a))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((c*sec(b*x + a))^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\sqrt{c\sec(bx+a)}c\sec(bx+a),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sec(b*x+a))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*sec(b*x + a))*c*sec(b*x + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (c\sec(a+bx))^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sec(b*x+a))**(3/2),x)
```

```
[Out] Integral((c*sec(a + b*x))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (c\sec(bx+a))^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sec(b*x+a))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((c*sec(b*x + a))^(3/2), x)
```

3.20 $\int \sqrt{c\sec(a+bx)}\,dx$

Optimal. Leaf size=38

$$\frac{2\sqrt{\cos(a+bx)}\text{EllipticF}\left(\frac{1}{2}(a+bx),2\right)\sqrt{c\sec(a+bx)}}{b}$$

```
[Out] (2*Sqrt[Cos[a + b*x]]*EllipticF[(a + b*x)/2, 2]*Sqrt[c*Sec[a + b*x]])/b
```

Rubi [A] time = 0.0192504, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3771, 2641}

$$\frac{2\sqrt{\cos(a+bx)}F\left(\frac{1}{2}(a+bx)\middle|2\right)\sqrt{c\sec(a+bx)}}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c*Sec[a + b*x]],x]
```

```
[Out] (2*Sqrt[Cos[a + b*x]]*EllipticF[(a + b*x)/2, 2]*Sqrt[c*Sec[a + b*x]])/b
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \sqrt{c\sec(a+bx)}\,dx &= \left(\sqrt{\cos(a+bx)}\sqrt{c\sec(a+bx)}\right)\int \frac{1}{\sqrt{\cos(a+bx)}}\,dx\\ &= \frac{2\sqrt{\cos(a+bx)}F\left(\frac{1}{2}(a+bx)\middle|2\right)\sqrt{c\sec(a+bx)}}{b}\end{aligned}$$

Mathematica [A] time = 0.0197674, size = 38, normalized size = 1.

$$\frac{2\sqrt{\cos(a+bx)}\text{EllipticF}\left(\frac{1}{2}(a+bx),2\right)\sqrt{c\sec(a+bx)}}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c*Sec[a + b*x]],x]
```

```
[Out] (2*Sqrt[Cos[a + b*x]]*EllipticF[(a + b*x)/2, 2]*Sqrt[c*Sec[a + b*x]])/b
```

Maple [C] time = 0.153, size = 98, normalized size = 2.6

$$\frac{-2\,i\,(-1+\cos(bx+a))\,(\cos(bx+a)+1)^2}{b\,(\sin(bx+a))^2}\sqrt{\frac{c}{\cos(bx+a)}}\sqrt{(\cos(bx+a)+1)^{-1}}\sqrt{\frac{\cos(bx+a)}{\cos(bx+a)+1}}\,\mathrm{EllipticF}\left(\frac{i\,(-1+\cos}{\sin(bx}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*sec(b*x+a))^(1/2),x)
```

```
[Out] -2*I/b*(c/cos(b*x+a))^(1/2)*(-1+cos(b*x+a))*(1/(cos(b*x+a)+1))^(1/2)*(cos(b
*x+a)/(cos(b*x+a)+1))^(1/2)*EllipticF(I*(-1+cos(b*x+a))/sin(b*x+a),I)*(cos(
b*x+a)+1)^2/sin(b*x+a)^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{c\sec(bx+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sec(b*x+a))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*sec(b*x + a)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\sqrt{c\sec(bx+a)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sec(b*x+a))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*sec(b*x + a)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{c\sec(a+bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sec(b*x+a))**(1/2),x)
```

```
[Out] Integral(sqrt(c*sec(a + b*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{c\sec(bx+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sec(b*x+a))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*sec(b*x + a)), x)
```

3.21 $\int \frac{1}{\sqrt{c\sec(a+bx)}}\,dx$

Optimal. Leaf size=38

$$\frac{2E\left(\frac{1}{2}(a+bx)\middle|2\right)}{b\sqrt{\cos(a+bx)}\sqrt{c\sec(a+bx)}}$$

```
[Out] (2*EllipticE[(a + b*x)/2, 2])/(b*Sqrt[Cos[a + b*x]]*Sqrt[c*Sec[a + b*x]])
```

Rubi [A] time = 0.0272559, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3771, 2639}

$$\frac{2E\left(\frac{1}{2}(a+bx)\middle|2\right)}{b\sqrt{\cos(a+bx)}\sqrt{c\sec(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[c*Sec[a + b*x]],x]
```

```
[Out] (2*EllipticE[(a + b*x)/2, 2])/(b*Sqrt[Cos[a + b*x]]*Sqrt[c*Sec[a + b*x]])
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{c\sec(a+bx)}}\,dx &= \frac{\int \sqrt{\cos(a+bx)}\,dx}{\sqrt{\cos(a+bx)}\sqrt{c\sec(a+bx)}}\\ &= \frac{2E\left(\frac{1}{2}(a+bx)\middle|2\right)}{b\sqrt{\cos(a+bx)}\sqrt{c\sec(a+bx)}}\end{aligned}$$

Mathematica [A] time = 0.0290382, size = 38, normalized size = 1.

$$\frac{2E\left(\frac{1}{2}(a+bx)\middle|2\right)}{b\sqrt{\cos(a+bx)}\sqrt{c\sec(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[c*Sec[a + b*x]],x]
```

```
[Out] (2*EllipticE[(a + b*x)/2, 2])/(b*Sqrt[Cos[a + b*x]]*Sqrt[c*Sec[a + b*x]])
```

Maple [C] time = 0.164, size = 306, normalized size = 8.1

$$2\frac{1}{b\sin(bx+a)c}\left(i\sqrt{(\cos(bx+a)+1)^{-1}}\sqrt{\frac{\cos(bx+a)}{\cos(bx+a)+1}}\text{EllipticF}\left(\frac{i(-1+\cos(bx+a))}{\sin(bx+a)},i\right)\cos(bx+a)\sin(bx+a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*sec(b*x+a))^(1/2),x)
```

```
[Out] 2/b*(I*(1/(cos(b*x+a)+1))^(1/2)*(cos(b*x+a)/(cos(b*x+a)+1))^(1/2)*EllipticF
(I*(-1+cos(b*x+a))/sin(b*x+a),I)*cos(b*x+a)*sin(b*x+a)-I*EllipticE(I*(-1+co
s(b*x+a))/sin(b*x+a),I)*cos(b*x+a)*sin(b*x+a)*(1/(cos(b*x+a)+1))^(1/2)*(cos
(b*x+a)/(cos(b*x+a)+1))^(1/2)+I*EllipticF(I*(-1+cos(b*x+a))/sin(b*x+a),I)*(
1/(cos(b*x+a)+1))^(1/2)*(cos(b*x+a)/(cos(b*x+a)+1))^(1/2)*sin(b*x+a)-I*Elli
pticE(I*(-1+cos(b*x+a))/sin(b*x+a),I)*sin(b*x+a)*(1/(cos(b*x+a)+1))^(1/2)*(
cos(b*x+a)/(cos(b*x+a)+1))^(1/2)-cos(b*x+a)^2+cos(b*x+a))*(c/cos(b*x+a))^(1
/2)/sin(b*x+a)/c
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{c\sec(bx+a)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*sec(b*x+a))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(c*sec(b*x + a)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{c\sec(bx+a)}}{c\sec(bx+a)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*sec(b*x+a))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*sec(b*x + a))/(c*sec(b*x + a)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{c\sec(a+bx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*sec(b*x+a))**(1/2),x)
```

```
[Out] Integral(1/sqrt(c*sec(a + b*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{c \sec (bx + a)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*sec(b*x+a))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(c*sec(b*x + a)), x)
```

3.22 $\int \frac{1}{(c\sec(a+bx))^{3/2}}\,dx$

Optimal. Leaf size=72

$$\frac{2\sqrt{\cos(a+bx)}\,\text{EllipticF}\left(\frac{1}{2}(a+bx),2\right)\sqrt{c\sec(a+bx)}}{3bc^2}+\frac{2\sin(a+bx)}{3bc\sqrt{c\sec(a+bx)}}$$

```
[Out] (2*Sqrt[Cos[a + b*x]]*EllipticF[(a + b*x)/2, 2]*Sqrt[c*Sec[a + b*x]])/(3*b*
c^2) + (2*Sin[a + b*x])/(3*b*c*Sqrt[c*Sec[a + b*x]])
```

Rubi [A] time = 0.0461409, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {3769, 3771, 2641}

$$\frac{2\sqrt{\cos(a+bx)}\,F\left(\frac{1}{2}(a+bx)\middle|2\right)\sqrt{c\sec(a+bx)}}{3bc^2}+\frac{2\sin(a+bx)}{3bc\sqrt{c\sec(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*Sec[a + b*x])^(-3/2),x]
```

```
[Out] (2*Sqrt[Cos[a + b*x]]*EllipticF[(a + b*x)/2, 2]*Sqrt[c*Sec[a + b*x]])/(3*b*
c^2) + (2*Sin[a + b*x])/(3*b*c*Sqrt[c*Sec[a + b*x]])
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(c\sec(a+bx))^{3/2}}\,dx &= \frac{2\sin(a+bx)}{3bc\sqrt{c\sec(a+bx)}}+\frac{\int \sqrt{c\sec(a+bx)}\,dx}{3c^2}\\ &= \frac{2\sin(a+bx)}{3bc\sqrt{c\sec(a+bx)}}+\frac{\left(\sqrt{\cos(a+bx)}\sqrt{c\sec(a+bx)}\right)\int \frac{1}{\sqrt{\cos(a+bx)}}\,dx}{3c^2}\\ &= \frac{2\sqrt{\cos(a+bx)}\,F\left(\frac{1}{2}(a+bx)\middle|2\right)\sqrt{c\sec(a+bx)}}{3bc^2}+\frac{2\sin(a+bx)}{3bc\sqrt{c\sec(a+bx)}}\end{aligned}$$

Mathematica [A] time = 0.0579416, size = 59, normalized size = 0.82

$$\frac{\sec^2(a+bx)\left(2\sqrt{\cos(a+bx)}\text{EllipticF}\left(\frac{1}{2}(a+bx),2\right)+\sin(2(a+bx))\right)}{3b(c\sec(a+bx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*Sec[a + b*x])^(-3/2),x]
```

```
[Out] (Sec[a + b*x]^2*(2*Sqrt[Cos[a + b*x]]*EllipticF[(a + b*x)/2, 2] + Sin[2*(a
+ b*x)]))/(3*b*(c*Sec[a + b*x])^(3/2))
```

Maple [C] time = 0.155, size = 131, normalized size = 1.8

$$-\frac{2\left(\cos(bx+a)+1\right)^2\left(-1+\cos(bx+a)\right)}{3b\left(\cos(bx+a)\right)^2\left(\sin(bx+a)\right)^3}\left(i\text{EllipticF}\left(\frac{i\left(-1+\cos(bx+a)\right)}{\sin(bx+a)},i\right)\sqrt{\left(\cos(bx+a)+1\right)^{-1}}\sqrt{\frac{\cos(bx+a)}{\cos(bx+a)+1}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*sec(b*x+a))^(3/2),x)
```

```
[Out] -2/3/b*(cos(b*x+a)+1)^2*(-1+cos(b*x+a))*(I*EllipticF(I*(-1+cos(b*x+a))/sin(
b*x+a),I)*(1/(cos(b*x+a)+1))^(1/2)*(cos(b*x+a)/(cos(b*x+a)+1))^(1/2)*sin(b*
x+a)-cos(b*x+a)^2+cos(b*x+a))/(c/cos(b*x+a))^(3/2)/cos(b*x+a)^2/sin(b*x+a)^
3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(c\sec(bx+a)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*sec(b*x+a))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((c*sec(b*x + a))^(-3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{c\sec(bx+a)}}{c^2\sec(bx+a)^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*sec(b*x+a))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*sec(b*x + a))/(c^2*sec(b*x + a)^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(c\sec(a+bx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*sec(b*x+a))**(3/2),x)
```

```
[Out] Integral((c*sec(a + b*x))**(-3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(c\sec(bx+a))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*sec(b*x+a))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((c*sec(b*x + a))^(-3/2), x)
```

3.23 $\int \frac{1}{(c\sec(a+bx))^{5/2}}\,dx$

Optimal. Leaf size=72

$$\frac{6E\left(\frac{1}{2}(a+bx)\middle|2\right)}{5bc^2\sqrt{\cos(a+bx)}\sqrt{c\sec(a+bx)}}+\frac{2\sin(a+bx)}{5bc(c\sec(a+bx))^{3/2}}$$

```
[Out] (6*EllipticE[(a + b*x)/2, 2])/(5*b*c^2*Sqrt[Cos[a + b*x]]*Sqrt[c*Sec[a + b*
x]]) + (2*Sin[a + b*x])/(5*b*c*(c*Sec[a + b*x])^(3/2))
```

Rubi [A] time = 0.0378077, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {3769, 3771, 2639}

$$\frac{6E\left(\frac{1}{2}(a+bx)\middle|2\right)}{5bc^2\sqrt{\cos(a+bx)}\sqrt{c\sec(a+bx)}}+\frac{2\sin(a+bx)}{5bc(c\sec(a+bx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*Sec[a + b*x])^(-5/2),x]
```

```
[Out] (6*EllipticE[(a + b*x)/2, 2])/(5*b*c^2*Sqrt[Cos[a + b*x]]*Sqrt[c*Sec[a + b*
x]]) + (2*Sin[a + b*x])/(5*b*c*(c*Sec[a + b*x])^(3/2))
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(c\sec(a+bx))^{5/2}}\,dx &= \frac{2\sin(a+bx)}{5bc(c\sec(a+bx))^{3/2}}+\frac{3\int \frac{1}{\sqrt{c\sec(a+bx)}}\,dx}{5c^2}\\ &= \frac{2\sin(a+bx)}{5bc(c\sec(a+bx))^{3/2}}+\frac{3\int \sqrt{\cos(a+bx)}\,dx}{5c^2\sqrt{\cos(a+bx)}\sqrt{c\sec(a+bx)}}\\ &= \frac{6E\left(\frac{1}{2}(a+bx)\middle|2\right)}{5bc^2\sqrt{\cos(a+bx)}\sqrt{c\sec(a+bx)}}+\frac{2\sin(a+bx)}{5bc(c\sec(a+bx))^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.0679626, size = 60, normalized size = 0.83

$$\frac{\sqrt{c\sec(a+bx)}\left(\sin(a+bx)+\sin(3(a+bx))+12\sqrt{\cos(a+bx)}E\left(\frac{1}{2}(a+bx)\middle|2\right)\right)}{10bc^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*Sec[a + b*x])^(-5/2),x]
```

```
[Out] (Sqrt[c*Sec[a + b*x]]*(12*Sqrt[Cos[a + b*x]]*EllipticE[(a + b*x)/2, 2] + Si
n[a + b*x] + Sin[3*(a + b*x)]))/(10*b*c^3)
```

Maple [C] time = 0.2, size = 323, normalized size = 4.5

$$\frac{2}{5b(\cos(bx+a))^3\sin(bx+a)}\left(3i\sqrt{(\cos(bx+a)+1)^{-1}}\sqrt{\frac{\cos(bx+a)}{\cos(bx+a)+1}}\text{EllipticF}\left(\frac{i(-1+\cos(bx+a))}{\sin(bx+a)},i\right)\cos(bx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*sec(b*x+a))^(5/2),x)
```

```
[Out] 2/5/b*(3*I*(1/(cos(b*x+a)+1))^(1/2)*(cos(b*x+a)/(cos(b*x+a)+1))^(1/2)*Ellip
ticF(I*(-1+cos(b*x+a))/sin(b*x+a),I)*cos(b*x+a)*sin(b*x+a)-3*I*EllipticE(I*
(-1+cos(b*x+a))/sin(b*x+a),I)*cos(b*x+a)*sin(b*x+a)*(1/(cos(b*x+a)+1))^(1/2
)*(cos(b*x+a)/(cos(b*x+a)+1))^(1/2)+3*I*EllipticF(I*(-1+cos(b*x+a))/sin(b*x
+a),I)*(1/(cos(b*x+a)+1))^(1/2)*(cos(b*x+a)/(cos(b*x+a)+1))^(1/2)*sin(b*x+a
)-3*I*EllipticE(I*(-1+cos(b*x+a))/sin(b*x+a),I)*sin(b*x+a)*(1/(cos(b*x+a)+1
))^(1/2)*(cos(b*x+a)/(cos(b*x+a)+1))^(1/2)-cos(b*x+a)^4-2*cos(b*x+a)^2+3*co
s(b*x+a))/(c/cos(b*x+a))^(5/2)/cos(b*x+a)^3/sin(b*x+a)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(c\sec(bx+a))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*sec(b*x+a))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((c*sec(b*x + a))^(-5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{c\sec(bx+a)}}{c^3\sec(bx+a)^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*sec(b*x+a))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*sec(b*x + a))/(c^3*sec(b*x + a)^3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(c\sec{(a+bx)})^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*sec(b*x+a))**(5/2),x)
```

```
[Out] Integral((c*sec(a + b*x))**(-5/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(c\sec{(bx+a)})^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*sec(b*x+a))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((c*sec(b*x + a))^(-5/2), x)
```

3.24 $\int \frac{1}{(c\sec(a+bx))^{7/2}}\,dx$

Optimal. Leaf size=100

$$\frac{10\sqrt{\cos(a+bx)}\text{EllipticF}\left(\frac{1}{2}(a+bx),2\right)\sqrt{c\sec(a+bx)}}{21bc^4}+\frac{10\sin(a+bx)}{21bc^3\sqrt{c\sec(a+bx)}}+\frac{2\sin(a+bx)}{7bc(c\sec(a+bx))^{5/2}}$$

```
[Out] (10*Sqrt[Cos[a + b*x]]*EllipticF[(a + b*x)/2, 2]*Sqrt[c*Sec[a + b*x]])/(21*
b*c^4) + (2*Sin[a + b*x])/(7*b*c*(c*Sec[a + b*x])^(5/2)) + (10*Sin[a + b*x]
)/(21*b*c^3*Sqrt[c*Sec[a + b*x]])
```

Rubi [A] time = 0.059252, antiderivative size = 100, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {3769, 3771, 2641}

$$\frac{10\sin(a+bx)}{21bc^3\sqrt{c\sec(a+bx)}}+\frac{10\sqrt{\cos(a+bx)}F\left(\frac{1}{2}(a+bx)\middle|2\right)\sqrt{c\sec(a+bx)}}{21bc^4}+\frac{2\sin(a+bx)}{7bc(c\sec(a+bx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*Sec[a + b*x])^(-7/2),x]
```

```
[Out] (10*Sqrt[Cos[a + b*x]]*EllipticF[(a + b*x)/2, 2]*Sqrt[c*Sec[a + b*x]])/(21*
b*c^4) + (2*Sin[a + b*x])/(7*b*c*(c*Sec[a + b*x])^(5/2)) + (10*Sin[a + b*x]
)/(21*b*c^3*Sqrt[c*Sec[a + b*x]])
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{1}{(c\sec(a+bx))^{7/2}}\,dx = \frac{2\sin(a+bx)}{7bc(c\sec(a+bx))^{5/2}} + \frac{5\int \frac{1}{(c\sec(a+bx))^{3/2}}\,dx}{7c^2}$$
$$= \frac{2\sin(a+bx)}{7bc(c\sec(a+bx))^{5/2}} + \frac{10\sin(a+bx)}{21bc^3\sqrt{c\sec(a+bx)}} + \frac{5\int \sqrt{c\sec(a+bx)}\,dx}{21c^4}$$
$$= \frac{2\sin(a+bx)}{7bc(c\sec(a+bx))^{5/2}} + \frac{10\sin(a+bx)}{21bc^3\sqrt{c\sec(a+bx)}} + \frac{\left(5\sqrt{\cos(a+bx)}\sqrt{c\sec(a+bx)}\right)\int \frac{1}{\sqrt{\cos(a+bx)}}\,dx}{21c^4}$$
$$= \frac{10\sqrt{\cos(a+bx)}F\left(\frac{1}{2}(a+bx)\middle|2\right)\sqrt{c\sec(a+bx)}}{21bc^4} + \frac{2\sin(a+bx)}{7bc(c\sec(a+bx))^{5/2}} + \frac{10\sin(a+bx)}{21bc^3\sqrt{c\sec(a+bx)}}$$

Mathematica [A] time = 0.0960934, size = 66, normalized size = 0.66

$$\frac{\sqrt{c\sec(a+bx)}\left(40\sqrt{\cos(a+bx)}\text{EllipticF}\left(\frac{1}{2}(a+bx),2\right)+26\sin(2(a+bx))+3\sin(4(a+bx))\right)}{84bc^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*Sec[a + b*x])^(-7/2),x]
```

```
[Out] (Sqrt[c*Sec[a + b*x]]*(40*Sqrt[Cos[a + b*x]]*EllipticF[(a + b*x)/2, 2] + 26
*Sin[2*(a + b*x)] + 3*Sin[4*(a + b*x)]))/(84*b*c^4)
```

Maple [C] time = 0.19, size = 153, normalized size = 1.5

$$-\frac{2\left(\cos(bx+a)+1\right)^2\left(-1+\cos(bx+a)\right)}{21\,b\left(\cos(bx+a)\right)^4\left(\sin(bx+a)\right)^3}\left(5\,i\text{EllipticF}\left(\frac{i\left(-1+\cos(bx+a)\right)}{\sin(bx+a)},i\right)\sqrt{\left(\cos(bx+a)+1\right)^{-1}}\sqrt{\frac{\cos(bx+a)}{\cos(bx+a)+1}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*sec(b*x+a))^(7/2),x)
```

```
[Out] -2/21/b*(cos(b*x+a)+1)^2*(-1+cos(b*x+a))*(5*I*EllipticF(I*(-1+cos(b*x+a))/s
in(b*x+a),I)*(1/(cos(b*x+a)+1))^(1/2)*(cos(b*x+a)/(cos(b*x+a)+1))^(1/2)*sin
(b*x+a)-3*cos(b*x+a)^4+3*cos(b*x+a)^3-5*cos(b*x+a)^2+5*cos(b*x+a))/(c/cos(b
*x+a))^(7/2)/cos(b*x+a)^4/sin(b*x+a)^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(c\sec(bx+a))^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*sec(b*x+a))^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((c*sec(b*x + a))^(-7/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{c\sec(bx+a)}}{c^4\sec(bx+a)^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*sec(b*x+a))^(7/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*sec(b*x + a))/(c^4*sec(b*x + a)^4), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*sec(b*x+a))**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(c\sec(bx+a))^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*sec(b*x+a))^(7/2),x, algorithm="giac")
```

```
[Out] integrate((c*sec(b*x + a))^(-7/2), x)
```

3.25 $\int \sec^{\frac{4}{3}}(a+bx)\,dx$

Optimal. Leaf size=51

$$\frac{3\sin(a+bx)\sqrt[3]{\sec(a+bx)}\text{Hypergeometric2F1}\left(-\frac{1}{6},\frac{1}{2},\frac{5}{6},\cos^2(a+bx)\right)}{b\sqrt{\sin^2(a+bx)}}$$

```
[Out] (3*Hypergeometric2F1[-1/6, 1/2, 5/6, Cos[a + b*x]^2]*Sec[a + b*x]^(1/3)*Sin
[a + b*x])/(b*Sqrt[Sin[a + b*x]^2])
```

Rubi [A] time = 0.023147, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {3772, 2643}

$$\frac{3\sin(a+bx)\sqrt[3]{\sec(a+bx)}\,{}_2F_1\left(-\frac{1}{6},\frac{1}{2};\frac{5}{6};\cos^2(a+bx)\right)}{b\sqrt{\sin^2(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[a + b*x]^(4/3),x]
```

```
[Out] (3*Hypergeometric2F1[-1/6, 1/2, 5/6, Cos[a + b*x]^2]*Sec[a + b*x]^(1/3)*Sin
[a + b*x])/(b*Sqrt[Sin[a + b*x]^2])
```

Rule 3772

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] && !IntegerQ[n]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}\int \sec^{\frac{4}{3}}(a+bx)\,dx &= \sqrt[3]{\cos(a+bx)}\sqrt[3]{\sec(a+bx)}\int \frac{1}{\cos^{\frac{4}{3}}(a+bx)}\,dx\\ &= \frac{3\,{}_2F_1\left(-\frac{1}{6},\frac{1}{2};\frac{5}{6};\cos^2(a+bx)\right)\sqrt[3]{\sec(a+bx)}\sin(a+bx)}{b\sqrt{\sin^2(a+bx)}}\end{aligned}$$

Mathematica [A] time = 0.0510717, size = 55, normalized size = 1.08

$$\frac{3\sqrt{-\tan^2(a+bx)}\csc(a+bx)\sqrt[3]{\sec(a+bx)}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{5}{3},\sec^2(a+bx)\right)}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[a + b*x]^(4/3),x]
```

```
[Out] (3*Csc[a + b*x]*Hypergeometric2F1[1/2, 2/3, 5/3, Sec[a + b*x]^2]*Sec[a + b*
x]^(1/3)*Sqrt[-Tan[a + b*x]^2])/(4*b)
```

Maple [F] time = 0.068, size = 0, normalized size = 0.

$$\int (\sec(bx+a))^{\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)^(4/3),x)
```

```
[Out] int(sec(b*x+a)^(4/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sec(bx+a)^{\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^(4/3),x, algorithm="maxima")
```

```
[Out] integrate(sec(b*x + a)^(4/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sec(bx+a)^{\frac{4}{3}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^(4/3),x, algorithm="fricas")
```

```
[Out] integral(sec(b*x + a)^(4/3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)**(4/3),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sec (bx + a)^{\frac{4}{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^(4/3),x, algorithm="giac")
```

```
[Out] integrate(sec(b*x + a)^(4/3), x)
```

3.26 $\int \sec^{\frac{2}{3}}(a+bx)\,dx$

Optimal. Leaf size=51

$$-\frac{3\sin(a+bx)\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{1}{2},\frac{7}{6},\cos^2(a+bx)\right)}{b\sqrt{\sin^2(a+bx)}\sqrt[3]{\sec(a+bx)}}$$

```
[Out] (-3*Hypergeometric2F1[1/6, 1/2, 7/6, Cos[a + b*x]^2]*Sin[a + b*x])/(b*Sec[a
 + b*x]^(1/3)*Sqrt[Sin[a + b*x]^2])
```

Rubi [A] time = 0.0276046, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {3772, 2643}

$$-\frac{3\sin(a+bx)\,{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{7}{6};\cos^2(a+bx)\right)}{b\sqrt{\sin^2(a+bx)}\sqrt[3]{\sec(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[a + b*x]^(2/3),x]
```

```
[Out] (-3*Hypergeometric2F1[1/6, 1/2, 7/6, Cos[a + b*x]^2]*Sin[a + b*x])/(b*Sec[a
 + b*x]^(1/3)*Sqrt[Sin[a + b*x]^2])
```

Rule 3772

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] && !IntegerQ[n]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}\int \sec^{\frac{2}{3}}(a+bx)\,dx &= \cos^{\frac{2}{3}}(a+bx)\sec^{\frac{2}{3}}(a+bx)\int \frac{1}{\cos^{\frac{2}{3}}(a+bx)}\,dx\\ &= -\frac{3\,{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{7}{6};\cos^2(a+bx)\right)\sin(a+bx)}{b\sqrt[3]{\sec(a+bx)}\sqrt{\sin^2(a+bx)}}\end{aligned}$$

Mathematica [A] time = 0.0356083, size = 55, normalized size = 1.08

$$\frac{3\sqrt{-\tan^2(a+bx)}\csc(a+bx)\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},\sec^2(a+bx)\right)}{2b\sqrt[3]{\sec(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[a + b*x]^(2/3),x]
```

```
[Out] (3*Csc[a + b*x]*Hypergeometric2F1[1/3, 1/2, 4/3, Sec[a + b*x]^2]*Sqrt[-Tan[
a + b*x]^2])/(2*b*Sec[a + b*x]^(1/3))
```

Maple [F] time = 0.094, size = 0, normalized size = 0.

$$\int (\sec (bx+a))^{\frac{2}{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)^(2/3),x)
```

```
[Out] int(sec(b*x+a)^(2/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sec (bx+a)^{\frac{2}{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^(2/3),x, algorithm="maxima")
```

```
[Out] integrate(sec(b*x + a)^(2/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\sec (bx+a)^{\frac{2}{3}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^(2/3),x, algorithm="fricas")
```

```
[Out] integral(sec(b*x + a)^(2/3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sec^{\frac{2}{3}}(a+bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)**(2/3),x)
```

```
[Out] Integral(sec(a + b*x)**(2/3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sec (bx + a)^{\frac{2}{3}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^(2/3),x, algorithm="giac")
```

```
[Out] integrate(sec(b*x + a)^(2/3), x)
```

3.27 $\int \sqrt[3]{\sec(a+bx)}\,dx$

Optimal. Leaf size=53

$$-\frac{3\sin(a+bx)\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},\cos^2(a+bx)\right)}{2b\sqrt{\sin^2(a+bx)}\sec^{\frac{2}{3}}(a+bx)}$$

```
[Out] (-3*Hypergeometric2F1[1/3, 1/2, 4/3, Cos[a + b*x]^2]*Sin[a + b*x])/(2*b*Sec
[a + b*x]^(2/3)*Sqrt[Sin[a + b*x]^2])
```

Rubi [A] time = 0.0332359, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {3772, 2643}

$$-\frac{3\sin(a+bx)\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};\cos^2(a+bx)\right)}{2b\sqrt{\sin^2(a+bx)}\sec^{\frac{2}{3}}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[a + b*x]^(1/3),x]
```

```
[Out] (-3*Hypergeometric2F1[1/3, 1/2, 4/3, Cos[a + b*x]^2]*Sin[a + b*x])/(2*b*Sec
[a + b*x]^(2/3)*Sqrt[Sin[a + b*x]^2])
```

Rule 3772

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] && !IntegerQ[n]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}\int \sqrt[3]{\sec(a+bx)}\,dx &= \sqrt[3]{\cos(a+bx)}\sqrt[3]{\sec(a+bx)}\int \frac{1}{\sqrt[3]{\cos(a+bx)}}\,dx\\ &= -\frac{3\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};\cos^2(a+bx)\right)\sin(a+bx)}{2b\sec^{\frac{2}{3}}(a+bx)\sqrt{\sin^2(a+bx)}}\end{aligned}$$

Mathematica [A] time = 0.0336867, size = 53, normalized size = 1.

$$\frac{3\sqrt{-\tan^2(a+bx)}\csc(a+bx)\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{1}{2},\frac{7}{6},\sec^2(a+bx)\right)}{b\sec^{\frac{2}{3}}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[a + b*x]^(1/3),x]
```

```
[Out] (3*Csc[a + b*x]*Hypergeometric2F1[1/6, 1/2, 7/6, Sec[a + b*x]^2]*Sqrt[-Tan[
a + b*x]^2])/(b*Sec[a + b*x]^(2/3))
```

Maple [F] time = 0.1, size = 0, normalized size = 0.

$$\int \sqrt[3]{\sec \left(bx+a\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)^(1/3),x)
```

```
[Out] int(sec(b*x+a)^(1/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sec \left(bx+a\right)^{\frac{1}{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate(sec(b*x + a)^(1/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\sec \left(bx+a\right)^{\frac{1}{3}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^(1/3),x, algorithm="fricas")
```

```
[Out] integral(sec(b*x + a)^(1/3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt[3]{\sec \left(a+bx\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)**(1/3),x)
```

```
[Out] Integral(sec(a + b*x)**(1/3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sec (bx+a)^{\frac{1}{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^(1/3),x, algorithm="giac")
```

```
[Out] integrate(sec(b*x + a)^(1/3), x)
```

3.28 $\int \frac{1}{\sqrt[3]{\sec(a+bx)}}\,dx$

Optimal. Leaf size=53

$$-\frac{3\sin(a+bx)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{5}{3},\cos^2(a+bx)\right)}{4b\sqrt{\sin^2(a+bx)}\sec^{\frac{4}{3}}(a+bx)}$$

```
[Out] (-3*Hypergeometric2F1[1/2, 2/3, 5/3, Cos[a + b*x]^2]*Sin[a + b*x])/(4*b*Sec
[a + b*x]^(4/3)*Sqrt[Sin[a + b*x]^2])
```

Rubi [A] time = 0.0254906, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {3772, 2643}

$$-\frac{3\sin(a+bx)\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};\cos^2(a+bx)\right)}{4b\sqrt{\sin^2(a+bx)}\sec^{\frac{4}{3}}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[a + b*x]^(-1/3),x]
```

```
[Out] (-3*Hypergeometric2F1[1/2, 2/3, 5/3, Cos[a + b*x]^2]*Sin[a + b*x])/(4*b*Sec
[a + b*x]^(4/3)*Sqrt[Sin[a + b*x]^2])
```

Rule 3772

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] && !IntegerQ[n]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt[3]{\sec(a+bx)}}\,dx &= \cos^{\frac{2}{3}}(a+bx)\sec^{\frac{2}{3}}(a+bx)\int \sqrt[3]{\cos(a+bx)}\,dx\\ &= -\frac{3\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};\cos^2(a+bx)\right)\sin(a+bx)}{4b\sec^{\frac{4}{3}}(a+bx)\sqrt{\sin^2(a+bx)}}\end{aligned}$$

Mathematica [A] time = 0.0815557, size = 53, normalized size = 1.

$$-\frac{3\sqrt{-\tan^2(a+bx)}\csc(a+bx)\text{Hypergeometric2F1}\left(-\frac{1}{6},\frac{1}{2},\frac{5}{6},\sec^2(a+bx)\right)}{b\sec^{\frac{4}{3}}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[a + b*x]^(-1/3),x]
```

```
[Out] (-3*Csc[a + b*x]*Hypergeometric2F1[-1/6, 1/2, 5/6, Sec[a + b*x]^2]*Sqrt[-Ta
n[a + b*x]^2])/(b*Sec[a + b*x]^(4/3))
```

Maple [F] time = 0.093, size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt[3]{\sec(bx+a)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sec(b*x+a)^(1/3),x)
```

```
[Out] int(1/sec(b*x+a)^(1/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sec(bx+a)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(b*x+a)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate(sec(b*x + a)^(-1/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{1}{\sec(bx+a)^{\frac{1}{3}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(b*x+a)^(1/3),x, algorithm="fricas")
```

```
[Out] integral(sec(b*x + a)^(-1/3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt[3]{\sec(a+bx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(b*x+a)**(1/3),x)
```

```
[Out] Integral(sec(a + b*x)**(-1/3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sec (bx + a)^{\frac{1}{3}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(b*x+a)^(1/3),x, algorithm="giac")
```

```
[Out] integrate(sec(b*x + a)^(-1/3), x)
```

3.29 $\int \frac{1}{\sec^{\frac{2}{3}}(a+bx)}\,dx$

Optimal. Leaf size=53

$$-\frac{3\sin(a+bx)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{6},\frac{11}{6},\cos^2(a+bx)\right)}{5b\sqrt{\sin^2(a+bx)}\sec^{\frac{5}{3}}(a+bx)}$$

```
[Out] (-3*Hypergeometric2F1[1/2, 5/6, 11/6, Cos[a + b*x]^2]*Sin[a + b*x])/(5*b*Se
c[a + b*x]^(5/3)*Sqrt[Sin[a + b*x]^2])
```

Rubi [A] time = 0.0254632, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {3772, 2643}

$$-\frac{3\sin(a+bx)\,{}_2F_1\left(\frac{1}{2},\frac{5}{6};\frac{11}{6};\cos^2(a+bx)\right)}{5b\sqrt{\sin^2(a+bx)}\sec^{\frac{5}{3}}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[a + b*x]^(-2/3),x]
```

```
[Out] (-3*Hypergeometric2F1[1/2, 5/6, 11/6, Cos[a + b*x]^2]*Sin[a + b*x])/(5*b*Se
c[a + b*x]^(5/3)*Sqrt[Sin[a + b*x]^2])
```

Rule 3772

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] && !IntegerQ[n]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sec^{\frac{2}{3}}(a+bx)}\,dx &= \sqrt[3]{\cos(a+bx)}\sqrt[3]{\sec(a+bx)}\int \cos^{\frac{2}{3}}(a+bx)\,dx\\ &= -\frac{3\,{}_2F_1\left(\frac{1}{2},\frac{5}{6};\frac{11}{6};\cos^2(a+bx)\right)\sin(a+bx)}{5b\sec^{\frac{5}{3}}(a+bx)\sqrt{\sin^2(a+bx)}}\end{aligned}$$

Mathematica [A] time = 0.0829981, size = 55, normalized size = 1.04

$$-\frac{3\sqrt{-\tan^2(a+bx)}\csc(a+bx)\text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{1}{2},\frac{2}{3},\sec^2(a+bx)\right)}{2b\sec^{\frac{5}{3}}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[a + b*x]^(-2/3),x]
```

```
[Out] (-3*Csc[a + b*x]*Hypergeometric2F1[-1/3, 1/2, 2/3, Sec[a + b*x]^2]*Sqrt[-Ta
n[a + b*x]^2])/(2*b*Sec[a + b*x]^(5/3))
```

Maple [F] time = 0.093, size = 0, normalized size = 0.

$$\int (\sec (bx + a))^{-\frac{2}{3}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sec(b*x+a)^(2/3),x)
```

```
[Out] int(1/sec(b*x+a)^(2/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sec (bx + a)^{\frac{2}{3}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(b*x+a)^(2/3),x, algorithm="maxima")
```

```
[Out] integrate(sec(b*x + a)^(-2/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\sec (bx + a)^{\frac{2}{3}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(b*x+a)^(2/3),x, algorithm="fricas")
```

```
[Out] integral(sec(b*x + a)^(-2/3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sec^{\frac{2}{3}} (a + bx)} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(b*x+a)**(2/3),x)
```

```
[Out] Integral(sec(a + b*x)**(-2/3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sec (bx+a)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(b*x+a)^(2/3),x, algorithm="giac")
```

```
[Out] integrate(sec(b*x + a)^(-2/3), x)
```

3.30 $\int \frac{1}{\sec^{\frac{4}{3}}(a+bx)}\,dx$

Optimal. Leaf size=53

$$-\frac{3\sin(a+bx)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{7}{6},\frac{13}{6},\cos^2(a+bx)\right)}{7b\sqrt{\sin^2(a+bx)}\sec^{\frac{7}{3}}(a+bx)}$$

```
[Out] (-3*Hypergeometric2F1[1/2, 7/6, 13/6, Cos[a + b*x]^2]*Sin[a + b*x])/(7*b*Se
c[a + b*x]^(7/3)*Sqrt[Sin[a + b*x]^2])
```

Rubi [A] time = 0.0247945, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {3772, 2643}

$$-\frac{3\sin(a+bx)\,{}_2F_1\left(\frac{1}{2},\frac{7}{6};\frac{13}{6};\cos^2(a+bx)\right)}{7b\sqrt{\sin^2(a+bx)}\sec^{\frac{7}{3}}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[a + b*x]^(-4/3),x]
```

```
[Out] (-3*Hypergeometric2F1[1/2, 7/6, 13/6, Cos[a + b*x]^2]*Sin[a + b*x])/(7*b*Se
c[a + b*x]^(7/3)*Sqrt[Sin[a + b*x]^2])
```

Rule 3772

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] && !IntegerQ[n]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sec^{\frac{4}{3}}(a+bx)}\,dx &= \cos^{\frac{2}{3}}(a+bx)\sec^{\frac{2}{3}}(a+bx)\int \cos^{\frac{4}{3}}(a+bx)\,dx\\ &= -\frac{3\,{}_2F_1\left(\frac{1}{2},\frac{7}{6};\frac{13}{6};\cos^2(a+bx)\right)\sin(a+bx)}{7b\sec^{\frac{7}{3}}(a+bx)\sqrt{\sin^2(a+bx)}}\end{aligned}$$

Mathematica [A] time = 0.0656554, size = 55, normalized size = 1.04

$$-\frac{3\sqrt{-\tan^2(a+bx)}\csc(a+bx)\text{Hypergeometric2F1}\left(-\frac{2}{3},\frac{1}{2},\frac{1}{3},\sec^2(a+bx)\right)}{4b\sec^{\frac{7}{3}}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[a + b*x]^(-4/3),x]
```

```
[Out] (-3*Csc[a + b*x]*Hypergeometric2F1[-2/3, 1/2, 1/3, Sec[a + b*x]^2]*Sqrt[-Ta
n[a + b*x]^2])/(4*b*Sec[a + b*x]^(7/3))
```

Maple [F] time = 0.055, size = 0, normalized size = 0.

$$\int (\sec (bx + a))^{-\frac{4}{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sec(b*x+a)^(4/3),x)
```

```
[Out] int(1/sec(b*x+a)^(4/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sec (bx + a)^{\frac{4}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(b*x+a)^(4/3),x, algorithm="maxima")
```

```
[Out] integrate(sec(b*x + a)^(-4/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\sec (bx + a)^{\frac{4}{3}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(b*x+a)^(4/3),x, algorithm="fricas")
```

```
[Out] integral(sec(b*x + a)^(-4/3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sec^{\frac{4}{3}} (a + bx)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(b*x+a)**(4/3),x)
```

```
[Out] Integral(sec(a + b*x)**(-4/3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sec(bx+a)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(b*x+a)^(4/3),x, algorithm="giac")
```

```
[Out] integrate(sec(b*x + a)^(-4/3), x)
```

3.31 $\int (c\sec(a+bx))^{4/3}\,dx$

Optimal. Leaf size=54

$$\frac{3c\sin(a+bx)\sqrt[3]{c\sec(a+bx)}\text{Hypergeometric2F1}\left(-\frac{1}{6},\frac{1}{2},\frac{5}{6},\cos^2(a+bx)\right)}{b\sqrt{\sin^2(a+bx)}}$$

```
[Out] (3*c*Hypergeometric2F1[-1/6, 1/2, 5/6, Cos[a + b*x]^2]*(c*Sec[a + b*x])^(1/
3)*Sin[a + b*x])/(b*Sqrt[Sin[a + b*x]^2])
```

Rubi [A] time = 0.0335746, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3772, 2643}

$$\frac{3c\sin(a+bx)\sqrt[3]{c\sec(a+bx)}\,{}_2F_1\left(-\frac{1}{6},\frac{1}{2};\frac{5}{6};\cos^2(a+bx)\right)}{b\sqrt{\sin^2(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*Sec[a + b*x])^(4/3),x]
```

```
[Out] (3*c*Hypergeometric2F1[-1/6, 1/2, 5/6, Cos[a + b*x]^2]*(c*Sec[a + b*x])^(1/
3)*Sin[a + b*x])/(b*Sqrt[Sin[a + b*x]^2])
```

Rule 3772

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] && !IntegerQ[n]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}\int (c\sec(a+bx))^{4/3}\,dx &= \sqrt[3]{\frac{\cos(a+bx)}{c}}\sqrt[3]{c\sec(a+bx)}\int \frac{1}{\left(\frac{\cos(a+bx)}{c}\right)^{4/3}}\,dx\\ &= \frac{3c\,{}_2F_1\left(-\frac{1}{6},\frac{1}{2};\frac{5}{6};\cos^2(a+bx)\right)\sqrt[3]{c\sec(a+bx)}\sin(a+bx)}{b\sqrt{\sin^2(a+bx)}}\end{aligned}$$

Mathematica [A] time = 0.0494979, size = 57, normalized size = 1.06

$$\frac{3\sqrt{-\tan^2(a+bx)}\cot(a+bx)(c\sec(a+bx))^{4/3}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{5}{3},\sec^2(a+bx)\right)}{4b}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c*Sec[a + b*x])^(4/3),x]
```

```
[Out] (3*Cot[a + b*x]*Hypergeometric2F1[1/2, 2/3, 5/3, Sec[a + b*x]^2]*(c*Sec[a +
 b*x])^(4/3)*Sqrt[-Tan[a + b*x]^2])/(4*b)
```

Maple [F] time = 0.059, size = 0, normalized size = 0.

$$\int (c\sec(bx+a))^{\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*sec(b*x+a))^(4/3),x)
```

```
[Out] int((c*sec(b*x+a))^(4/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (c\sec(bx+a))^{\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sec(b*x+a))^(4/3),x, algorithm="maxima")
```

```
[Out] integrate((c*sec(b*x + a))^(4/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((c\sec(bx+a))^{\frac{1}{3}}c\sec(bx+a),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sec(b*x+a))^(4/3),x, algorithm="fricas")
```

```
[Out] integral((c*sec(b*x + a))^(1/3)*c*sec(b*x + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sec(b*x+a))**(4/3),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (c\sec(bx+a))^{\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sec(b*x+a))^(4/3),x, algorithm="giac")
```

```
[Out] integrate((c*sec(b*x + a))^(4/3), x)
```

3.32 $\int (c\sec(a+bx))^{2/3}\,dx$

Optimal. Leaf size=54

$$-\frac{3c\sin(a+bx)\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{1}{2},\frac{7}{6},\cos^2(a+bx)\right)}{b\sqrt{\sin^2(a+bx)}\sqrt[3]{c\sec(a+bx)}}$$

```
[Out] (-3*c*Hypergeometric2F1[1/6, 1/2, 7/6, Cos[a + b*x]^2]*Sin[a + b*x])/(b*(c*
Sec[a + b*x])^(1/3)*Sqrt[Sin[a + b*x]^2])
```

Rubi [A] time = 0.0295325, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3772, 2643}

$$-\frac{3c\sin(a+bx)\,{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{7}{6};\cos^2(a+bx)\right)}{b\sqrt{\sin^2(a+bx)}\sqrt[3]{c\sec(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*Sec[a + b*x])^(2/3),x]
```

```
[Out] (-3*c*Hypergeometric2F1[1/6, 1/2, 7/6, Cos[a + b*x]^2]*Sin[a + b*x])/(b*(c*
Sec[a + b*x])^(1/3)*Sqrt[Sin[a + b*x]^2])
```

Rule 3772

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] && !IntegerQ[n]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}\int (c\sec(a+bx))^{2/3}\,dx &= \left(\frac{\cos(a+bx)}{c}\right)^{2/3}(c\sec(a+bx))^{2/3}\int \frac{1}{\left(\frac{\cos(a+bx)}{c}\right)^{2/3}}\,dx\\ &= -\frac{3\cos(a+bx)\,{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{7}{6};\cos^2(a+bx)\right)(c\sec(a+bx))^{2/3}\sin(a+bx)}{b\sqrt{\sin^2(a+bx)}}\end{aligned}$$

Mathematica [A] time = 0.0410417, size = 57, normalized size = 1.06

$$\frac{3\sqrt{-\tan^2(a+bx)}\cot(a+bx)(c\sec(a+bx))^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},\sec^2(a+bx)\right)}{2b}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c*Sec[a + b*x])^(2/3),x]
```

```
[Out] (3*Cot[a + b*x]*Hypergeometric2F1[1/3, 1/2, 4/3, Sec[a + b*x]^2]*(c*Sec[a +
 b*x])^(2/3)*Sqrt[-Tan[a + b*x]^2])/(2*b)
```

Maple [F] time = 0.084, size = 0, normalized size = 0.

$$\int \left(c \sec \left(bx+a\right)\right)^{\frac{2}{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*sec(b*x+a))^(2/3),x)
```

```
[Out] int((c*sec(b*x+a))^(2/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(c \sec \left(bx+a\right)\right)^{\frac{2}{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sec(b*x+a))^(2/3),x, algorithm="maxima")
```

```
[Out] integrate((c*sec(b*x + a))^(2/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(c \sec \left(bx+a\right)\right)^{\frac{2}{3}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sec(b*x+a))^(2/3),x, algorithm="fricas")
```

```
[Out] integral((c*sec(b*x + a))^(2/3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(c \sec \left(a+bx\right)\right)^{\frac{2}{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sec(b*x+a))**(2/3),x)
```

```
[Out] Integral((c*sec(a + b*x))**(2/3), x)
```

<u>Giac</u> [F] time = 0., size = 0, normalized size = 0.

$$\int (c\sec(bx+a))^{\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sec(b*x+a))^(2/3),x, algorithm="giac")
```

```
[Out] integrate((c*sec(b*x + a))^(2/3), x)
```

3.33 $\int \sqrt[3]{c \sec(a+bx)}\,dx$

Optimal. Leaf size=56

$$-\frac{3c\sin(a+bx)\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},\cos^2(a+bx)\right)}{2b\sqrt{\sin^2(a+bx)}(c\sec(a+bx))^{2/3}}$$

```
[Out] (-3*c*Hypergeometric2F1[1/3, 1/2, 4/3, Cos[a + b*x]^2]*Sin[a + b*x])/(2*b*(
c*Sec[a + b*x])^(2/3)*Sqrt[Sin[a + b*x]^2])
```

Rubi [A] time = 0.0349386, antiderivative size = 56, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3772, 2643}

$$-\frac{3c\sin(a+bx)\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};\cos^2(a+bx)\right)}{2b\sqrt{\sin^2(a+bx)}(c\sec(a+bx))^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*Sec[a + b*x])^(1/3),x]
```

```
[Out] (-3*c*Hypergeometric2F1[1/3, 1/2, 4/3, Cos[a + b*x]^2]*Sin[a + b*x])/(2*b*(
c*Sec[a + b*x])^(2/3)*Sqrt[Sin[a + b*x]^2])
```

Rule 3772

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] && !IntegerQ[n]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}\int \sqrt[3]{c\sec(a+bx)}\,dx &= \sqrt[3]{\frac{\cos(a+bx)}{c}}\sqrt[3]{c\sec(a+bx)}\int \frac{1}{\sqrt[3]{\frac{\cos(a+bx)}{c}}}\,dx\\ &= -\frac{3\cos(a+bx)\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};\cos^2(a+bx)\right)\sqrt[3]{c\sec(a+bx)}\sin(a+bx)}{2b\sqrt{\sin^2(a+bx)}}\end{aligned}$$

Mathematica [A] time = 0.0395714, size = 55, normalized size = 0.98

$$\frac{3\sqrt{-\tan^2(a+bx)}\cot(a+bx)\sqrt[3]{c\sec(a+bx)}\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{1}{2},\frac{7}{6},\sec^2(a+bx)\right)}{b}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c*Sec[a + b*x])^(1/3),x]
```

```
[Out] (3*Cot[a + b*x]*Hypergeometric2F1[1/6, 1/2, 7/6, Sec[a + b*x]^2]*(c*Sec[a +
 b*x])^(1/3)*Sqrt[-Tan[a + b*x]^2])/b
```

Maple [F] time = 0.105, size = 0, normalized size = 0.

$$\int \sqrt[3]{c \sec (bx + a)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*sec(b*x+a))^(1/3),x)
```

```
[Out] int((c*sec(b*x+a))^(1/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (c \sec (bx + a))^{\frac{1}{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sec(b*x+a))^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((c*sec(b*x + a))^(1/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((c \sec (bx + a))^{\frac{1}{3}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sec(b*x+a))^(1/3),x, algorithm="fricas")
```

```
[Out] integral((c*sec(b*x + a))^(1/3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt[3]{c \sec (a + bx)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sec(b*x+a))**(1/3),x)
```

```
[Out] Integral((c*sec(a + b*x))**(1/3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (c \sec (bx + a))^{\frac{1}{3}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sec(b*x+a))^(1/3),x, algorithm="giac")
```

```
[Out] integrate((c*sec(b*x + a))^(1/3), x)
```

3.34 $\int \frac{1}{\sqrt[3]{c\sec(a+bx)}}\,dx$

Optimal. Leaf size=56

$$-\frac{3c\sin(a+bx)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{5}{3},\cos^2(a+bx)\right)}{4b\sqrt{\sin^2(a+bx)}(c\sec(a+bx))^{4/3}}$$

```
[Out] (-3*c*Hypergeometric2F1[1/2, 2/3, 5/3, Cos[a + b*x]^2]*Sin[a + b*x])/(4*b*(
c*Sec[a + b*x])^(4/3)*Sqrt[Sin[a + b*x]^2])
```

Rubi [A] time = 0.0324897, antiderivative size = 56, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3772, 2643}

$$-\frac{3c\sin(a+bx)\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};\cos^2(a+bx)\right)}{4b\sqrt{\sin^2(a+bx)}(c\sec(a+bx))^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*Sec[a + b*x])^(-1/3),x]
```

```
[Out] (-3*c*Hypergeometric2F1[1/2, 2/3, 5/3, Cos[a + b*x]^2]*Sin[a + b*x])/(4*b*(
c*Sec[a + b*x])^(4/3)*Sqrt[Sin[a + b*x]^2])
```

Rule 3772

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] && !IntegerQ[n]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt[3]{c\sec(a+bx)}}\,dx &= \left(\frac{\cos(a+bx)}{c}\right)^{2/3}(c\sec(a+bx))^{2/3}\int \sqrt[3]{\frac{\cos(a+bx)}{c}}\,dx\\ &= -\frac{3\cos^2(a+bx)\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};\cos^2(a+bx)\right)(c\sec(a+bx))^{2/3}\sin(a+bx)}{4bc\sqrt{\sin^2(a+bx)}}\end{aligned}$$

Mathematica [A] time = 0.0521933, size = 55, normalized size = 0.98

$$-\frac{3\sqrt{-\tan^2(a+bx)}\cot(a+bx)\text{Hypergeometric2F1}\left(-\frac{1}{6},\frac{1}{2},\frac{5}{6},\sec^2(a+bx)\right)}{b\sqrt[3]{c\sec(a+bx)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c*Sec[a + b*x])^(-1/3),x]
```

```
[Out] (-3*Cot[a + b*x]*Hypergeometric2F1[-1/6, 1/2, 5/6, Sec[a + b*x]^2]*Sqrt[-Ta
n[a + b*x]^2])/(b*(c*Sec[a + b*x])^(1/3))
```

Maple [F] time = 0.094, size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt[3]{c\sec\left(bx+a\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*sec(b*x+a))^(1/3),x)
```

```
[Out] int(1/(c*sec(b*x+a))^(1/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(c\sec\left(bx+a\right)\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*sec(b*x+a))^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((c*sec(b*x + a))^(-1/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(c\sec\left(bx+a\right)\right)^{\frac{2}{3}}}{c\sec\left(bx+a\right)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*sec(b*x+a))^(1/3),x, algorithm="fricas")
```

```
[Out] integral((c*sec(b*x + a))^(2/3)/(c*sec(b*x + a)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt[3]{c\sec\left(a+bx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*sec(b*x+a))**(1/3),x)
```

```
[Out] Integral((c*sec(a + b*x))**(-1/3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(c\sec(bx+a))^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*sec(b*x+a))^(1/3),x, algorithm="giac")
```

```
[Out] integrate((c*sec(b*x + a))^(-1/3), x)
```

3.35 $\int \frac{1}{(c\sec(a+bx))^{2/3}}\,dx$

Optimal. Leaf size=56

$$-\frac{3c\sin(a+bx)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{6},\frac{11}{6},\cos^2(a+bx)\right)}{5b\sqrt{\sin^2(a+bx)}(c\sec(a+bx))^{5/3}}$$

```
[Out] (-3*c*Hypergeometric2F1[1/2, 5/6, 11/6, Cos[a + b*x]^2]*Sin[a + b*x])/(5*b*
(c*Sec[a + b*x])^(5/3)*Sqrt[Sin[a + b*x]^2])
```

Rubi [A] time = 0.0282795, antiderivative size = 56, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3772, 2643}

$$-\frac{3c\sin(a+bx)\,{}_2F_1\left(\frac{1}{2},\frac{5}{6};\frac{11}{6};\cos^2(a+bx)\right)}{5b\sqrt{\sin^2(a+bx)}(c\sec(a+bx))^{5/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*Sec[a + b*x])^(-2/3),x]
```

```
[Out] (-3*c*Hypergeometric2F1[1/2, 5/6, 11/6, Cos[a + b*x]^2]*Sin[a + b*x])/(5*b*
(c*Sec[a + b*x])^(5/3)*Sqrt[Sin[a + b*x]^2])
```

Rule 3772

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] && !IntegerQ[n]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(c\sec(a+bx))^{2/3}}\,dx &= \sqrt[3]{\frac{\cos(a+bx)}{c}}\sqrt[3]{c\sec(a+bx)}\int \left(\frac{\cos(a+bx)}{c}\right)^{2/3}\,dx\\ &= -\frac{3\cos^2(a+bx)\,{}_2F_1\left(\frac{1}{2},\frac{5}{6};\frac{11}{6};\cos^2(a+bx)\right)\sqrt[3]{c\sec(a+bx)}\sin(a+bx)}{5bc\sqrt{\sin^2(a+bx)}}\end{aligned}$$

Mathematica [A] time = 0.0526139, size = 57, normalized size = 1.02

$$-\frac{3\sqrt{-\tan^2(a+bx)}\cot(a+bx)\text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{1}{2},\frac{2}{3},\sec^2(a+bx)\right)}{2b(c\sec(a+bx))^{2/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c*Sec[a + b*x])^(-2/3),x]
```

```
[Out] (-3*Cot[a + b*x]*Hypergeometric2F1[-1/3, 1/2, 2/3, Sec[a + b*x]^2]*Sqrt[-Ta
n[a + b*x]^2])/(2*b*(c*Sec[a + b*x])^(2/3))
```

Maple [F] time = 0.097, size = 0, normalized size = 0.

$$\int (c\sec(bx+a))^{-\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*sec(b*x+a))^(2/3),x)
```

```
[Out] int(1/(c*sec(b*x+a))^(2/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(c\sec(bx+a))^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*sec(b*x+a))^(2/3),x, algorithm="maxima")
```

```
[Out] integrate((c*sec(b*x + a))^(-2/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{(c\sec(bx+a))^{\frac{1}{3}}}{c\sec(bx+a)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*sec(b*x+a))^(2/3),x, algorithm="fricas")
```

```
[Out] integral((c*sec(b*x + a))^(1/3)/(c*sec(b*x + a)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(c\sec(a+bx))^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*sec(b*x+a))**(2/3),x)
```

```
[Out] Integral((c*sec(a + b*x))**(-2/3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(c\sec(bx+a))^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*sec(b*x+a))^(2/3),x, algorithm="giac")
```

```
[Out] integrate((c*sec(b*x + a))^(-2/3), x)
```

3.36 $\int \frac{1}{(c\sec(a+bx))^{4/3}}\,dx$

Optimal. Leaf size=56

$$-\frac{3c\sin(a+bx)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{7}{6},\frac{13}{6},\cos^2(a+bx)\right)}{7b\sqrt{\sin^2(a+bx)}(c\sec(a+bx))^{7/3}}$$

```
[Out] (-3*c*Hypergeometric2F1[1/2, 7/6, 13/6, Cos[a + b*x]^2]*Sin[a + b*x])/(7*b*
(c*Sec[a + b*x])^(7/3)*Sqrt[Sin[a + b*x]^2])
```

Rubi [A] time = 0.0286994, antiderivative size = 56, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3772, 2643}

$$-\frac{3c\sin(a+bx)\,{}_2F_1\left(\frac{1}{2},\frac{7}{6};\frac{13}{6};\cos^2(a+bx)\right)}{7b\sqrt{\sin^2(a+bx)}(c\sec(a+bx))^{7/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*Sec[a + b*x])^(-4/3),x]
```

```
[Out] (-3*c*Hypergeometric2F1[1/2, 7/6, 13/6, Cos[a + b*x]^2]*Sin[a + b*x])/(7*b*
(c*Sec[a + b*x])^(7/3)*Sqrt[Sin[a + b*x]^2])
```

Rule 3772

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(c\sec(a+bx))^{4/3}}\,dx &= \left(\frac{\cos(a+bx)}{c}\right)^{2/3}(c\sec(a+bx))^{2/3}\int \left(\frac{\cos(a+bx)}{c}\right)^{4/3}dx\\ &= -\frac{3\cos^3(a+bx)\,{}_2F_1\left(\frac{1}{2},\frac{7}{6};\frac{13}{6};\cos^2(a+bx)\right)(c\sec(a+bx))^{2/3}\sin(a+bx)}{7bc^2\sqrt{\sin^2(a+bx)}}\end{aligned}$$

Mathematica [A] time = 0.0688611, size = 57, normalized size = 1.02

$$-\frac{3\sqrt{-\tan^2(a+bx)}\cot(a+bx)\text{Hypergeometric2F1}\left(-\frac{2}{3},\frac{1}{2},\frac{1}{3},\sec^2(a+bx)\right)}{4b(c\sec(a+bx))^{4/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c*Sec[a + b*x])^(-4/3),x]
```

```
[Out] (-3*Cot[a + b*x]*Hypergeometric2F1[-2/3, 1/2, 1/3, Sec[a + b*x]^2]*Sqrt[-Ta
n[a + b*x]^2])/(4*b*(c*Sec[a + b*x])^(4/3))
```

Maple [F] time = 0.054, size = 0, normalized size = 0.

$$\int (c\sec(bx+a))^{-\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*sec(b*x+a))^(4/3),x)
```

```
[Out] int(1/(c*sec(b*x+a))^(4/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(c\sec(bx+a))^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*sec(b*x+a))^(4/3),x, algorithm="maxima")
```

```
[Out] integrate((c*sec(b*x + a))^(-4/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(c\sec(bx+a))^{\frac{2}{3}}}{c^2\sec(bx+a)^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*sec(b*x+a))^(4/3),x, algorithm="fricas")
```

```
[Out] integral((c*sec(b*x + a))^(2/3)/(c^2*sec(b*x + a)^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(c\sec(a+bx))^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*sec(b*x+a))**(4/3),x)
```

```
[Out] Integral((c*sec(a + b*x))**(-4/3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(c\sec(bx+a))^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*sec(b*x+a))^(4/3),x, algorithm="giac")
```

```
[Out] integrate((c*sec(b*x + a))^(-4/3), x)
```

3.37 $\int \sec^n(a+bx)\,dx$

Optimal. Leaf size=70

$$-\frac{\sin(a+bx)\sec^{n-1}(a+bx)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1-n}{2},\frac{3-n}{2},\cos^2(a+bx)\right)}{b(1-n)\sqrt{\sin^2(a+bx)}}$$

```
[Out] -((Hypergeometric2F1[1/2, (1 - n)/2, (3 - n)/2, Cos[a + b*x]^2]*Sec[a + b*x
]^(-1 + n)*Sin[a + b*x])/(b*(1 - n)*Sqrt[Sin[a + b*x]^2]))
```

Rubi [A] time = 0.0328775, antiderivative size = 70, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {3772, 2643}

$$-\frac{\sin(a+bx)\sec^{n-1}(a+bx)\,{}_2F_1\left(\frac{1}{2},\frac{1-n}{2};\frac{3-n}{2};\cos^2(a+bx)\right)}{b(1-n)\sqrt{\sin^2(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[a + b*x]^n,x]
```

```
[Out] -((Hypergeometric2F1[1/2, (1 - n)/2, (3 - n)/2, Cos[a + b*x]^2]*Sec[a + b*x
]^(-1 + n)*Sin[a + b*x])/(b*(1 - n)*Sqrt[Sin[a + b*x]^2]))
```

Rule 3772

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] && !IntegerQ[n]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}\int \sec^n(a+bx)\,dx &= \cos^n(a+bx)\sec^n(a+bx)\int \cos^{-n}(a+bx)\,dx\\ &= -\frac{{}_2F_1\left(\frac{1}{2},\frac{1-n}{2};\frac{3-n}{2};\cos^2(a+bx)\right)\sec^{-1+n}(a+bx)\sin(a+bx)}{b(1-n)\sqrt{\sin^2(a+bx)}}\end{aligned}$$

Mathematica [A] time = 0.0495195, size = 61, normalized size = 0.87

$$\frac{\sqrt{-\tan^2(a+bx)}\csc(a+bx)\sec^{n-1}(a+bx)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{n}{2},\frac{n+2}{2},\sec^2(a+bx)\right)}{bn}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[a + b*x]^n,x]
```

```
[Out] (Csc[a + b*x]*Hypergeometric2F1[1/2, n/2, (2 + n)/2, Sec[a + b*x]^2]*Sec[a
+ b*x]^(-1 + n)*Sqrt[-Tan[a + b*x]^2])/(b*n)
```

Maple [F] time = 0.291, size = 0, normalized size = 0.

$$\int (\sec (bx + a))^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)^n,x)
```

```
[Out] int(sec(b*x+a)^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sec (bx + a)^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^n,x, algorithm="maxima")
```

```
[Out] integrate(sec(b*x + a)^n, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sec (bx + a)^n, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^n,x, algorithm="fricas")
```

```
[Out] integral(sec(b*x + a)^n, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sec^n (a + bx) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)**n,x)
```

```
[Out] Integral(sec(a + b*x)**n, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sec(bx+a)^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^n,x, algorithm="giac")
```

```
[Out] integrate(sec(b*x + a)^n, x)
```

3.38 $\int (c\sec(a+bx))^n\,dx$

Optimal. Leaf size=73

$$-\frac{c\sin(a+bx)(c\sec(a+bx))^{n-1}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1-n}{2},\frac{3-n}{2},\cos^2(a+bx)\right)}{b(1-n)\sqrt{\sin^2(a+bx)}}$$

```
[Out] -((c*Hypergeometric2F1[1/2, (1 - n)/2, (3 - n)/2, Cos[a + b*x]^2]*(c*Sec[a
+ b*x])^(-1 + n)*Sin[a + b*x])/(b*(1 - n)*Sqrt[Sin[a + b*x]^2]))
```

Rubi [A] time = 0.0311051, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {3772, 2643}

$$-\frac{c\sin(a+bx)(c\sec(a+bx))^{n-1}\,{}_2F_1\left(\frac{1}{2},\frac{1-n}{2};\frac{3-n}{2};\cos^2(a+bx)\right)}{b(1-n)\sqrt{\sin^2(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*Sec[a + b*x])^n,x]
```

```
[Out] -((c*Hypergeometric2F1[1/2, (1 - n)/2, (3 - n)/2, Cos[a + b*x]^2]*(c*Sec[a
+ b*x])^(-1 + n)*Sin[a + b*x])/(b*(1 - n)*Sqrt[Sin[a + b*x]^2]))
```

Rule 3772

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}\int (c\sec(a+bx))^n\,dx &= \left(\frac{\cos(a+bx)}{c}\right)^n (c\sec(a+bx))^n \int \left(\frac{\cos(a+bx)}{c}\right)^{-n} dx\\ &= -\frac{\cos(a+bx)\,{}_2F_1\left(\frac{1}{2},\frac{1-n}{2};\frac{3-n}{2};\cos^2(a+bx)\right)(c\sec(a+bx))^n\sin(a+bx)}{b(1-n)\sqrt{\sin^2(a+bx)}}\end{aligned}$$

Mathematica [A] time = 0.0469835, size = 61, normalized size = 0.84

$$\frac{\sqrt{-\tan^2(a+bx)}\cot(a+bx)(c\sec(a+bx))^n\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{n}{2},\frac{n+2}{2},\sec^2(a+bx)\right)}{bn}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c*Sec[a + b*x])^n,x]
```

```
[Out] (Cot[a + b*x]*Hypergeometric2F1[1/2, n/2, (2 + n)/2, Sec[a + b*x]^2]*(c*Sec
[a + b*x])^n*Sqrt[-Tan[a + b*x]^2])/(b*n)
```

Maple [F] time = 0.265, size = 0, normalized size = 0.

$$\int (c \sec (bx + a))^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*sec(b*x+a))^n,x)
```

```
[Out] int((c*sec(b*x+a))^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (c \sec (bx + a))^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sec(b*x+a))^n,x, algorithm="maxima")
```

```
[Out] integrate((c*sec(b*x + a))^n, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((c \sec (bx + a))^n, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sec(b*x+a))^n,x, algorithm="fricas")
```

```
[Out] integral((c*sec(b*x + a))^n, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (c \sec (a + bx))^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sec(b*x+a))**n,x)
```

```
[Out] Integral((c*sec(a + b*x))**n, x)
```

<u>Giac</u> [F] time = 0., size = 0, normalized size = 0.

$$\int (c\sec(bx+a))^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sec(b*x+a))^n,x, algorithm="giac")
```

```
[Out] integrate((c*sec(b*x + a))^n, x)
```

3.39 $\int \sec^2(x)^{7/2} \, dx$

Optimal. Leaf size=50

$$\frac{1}{6}\tan(x)\sec^2(x)^{5/2} + \frac{5}{24}\tan(x)\sec^2(x)^{3/2} + \frac{5}{16}\tan(x)\sqrt{\sec^2(x)} + \frac{5}{16}\sinh^{-1}(\tan(x))$$

```
[Out] (5*ArcSinh[Tan[x]])/16 + (5*Sqrt[Sec[x]^2]*Tan[x])/16 + (5*(Sec[x]^2)^(3/2)
*Tan[x])/24 + ((Sec[x]^2)^(5/2)*Tan[x])/6
```

Rubi [A] time = 0.016905, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {4122, 195, 215}

$$\frac{1}{6}\tan(x)\sec^2(x)^{5/2} + \frac{5}{24}\tan(x)\sec^2(x)^{3/2} + \frac{5}{16}\tan(x)\sqrt{\sec^2(x)} + \frac{5}{16}\sinh^{-1}(\tan(x))$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[x]^2)^(7/2),x]
```

```
[Out] (5*ArcSinh[Tan[x]])/16 + (5*Sqrt[Sec[x]^2]*Tan[x])/16 + (5*(Sec[x]^2)^(3/2)
*Tan[x])/24 + ((Sec[x]^2)^(5/2)*Tan[x])/6
```

Rule 4122

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFac
tors[Tan[e + f*x], x]}, Dist[(b*ff)/f, Subst[Int[(b + b*ff^2*x^2)^(p - 1),
x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{b, e, f, p}, x] &&  !IntegerQ[p]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}
\int \sec^2(x)^{7/2} \, dx &= \text{Subst}\left(\int \left(1+x^2\right)^{5/2} \, dx, x, \tan(x)\right)\\
&= \frac{1}{6}\sec^2(x)^{5/2}\tan(x) + \frac{5}{6}\text{Subst}\left(\int \left(1+x^2\right)^{3/2} \, dx, x, \tan(x)\right)\\
&= \frac{5}{24}\sec^2(x)^{3/2}\tan(x) + \frac{1}{6}\sec^2(x)^{5/2}\tan(x) + \frac{5}{8}\text{Subst}\left(\int \sqrt{1+x^2} \, dx, x, \tan(x)\right)\\
&= \frac{5}{16}\sqrt{\sec^2(x)}\tan(x) + \frac{5}{24}\sec^2(x)^{3/2}\tan(x) + \frac{1}{6}\sec^2(x)^{5/2}\tan(x) + \frac{5}{16}\text{Subst}\left(\int \frac{1}{\sqrt{1+x^2}} \, dx, x, \tan(x)\right)\\
&= \frac{5}{16}\sinh^{-1}(\tan(x)) + \frac{5}{16}\sqrt{\sec^2(x)}\tan(x) + \frac{5}{24}\sec^2(x)^{3/2}\tan(x) + \frac{1}{6}\sec^2(x)^{5/2}\tan(x)
\end{aligned}$$

Mathematica [A] time = 0.290101, size = 74, normalized size = 1.48

$$\frac{1}{96}\cos(x)\sqrt{\sec^2(x)}\left(\frac{1}{8}(198\sin(x)+85\sin(3x)+15\sin(5x))\sec^6(x)-30\log\left(\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right)+30\log\left(\sin\left(\frac{x}{2}\right)+\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[x]^2)^(7/2),x]
```

```
[Out] (Cos[x]*Sqrt[Sec[x]^2]*(-30*Log[Cos[x/2] - Sin[x/2]] + 30*Log[Cos[x/2] + Si
n[x/2]] + (Sec[x]^6*(198*Sin[x] + 85*Sin[3*x] + 15*Sin[5*x]))/8))/96
```

Maple [A] time = 0.159, size = 72, normalized size = 1.4

$$-\frac{\cos(x)}{48}\left(15\ln\left(-\frac{-1+\cos(x)+\sin(x)}{\sin(x)}\right)(\cos(x))^6-15\ln\left(-\frac{-1+\cos(x)-\sin(x)}{\sin(x)}\right)(\cos(x))^6-15(\cos(x))^4\sin(x\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sec(x)^2)^(7/2),x)
```

```
[Out] -1/48*(15*ln(-(-1+cos(x)+sin(x))/sin(x))*cos(x)^6-15*ln(-(-1+cos(x)-sin(x))
/sin(x))*cos(x)^6-15*cos(x)^4*sin(x)-10*cos(x)^2*sin(x)-8*sin(x))*cos(x)*(1
/cos(x)^2)^(7/2)
```

Maxima [A] time = 1.74061, size = 57, normalized size = 1.14

$$\frac{1}{6}\left(\tan(x)^2+1\right)^{\frac{5}{2}}\tan(x)+\frac{5}{24}\left(\tan(x)^2+1\right)^{\frac{3}{2}}\tan(x)+\frac{5}{16}\sqrt{\tan(x)^2+1}\tan(x)+\frac{5}{16}\operatorname{arsinh}(\tan(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sec(x)^2)^(7/2),x, algorithm="maxima")
```

```
[Out] 1/6*(tan(x)^2 + 1)^(5/2)*tan(x) + 5/24*(tan(x)^2 + 1)^(3/2)*tan(x) + 5/16*s
qrt(tan(x)^2 + 1)*tan(x) + 5/16*arcsinh(tan(x))
```

Fricas [A] time = 1.36736, size = 162, normalized size = 3.24

$$-\frac{15\cos(x)^6\log(\sin(x)+1)-15\cos(x)^6\log(-\sin(x)+1)+2\left(15\cos(x)^4+10\cos(x)^2+8\right)\sin(x)}{96\cos(x)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sec(x)^2)^(7/2),x, algorithm="fricas")
```

```
[Out] -1/96*(15*cos(x)^6*log(sin(x) + 1) - 15*cos(x)^6*log(-sin(x) + 1) + 2*(15*c
os(x)^4 + 10*cos(x)^2 + 8)*sin(x))/cos(x)^6
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sec(x)**2)**(7/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.28619, size = 80, normalized size = 1.6

$$\frac{5\,\log\left(\sin\left(x\right)+1\right)}{32\,\mathrm{sgn}\left(\cos\left(x\right)\right)}-\frac{5\,\log\left(-\sin\left(x\right)+1\right)}{32\,\mathrm{sgn}\left(\cos\left(x\right)\right)}-\frac{15\,\sin\left(x\right)^{5}-40\,\sin\left(x\right)^{3}+33\,\sin\left(x\right)}{48\left(\sin\left(x\right)^{2}-1\right)^{3}\mathrm{sgn}\left(\cos\left(x\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sec(x)^2)^(7/2),x, algorithm="giac")
```

```
[Out] 5/32*log(sin(x) + 1)/sgn(cos(x)) - 5/32*log(-sin(x) + 1)/sgn(cos(x)) - 1/48
*(15*sin(x)^5 - 40*sin(x)^3 + 33*sin(x))/((sin(x)^2 - 1)^3*sgn(cos(x)))
```

3.40 $\int \sec^2(x)^{5/2}\,dx$

Optimal. Leaf size=36

$$\frac{1}{4}\tan(x)\sec^2(x)^{3/2}+\frac{3}{8}\tan(x)\sqrt{\sec^2(x)}+\frac{3}{8}\sinh^{-1}(\tan(x))$$

```
[Out] (3*ArcSinh[Tan[x]])/8 + (3*Sqrt[Sec[x]^2]*Tan[x])/8 + ((Sec[x]^2)^(3/2)*Tan
[x])/4
```

Rubi [A] time = 0.0124014, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {4122, 195, 215}

$$\frac{1}{4}\tan(x)\sec^2(x)^{3/2}+\frac{3}{8}\tan(x)\sqrt{\sec^2(x)}+\frac{3}{8}\sinh^{-1}(\tan(x))$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[x]^2)^(5/2),x]
```

```
[Out] (3*ArcSinh[Tan[x]])/8 + (3*Sqrt[Sec[x]^2]*Tan[x])/8 + ((Sec[x]^2)^(3/2)*Tan
[x])/4
```

Rule 4122

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFac
tors[Tan[e + f*x], x]}, Dist[(b*ff)/f, Subst[Int[(b + b*ff^2*x^2)^(p - 1),
x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{b, e, f, p}, x] && !IntegerQ[p]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}
\int \sec^2(x)^{5/2}\,dx &= \text{Subst}\left(\int \left(1+x^2\right)^{3/2}\,dx,x,\tan(x)\right)\\
&= \frac{1}{4}\sec^2(x)^{3/2}\tan(x)+\frac{3}{4}\text{Subst}\left(\int \sqrt{1+x^2}\,dx,x,\tan(x)\right)\\
&= \frac{3}{8}\sqrt{\sec^2(x)}\tan(x)+\frac{1}{4}\sec^2(x)^{3/2}\tan(x)+\frac{3}{8}\text{Subst}\left(\int \frac{1}{\sqrt{1+x^2}}\,dx,x,\tan(x)\right)\\
&= \frac{3}{8}\sinh^{-1}(\tan(x))+\frac{3}{8}\sqrt{\sec^2(x)}\tan(x)+\frac{1}{4}\sec^2(x)^{3/2}\tan(x)
\end{aligned}$$

Mathematica [A] time = 0.1402, size = 68, normalized size = 1.89

$$\frac{1}{16}\cos(x)\sqrt{\sec^2(x)}\left(\frac{1}{2}(11\sin(x)+3\sin(3x))\sec^4(x)-6\log\left(\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right)+6\log\left(\sin\left(\frac{x}{2}\right)+\cos\left(\frac{x}{2}\right)\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[x]^2)^(5/2),x]
```

```
[Out] (Cos[x]*Sqrt[Sec[x]^2]*(-6*Log[Cos[x/2] - Sin[x/2]] + 6*Log[Cos[x/2] + Sin[
x/2]] + (Sec[x]^4*(11*Sin[x] + 3*Sin[3*x]))/2))/16
```

Maple [B] time = 0.091, size = 64, normalized size = 1.8

$$\frac{\cos(x)}{8}\left(3\,(\cos(x))^4\ln\left(-\frac{-1+\cos(x)-\sin(x)}{\sin(x)}\right)-3\,(\cos(x))^4\ln\left(-\frac{-1+\cos(x)+\sin(x)}{\sin(x)}\right)+3\,(\cos(x))^2\sin(x)+2\sin$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sec(x)^2)^(5/2),x)
```

```
[Out] 1/8*(3*cos(x)^4*ln(-(-1+cos(x)-sin(x))/sin(x))-3*cos(x)^4*ln(-(-1+cos(x)+si
n(x))/sin(x))+3*cos(x)^2*sin(x)+2*sin(x))*cos(x)*(1/cos(x)^2)^(5/2)
```

Maxima [A] time = 1.64798, size = 41, normalized size = 1.14

$$\frac{1}{4}\left(\tan(x)^2+1\right)^{\frac{3}{2}}\tan(x)+\frac{3}{8}\sqrt{\tan(x)^2+1}\tan(x)+\frac{3}{8}\operatorname{arsinh}(\tan(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sec(x)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/4*(tan(x)^2 + 1)^(3/2)*tan(x) + 3/8*sqrt(tan(x)^2 + 1)*tan(x) + 3/8*arcsi
nh(tan(x))
```

Fricas [A] time = 1.40946, size = 139, normalized size = 3.86

$$-\frac{3\cos(x)^4\log(\sin(x)+1)-3\cos(x)^4\log(-\sin(x)+1)+2\left(3\cos(x)^2+2\right)\sin(x)}{16\cos(x)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sec(x)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/16*(3*cos(x)^4*log(sin(x) + 1) - 3*cos(x)^4*log(-sin(x) + 1) + 2*(3*cos(
x)^2 + 2)*sin(x))/cos(x)^4
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sec(x)**2)**(5/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.27814, size = 72, normalized size = 2.

$$\frac{3 \log(\sin(x)+1)}{16 \operatorname{sgn}(\cos(x))} - \frac{3 \log(-\sin(x)+1)}{16 \operatorname{sgn}(\cos(x))} - \frac{3 \sin(x)^3 - 5 \sin(x)}{8 \left(\sin(x)^2 - 1\right)^2 \operatorname{sgn}(\cos(x))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sec(x)^2)^(5/2),x, algorithm="giac")
```

```
[Out] 3/16*log(sin(x) + 1)/sgn(cos(x)) - 3/16*log(-sin(x) + 1)/sgn(cos(x)) - 1/8*
(3*sin(x)^3 - 5*sin(x))/((sin(x)^2 - 1)^2*sgn(cos(x)))
```

3.41 $\int \sec^2(x)^{3/2}\,dx$

Optimal. Leaf size=22

$$\frac{1}{2}\tan(x)\sqrt{\sec^2(x)}+\frac{1}{2}\sinh^{-1}(\tan(x))$$

```
[Out] ArcSinh[Tan[x]]/2 + (Sqrt[Sec[x]^2]*Tan[x])/2
```

Rubi [A] time = 0.0085539, antiderivative size = 22, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {4122, 195, 215}

$$\frac{1}{2}\tan(x)\sqrt{\sec^2(x)}+\frac{1}{2}\sinh^{-1}(\tan(x))$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[x]^2)^(3/2),x]

[Out] ArcSinh[Tan[x]]/2 + (Sqrt[Sec[x]^2]*Tan[x])/2
```

Rule 4122

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFac
tors[Tan[e + f*x], x]}, Dist[(b*ff)/f, Subst[Int[(b + b*ff^2*x^2)^(p - 1),
x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{b, e, f, p}, x] && !IntegerQ[p]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}\int \sec^2(x)^{3/2}\,dx &= \text{Subst}\left(\int \sqrt{1+x^2}\,dx, x, \tan(x)\right)\\ &= \frac{1}{2}\sqrt{\sec^2(x)}\tan(x)+\frac{1}{2}\text{Subst}\left(\int \frac{1}{\sqrt{1+x^2}}\,dx, x, \tan(x)\right)\\ &= \frac{1}{2}\sinh^{-1}(\tan(x))+\frac{1}{2}\sqrt{\sec^2(x)}\tan(x)\end{aligned}$$

Mathematica [B] time = 0.0565225, size = 52, normalized size = 2.36

$$\frac{1}{2}\cos(x)\sqrt{\sec^2(x)}\left(\tan(x)\sec(x)-\log\left(\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right)+\log\left(\sin\left(\frac{x}{2}\right)+\cos\left(\frac{x}{2}\right)\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[x]^2)^(3/2),x]
```

```
[Out] (Cos[x]*Sqrt[Sec[x]^2]*(-Log[Cos[x/2] - Sin[x/2]] + Log[Cos[x/2] + Sin[x/2]
] + Sec[x]*Tan[x]))/2
```

Maple [B] time = 0.065, size = 53, normalized size = 2.4

$$\frac{\cos(x)}{2}\left((\cos(x))^2\ln\left(-\frac{-1+\cos(x)-\sin(x)}{\sin(x)}\right)-(\cos(x))^2\ln\left(-\frac{-1+\cos(x)+\sin(x)}{\sin(x)}\right)+\sin(x)\right)\left((\cos(x))^{-2}\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sec(x)^2)^(3/2),x)
```

```
[Out] 1/2*(cos(x)^2*ln(-(-1+cos(x)-sin(x))/sin(x))-cos(x)^2*ln(-(-1+cos(x)+sin(x)
)/sin(x))+sin(x))*cos(x)*(1/cos(x)^2)^(3/2)
```

Maxima [A] time = 1.7188, size = 24, normalized size = 1.09

$$\frac{1}{2}\sqrt{\tan(x)^2+1}\tan(x)+\frac{1}{2}\,\mathrm{arsinh}(\tan(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sec(x)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(tan(x)^2 + 1)*tan(x) + 1/2*arcsinh(tan(x))
```

Fricas [B] time = 1.33071, size = 109, normalized size = 4.95

$$-\frac{\cos(x)^2\log(\sin(x)+1)-\cos(x)^2\log(-\sin(x)+1)+2\sin(x)}{4\cos(x)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sec(x)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/4*(cos(x)^2*log(sin(x) + 1) - cos(x)^2*log(-sin(x) + 1) + 2*sin(x))/cos(
x)^2
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\left(\sec^2(x)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sec(x)**2)**(3/2),x)
```

```
[Out] Integral((sec(x)**2)**(3/2), x)
```

Giac [B] time = 1.36668, size = 59, normalized size = 2.68

$$\frac{\log\left(\sin\left(x\right)+1\right)}{4\,\mathrm{sgn}\left(\cos\left(x\right)\right)}-\frac{\log\left(-\sin\left(x\right)+1\right)}{4\,\mathrm{sgn}\left(\cos\left(x\right)\right)}-\frac{\sin\left(x\right)}{2\left(\sin\left(x\right)^{2}-1\right)\mathrm{sgn}\left(\cos\left(x\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sec(x)^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/4*log(sin(x) + 1)/sgn(cos(x)) - 1/4*log(-sin(x) + 1)/sgn(cos(x)) - 1/2*si
n(x)/((sin(x)^2 - 1)*sgn(cos(x)))
```

3.42 $\int \sqrt{\sec^2(x)}\, dx$

Optimal. Leaf size=3

$$\sinh^{-1}(\tan(x))$$

```
[Out] ArcSinh[Tan[x]]
```

Rubi [A] time = 0.005895, antiderivative size = 3, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {4122, 215}

$$\sinh^{-1}(\tan(x))$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Sec[x]^2],x]
```

```
[Out] ArcSinh[Tan[x]]
```

Rule 4122

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFac
tors[Tan[e + f*x], x]}, Dist[(b*ff)/f, Subst[Int[(b + b*ff^2*x^2)^(p - 1),
x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{b, e, f, p}, x] &&  !IntegerQ[p]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}\int \sqrt{\sec^2(x)}\, dx &= \text{Subst}\left(\int \frac{1}{\sqrt{1+x^2}}\, dx, x, \tan(x)\right)\\ &= \sinh^{-1}(\tan(x))\end{aligned}$$

Mathematica [B] time = 0.0093418, size = 44, normalized size = 14.67

$$\cos(x)\sqrt{\sec^2(x)}\left(\log\left(\sin\left(\frac{x}{2}\right)+\cos\left(\frac{x}{2}\right)\right)-\log\left(\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Sec[x]^2],x]
```

```
[Out] Cos[x]*(-Log[Cos[x/2] - Sin[x/2]] + Log[Cos[x/2] + Sin[x/2]])*Sqrt[Sec[x]^2
]
```

Maple [B] time = 0.071, size = 21, normalized size = 7.

$$-2\cos(x)\operatorname{Artanh}\left(\frac{-1+\cos(x)}{\sin(x)}\right)\sqrt{(\cos(x))^{-2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sec(x)^2)^(1/2),x)
```

```
[Out] -2*cos(x)*arctanh((-1+cos(x))/sin(x))*(1/cos(x)^2)^(1/2)
```

Maxima [A] time = 1.73177, size = 4, normalized size = 1.33

$$\operatorname{arsinh}(\tan(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sec(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] arcsinh(tan(x))
```

Fricas [B] time = 1.42133, size = 61, normalized size = 20.33

$$-\frac{1}{2}\log(\sin(x)+1)+\frac{1}{2}\log(-\sin(x)+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sec(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/2*log(sin(x) + 1) + 1/2*log(-sin(x) + 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{\sec^2(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sec(x)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(sec(x)**2), x)
```

Giac [B] time = 1.33872, size = 47, normalized size = 15.67

$$\frac{\log\left(\left|\frac{1}{\sin(x)}+\sin(x)+2\right|\right)}{4\operatorname{sgn}(\cos(x))}-\frac{\log\left(\left|\frac{1}{\sin(x)}+\sin(x)-2\right|\right)}{4\operatorname{sgn}(\cos(x))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sec(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*log(abs(1/sin(x) + sin(x) + 2))/sgn(cos(x)) - 1/4*log(abs(1/sin(x) + si
n(x) - 2))/sgn(cos(x))
```

3.43 $\int \frac{1}{\sqrt{\sec^2(x)}} \, dx$

Optimal. Leaf size=11

$$\frac{\tan(x)}{\sqrt{\sec^2(x)}}$$

```
[Out] Tan[x]/Sqrt[Sec[x]^2]
```

Rubi [A] time = 0.0073548, antiderivative size = 11, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {4122, 191}

$$\frac{\tan(x)}{\sqrt{\sec^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[Sec[x]^2],x]
```

```
[Out] Tan[x]/Sqrt[Sec[x]^2]
```

Rule 4122

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFac
tors[Tan[e + f*x], x]}, Dist[(b*ff)/f, Subst[Int[(b + b*ff^2*x^2)^(p - 1),
x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{b, e, f, p}, x] && !IntegerQ[p]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$\begin{aligned} \int \frac{1}{\sqrt{\sec^2(x)}} \, dx &= \text{Subst}\left(\int \frac{1}{\left(1+x^2\right)^{3/2}} \, dx, x, \tan(x)\right) \\ &= \frac{\tan(x)}{\sqrt{\sec^2(x)}} \end{aligned}$$

Mathematica [A] time = 0.0058971, size = 11, normalized size = 1.

$$\frac{\tan(x)}{\sqrt{\sec^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[Sec[x]^2],x]
```

```
[Out] Tan[x]/Sqrt[Sec[x]^2]
```

Maple [A] time = 0.074, size = 14, normalized size = 1.3

$$\frac{\sin\left(x\right)}{\cos\left(x\right)}\frac{1}{\sqrt{\left(\cos\left(x\right)\right)^{-2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sec(x)^2)^(1/2),x)
```

```
[Out] sin(x)/(1/cos(x)^2)^(1/2)/cos(x)
```

Maxima [A] time = 1.18009, size = 15, normalized size = 1.36

$$\frac{\tan\left(x\right)}{\sqrt{\tan\left(x\right)^{2}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sec(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] tan(x)/sqrt(tan(x)^2 + 1)
```

Fricas [A] time = 1.2399, size = 12, normalized size = 1.09

$$-\sin\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sec(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -sin(x)
```

Sympy [A] time = 0.482606, size = 10, normalized size = 0.91

$$\frac{\tan\left(x\right)}{\sqrt{\sec^{2}\left(x\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sec(x)**2)**(1/2),x)
```

```
[Out] tan(x)/sqrt(sec(x)**2)
```

Giac [A] time = 1.27162, size = 8, normalized size = 0.73

$$\operatorname{sgn}\left(\cos\left(x\right)\right)\sin\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sec(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] sgn(cos(x))*sin(x)
```

3.44 $\int \frac{1}{\sec^2(x)^{3/2}}\,dx$

Optimal. Leaf size=29

$$\frac{2\tan(x)}{3\sqrt{\sec^2(x)}}+\frac{\tan(x)}{3\sec^2(x)^{3/2}}$$

```
[Out] Tan[x]/(3*(Sec[x]^2)^(3/2)) + (2*Tan[x])/(3*Sqrt[Sec[x]^2])
```

Rubi [A] time = 0.0105769, antiderivative size = 29, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {4122, 192, 191}

$$\frac{2\tan(x)}{3\sqrt{\sec^2(x)}}+\frac{\tan(x)}{3\sec^2(x)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[x]^2)^(-3/2),x]

[Out] Tan[x]/(3*(Sec[x]^2)^(3/2)) + (2*Tan[x])/(3*Sqrt[Sec[x]^2])
```

Rule 4122

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFac
tors[Tan[e + f*x], x]}, Dist[(b*ff)/f, Subst[Int[(b + b*ff^2*x^2)^(p - 1),
x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{b, e, f, p}, x] &&  !IntegerQ[p]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sec^2(x)^{3/2}}\,dx &= \text{Subst}\left(\int \frac{1}{\left(1+x^2\right)^{5/2}}\,dx,x,\tan(x)\right)\\ &= \frac{\tan(x)}{3\sec^2(x)^{3/2}}+\frac{2}{3}\text{Subst}\left(\int \frac{1}{\left(1+x^2\right)^{3/2}}\,dx,x,\tan(x)\right)\\ &= \frac{\tan(x)}{3\sec^2(x)^{3/2}}+\frac{2\tan(x)}{3\sqrt{\sec^2(x)}}\end{aligned}$$

Mathematica [A] time = 0.0149289, size = 23, normalized size = 0.79

$$\frac{(9\sin(x)+\sin(3x))\sec(x)}{12\sqrt{\sec^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[x]^2)^(-3/2),x]
```

```
[Out] (Sec[x]*(9*Sin[x] + Sin[3*x]))/(12*Sqrt[Sec[x]^2])
```

Maple [A] time = 0.059, size = 21, normalized size = 0.7

$$\frac{\sin(x)\left((\cos(x))^2+2\right)}{3\,(\cos(x))^3}\left((\cos(x))^{-2}\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sec(x)^2)^(3/2),x)
```

```
[Out] 1/3*sin(x)*(cos(x)^2+2)/cos(x)^3/(1/cos(x)^2)^(3/2)
```

Maxima [A] time = 1.08934, size = 34, normalized size = 1.17

$$\frac{2\,\tan(x)}{3\,\sqrt{\tan(x)^2+1}}+\frac{\tan(x)}{3\,\left(\tan(x)^2+1\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sec(x)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 2/3*tan(x)/sqrt(tan(x)^2 + 1) + 1/3*tan(x)/(tan(x)^2 + 1)^(3/2)
```

Fricas [A] time = 1.31181, size = 38, normalized size = 1.31

$$-\frac{1}{3}\left(\cos(x)^2+2\right)\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sec(x)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/3*(cos(x)^2 + 2)*sin(x)
```

Sympy [A] time = 1.5059, size = 27, normalized size = 0.93

$$\frac{2\tan^3(x)}{3\left(\sec^2(x)\right)^{\frac{3}{2}}}+\frac{\tan(x)}{\left(\sec^2(x)\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sec(x)**2)**(3/2),x)
```

```
[Out] 2*tan(x)**3/(3*(sec(x)**2)**(3/2)) + tan(x)/(sec(x)**2)**(3/2)
```

Giac [A] time = 1.35019, size = 22, normalized size = 0.76

$$-\frac{1}{3}\,\mathrm{sgn}\left(\cos\left(x\right)\right)\sin\left(x\right)^{3}+\mathrm{sgn}\left(\cos\left(x\right)\right)\sin\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sec(x)^2)^(3/2),x, algorithm="giac")
```

```
[Out] -1/3*sgn(cos(x))*sin(x)^3 + sgn(cos(x))*sin(x)
```

3.45 $\int \frac{1}{\sec^2(x)^{5/2}}\,dx$

Optimal. Leaf size=43

$$\frac{8\tan(x)}{15\sqrt{\sec^2(x)}}+\frac{4\tan(x)}{15\sec^2(x)^{3/2}}+\frac{\tan(x)}{5\sec^2(x)^{5/2}}$$

```
[Out] Tan[x]/(5*(Sec[x]^2)^(5/2)) + (4*Tan[x])/(15*(Sec[x]^2)^(3/2)) + (8*Tan[x])
/(15*Sqrt[Sec[x]^2])
```

Rubi [A] time = 0.0149745, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {4122, 192, 191}

$$\frac{8\tan(x)}{15\sqrt{\sec^2(x)}}+\frac{4\tan(x)}{15\sec^2(x)^{3/2}}+\frac{\tan(x)}{5\sec^2(x)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[x]^2)^(-5/2),x]
```

```
[Out] Tan[x]/(5*(Sec[x]^2)^(5/2)) + (4*Tan[x])/(15*(Sec[x]^2)^(3/2)) + (8*Tan[x])
/(15*Sqrt[Sec[x]^2])
```

Rule 4122

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFac
tors[Tan[e + f*x], x]}, Dist[(b*ff)/f, Subst[Int[(b + b*ff^2*x^2)^(p - 1),
x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{b, e, f, p}, x] && !IntegerQ[p]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{\sec^2(x)^{5/2}}\,dx &= \text{Subst}\left(\int \frac{1}{\left(1+x^2\right)^{7/2}}\,dx, x, \tan(x)\right) \\
&= \frac{\tan(x)}{5\sec^2(x)^{5/2}} + \frac{4}{5}\text{Subst}\left(\int \frac{1}{\left(1+x^2\right)^{5/2}}\,dx, x, \tan(x)\right) \\
&= \frac{\tan(x)}{5\sec^2(x)^{5/2}} + \frac{4\tan(x)}{15\sec^2(x)^{3/2}} + \frac{8}{15}\text{Subst}\left(\int \frac{1}{\left(1+x^2\right)^{3/2}}\,dx, x, \tan(x)\right) \\
&= \frac{\tan(x)}{5\sec^2(x)^{5/2}} + \frac{4\tan(x)}{15\sec^2(x)^{3/2}} + \frac{8\tan(x)}{15\sqrt{\sec^2(x)}}
\end{aligned}
$$

Mathematica [A] time = 0.0249409, size = 31, normalized size = 0.72

$$\frac{(150\sin(x) + 25\sin(3x) + 3\sin(5x))\sec(x)}{240\sqrt{\sec^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[x]^2)^(-5/2),x]
```

```
[Out] (Sec[x]*(150*Sin[x] + 25*Sin[3*x] + 3*Sin[5*x]))/(240*Sqrt[Sec[x]^2])
```

Maple [A] time = 0.072, size = 29, normalized size = 0.7

$$\frac{\sin(x)\left(3\,(\cos(x))^4 + 4\,(\cos(x))^2 + 8\right)}{15\,(\cos(x))^5}\left((\cos(x))^{-2}\right)^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sec(x)^2)^(5/2),x)
```

```
[Out] 1/15*sin(x)*(3*cos(x)^4+4*cos(x)^2+8)/cos(x)^5/(1/cos(x)^2)^(5/2)
```

Maxima [A] time = 1.16518, size = 50, normalized size = 1.16

$$\frac{8\tan(x)}{15\sqrt{\tan(x)^2+1}} + \frac{4\tan(x)}{15\left(\tan(x)^2+1\right)^{\frac{3}{2}}} + \frac{\tan(x)}{5\left(\tan(x)^2+1\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sec(x)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 8/15*tan(x)/sqrt(tan(x)^2 + 1) + 4/15*tan(x)/(tan(x)^2 + 1)^(3/2) + 1/5*tan
(x)/(tan(x)^2 + 1)^(5/2)
```

Fricas [A] time = 1.44419, size = 59, normalized size = 1.37

$$-\frac{1}{15}\left(3\cos(x)^4 + 4\cos(x)^2 + 8\right)\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sec(x)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/15*(3*cos(x)^4 + 4*cos(x)^2 + 8)*sin(x)
```

Sympy [A] time = 17.704, size = 44, normalized size = 1.02

$$\frac{8\tan^5(x)}{15\left(\sec^2(x)\right)^{\frac{5}{2}}} + \frac{4\tan^3(x)}{3\left(\sec^2(x)\right)^{\frac{5}{2}}} + \frac{\tan(x)}{\left(\sec^2(x)\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sec(x)**2)**(5/2),x)
```

```
[Out] 8*tan(x)**5/(15*(sec(x)**2)**(5/2)) + 4*tan(x)**3/(3*(sec(x)**2)**(5/2)) +
tan(x)/(sec(x)**2)**(5/2)
```

Giac [A] time = 1.25735, size = 34, normalized size = 0.79

$$\frac{1}{5}\,\mathrm{sgn}(\cos(x))\sin(x)^5 - \frac{2}{3}\,\mathrm{sgn}(\cos(x))\sin(x)^3 + \mathrm{sgn}(\cos(x))\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sec(x)^2)^(5/2),x, algorithm="giac")
```

```
[Out] 1/5*sgn(cos(x))*sin(x)^5 - 2/3*sgn(cos(x))*sin(x)^3 + sgn(cos(x))*sin(x)
```

3.46 $\int \frac{1}{\sec^2(x)^{7/2}} \, dx$

Optimal. Leaf size=57

$$\frac{16\tan(x)}{35\sqrt{\sec^2(x)}}+\frac{8\tan(x)}{35\sec^2(x)^{3/2}}+\frac{6\tan(x)}{35\sec^2(x)^{5/2}}+\frac{\tan(x)}{7\sec^2(x)^{7/2}}$$

```
[Out] Tan[x]/(7*(Sec[x]^2)^(7/2)) + (6*Tan[x])/(35*(Sec[x]^2)^(5/2)) + (8*Tan[x])
/(35*(Sec[x]^2)^(3/2)) + (16*Tan[x])/(35*Sqrt[Sec[x]^2])
```

Rubi [A] time = 0.0188078, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {4122, 192, 191}

$$\frac{16\tan(x)}{35\sqrt{\sec^2(x)}}+\frac{8\tan(x)}{35\sec^2(x)^{3/2}}+\frac{6\tan(x)}{35\sec^2(x)^{5/2}}+\frac{\tan(x)}{7\sec^2(x)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[x]^2)^(-7/2),x]
```

```
[Out] Tan[x]/(7*(Sec[x]^2)^(7/2)) + (6*Tan[x])/(35*(Sec[x]^2)^(5/2)) + (8*Tan[x])
/(35*(Sec[x]^2)^(3/2)) + (16*Tan[x])/(35*Sqrt[Sec[x]^2])
```

Rule 4122

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFac
tors[Tan[e + f*x], x]}, Dist[(b*ff)/f, Subst[Int[(b + b*ff^2*x^2)^(p - 1),
x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{b, e, f, p}, x] &&  !IntegerQ[p]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{\sec^2(x)^{7/2}}\,dx &= \text{Subst}\left(\int \frac{1}{\left(1+x^2\right)^{9/2}}\,dx, x, \tan(x)\right) \\
&= \frac{\tan(x)}{7\sec^2(x)^{7/2}} + \frac{6}{7}\text{Subst}\left(\int \frac{1}{\left(1+x^2\right)^{7/2}}\,dx, x, \tan(x)\right) \\
&= \frac{\tan(x)}{7\sec^2(x)^{7/2}} + \frac{6\tan(x)}{35\sec^2(x)^{5/2}} + \frac{24}{35}\text{Subst}\left(\int \frac{1}{\left(1+x^2\right)^{5/2}}\,dx, x, \tan(x)\right) \\
&= \frac{\tan(x)}{7\sec^2(x)^{7/2}} + \frac{6\tan(x)}{35\sec^2(x)^{5/2}} + \frac{8\tan(x)}{35\sec^2(x)^{3/2}} + \frac{16}{35}\text{Subst}\left(\int \frac{1}{\left(1+x^2\right)^{3/2}}\,dx, x, \tan(x)\right) \\
&= \frac{\tan(x)}{7\sec^2(x)^{7/2}} + \frac{6\tan(x)}{35\sec^2(x)^{5/2}} + \frac{8\tan(x)}{35\sec^2(x)^{3/2}} + \frac{16\tan(x)}{35\sqrt{\sec^2(x)}}
\end{aligned}
$$

Mathematica [A] time = 0.0376147, size = 37, normalized size = 0.65

$$
\frac{(1225\sin(x) + 245\sin(3x) + 49\sin(5x) + 5\sin(7x))\sec(x)}{2240\sqrt{\sec^2(x)}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[x]^2)^(-7/2),x]
```

```
[Out] (Sec[x]*(1225*Sin[x] + 245*Sin[3*x] + 49*Sin[5*x] + 5*Sin[7*x]))/(2240*Sqrt
[Sec[x]^2])
```

Maple [A] time = 0.088, size = 35, normalized size = 0.6

$$
\frac{\sin(x)\left(5\,(\cos(x))^6 + 6\,(\cos(x))^4 + 8\,(\cos(x))^2 + 16\right)}{35\,(\cos(x))^7}\left((\cos(x))^{-2}\right)^{-\frac{7}{2}}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sec(x)^2)^(7/2),x)
```

```
[Out] 1/35*sin(x)*(5*cos(x)^6+6*cos(x)^4+8*cos(x)^2+16)/cos(x)^7/(1/cos(x)^2)^(7/
2)
```

Maxima [A] time = 1.06939, size = 66, normalized size = 1.16

$$
\frac{16\tan(x)}{35\sqrt{\tan(x)^2+1}} + \frac{8\tan(x)}{35\left(\tan(x)^2+1\right)^{\frac{3}{2}}} + \frac{6\tan(x)}{35\left(\tan(x)^2+1\right)^{\frac{5}{2}}} + \frac{\tan(x)}{7\left(\tan(x)^2+1\right)^{\frac{7}{2}}}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sec(x)^2)^(7/2),x, algorithm="maxima")
```

```
[Out] 16/35*tan(x)/sqrt(tan(x)^2 + 1) + 8/35*tan(x)/(tan(x)^2 + 1)^(3/2) + 6/35*t
an(x)/(tan(x)^2 + 1)^(5/2) + 1/7*tan(x)/(tan(x)^2 + 1)^(7/2)
```

Fricas [A] time = 1.4212, size = 78, normalized size = 1.37

$$-\frac{1}{35}\left(5\cos\left(x\right)^{6}+6\cos\left(x\right)^{4}+8\cos\left(x\right)^{2}+16\right)\sin\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sec(x)^2)^(7/2),x, algorithm="fricas")
```

```
[Out] -1/35*(5*cos(x)^6 + 6*cos(x)^4 + 8*cos(x)^2 + 16)*sin(x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sec(x)**2)**(7/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.4046, size = 46, normalized size = 0.81

$$-\frac{1}{7}\,\mathrm{sgn}\left(\cos\left(x\right)\right)\sin\left(x\right)^{7}+\frac{3}{5}\,\mathrm{sgn}\left(\cos\left(x\right)\right)\sin\left(x\right)^{5}-\mathrm{sgn}\left(\cos\left(x\right)\right)\sin\left(x\right)^{3}+\mathrm{sgn}\left(\cos\left(x\right)\right)\sin\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sec(x)^2)^(7/2),x, algorithm="giac")
```

```
[Out] -1/7*sgn(cos(x))*sin(x)^7 + 3/5*sgn(cos(x))*sin(x)^5 - sgn(cos(x))*sin(x)^3
 + sgn(cos(x))*sin(x)
```

3.47 $\int \left(a \sec^2(x)\right)^{7/2} dx$

Optimal. Leaf size=84

$$\frac{5}{16}a^3\tan(x)\sqrt{a\sec^2(x)}+\frac{5}{24}a^2\tan(x)\left(a\sec^2(x)\right)^{3/2}+\frac{5}{16}a^{7/2}\tanh^{-1}\left(\frac{\sqrt{a}\tan(x)}{\sqrt{a\sec^2(x)}}\right)+\frac{1}{6}a\tan(x)\left(a\sec^2(x)\right)^{5/2}$$

```
[Out] (5*a^(7/2)*ArcTanh[(Sqrt[a]*Tan[x])/Sqrt[a*Sec[x]^2]])/16 + (5*a^3*Sqrt[a*S
ec[x]^2]*Tan[x])/16 + (5*a^2*(a*Sec[x]^2)^(3/2)*Tan[x])/24 + (a*(a*Sec[x]^2
)^(5/2)*Tan[x])/6
```

Rubi [A] time = 0.0413025, antiderivative size = 84, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {4122, 195, 217, 206}

$$\frac{5}{16}a^3\tan(x)\sqrt{a\sec^2(x)}+\frac{5}{24}a^2\tan(x)\left(a\sec^2(x)\right)^{3/2}+\frac{5}{16}a^{7/2}\tanh^{-1}\left(\frac{\sqrt{a}\tan(x)}{\sqrt{a\sec^2(x)}}\right)+\frac{1}{6}a\tan(x)\left(a\sec^2(x)\right)^{5/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Sec[x]^2)^(7/2),x]
```

```
[Out] (5*a^(7/2)*ArcTanh[(Sqrt[a]*Tan[x])/Sqrt[a*Sec[x]^2]])/16 + (5*a^3*Sqrt[a*S
ec[x]^2]*Tan[x])/16 + (5*a^2*(a*Sec[x]^2)^(3/2)*Tan[x])/24 + (a*(a*Sec[x]^2
)^(5/2)*Tan[x])/6
```

Rule 4122

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFac
tors[Tan[e + f*x], x]}, Dist[(b*ff)/f, Subst[Int[(b + b*ff^2*x^2)^(p - 1),
x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{b, e, f, p}, x] && !IntegerQ[p]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \left(a\sec^2(x)\right)^{7/2} dx = a\,\text{Subst}\left(\int \left(a+ax^2\right)^{5/2} dx, x, \tan(x)\right)$$
$$= \frac{1}{6}a\left(a\sec^2(x)\right)^{5/2}\tan(x) + \frac{1}{6}\left(5a^2\right)\text{Subst}\left(\int \left(a+ax^2\right)^{3/2} dx, x, \tan(x)\right)$$
$$= \frac{5}{24}a^2\left(a\sec^2(x)\right)^{3/2}\tan(x) + \frac{1}{6}a\left(a\sec^2(x)\right)^{5/2}\tan(x) + \frac{1}{8}\left(5a^3\right)\text{Subst}\left(\int \sqrt{a+ax^2}\,dx, x, \tan(x\right.$$
$$= \frac{5}{16}a^3\sqrt{a\sec^2(x)}\tan(x) + \frac{5}{24}a^2\left(a\sec^2(x)\right)^{3/2}\tan(x) + \frac{1}{6}a\left(a\sec^2(x)\right)^{5/2}\tan(x) + \frac{1}{16}\left(5a^4\right)\text{Subs}$$
$$= \frac{5}{16}a^3\sqrt{a\sec^2(x)}\tan(x) + \frac{5}{24}a^2\left(a\sec^2(x)\right)^{3/2}\tan(x) + \frac{1}{6}a\left(a\sec^2(x)\right)^{5/2}\tan(x) + \frac{1}{16}\left(5a^4\right)\text{Subs}$$
$$= \frac{5}{16}a^{7/2}\tanh^{-1}\left(\frac{\sqrt{a}\tan(x)}{\sqrt{a\sec^2(x)}}\right) + \frac{5}{16}a^3\sqrt{a\sec^2(x)}\tan(x) + \frac{5}{24}a^2\left(a\sec^2(x)\right)^{3/2}\tan(x) + \frac{1}{6}a\left(a\sec\right.$$

Mathematica [A] time = 0.124033, size = 78, normalized size = 0.93

$$\frac{1}{96}\cos^7(x)\left(a\sec^2(x)\right)^{7/2}\left(\frac{1}{8}(198\sin(x)+85\sin(3x)+15\sin(5x))\sec^6(x) - 30\log\left(\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right) + 30\log\left(\sin\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Sec[x]^2)^(7/2),x]
```

```
[Out] (Cos[x]^7*(a*Sec[x]^2)^(7/2)*(-30*Log[Cos[x/2] - Sin[x/2]] + 30*Log[Cos[x/2
] + Sin[x/2]] + (Sec[x]^6*(198*Sin[x] + 85*Sin[3*x] + 15*Sin[5*x]))/8))/96
```

Maple [A] time = 0.145, size = 74, normalized size = 0.9

$$\frac{\cos(x)}{48}\left(15\ln\left(-\frac{-1+\cos(x)-\sin(x)}{\sin(x)}\right)(\cos(x))^6 - 15\ln\left(-\frac{-1+\cos(x)+\sin(x)}{\sin(x)}\right)(\cos(x))^6 + 15(\cos(x))^4\sin(x)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sec(x)^2)^(7/2),x)
```

```
[Out] 1/48*(15*ln(-(-1+cos(x)-sin(x))/sin(x))*cos(x)^6-15*ln(-(-1+cos(x)+sin(x))/
sin(x))*cos(x)^6+15*cos(x)^4*sin(x)+10*cos(x)^2*sin(x)+8*sin(x))*cos(x)*(a/
cos(x)^2)^(7/2)
```

Maxima [B] time = 16.7386, size = 2936, normalized size = 34.95

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sec(x)^2)^(7/2),x, algorithm="maxima")
```

```
[Out] 1/96*(2040*a^3*cos(3*x)*sin(2*x) + 360*a^3*cos(x)*sin(2*x) - 360*a^3*cos(2*
x)*sin(x) - 60*a^3*sin(x) + 4*(15*a^3*sin(11*x) + 85*a^3*sin(9*x) + 198*a^3
*sin(7*x) - 198*a^3*sin(5*x) - 85*a^3*sin(3*x) - 15*a^3*sin(x))*cos(12*x) -
 60*(6*a^3*sin(10*x) + 15*a^3*sin(8*x) + 20*a^3*sin(6*x) + 15*a^3*sin(4*x)
```

```
+ 6*a^3*sin(2*x))*cos(11*x) + 24*(85*a^3*sin(9*x) + 198*a^3*sin(7*x) - 198*
a^3*sin(5*x) - 85*a^3*sin(3*x) - 15*a^3*sin(x))*cos(10*x) - 340*(15*a^3*sin
(8*x) + 20*a^3*sin(6*x) + 15*a^3*sin(4*x) + 6*a^3*sin(2*x))*cos(9*x) + 60*(
198*a^3*sin(7*x) - 198*a^3*sin(5*x) - 85*a^3*sin(3*x) - 15*a^3*sin(x))*cos(
8*x) - 792*(20*a^3*sin(6*x) + 15*a^3*sin(4*x) + 6*a^3*sin(2*x))*cos(7*x) -
80*(198*a^3*sin(5*x) + 85*a^3*sin(3*x) + 15*a^3*sin(x))*cos(6*x) + 2376*(5*
a^3*sin(4*x) + 2*a^3*sin(2*x))*cos(5*x) - 300*(17*a^3*sin(3*x) + 3*a^3*sin(
x))*cos(4*x) + 15*(a^3*cos(12*x)^2 + 36*a^3*cos(10*x)^2 + 225*a^3*cos(8*x)^
2 + 400*a^3*cos(6*x)^2 + 225*a^3*cos(4*x)^2 + 36*a^3*cos(2*x)^2 + a^3*sin(1
2*x)^2 + 36*a^3*sin(10*x)^2 + 225*a^3*sin(8*x)^2 + 400*a^3*sin(6*x)^2 + 225
*a^3*sin(4*x)^2 + 180*a^3*sin(4*x)*sin(2*x) + 36*a^3*sin(2*x)^2 + 12*a^3*co
s(2*x) + a^3 + 2*(6*a^3*cos(10*x) + 15*a^3*cos(8*x) + 20*a^3*cos(6*x) + 15*
a^3*cos(4*x) + 6*a^3*cos(2*x) + a^3)*cos(12*x) + 12*(15*a^3*cos(8*x) + 20*a
^3*cos(6*x) + 15*a^3*cos(4*x) + 6*a^3*cos(2*x) + a^3)*cos(10*x) + 30*(20*a^
3*cos(6*x) + 15*a^3*cos(4*x) + 6*a^3*cos(2*x) + a^3)*cos(8*x) + 40*(15*a^3*
cos(4*x) + 6*a^3*cos(2*x) + a^3)*cos(6*x) + 30*(6*a^3*cos(2*x) + a^3)*cos(4
*x) + 2*(6*a^3*sin(10*x) + 15*a^3*sin(8*x) + 20*a^3*sin(6*x) + 15*a^3*sin(4
*x) + 6*a^3*sin(2*x))*sin(12*x) + 12*(15*a^3*sin(8*x) + 20*a^3*sin(6*x) + 1
5*a^3*sin(4*x) + 6*a^3*sin(2*x))*sin(10*x) + 30*(20*a^3*sin(6*x) + 15*a^3*s
in(4*x) + 6*a^3*sin(2*x))*sin(8*x) + 120*(5*a^3*sin(4*x) + 2*a^3*sin(2*x))*
sin(6*x))*log(cos(x)^2 + sin(x)^2 + 2*sin(x) + 1) - 15*(a^3*cos(12*x)^2 + 3
6*a^3*cos(10*x)^2 + 225*a^3*cos(8*x)^2 + 400*a^3*cos(6*x)^2 + 225*a^3*cos(4
*x)^2 + 36*a^3*cos(2*x)^2 + a^3*sin(12*x)^2 + 36*a^3*sin(10*x)^2 + 225*a^3*
sin(8*x)^2 + 400*a^3*sin(6*x)^2 + 225*a^3*sin(4*x)^2 + 180*a^3*sin(4*x)*sin
(2*x) + 36*a^3*sin(2*x)^2 + 12*a^3*cos(2*x) + a^3 + 2*(6*a^3*cos(10*x) + 15
*a^3*cos(8*x) + 20*a^3*cos(6*x) + 15*a^3*cos(4*x) + 6*a^3*cos(2*x) + a^3)*c
os(12*x) + 12*(15*a^3*cos(8*x) + 20*a^3*cos(6*x) + 15*a^3*cos(4*x) + 6*a^3*
cos(2*x) + a^3)*cos(10*x) + 30*(20*a^3*cos(6*x) + 15*a^3*cos(4*x) + 6*a^3*c
os(2*x) + a^3)*cos(8*x) + 40*(15*a^3*cos(4*x) + 6*a^3*cos(2*x) + a^3)*cos(6
*x) + 30*(6*a^3*cos(2*x) + a^3)*cos(4*x) + 2*(6*a^3*sin(10*x) + 15*a^3*sin(
8*x) + 20*a^3*sin(6*x) + 15*a^3*sin(4*x) + 6*a^3*sin(2*x))*sin(12*x) + 12*(
15*a^3*sin(8*x) + 20*a^3*sin(6*x) + 15*a^3*sin(4*x) + 6*a^3*sin(2*x))*sin(1
0*x) + 30*(20*a^3*sin(6*x) + 15*a^3*sin(4*x) + 6*a^3*sin(2*x))*sin(8*x) + 1
20*(5*a^3*sin(4*x) + 2*a^3*sin(2*x))*sin(6*x))*log(cos(x)^2 + sin(x)^2 - 2*
sin(x) + 1) - 4*(15*a^3*cos(11*x) + 85*a^3*cos(9*x) + 198*a^3*cos(7*x) - 19
8*a^3*cos(5*x) - 85*a^3*cos(3*x) - 15*a^3*cos(x))*sin(12*x) + 60*(6*a^3*cos
(10*x) + 15*a^3*cos(8*x) + 20*a^3*cos(6*x) + 15*a^3*cos(4*x) + 6*a^3*cos(2*
x) + a^3)*sin(11*x) - 24*(85*a^3*cos(9*x) + 198*a^3*cos(7*x) - 198*a^3*cos(
5*x) - 85*a^3*cos(3*x) - 15*a^3*cos(x))*sin(10*x) + 340*(15*a^3*cos(8*x) +
20*a^3*cos(6*x) + 15*a^3*cos(4*x) + 6*a^3*cos(2*x) + a^3)*sin(9*x) - 60*(19
8*a^3*cos(7*x) - 198*a^3*cos(5*x) - 85*a^3*cos(3*x) - 15*a^3*cos(x))*sin(8*
x) + 792*(20*a^3*cos(6*x) + 15*a^3*cos(4*x) + 6*a^3*cos(2*x) + a^3)*sin(7*x
) + 80*(198*a^3*cos(5*x) + 85*a^3*cos(3*x) + 15*a^3*cos(x))*sin(6*x) - 792*
(15*a^3*cos(4*x) + 6*a^3*cos(2*x) + a^3)*sin(5*x) + 300*(17*a^3*cos(3*x) +
3*a^3*cos(x))*sin(4*x) - 340*(6*a^3*cos(2*x) + a^3)*sin(3*x))*sqrt(a)/(2*(6
*cos(10*x) + 15*cos(8*x) + 20*cos(6*x) + 15*cos(4*x) + 6*cos(2*x) + 1)*cos(
12*x) + cos(12*x)^2 + 12*(15*cos(8*x) + 20*cos(6*x) + 15*cos(4*x) + 6*cos(2
*x) + 1)*cos(10*x) + 36*cos(10*x)^2 + 30*(20*cos(6*x) + 15*cos(4*x) + 6*cos
(2*x) + 1)*cos(8*x) + 225*cos(8*x)^2 + 40*(15*cos(4*x) + 6*cos(2*x) + 1)*co
s(6*x) + 400*cos(6*x)^2 + 30*(6*cos(2*x) + 1)*cos(4*x) + 225*cos(4*x)^2 + 3
6*cos(2*x)^2 + 2*(6*sin(10*x) + 15*sin(8*x) + 20*sin(6*x) + 15*sin(4*x) + 6
*sin(2*x))*sin(12*x) + sin(12*x)^2 + 12*(15*sin(8*x) + 20*sin(6*x) + 15*sin
(4*x) + 6*sin(2*x))*sin(10*x) + 36*sin(10*x)^2 + 30*(20*sin(6*x) + 15*sin(4
*x) + 6*sin(2*x))*sin(8*x) + 225*sin(8*x)^2 + 120*(5*sin(4*x) + 2*sin(2*x))
*sin(6*x) + 400*sin(6*x)^2 + 225*sin(4*x)^2 + 180*sin(4*x)*sin(2*x) + 36*si
n(2*x)^2 + 12*cos(2*x) + 1)
```

Fricas [A] time = 1.47792, size = 186, normalized size = 2.21

$$-\frac{\left(15\,a^3\cos\left(x\right)^6\log\left(-\frac{\sin(x)-1}{\sin(x)+1}\right)-2\left(15\,a^3\cos\left(x\right)^4+10\,a^3\cos\left(x\right)^2+8\,a^3\right)\sin\left(x\right)\right)\sqrt{\frac{a}{\cos(x)^2}}}{96\,\cos\left(x\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sec(x)^2)^(7/2),x, algorithm="fricas")
```

```
[Out] -1/96*(15*a^3*cos(x)^6*log(-(sin(x) - 1)/(sin(x) + 1)) - 2*(15*a^3*cos(x)^4
 + 10*a^3*cos(x)^2 + 8*a^3)*sin(x))*sqrt(a/cos(x)^2)/cos(x)^5
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sec(x)**2)**(7/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.30087, size = 107, normalized size = 1.27

$$\frac{1}{96}\left(15\,a^3\log\left(\sin\left(x\right)+1\right)\mathrm{sgn}\left(\cos\left(x\right)\right)-15\,a^3\log\left(-\sin\left(x\right)+1\right)\mathrm{sgn}\left(\cos\left(x\right)\right)-\frac{2\left(15\,a^3\mathrm{sgn}\left(\cos\left(x\right)\right)\sin\left(x\right)^5-40\,a^3\mathrm{s}\right.}{\left(\mathrm{s}\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sec(x)^2)^(7/2),x, algorithm="giac")
```

```
[Out] 1/96*(15*a^3*log(sin(x) + 1)*sgn(cos(x)) - 15*a^3*log(-sin(x) + 1)*sgn(cos(
x)) - 2*(15*a^3*sgn(cos(x))*sin(x)^5 - 40*a^3*sgn(cos(x))*sin(x)^3 + 33*a^3
*sgn(cos(x))*sin(x))/(sin(x)^2 - 1)^3)*sqrt(a)
```

3.48 $\int \left(a\sec^2(x)\right)^{5/2}\,dx$

Optimal. Leaf size=65

$$\frac{3}{8}a^2\tan(x)\sqrt{a\sec^2(x)}+\frac{3}{8}a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a}\tan(x)}{\sqrt{a\sec^2(x)}}\right)+\frac{1}{4}a\tan(x)\left(a\sec^2(x)\right)^{3/2}$$

```
[Out] (3*a^(5/2)*ArcTanh[(Sqrt[a]*Tan[x])/Sqrt[a*Sec[x]^2]])/8 + (3*a^2*Sqrt[a*Se
c[x]^2]*Tan[x])/8 + (a*(a*Sec[x]^2)^(3/2)*Tan[x])/4
```

Rubi [A] time = 0.0313453, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {4122, 195, 217, 206}

$$\frac{3}{8}a^2\tan(x)\sqrt{a\sec^2(x)}+\frac{3}{8}a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a}\tan(x)}{\sqrt{a\sec^2(x)}}\right)+\frac{1}{4}a\tan(x)\left(a\sec^2(x)\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Sec[x]^2)^(5/2),x]
```

```
[Out] (3*a^(5/2)*ArcTanh[(Sqrt[a]*Tan[x])/Sqrt[a*Sec[x]^2]])/8 + (3*a^2*Sqrt[a*Se
c[x]^2]*Tan[x])/8 + (a*(a*Sec[x]^2)^(3/2)*Tan[x])/4
```

Rule 4122

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFac
tors[Tan[e + f*x], x]}, Dist[(b*ff)/f, Subst[Int[(b + b*ff^2*x^2)^(p - 1),
x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{b, e, f, p}, x] && !IntegerQ[p]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \left(a\sec^2(x)\right)^{5/2} dx = a\,\text{Subst}\left(\int \left(a+ax^2\right)^{3/2} dx, x, \tan(x)\right)$$
$$= \frac{1}{4}a\left(a\sec^2(x)\right)^{3/2}\tan(x) + \frac{1}{4}\left(3a^2\right)\text{Subst}\left(\int \sqrt{a+ax^2}\,dx, x, \tan(x)\right)$$
$$= \frac{3}{8}a^2\sqrt{a\sec^2(x)}\tan(x) + \frac{1}{4}a\left(a\sec^2(x)\right)^{3/2}\tan(x) + \frac{1}{8}\left(3a^3\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+ax^2}}dx, x, \tan(x)\right)$$
$$= \frac{3}{8}a^2\sqrt{a\sec^2(x)}\tan(x) + \frac{1}{4}a\left(a\sec^2(x)\right)^{3/2}\tan(x) + \frac{1}{8}\left(3a^3\right)\text{Subst}\left(\int \frac{1}{1-ax^2}dx, x, \frac{\tan(x)}{\sqrt{a\sec^2(x)}}\right)$$
$$= \frac{3}{8}a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a}\tan(x)}{\sqrt{a\sec^2(x)}}\right) + \frac{3}{8}a^2\sqrt{a\sec^2(x)}\tan(x) + \frac{1}{4}a\left(a\sec^2(x)\right)^{3/2}\tan(x)$$

Mathematica [A] time = 0.120652, size = 72, normalized size = 1.11

$$\frac{1}{16}\cos^5(x)\left(a\sec^2(x)\right)^{5/2}\left(\frac{1}{2}(11\sin(x)+3\sin(3x))\sec^4(x) - 6\log\left(\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right) + 6\log\left(\sin\left(\frac{x}{2}\right)+\cos\left(\frac{x}{2}\right)\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Sec[x]^2)^(5/2),x]
```

```
[Out] (Cos[x]^5*(a*Sec[x]^2)^(5/2)*(-6*Log[Cos[x/2] - Sin[x/2]] + 6*Log[Cos[x/2]
+ Sin[x/2]] + (Sec[x]^4*(11*Sin[x] + 3*Sin[3*x]))/2))/16
```

Maple [A] time = 0.087, size = 66, normalized size = 1.

$$\frac{\cos(x)}{8}\left(3\,(\cos(x))^4\ln\left(-\frac{-1+\cos(x)-\sin(x)}{\sin(x)}\right) - 3\,(\cos(x))^4\ln\left(-\frac{-1+\cos(x)+\sin(x)}{\sin(x)}\right) + 3\,(\cos(x))^2\sin(x) + 2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sec(x)^2)^(5/2),x)
```

```
[Out] 1/8*(3*cos(x)^4*ln(-(-1+cos(x)-sin(x))/sin(x))-3*cos(x)^4*ln(-(-1+cos(x)+si
n(x))/sin(x))+3*cos(x)^2*sin(x)+2*sin(x))*cos(x)*(a/cos(x)^2)^(5/2)
```

Maxima [B] time = 2.67276, size = 1500, normalized size = 23.08

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sec(x)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/16*(176*a^2*cos(3*x)*sin(2*x) + 48*a^2*cos(x)*sin(2*x) - 48*a^2*cos(2*x)*
sin(x) - 12*a^2*sin(x) + 4*(3*a^2*sin(7*x) + 11*a^2*sin(5*x) - 11*a^2*sin(3
*x) - 3*a^2*sin(x))*cos(8*x) - 24*(2*a^2*sin(6*x) + 3*a^2*sin(4*x) + 2*a^2*
sin(2*x))*cos(7*x) + 16*(11*a^2*sin(5*x) - 11*a^2*sin(3*x) - 3*a^2*sin(x))*
cos(6*x) - 88*(3*a^2*sin(4*x) + 2*a^2*sin(2*x))*cos(5*x) - 24*(11*a^2*sin(3
*x) + 3*a^2*sin(x))*cos(4*x) + 3*(a^2*cos(8*x)^2 + 16*a^2*cos(6*x)^2 + 36*a
^2*cos(4*x)^2 + 16*a^2*cos(2*x)^2 + a^2*sin(8*x)^2 + 16*a^2*sin(6*x)^2 + 36
```

```
*a^2*sin(4*x)^2 + 48*a^2*sin(4*x)*sin(2*x) + 16*a^2*sin(2*x)^2 + 8*a^2*cos(
2*x) + a^2 + 2*(4*a^2*cos(6*x) + 6*a^2*cos(4*x) + 4*a^2*cos(2*x) + a^2)*cos
(8*x) + 8*(6*a^2*cos(4*x) + 4*a^2*cos(2*x) + a^2)*cos(6*x) + 12*(4*a^2*cos(
2*x) + a^2)*cos(4*x) + 4*(2*a^2*sin(6*x) + 3*a^2*sin(4*x) + 2*a^2*sin(2*x))
*sin(8*x) + 16*(3*a^2*sin(4*x) + 2*a^2*sin(2*x))*sin(6*x))*log(cos(x)^2 + s
in(x)^2 + 2*sin(x) + 1) - 3*(a^2*cos(8*x)^2 + 16*a^2*cos(6*x)^2 + 36*a^2*co
s(4*x)^2 + 16*a^2*cos(2*x)^2 + a^2*sin(8*x)^2 + 16*a^2*sin(6*x)^2 + 36*a^2*
sin(4*x)^2 + 48*a^2*sin(4*x)*sin(2*x) + 16*a^2*sin(2*x)^2 + 8*a^2*cos(2*x)
+ a^2 + 2*(4*a^2*cos(6*x) + 6*a^2*cos(4*x) + 4*a^2*cos(2*x) + a^2)*cos(8*x)
 + 8*(6*a^2*cos(4*x) + 4*a^2*cos(2*x) + a^2)*cos(6*x) + 12*(4*a^2*cos(2*x)
+ a^2)*cos(4*x) + 4*(2*a^2*sin(6*x) + 3*a^2*sin(4*x) + 2*a^2*sin(2*x))*sin(
8*x) + 16*(3*a^2*sin(4*x) + 2*a^2*sin(2*x))*sin(6*x))*log(cos(x)^2 + sin(x)
^2 - 2*sin(x) + 1) - 4*(3*a^2*cos(7*x) + 11*a^2*cos(5*x) - 11*a^2*cos(3*x)
- 3*a^2*cos(x))*sin(8*x) + 12*(4*a^2*cos(6*x) + 6*a^2*cos(4*x) + 4*a^2*cos(
2*x) + a^2)*sin(7*x) - 16*(11*a^2*cos(5*x) - 11*a^2*cos(3*x) - 3*a^2*cos(x)
)*sin(6*x) + 44*(6*a^2*cos(4*x) + 4*a^2*cos(2*x) + a^2)*sin(5*x) + 24*(11*a
^2*cos(3*x) + 3*a^2*cos(x))*sin(4*x) - 44*(4*a^2*cos(2*x) + a^2)*sin(3*x))*
sqrt(a)/(2*(4*cos(6*x) + 6*cos(4*x) + 4*cos(2*x) + 1)*cos(8*x) + cos(8*x)^2
 + 8*(6*cos(4*x) + 4*cos(2*x) + 1)*cos(6*x) + 16*cos(6*x)^2 + 12*(4*cos(2*x
) + 1)*cos(4*x) + 36*cos(4*x)^2 + 16*cos(2*x)^2 + 4*(2*sin(6*x) + 3*sin(4*x
) + 2*sin(2*x))*sin(8*x) + sin(8*x)^2 + 16*(3*sin(4*x) + 2*sin(2*x))*sin(6*
x) + 16*sin(6*x)^2 + 36*sin(4*x)^2 + 48*sin(4*x)*sin(2*x) + 16*sin(2*x)^2 +
 8*cos(2*x) + 1)
```

Fricas [A] time = 1.56971, size = 159, normalized size = 2.45

$$-\frac{\left(3\,a^2\cos\left(x\right)^4\log\left(-\frac{\sin(x)-1}{\sin(x)+1}\right)-2\left(3\,a^2\cos\left(x\right)^2+2\,a^2\right)\sin\left(x\right)\right)\sqrt{\frac{a}{\cos(x)^2}}}{16\,\cos\left(x\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sec(x)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/16*(3*a^2*cos(x)^4*log(-(sin(x) - 1)/(sin(x) + 1)) - 2*(3*a^2*cos(x)^2 +
 2*a^2)*sin(x))*sqrt(a/cos(x)^2)/cos(x)^3
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sec(x)**2)**(5/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.40925, size = 90, normalized size = 1.38

$$\frac{1}{16}\Bigg(3\,a^2\log\left(\sin\left(x\right)+1\right)\mathrm{sgn}\left(\cos\left(x\right)\right)-3\,a^2\log\left(-\sin\left(x\right)+1\right)\mathrm{sgn}\left(\cos\left(x\right)\right)-\frac{2\left(3\,a^2\mathrm{sgn}\left(\cos\left(x\right)\right)\sin\left(x\right)^3-5\,a^2\mathrm{sgn}\left(\cos\right.\right.}{\left(\sin\left(x\right)^2-1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sec(x)^2)^(5/2),x, algorithm="giac")
```

```
[Out] 1/16*(3*a^2*log(sin(x) + 1)*sgn(cos(x)) - 3*a^2*log(-sin(x) + 1)*sgn(cos(x)
) - 2*(3*a^2*sgn(cos(x))*sin(x)^3 - 5*a^2*sgn(cos(x))*sin(x))/(sin(x)^2 - 1
)^2)*sqrt(a)
```

3.49 $\int \left(a\sec^2(x)\right)^{3/2} dx$

Optimal. Leaf size=46

$$\frac{1}{2}a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\tan(x)}{\sqrt{a\sec^2(x)}}\right)+\frac{1}{2}a\tan(x)\sqrt{a\sec^2(x)}$$

```
[Out] (a^(3/2)*ArcTanh[(Sqrt[a]*Tan[x])/Sqrt[a*Sec[x]^2]])/2 + (a*Sqrt[a*Sec[x]^2
]*Tan[x])/2
```

Rubi [A] time = 0.0223066, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {4122, 195, 217, 206}

$$\frac{1}{2}a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\tan(x)}{\sqrt{a\sec^2(x)}}\right)+\frac{1}{2}a\tan(x)\sqrt{a\sec^2(x)}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Sec[x]^2)^(3/2),x]
```

```
[Out] (a^(3/2)*ArcTanh[(Sqrt[a]*Tan[x])/Sqrt[a*Sec[x]^2]])/2 + (a*Sqrt[a*Sec[x]^2
]*Tan[x])/2
```

Rule 4122

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFac
tors[Tan[e + f*x], x]}, Dist[(b*ff)/f, Subst[Int[(b + b*ff^2*x^2)^(p - 1),
x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{b, e, f, p}, x] && !IntegerQ[p]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \left(a\sec^2(x)\right)^{3/2} dx &= a\,\text{Subst}\left(\int \sqrt{a+ax^2}\,dx, x, \tan(x)\right)\\
&= \frac{1}{2}a\sqrt{a\sec^2(x)}\tan(x) + \frac{1}{2}a^2\,\text{Subst}\left(\int \frac{1}{\sqrt{a+ax^2}}\,dx, x, \tan(x)\right)\\
&= \frac{1}{2}a\sqrt{a\sec^2(x)}\tan(x) + \frac{1}{2}a^2\,\text{Subst}\left(\int \frac{1}{1-ax^2}\,dx, x, \frac{\tan(x)}{\sqrt{a\sec^2(x)}}\right)\\
&= \frac{1}{2}a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\tan(x)}{\sqrt{a\sec^2(x)}}\right) + \frac{1}{2}a\sqrt{a\sec^2(x)}\tan(x)
\end{aligned}
$$

Mathematica [A] time = 0.050598, size = 55, normalized size = 1.2

$$
\frac{1}{2}a\cos(x)\sqrt{a\sec^2(x)}\left(\tan(x)\sec(x) - \log\left(\cos\left(\frac{x}{2}\right) - \sin\left(\frac{x}{2}\right)\right) + \log\left(\sin\left(\frac{x}{2}\right) + \cos\left(\frac{x}{2}\right)\right)\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Sec[x]^2)^(3/2),x]
```

```
[Out] (a*Cos[x]*Sqrt[a*Sec[x]^2]*(-Log[Cos[x/2] - Sin[x/2]] + Log[Cos[x/2] + Sin[
x/2]] + Sec[x]*Tan[x]))/2
```

Maple [A] time = 0.061, size = 55, normalized size = 1.2

$$
\frac{\cos(x)}{2}\left((\cos(x))^2\ln\left(-\frac{-1+\cos(x)-\sin(x)}{\sin(x)}\right) - (\cos(x))^2\ln\left(-\frac{-1+\cos(x)+\sin(x)}{\sin(x)}\right) + \sin(x)\right)\left(\frac{a}{(\cos(x))^2}\right)^{\frac{3}{2}}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sec(x)^2)^(3/2),x)
```

```
[Out] 1/2*(cos(x)^2*ln(-(-1+cos(x)-sin(x))/sin(x))-cos(x)^2*ln(-(-1+cos(x)+sin(x)
)/sin(x))+sin(x))*cos(x)*(a/cos(x)^2)^(3/2)
```

Maxima [B] time = 1.94571, size = 437, normalized size = 9.5

$$
-\frac{\left(8\,a\cos(3\,x)\sin(2\,x) - 8\,a\cos(x)\sin(2\,x) + 8\,a\cos(2\,x)\sin(x) - 4\,(a\sin(3\,x) - a\sin(x))\cos(4\,x) - \left(a\cos(4\,x)^2 + \right.\right.}{}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sec(x)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/4*(8*a*cos(3*x)*sin(2*x) - 8*a*cos(x)*sin(2*x) + 8*a*cos(2*x)*sin(x) - 4
*(a*sin(3*x) - a*sin(x))*cos(4*x) - (a*cos(4*x)^2 + 4*a*cos(2*x)^2 + a*sin(
4*x)^2 + 4*a*sin(4*x)*sin(2*x) + 4*a*sin(2*x)^2 + 2*(2*a*cos(2*x) + a)*cos(
4*x) + 4*a*cos(2*x) + a)*log(cos(x)^2 + sin(x)^2 + 2*sin(x) + 1) + (a*cos(4
*x)^2 + 4*a*cos(2*x)^2 + a*sin(4*x)^2 + 4*a*sin(4*x)*sin(2*x) + 4*a*sin(2*x
)^2 + 2*(2*a*cos(2*x) + a)*cos(4*x) + 4*a*cos(2*x) + a)*log(cos(x)^2 + sin(
x)^2 - 2*sin(x) + 1) + 4*(a*cos(3*x) - a*cos(x))*sin(4*x) - 4*(2*a*cos(2*x)
```

```
 + a)*sin(3*x) + 4*a*sin(x))*sqrt(a)/(2*(2*cos(2*x) + 1)*cos(4*x) + cos(4*x
)^2 + 4*cos(2*x)^2 + sin(4*x)^2 + 4*sin(4*x)*sin(2*x) + 4*sin(2*x)^2 + 4*co
s(2*x) + 1)
```

Fricas [A] time = 1.47143, size = 119, normalized size = 2.59

$$-\frac{\left(a\cos\left(x\right)^{2}\log\left(-\frac{\sin(x)-1}{\sin(x)+1}\right)-2\,a\sin\left(x\right)\right)\sqrt{\frac{a}{\cos(x)^{2}}}}{4\,\cos\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sec(x)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/4*(a*cos(x)^2*log(-(sin(x) - 1)/(sin(x) + 1)) - 2*a*sin(x))*sqrt(a/cos(x
)^2)/cos(x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\left(a\sec^{2}\left(x\right)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sec(x)**2)**(3/2),x)
```

```
[Out] Integral((a*sec(x)**2)**(3/2), x)
```

Giac [A] time = 1.30607, size = 57, normalized size = 1.24

$$\frac{1}{4}\left(\log\left(\sin\left(x\right)+1\right)\operatorname{sgn}\left(\cos\left(x\right)\right)-\log\left(-\sin\left(x\right)+1\right)\operatorname{sgn}\left(\cos\left(x\right)\right)-\frac{2\operatorname{sgn}\left(\cos\left(x\right)\right)\sin\left(x\right)}{\sin\left(x\right)^{2}-1}\right)a^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sec(x)^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/4*(log(sin(x) + 1)*sgn(cos(x)) - log(-sin(x) + 1)*sgn(cos(x)) - 2*sgn(cos
(x))*sin(x)/(sin(x)^2 - 1))*a^(3/2)
```

3.50 $\int \sqrt{a\sec^2(x)}\,dx$

Optimal. Leaf size=25

$$\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\tan(x)}{\sqrt{a\sec^2(x)}}\right)$$

```
[Out] Sqrt[a]*ArcTanh[(Sqrt[a]*Tan[x])/Sqrt[a*Sec[x]^2]]
```

Rubi [A] time = 0.0148376, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {4122, 217, 206}

$$\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\tan(x)}{\sqrt{a\sec^2(x)}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*Sec[x]^2],x]
```

```
[Out] Sqrt[a]*ArcTanh[(Sqrt[a]*Tan[x])/Sqrt[a*Sec[x]^2]]
```

Rule 4122

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFac
tors[Tan[e + f*x], x]}, Dist[(b*ff)/f, Subst[Int[(b + b*ff^2*x^2)^(p - 1),
x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{b, e, f, p}, x] && !IntegerQ[p]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \sqrt{a\sec^2(x)}\,dx &= a\,\text{Subst}\left(\int \frac{1}{\sqrt{a+ax^2}}\,dx, x, \tan(x)\right)\\
&= a\,\text{Subst}\left(\int \frac{1}{1-ax^2}\,dx, x, \frac{\tan(x)}{\sqrt{a\sec^2(x)}}\right)\\
&= \sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\tan(x)}{\sqrt{a\sec^2(x)}}\right)
\end{aligned}$$

Mathematica [A] time = 0.0078522, size = 46, normalized size = 1.84

$$\cos(x)\sqrt{a\sec^2(x)}\left(\log\left(\sin\left(\frac{x}{2}\right)+\cos\left(\frac{x}{2}\right)\right)-\log\left(\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*Sec[x]^2],x]
```

```
[Out] Cos[x]*(-Log[Cos[x/2] - Sin[x/2]] + Log[Cos[x/2] + Sin[x/2]])*Sqrt[a*Sec[x]
^2]
```

Maple [A] time = 0.056, size = 23, normalized size = 0.9

$$-2\cos(x)\operatorname{Artanh}\left(\frac{-1+\cos(x)}{\sin(x)}\right)\sqrt{\frac{a}{(\cos(x))^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sec(x)^2)^(1/2),x)
```

```
[Out] -2*cos(x)*arctanh((-1+cos(x))/sin(x))*(a/cos(x)^2)^(1/2)
```

Maxima [A] time = 1.93595, size = 51, normalized size = 2.04

$$\frac{1}{2}\sqrt{a}\left(\log\left(\cos(x)^2+\sin(x)^2+2\sin(x)+1\right)-\log\left(\cos(x)^2+\sin(x)^2-2\sin(x)+1\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sec(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(a)*(log(cos(x)^2 + sin(x)^2 + 2*sin(x) + 1) - log(cos(x)^2 + sin(x
)^2 - 2*sin(x) + 1))
```

Fricas [A] time = 1.46416, size = 171, normalized size = 6.84

$$\left[-\frac{1}{2}\sqrt{\frac{a}{\cos(x)^2}}\cos(x)\log\left(-\frac{\sin(x)-1}{\sin(x)+1}\right), -\sqrt{-a}\arctan\left(\frac{\sqrt{-a}\sqrt{\frac{a}{\cos(x)^2}}\cos(x)\sin(x)}{a}\right)\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sec(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/2*sqrt(a/cos(x)^2)*cos(x)*log(-(sin(x) - 1)/(sin(x) + 1)), -sqrt(-a)*ar
ctan(sqrt(-a)*sqrt(a/cos(x)^2)*cos(x)*sin(x)/a)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{a\sec^2(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sec(x)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(a*sec(x)**2), x)
```

Giac [A] time = 1.34264, size = 42, normalized size = 1.68

$$\frac{1}{4}\sqrt{a}\left(\log\left(\left|\frac{1}{\sin(x)}+\sin(x)+2\right|\right)-\log\left(\left|\frac{1}{\sin(x)}+\sin(x)-2\right|\right)\right)\operatorname{sgn}(\cos(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sec(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(a)*(log(abs(1/sin(x) + sin(x) + 2)) - log(abs(1/sin(x) + sin(x) -
2)))*sgn(cos(x))
```

3.51 $\int \frac{1}{\sqrt{a\sec^2(x)}}\,dx$

Optimal. Leaf size=13

$$\frac{\tan(x)}{\sqrt{a\sec^2(x)}}$$

```
[Out] Tan[x]/Sqrt[a*Sec[x]^2]
```

Rubi [A] time = 0.0286669, antiderivative size = 13, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {4122, 191}

$$\frac{\tan(x)}{\sqrt{a\sec^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a*Sec[x]^2],x]
```

```
[Out] Tan[x]/Sqrt[a*Sec[x]^2]
```

Rule 4122

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFac
tors[Tan[e + f*x], x]}, Dist[(b*ff)/f, Subst[Int[(b + b*ff^2*x^2)^(p - 1),
x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{b, e, f, p}, x] && !IntegerQ[p]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{a\sec^2(x)}}\,dx &= a\,\text{Subst}\left(\int \frac{1}{\left(a+ax^2\right)^{3/2}}\,dx, x, \tan(x)\right)\\ &= \frac{\tan(x)}{\sqrt{a\sec^2(x)}}\end{aligned}$$

Mathematica [A] time = 0.0053793, size = 13, normalized size = 1.

$$\frac{\tan(x)}{\sqrt{a\sec^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a*Sec[x]^2],x]
```

```
[Out] Tan[x]/Sqrt[a*Sec[x]^2]
```

Maple [A] time = 0.069, size = 16, normalized size = 1.2

$$\frac{\sin(x)}{\cos(x)}\frac{1}{\sqrt{\frac{a}{(\cos(x))^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*sec(x)^2)^(1/2),x)
```

```
[Out] sin(x)/(a/cos(x)^2)^(1/2)/cos(x)
```

Maxima [A] time = 1.88085, size = 8, normalized size = 0.62

$$\frac{\sin(x)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sec(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] sin(x)/sqrt(a)
```

Fricas [A] time = 1.44531, size = 46, normalized size = 3.54

$$\frac{\sqrt{\frac{a}{\cos(x)^2}}\cos(x)\sin(x)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sec(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] sqrt(a/cos(x)^2)*cos(x)*sin(x)/a
```

Sympy [A] time = 0.53778, size = 15, normalized size = 1.15

$$\frac{\tan(x)}{\sqrt{a}\sqrt{\sec^2(x)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sec(x)**2)**(1/2),x)
```

```
[Out] tan(x)/(sqrt(a)*sqrt(sec(x)**2))
```

Giac [A] time = 1.22898, size = 15, normalized size = 1.15

$$\frac{\sin(x)}{\sqrt{a}\,\mathrm{sgn}(\cos(x))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sec(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] sin(x)/(sqrt(a)*sgn(cos(x)))
```

3.52 $\int \frac{1}{\left(a\sec^2(x)\right)^{3/2}}\,dx$

Optimal. Leaf size=36

$$\frac{2\tan(x)}{3a\sqrt{a\sec^2(x)}}+\frac{\tan(x)}{3\left(a\sec^2(x)\right)^{3/2}}$$

```
[Out] Tan[x]/(3*(a*Sec[x]^2)^(3/2)) + (2*Tan[x])/(3*a*Sqrt[a*Sec[x]^2])
```

Rubi [A] time = 0.0182093, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {4122, 192, 191}

$$\frac{2\tan(x)}{3a\sqrt{a\sec^2(x)}}+\frac{\tan(x)}{3\left(a\sec^2(x)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Sec[x]^2)^(-3/2),x]
```

```
[Out] Tan[x]/(3*(a*Sec[x]^2)^(3/2)) + (2*Tan[x])/(3*a*Sqrt[a*Sec[x]^2])
```

Rule 4122

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFac
tors[Tan[e + f*x], x]}, Dist[(b*ff)/f, Subst[Int[(b + b*ff^2*x^2)^(p - 1),
x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{b, e, f, p}, x] && !IntegerQ[p]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\left(a\sec^2(x)\right)^{3/2}}\,dx &= a\,\text{Subst}\left(\int \frac{1}{\left(a+ax^2\right)^{5/2}}\,dx,x,\tan(x)\right)\\
&= \frac{\tan(x)}{3\left(a\sec^2(x)\right)^{3/2}}+\frac{2}{3}\text{Subst}\left(\int \frac{1}{\left(a+ax^2\right)^{3/2}}\,dx,x,\tan(x)\right)\\
&= \frac{\tan(x)}{3\left(a\sec^2(x)\right)^{3/2}}+\frac{2\tan(x)}{3a\sqrt{a\sec^2(x)}}
\end{aligned}$$

Mathematica [A] time = 0.0178713, size = 27, normalized size = 0.75

$$\frac{(9\sin(x)+\sin(3x))\sec^3(x)}{12\left(a\sec^2(x)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Sec[x]^2)^(-3/2),x]
```

```
[Out] (Sec[x]^3*(9*Sin[x] + Sin[3*x]))/(12*(a*Sec[x]^2)^(3/2))
```

Maple [A] time = 0.057, size = 23, normalized size = 0.6

$$\frac{\sin(x)\left((\cos(x))^2+2\right)}{3\,(\cos(x))^3}\left(\frac{a}{(\cos(x))^2}\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*sec(x)^2)^(3/2),x)
```

```
[Out] 1/3*sin(x)*(cos(x)^2+2)/cos(x)^3/(a/cos(x)^2)^(3/2)
```

Maxima [A] time = 1.94539, size = 19, normalized size = 0.53

$$\frac{\sin(3\,x)+9\,\sin(x)}{12\,a^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sec(x)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/12*(sin(3*x) + 9*sin(x))/a^(3/2)
```

Fricas [A] time = 1.4321, size = 74, normalized size = 2.06

$$\frac{\left(\cos(x)^3+2\,\cos(x)\right)\sqrt{\frac{a}{\cos(x)^2}}\,\sin(x)}{3\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sec(x)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/3*(cos(x)^3 + 2*cos(x))*sqrt(a/cos(x)^2)*sin(x)/a^2
```

Sympy [A] time = 1.48685, size = 37, normalized size = 1.03

$$\frac{2\tan^3(x)}{3a^{\frac{3}{2}}\left(\sec^2(x)\right)^{\frac{3}{2}}}+\frac{\tan(x)}{a^{\frac{3}{2}}\left(\sec^2(x)\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sec(x)**2)**(3/2),x)
```

```
[Out] 2*tan(x)**3/(3*a**(3/2)*(sec(x)**2)**(3/2)) + tan(x)/(a**(3/2)*(sec(x)**2)*
*(3/2))
```

Giac [B] time = 1.37879, size = 78, normalized size = 2.17

$$\frac{2\left(3\left(\frac{1}{\tan\left(\frac{1}{2}x\right)}+\tan\left(\frac{1}{2}x\right)\right)^2\operatorname{sgn}\left(-\tan\left(\frac{1}{2}x\right)^2+1\right)-4\operatorname{sgn}\left(-\tan\left(\frac{1}{2}x\right)^2+1\right)\right)}{3a^{\frac{3}{2}}\left(\frac{1}{\tan\left(\frac{1}{2}x\right)}+\tan\left(\frac{1}{2}x\right)\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sec(x)^2)^(3/2),x, algorithm="giac")
```

```
[Out] 2/3*(3*(1/tan(1/2*x) + tan(1/2*x))^2*sgn(-tan(1/2*x)^2 + 1) - 4*sgn(-tan(1/
2*x)^2 + 1))/(a^(3/2)*(1/tan(1/2*x) + tan(1/2*x))^3)
```

3.53 $\int \frac{1}{\left(a\sec^2(x)\right)^{5/2}}\,dx$

Optimal. Leaf size=55

$$\frac{8\tan(x)}{15a^2\sqrt{a\sec^2(x)}}+\frac{4\tan(x)}{15a\left(a\sec^2(x)\right)^{3/2}}+\frac{\tan(x)}{5\left(a\sec^2(x)\right)^{5/2}}$$

```
[Out] Tan[x]/(5*(a*Sec[x]^2)^(5/2)) + (4*Tan[x])/(15*a*(a*Sec[x]^2)^(3/2)) + (8*T
an[x])/(15*a^2*Sqrt[a*Sec[x]^2])
```

Rubi [A] time = 0.0266371, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {4122, 192, 191}

$$\frac{8\tan(x)}{15a^2\sqrt{a\sec^2(x)}}+\frac{4\tan(x)}{15a\left(a\sec^2(x)\right)^{3/2}}+\frac{\tan(x)}{5\left(a\sec^2(x)\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Sec[x]^2)^(-5/2),x]
```

```
[Out] Tan[x]/(5*(a*Sec[x]^2)^(5/2)) + (4*Tan[x])/(15*a*(a*Sec[x]^2)^(3/2)) + (8*T
an[x])/(15*a^2*Sqrt[a*Sec[x]^2])
```

Rule 4122

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFac
tors[Tan[e + f*x], x]}, Dist[(b*ff)/f, Subst[Int[(b + b*ff^2*x^2)^(p - 1),
x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{b, e, f, p}, x] &&  !IntegerQ[p]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$\int \frac{1}{\left(a\sec^2(x)\right)^{5/2}}\,dx = a\,\text{Subst}\left(\int \frac{1}{\left(a+ax^2\right)^{7/2}}\,dx, x, \tan(x)\right)$$
$$= \frac{\tan(x)}{5\left(a\sec^2(x)\right)^{5/2}} + \frac{4}{5}\text{Subst}\left(\int \frac{1}{\left(a+ax^2\right)^{5/2}}\,dx, x, \tan(x)\right)$$
$$= \frac{\tan(x)}{5\left(a\sec^2(x)\right)^{5/2}} + \frac{4\tan(x)}{15a\left(a\sec^2(x)\right)^{3/2}} + \frac{8\,\text{Subst}\left(\int \frac{1}{\left(a+ax^2\right)^{3/2}}\,dx, x, \tan(x)\right)}{15a}$$
$$= \frac{\tan(x)}{5\left(a\sec^2(x)\right)^{5/2}} + \frac{4\tan(x)}{15a\left(a\sec^2(x)\right)^{3/2}} + \frac{8\tan(x)}{15a^2\sqrt{a\sec^2(x)}}$$

Mathematica [A] time = 0.0278019, size = 36, normalized size = 0.65

$$\frac{(150\sin(x) + 25\sin(3x) + 3\sin(5x))\cos(x)\sqrt{a\sec^2(x)}}{240a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Sec[x]^2)^(-5/2),x]
```

```
[Out] (Cos[x]*Sqrt[a*Sec[x]^2]*(150*Sin[x] + 25*Sin[3*x] + 3*Sin[5*x]))/(240*a^3)
```

Maple [A] time = 0.064, size = 31, normalized size = 0.6

$$\frac{\sin(x)\left(3\,(\cos(x))^4 + 4\,(\cos(x))^2 + 8\right)}{15\,(\cos(x))^5}\left(\frac{a}{(\cos(x))^2}\right)^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*sec(x)^2)^(5/2),x)
```

```
[Out] 1/15*sin(x)*(3*cos(x)^4+4*cos(x)^2+8)/cos(x)^5/(a/cos(x)^2)^(5/2)
```

Maxima [A] time = 1.83415, size = 30, normalized size = 0.55

$$\frac{3\sin(5x) + 25\sin(3x) + 150\sin(x)}{240\,a^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sec(x)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/240*(3*sin(5*x) + 25*sin(3*x) + 150*sin(x))/a^(5/2)
```

Fricas [A] time = 1.44471, size = 96, normalized size = 1.75

$$\frac{\left(3\,\cos\left(x\right)^{5}+4\,\cos\left(x\right)^{3}+8\,\cos\left(x\right)\right)\sqrt{\frac{a}{\cos\left(x\right)^{2}}}\sin\left(x\right)}{15\,a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sec(x)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/15*(3*cos(x)^5 + 4*cos(x)^3 + 8*cos(x))*sqrt(a/cos(x)^2)*sin(x)/a^3
```

Sympy [A] time = 17.4853, size = 60, normalized size = 1.09

$$\frac{8\tan^{5}\left(x\right)}{15a^{\frac{5}{2}}\left(\sec^{2}\left(x\right)\right)^{\frac{5}{2}}}+\frac{4\tan^{3}\left(x\right)}{3a^{\frac{5}{2}}\left(\sec^{2}\left(x\right)\right)^{\frac{5}{2}}}+\frac{\tan\left(x\right)}{a^{\frac{5}{2}}\left(\sec^{2}\left(x\right)\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sec(x)**2)**(5/2),x)
```

```
[Out] 8*tan(x)**5/(15*a**(5/2)*(sec(x)**2)**(5/2)) + 4*tan(x)**3/(3*a**(5/2)*(sec
(x)**2)**(5/2)) + tan(x)/(a**(5/2)*(sec(x)**2)**(5/2))
```

Giac [A] time = 1.44072, size = 113, normalized size = 2.05

$$\frac{2\left(15\left(\frac{1}{\tan\left(\frac{1}{2}\,x\right)}+\tan\left(\frac{1}{2}\,x\right)\right)^{4}\mathrm{sgn}\left(-\tan\left(\frac{1}{2}\,x\right)^{2}+1\right)-40\left(\frac{1}{\tan\left(\frac{1}{2}\,x\right)}+\tan\left(\frac{1}{2}\,x\right)\right)^{2}\mathrm{sgn}\left(-\tan\left(\frac{1}{2}\,x\right)^{2}+1\right)+48\,\mathrm{sgn}\left(-\tan\left(\cdots\right.\right.\right.}{15\,a^{\frac{5}{2}}\left(\frac{1}{\tan\left(\frac{1}{2}\,x\right)}+\tan\left(\frac{1}{2}\,x\right)\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sec(x)^2)^(5/2),x, algorithm="giac")
```

```
[Out] 2/15*(15*(1/tan(1/2*x) + tan(1/2*x))^4*sgn(-tan(1/2*x)^2 + 1) - 40*(1/tan(1
/2*x) + tan(1/2*x))^2*sgn(-tan(1/2*x)^2 + 1) + 48*sgn(-tan(1/2*x)^2 + 1))/(
a^(5/2)*(1/tan(1/2*x) + tan(1/2*x))^5)
```

3.54 $\int \frac{1}{\left(a\sec^2(x)\right)^{7/2}}\,dx$

Optimal. Leaf size=74

$$\frac{16\tan(x)}{35a^3\sqrt{a\sec^2(x)}}+\frac{8\tan(x)}{35a^2\left(a\sec^2(x)\right)^{3/2}}+\frac{6\tan(x)}{35a\left(a\sec^2(x)\right)^{5/2}}+\frac{\tan(x)}{7\left(a\sec^2(x)\right)^{7/2}}$$

```
[Out] Tan[x]/(7*(a*Sec[x]^2)^(7/2)) + (6*Tan[x])/(35*a*(a*Sec[x]^2)^(5/2)) + (8*T
an[x])/(35*a^2*(a*Sec[x]^2)^(3/2)) + (16*Tan[x])/(35*a^3*Sqrt[a*Sec[x]^2])
```

Rubi [A] time = 0.0349597, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {4122, 192, 191}

$$\frac{16\tan(x)}{35a^3\sqrt{a\sec^2(x)}}+\frac{8\tan(x)}{35a^2\left(a\sec^2(x)\right)^{3/2}}+\frac{6\tan(x)}{35a\left(a\sec^2(x)\right)^{5/2}}+\frac{\tan(x)}{7\left(a\sec^2(x)\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Sec[x]^2)^(-7/2),x]
```

```
[Out] Tan[x]/(7*(a*Sec[x]^2)^(7/2)) + (6*Tan[x])/(35*a*(a*Sec[x]^2)^(5/2)) + (8*T
an[x])/(35*a^2*(a*Sec[x]^2)^(3/2)) + (16*Tan[x])/(35*a^3*Sqrt[a*Sec[x]^2])
```

Rule 4122

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFac
tors[Tan[e + f*x], x]}, Dist[(b*ff)/f, Subst[Int[(b + b*ff^2*x^2)^(p - 1),
x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{b, e, f, p}, x] && !IntegerQ[p]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$\int \frac{1}{\left(a\sec^2(x)\right)^{7/2}}\,dx = a\,\mathrm{Subst}\left(\int \frac{1}{\left(a+ax^2\right)^{9/2}}\,dx, x, \tan(x)\right)$$
$$= \frac{\tan(x)}{7\left(a\sec^2(x)\right)^{7/2}} + \frac{6}{7}\,\mathrm{Subst}\left(\int \frac{1}{\left(a+ax^2\right)^{7/2}}\,dx, x, \tan(x)\right)$$
$$= \frac{\tan(x)}{7\left(a\sec^2(x)\right)^{7/2}} + \frac{6\tan(x)}{35a\left(a\sec^2(x)\right)^{5/2}} + \frac{24\,\mathrm{Subst}\left(\int \frac{1}{\left(a+ax^2\right)^{5/2}}\,dx, x, \tan(x)\right)}{35a}$$
$$= \frac{\tan(x)}{7\left(a\sec^2(x)\right)^{7/2}} + \frac{6\tan(x)}{35a\left(a\sec^2(x)\right)^{5/2}} + \frac{8\tan(x)}{35a^2\left(a\sec^2(x)\right)^{3/2}} + \frac{16\,\mathrm{Subst}\left(\int \frac{1}{\left(a+ax^2\right)^{3/2}}\,dx, x, \tan(x)\right)}{35a^2}$$
$$= \frac{\tan(x)}{7\left(a\sec^2(x)\right)^{7/2}} + \frac{6\tan(x)}{35a\left(a\sec^2(x)\right)^{5/2}} + \frac{8\tan(x)}{35a^2\left(a\sec^2(x)\right)^{3/2}} + \frac{16\tan(x)}{35a^3\sqrt{a\sec^2(x)}}$$

Mathematica [A] time = 0.0347235, size = 42, normalized size = 0.57

$$\frac{(1225\sin(x)+245\sin(3x)+49\sin(5x)+5\sin(7x))\cos(x)\sqrt{a\sec^2(x)}}{2240a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Sec[x]^2)^(-7/2),x]
```

```
[Out] (Cos[x]*Sqrt[a*Sec[x]^2]*(1225*Sin[x] + 245*Sin[3*x] + 49*Sin[5*x] + 5*Sin[
7*x]))/(2240*a^4)
```

Maple [A] time = 0.081, size = 37, normalized size = 0.5

$$\frac{\sin(x)\left(5\,(\cos(x))^6+6\,(\cos(x))^4+8\,(\cos(x))^2+16\right)}{35\,(\cos(x))^7}\left(\frac{a}{(\cos(x))^2}\right)^{-\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*sec(x)^2)^(7/2),x)
```

```
[Out] 1/35*sin(x)*(5*cos(x)^6+6*cos(x)^4+8*cos(x)^2+16)/cos(x)^7/(a/cos(x)^2)^(7/
2)
```

Maxima [A] time = 1.7842, size = 38, normalized size = 0.51

$$\frac{5\sin(7x)+49\sin(5x)+245\sin(3x)+1225\sin(x)}{2240\,a^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sec(x)^2)^(7/2),x, algorithm="maxima")
```

```
[Out] 1/2240*(5*sin(7*x) + 49*sin(5*x) + 245*sin(3*x) + 1225*sin(x))/a^(7/2)
```

Fricas [A] time = 1.45123, size = 115, normalized size = 1.55

$$\frac{\left(5 \cos\left(x\right)^{7} + 6 \cos\left(x\right)^{5} + 8 \cos\left(x\right)^{3} + 16 \cos\left(x\right)\right) \sqrt{\frac{a}{\cos\left(x\right)^{2}}} \sin\left(x\right)}{35 \, a^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sec(x)^2)^(7/2),x, algorithm="fricas")
```

```
[Out] 1/35*(5*cos(x)^7 + 6*cos(x)^5 + 8*cos(x)^3 + 16*cos(x))*sqrt(a/cos(x)^2)*si
n(x)/a^4
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sec(x)**2)**(7/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.6085, size = 149, normalized size = 2.01

$$\frac{2 \left(35 \left(\frac{1}{\tan\left(\frac{1}{2} x\right)} + \tan\left(\frac{1}{2} x\right)\right)^{6} \operatorname{sgn}\left(-\tan\left(\frac{1}{2} x\right)^{2} + 1\right) - 140 \left(\frac{1}{\tan\left(\frac{1}{2} x\right)} + \tan\left(\frac{1}{2} x\right)\right)^{4} \operatorname{sgn}\left(-\tan\left(\frac{1}{2} x\right)^{2} + 1\right) + 336 \left(\frac{1}{\tan\left(\frac{1}{2} x\right.}\right.\right.}{35 \, a^{\frac{7}{2}} \left(\frac{1}{\tan\left(\frac{1}{2} x\right)} + \tan\left(\frac{1}{2} x\right)\right)^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sec(x)^2)^(7/2),x, algorithm="giac")
```

```
[Out] 2/35*(35*(1/tan(1/2*x) + tan(1/2*x))^6*sgn(-tan(1/2*x)^2 + 1) - 140*(1/tan(
1/2*x) + tan(1/2*x))^4*sgn(-tan(1/2*x)^2 + 1) + 336*(1/tan(1/2*x) + tan(1/2
*x))^2*sgn(-tan(1/2*x)^2 + 1) - 320*sgn(-tan(1/2*x)^2 + 1))/(a^(7/2)*(1/tan
(1/2*x) + tan(1/2*x))^7)
```

3.55 $\int \left(a\sec^3(x)\right)^{5/2}\,dx$

Optimal. Leaf size=117

$$\frac{2}{13}a^2\tan(x)\sec^4(x)\sqrt{a\sec^3(x)}+\frac{22}{117}a^2\tan(x)\sec^2(x)\sqrt{a\sec^3(x)}+\frac{154}{585}a^2\tan(x)\sqrt{a\sec^3(x)}-\frac{154}{195}a^2\cos^{\frac{3}{2}}(x)E\left(\frac{x}{2}\middle|2\right)\sqrt{a}$$

```
[Out] (-154*a^2*Cos[x]^(3/2)*EllipticE[x/2, 2]*Sqrt[a*Sec[x]^3])/195 + (154*a^2*C
os[x]*Sqrt[a*Sec[x]^3]*Sin[x])/195 + (154*a^2*Sqrt[a*Sec[x]^3]*Tan[x])/585
+ (22*a^2*Sec[x]^2*Sqrt[a*Sec[x]^3]*Tan[x])/117 + (2*a^2*Sec[x]^4*Sqrt[a*Se
c[x]^3]*Tan[x])/13
```

Rubi [A] time = 0.0522832, antiderivative size = 117, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {4123, 3768, 3771, 2639}

$$\frac{2}{13}a^2\tan(x)\sec^4(x)\sqrt{a\sec^3(x)}+\frac{22}{117}a^2\tan(x)\sec^2(x)\sqrt{a\sec^3(x)}+\frac{154}{585}a^2\tan(x)\sqrt{a\sec^3(x)}-\frac{154}{195}a^2\cos^{\frac{3}{2}}(x)E\left(\frac{x}{2}\middle|2\right)\sqrt{a}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Sec[x]^3)^(5/2),x]
```

```
[Out] (-154*a^2*Cos[x]^(3/2)*EllipticE[x/2, 2]*Sqrt[a*Sec[x]^3])/195 + (154*a^2*C
os[x]*Sqrt[a*Sec[x]^3]*Sin[x])/195 + (154*a^2*Sqrt[a*Sec[x]^3]*Tan[x])/585
+ (22*a^2*Sec[x]^2*Sqrt[a*Sec[x]^3]*Tan[x])/117 + (2*a^2*Sec[x]^4*Sqrt[a*Se
c[x]^3]*Tan[x])/13
```

Rule 4123

```
Int[((b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> Dist[(b
^IntPart[p]*(b*(c*Sec[e + f*x])^n)^FracPart[p])/(c*Sec[e + f*x])^(n*FracPar
t[p]), Int[(c*Sec[e + f*x])^(n*p), x], x] /; FreeQ[{b, c, e, f, n, p}, x] &
& !IntegerQ[p]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \left(a\sec^3(x)\right)^{5/2} dx = \frac{\left(a^2\sqrt{a\sec^3(x)}\right)\int \sec^{\frac{15}{2}}(x)\,dx}{\sec^{\frac{3}{2}}(x)}$$
$$= \frac{2}{13}a^2\sec^4(x)\sqrt{a\sec^3(x)}\tan(x) + \frac{\left(11a^2\sqrt{a\sec^3(x)}\right)\int \sec^{\frac{11}{2}}(x)\,dx}{13\sec^{\frac{3}{2}}(x)}$$
$$= \frac{22}{117}a^2\sec^2(x)\sqrt{a\sec^3(x)}\tan(x) + \frac{2}{13}a^2\sec^4(x)\sqrt{a\sec^3(x)}\tan(x) + \frac{\left(77a^2\sqrt{a\sec^3(x)}\right)\int \sec^{\frac{7}{2}}(x}{117\sec^{\frac{3}{2}}(x)}$$
$$= \frac{154}{585}a^2\sqrt{a\sec^3(x)}\tan(x) + \frac{22}{117}a^2\sec^2(x)\sqrt{a\sec^3(x)}\tan(x) + \frac{2}{13}a^2\sec^4(x)\sqrt{a\sec^3(x)}\tan(x) +$$
$$= \frac{154}{195}a^2\cos(x)\sqrt{a\sec^3(x)}\sin(x) + \frac{154}{585}a^2\sqrt{a\sec^3(x)}\tan(x) + \frac{22}{117}a^2\sec^2(x)\sqrt{a\sec^3(x)}\tan(x) +$$
$$= \frac{154}{195}a^2\cos(x)\sqrt{a\sec^3(x)}\sin(x) + \frac{154}{585}a^2\sqrt{a\sec^3(x)}\tan(x) + \frac{22}{117}a^2\sec^2(x)\sqrt{a\sec^3(x)}\tan(x) +$$
$$= -\frac{154}{195}a^2\cos^{\frac{3}{2}}(x)E\left(\frac{x}{2}\middle|2\right)\sqrt{a\sec^3(x)} + \frac{154}{195}a^2\cos(x)\sqrt{a\sec^3(x)}\sin(x) + \frac{154}{585}a^2\sqrt{a\sec^3(x)}\tan(x$$

Mathematica [A] time = 0.0933119, size = 59, normalized size = 0.5

$$-\frac{2}{585}a\sec(x)\left(a\sec^3(x)\right)^{3/2}\left(-45\tan(x) - 231\sin(x)\cos^5(x) - 77\sin(x)\cos^3(x) + 231\cos^{\frac{11}{2}}(x)E\left(\frac{x}{2}\middle|2\right) - 55\sin(x)\cos\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Sec[x]^3)^(5/2),x]
```

```
[Out] (-2*a*Sec[x]*(a*Sec[x]^3)^(3/2)*(231*Cos[x]^(11/2)*EllipticE[x/2, 2] - 55*C
os[x]*Sin[x] - 77*Cos[x]^3*Sin[x] - 231*Cos[x]^5*Sin[x] - 45*Tan[x]))/585
```

Maple [C] time = 0.368, size = 223, normalized size = 1.9

$$-\frac{2\left(\cos(x)+1\right)^2\left(-1+\cos(x)\right)^2\cos(x)}{585\left(\sin(x)\right)^5}\left(231\,i\left(\cos(x)\right)^7\sin(x)\sqrt{\frac{\cos(x)}{\cos(x)+1}}\sqrt{\left(\cos(x)+1\right)^{-1}}\mathrm{EllipticF}\left(\frac{i\left(-1+\cos(\right.}{\sin(x)}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sec(x)^3)^(5/2),x)
```

```
[Out] -2/585*(cos(x)+1)^2*(-1+cos(x))^2*(231*I*cos(x)^7*sin(x)*(cos(x)/(cos(x)+1)
)^(1/2)*(1/(cos(x)+1))^(1/2)*EllipticF(I*(-1+cos(x))/sin(x),I)-231*I*cos(x)
^7*sin(x)*(cos(x)/(cos(x)+1))^(1/2)*(1/(cos(x)+1))^(1/2)*EllipticE(I*(-1+co
s(x))/sin(x),I)+231*I*cos(x)^6*sin(x)*(cos(x)/(cos(x)+1))^(1/2)*(1/(cos(x)+
1))^(1/2)*EllipticF(I*(-1+cos(x))/sin(x),I)-231*I*cos(x)^6*sin(x)*(cos(x)/(
cos(x)+1))^(1/2)*(1/(cos(x)+1))^(1/2)*EllipticE(I*(-1+cos(x))/sin(x),I)+231
*cos(x)^7-154*cos(x)^6-22*cos(x)^4-10*cos(x)^2-45)*cos(x)*(a/cos(x)^3)^(5/2
)/sin(x)^5
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a \sec \left(x\right)^{3}\right)^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sec(x)^3)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sec(x)^3)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\sqrt{a \sec \left(x\right)^{3}} a^{2} \sec \left(x\right)^{6}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sec(x)^3)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a*sec(x)^3)*a^2*sec(x)^6, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sec(x)**3)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a \sec \left(x\right)^{3}\right)^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sec(x)^3)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((a*sec(x)^3)^(5/2), x)
```

3.56 $\int \left(a\sec^3(x)\right)^{3/2}\,dx$

Optimal. Leaf size=65

$$\frac{10}{21}a\cos^{\frac{3}{2}}(x)\text{EllipticF}\left(\frac{x}{2},2\right)\sqrt{a\sec^3(x)}+\frac{10}{21}a\sin(x)\sqrt{a\sec^3(x)}+\frac{2}{7}a\tan(x)\sec(x)\sqrt{a\sec^3(x)}$$

```
[Out] (10*a*Cos[x]^(3/2)*EllipticF[x/2, 2]*Sqrt[a*Sec[x]^3])/21 + (10*a*Sqrt[a*Se
c[x]^3]*Sin[x])/21 + (2*a*Sec[x]*Sqrt[a*Sec[x]^3]*Tan[x])/7
```

Rubi [A] time = 0.0353327, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {4123, 3768, 3771, 2641}

$$\frac{10}{21}a\sin(x)\sqrt{a\sec^3(x)}+\frac{2}{7}a\tan(x)\sec(x)\sqrt{a\sec^3(x)}+\frac{10}{21}a\cos^{\frac{3}{2}}(x)F\left(\frac{x}{2}\middle|2\right)\sqrt{a\sec^3(x)}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Sec[x]^3)^(3/2),x]
```

```
[Out] (10*a*Cos[x]^(3/2)*EllipticF[x/2, 2]*Sqrt[a*Sec[x]^3])/21 + (10*a*Sqrt[a*Se
c[x]^3]*Sin[x])/21 + (2*a*Sec[x]*Sqrt[a*Sec[x]^3]*Tan[x])/7
```

Rule 4123

```
Int[((b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> Dist[(b
^IntPart[p]*(b*(c*Sec[e + f*x])^n)^FracPart[p])/(c*Sec[e + f*x])^(n*FracPar
t[p]), Int[(c*Sec[e + f*x])^(n*p), x], x] /; FreeQ[{b, c, e, f, n, p}, x] &
&  !IntegerQ[p]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \left(a\sec^3(x)\right)^{3/2} dx = \frac{\left(a\sqrt{a\sec^3(x)}\right)\int \sec^{\frac{9}{2}}(x)\,dx}{\sec^{\frac{3}{2}}(x)}$$
$$= \frac{2}{7}a\sec(x)\sqrt{a\sec^3(x)}\tan(x) + \frac{\left(5a\sqrt{a\sec^3(x)}\right)\int \sec^{\frac{5}{2}}(x)\,dx}{7\sec^{\frac{3}{2}}(x)}$$
$$= \frac{10}{21}a\sqrt{a\sec^3(x)}\sin(x) + \frac{2}{7}a\sec(x)\sqrt{a\sec^3(x)}\tan(x) + \frac{\left(5a\sqrt{a\sec^3(x)}\right)\int \sqrt{\sec(x)}\,dx}{21\sec^{\frac{3}{2}}(x)}$$
$$= \frac{10}{21}a\sqrt{a\sec^3(x)}\sin(x) + \frac{2}{7}a\sec(x)\sqrt{a\sec^3(x)}\tan(x) + \frac{1}{21}\left(5a\cos^{\frac{3}{2}}(x)\sqrt{a\sec^3(x)}\right)\int \frac{1}{\sqrt{\cos(x)}}dx$$
$$= \frac{10}{21}a\cos^{\frac{3}{2}}(x)F\left(\frac{x}{2}\middle|2\right)\sqrt{a\sec^3(x)} + \frac{10}{21}a\sqrt{a\sec^3(x)}\sin(x) + \frac{2}{7}a\sec(x)\sqrt{a\sec^3(x)}\tan(x)$$

Mathematica [A] time = 0.0346709, size = 43, normalized size = 0.66

$$\frac{2}{21}a\sec(x)\sqrt{a\sec^3(x)}\left(5\cos^{\frac{5}{2}}(x)\text{EllipticF}\left(\frac{x}{2},2\right) + 3\tan(x) + 5\sin(x)\cos(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Sec[x]^3)^(3/2),x]
```

```
[Out] (2*a*Sec[x]*Sqrt[a*Sec[x]^3]*(5*Cos[x]^(5/2)*EllipticF[x/2, 2] + 5*Cos[x]*S
in[x] + 3*Tan[x]))/21
```

Maple [C] time = 0.182, size = 87, normalized size = 1.3

$$-\frac{2\,(\cos(x)+1)^2(-1+\cos(x))\cos(x)}{21\,(\sin(x))^3}\left(5\,i\,(\cos(x))^3\sin(x)\sqrt{(\cos(x)+1)^{-1}}\sqrt{\frac{\cos(x)}{\cos(x)+1}}\text{EllipticF}\left(\frac{i\,(-1+\cos(x))}{\sin(x)},i\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sec(x)^3)^(3/2),x)
```

```
[Out] -2/21*(cos(x)+1)^2*(-1+cos(x))*(5*I*cos(x)^3*sin(x)*(1/(cos(x)+1))^(1/2)*(c
os(x)/(cos(x)+1))^(1/2)*EllipticF(I*(-1+cos(x))/sin(x),I)-5*cos(x)^3+5*cos(
x)^2-3*cos(x)+3)*cos(x)*(a/cos(x)^3)^(3/2)/sin(x)^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\sec(x)^3\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sec(x)^3)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sec(x)^3)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{a\sec\left(x\right)^{3}}a\sec\left(x\right)^{3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sec(x)^3)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a*sec(x)^3)*a*sec(x)^3, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\left(a\sec^{3}\left(x\right)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sec(x)**3)**(3/2),x)
```

```
[Out] Integral((a*sec(x)**3)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(a\sec\left(x\right)^{3}\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sec(x)^3)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a*sec(x)^3)^(3/2), x)
```

3.57 $\int \sqrt{a\sec^3(x)}\,dx$

Optimal. Leaf size=42

$$2\sin(x)\cos(x)\sqrt{a\sec^3(x)} - 2\cos^{\frac{3}{2}}(x)E\left(\frac{x}{2}\middle|2\right)\sqrt{a\sec^3(x)}$$

```
[Out] -2*Cos[x]^(3/2)*EllipticE[x/2, 2]*Sqrt[a*Sec[x]^3] + 2*Cos[x]*Sqrt[a*Sec[x]
^3]*Sin[x]
```

Rubi [A] time = 0.0256077, antiderivative size = 42, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {4123, 3768, 3771, 2639}

$$2\sin(x)\cos(x)\sqrt{a\sec^3(x)} - 2\cos^{\frac{3}{2}}(x)E\left(\frac{x}{2}\middle|2\right)\sqrt{a\sec^3(x)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*Sec[x]^3],x]
```

```
[Out] -2*Cos[x]^(3/2)*EllipticE[x/2, 2]*Sqrt[a*Sec[x]^3] + 2*Cos[x]*Sqrt[a*Sec[x]
^3]*Sin[x]
```

Rule 4123

```
Int[((b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> Dist[(b
^IntPart[p]*(b*(c*Sec[e + f*x])^n)^FracPart[p])/(c*Sec[e + f*x])^(n*FracPar
t[p]), Int[(c*Sec[e + f*x])^(n*p), x], x] /; FreeQ[{b, c, e, f, n, p}, x] &
& !IntegerQ[p]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sqrt{a\sec^3(x)}\,dx = \frac{\sqrt{a\sec^3(x)}\int \sec^{\frac{3}{2}}(x)\,dx}{\sec^{\frac{3}{2}}(x)}$$
$$= 2\cos(x)\sqrt{a\sec^3(x)}\sin(x) - \frac{\sqrt{a\sec^3(x)}\int \frac{1}{\sqrt{\sec(x)}}\,dx}{\sec^{\frac{3}{2}}(x)}$$
$$= 2\cos(x)\sqrt{a\sec^3(x)}\sin(x) - \left(\cos^{\frac{3}{2}}(x)\sqrt{a\sec^3(x)}\right)\int \sqrt{\cos(x)}\,dx$$
$$= -2\cos^{\frac{3}{2}}(x)E\left(\frac{x}{2}\middle|2\right)\sqrt{a\sec^3(x)} + 2\cos(x)\sqrt{a\sec^3(x)}\sin(x)$$

Mathematica [A] time = 0.0171513, size = 32, normalized size = 0.76

$$2\cos(x)\sqrt{a\sec^3(x)}\left(\sin(x) - \sqrt{\cos(x)}E\left(\frac{x}{2}\middle|2\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*Sec[x]^3],x]

[Out] 2*Cos[x]*Sqrt[a*Sec[x]^3]*(-(Sqrt[Cos[x]]*EllipticE[x/2, 2]) + Sin[x])
```

Maple [C] time = 0.205, size = 191, normalized size = 4.6

$$2\,\frac{(\cos(x)+1)^2(-1+\cos(x))^2\cos(x)}{(\sin(x))^5}\left(i\cos(x)\sin(x)\sqrt{(\cos(x)+1)^{-1}}\sqrt{\frac{\cos(x)}{\cos(x)+1}}\mathrm{EllipticE}\left(\frac{i(-1+\cos(x))}{\sin(x)},i\right)-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sec(x)^3)^(1/2),x)

[Out] 2*(cos(x)+1)^2*(-1+cos(x))^2*(I*cos(x)*sin(x)*(1/(cos(x)+1))^(1/2)*(cos(x)/
(cos(x)+1))^(1/2)*EllipticE(I*(-1+cos(x))/sin(x),I)-I*(1/(cos(x)+1))^(1/2)*
(cos(x)/(cos(x)+1))^(1/2)*EllipticF(I*(-1+cos(x))/sin(x),I)*cos(x)*sin(x)+I
*sin(x)*(1/(cos(x)+1))^(1/2)*(cos(x)/(cos(x)+1))^(1/2)*EllipticE(I*(-1+cos(
x))/sin(x),I)-I*EllipticF(I*(-1+cos(x))/sin(x),I)*(1/(cos(x)+1))^(1/2)*(cos
(x)/(cos(x)+1))^(1/2)*sin(x)-cos(x)+1)*cos(x)*(a/cos(x)^3)^(1/2)/sin(x)^5
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a\sec(x)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sec(x)^3)^(1/2),x, algorithm="maxima")

[Out] integrate(sqrt(a*sec(x)^3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{a\sec\left(x\right)^{3}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sec(x)^3)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a*sec(x)^3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a\sec^{3}\left(x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sec(x)**3)**(1/2),x)
```

```
[Out] Integral(sqrt(a*sec(x)**3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a\sec\left(x\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sec(x)^3)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a*sec(x)^3), x)
```

3.58 $\int \frac{1}{\sqrt{a \sec^3(x)}} \, dx$

Optimal. Leaf size=44

$$\frac{2\text{EllipticF}\left(\frac{x}{2},2\right)}{3\cos^{\frac{3}{2}}(x)\sqrt{a\sec^3(x)}}+\frac{2\tan(x)}{3\sqrt{a\sec^3(x)}}$$

```
[Out] (2*EllipticF[x/2, 2])/(3*Cos[x]^(3/2)*Sqrt[a*Sec[x]^3]) + (2*Tan[x])/(3*Sqr
t[a*Sec[x]^3])
```

Rubi [A] time = 0.0270201, antiderivative size = 44, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {4123, 3769, 3771, 2641}

$$\frac{2\tan(x)}{3\sqrt{a\sec^3(x)}}+\frac{2F\left(\frac{x}{2}\middle|2\right)}{3\cos^{\frac{3}{2}}(x)\sqrt{a\sec^3(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a*Sec[x]^3],x]
```

```
[Out] (2*EllipticF[x/2, 2])/(3*Cos[x]^(3/2)*Sqrt[a*Sec[x]^3]) + (2*Tan[x])/(3*Sqr
t[a*Sec[x]^3])
```

Rule 4123

```
Int[((b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> Dist[(b
^IntPart[p]*(b*(c*Sec[e + f*x])^n)^FracPart[p])/(c*Sec[e + f*x])^(n*FracPar
t[p]), Int[(c*Sec[e + f*x])^(n*p), x], x] /; FreeQ[{b, c, e, f, n, p}, x] &
&  !IntegerQ[p]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{1}{\sqrt{a\sec^3(x)}}\,dx = \frac{\sec^{\frac{3}{2}}(x)\int \frac{1}{\sec^{\frac{3}{2}}(x)}\,dx}{\sqrt{a\sec^3(x)}}$$
$$= \frac{2\tan(x)}{3\sqrt{a\sec^3(x)}} + \frac{\sec^{\frac{3}{2}}(x)\int \sqrt{\sec(x)}\,dx}{3\sqrt{a\sec^3(x)}}$$
$$= \frac{2\tan(x)}{3\sqrt{a\sec^3(x)}} + \frac{\int \frac{1}{\sqrt{\cos(x)}}\,dx}{3\cos^{\frac{3}{2}}(x)\sqrt{a\sec^3(x)}}$$
$$= \frac{2F\left(\frac{x}{2}\middle|2\right)}{3\cos^{\frac{3}{2}}(x)\sqrt{a\sec^3(x)}} + \frac{2\tan(x)}{3\sqrt{a\sec^3(x)}}$$

Mathematica [A] time = 0.0398245, size = 31, normalized size = 0.7

$$\frac{2\left(\frac{\text{EllipticF}\left(\frac{x}{2},2\right)}{\cos^{\frac{3}{2}}(x)} + \tan(x)\right)}{3\sqrt{a\sec^3(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a*Sec[x]^3],x]
```

```
[Out] (2*(EllipticF[x/2, 2]/Cos[x]^(3/2) + Tan[x]))/(3*Sqrt[a*Sec[x]^3])
```

Maple [C] time = 0.14, size = 76, normalized size = 1.7

$$\frac{(-2+2\cos(x))(\cos(x)+1)^2}{3(\cos(x))^2(\sin(x))^3}\left(-i\text{EllipticF}\left(\frac{i(-1+\cos(x))}{\sin(x)},i\right)\sqrt{(\cos(x)+1)^{-1}}\sqrt{\frac{\cos(x)}{\cos(x)+1}}\sin(x)+(\cos(x))^2-\cos($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*sec(x)^3)^(1/2),x)
```

```
[Out] 2/3*(-1+cos(x))*(-I*EllipticF(I*(-1+cos(x))/sin(x),I)*(1/(cos(x)+1))^(1/2)*
(cos(x)/(cos(x)+1))^(1/2)*sin(x)+cos(x)^2-cos(x))*(cos(x)+1)^2/cos(x)^2/sin
(x)^3/(a/cos(x)^3)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a\sec(x)^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sec(x)^3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(a*sec(x)^3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{a\sec\left(x\right)^{3}}}{a\sec\left(x\right)^{3}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sec(x)^3)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a*sec(x)^3)/(a*sec(x)^3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a\sec^{3}\left(x\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sec(x)**3)**(1/2),x)
```

```
[Out] Integral(1/sqrt(a*sec(x)**3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a\sec\left(x\right)^{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sec(x)^3)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(a*sec(x)^3), x)
```

3.59 $\int \frac{1}{\left(a\sec^3(x)\right)^{3/2}}\,dx$

Optimal. Leaf size=73

$$\frac{14\sin(x)}{45a\sqrt{a\sec^3(x)}}+\frac{2\sin(x)\cos^2(x)}{9a\sqrt{a\sec^3(x)}}+\frac{14E\left(\frac{x}{2}\middle|2\right)}{15a\cos^{\frac{3}{2}}(x)\sqrt{a\sec^3(x)}}$$

```
[Out] (14*EllipticE[x/2, 2])/(15*a*Cos[x]^(3/2)*Sqrt[a*Sec[x]^3]) + (14*Sin[x])/(
45*a*Sqrt[a*Sec[x]^3]) + (2*Cos[x]^2*Sin[x])/(9*a*Sqrt[a*Sec[x]^3])
```

Rubi [A] time = 0.0367949, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {4123, 3769, 3771, 2639}

$$\frac{14\sin(x)}{45a\sqrt{a\sec^3(x)}}+\frac{2\sin(x)\cos^2(x)}{9a\sqrt{a\sec^3(x)}}+\frac{14E\left(\frac{x}{2}\middle|2\right)}{15a\cos^{\frac{3}{2}}(x)\sqrt{a\sec^3(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Sec[x]^3)^(-3/2),x]
```

```
[Out] (14*EllipticE[x/2, 2])/(15*a*Cos[x]^(3/2)*Sqrt[a*Sec[x]^3]) + (14*Sin[x])/(
45*a*Sqrt[a*Sec[x]^3]) + (2*Cos[x]^2*Sin[x])/(9*a*Sqrt[a*Sec[x]^3])
```

Rule 4123

```
Int[((b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> Dist[(b
^IntPart[p]*(b*(c*Sec[e + f*x])^n)^FracPart[p])/(c*Sec[e + f*x])^(n*FracPar
t[p]), Int[(c*Sec[e + f*x])^(n*p), x], x] /; FreeQ[{b, c, e, f, n, p}, x] &
&  !IntegerQ[p]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{1}{\left(a\sec^3(x)\right)^{3/2}}\,dx = \frac{\sec^{\frac{3}{2}}(x)\int \frac{1}{\sec^{\frac{9}{2}}(x)}\,dx}{a\sqrt{a\sec^3(x)}}$$

$$= \frac{2\cos^2(x)\sin(x)}{9a\sqrt{a\sec^3(x)}} + \frac{\left(7\sec^{\frac{3}{2}}(x)\right)\int \frac{1}{\sec^{\frac{5}{2}}(x)}\,dx}{9a\sqrt{a\sec^3(x)}}$$

$$= \frac{14\sin(x)}{45a\sqrt{a\sec^3(x)}} + \frac{2\cos^2(x)\sin(x)}{9a\sqrt{a\sec^3(x)}} + \frac{\left(7\sec^{\frac{3}{2}}(x)\right)\int \frac{1}{\sqrt{\sec(x)}}\,dx}{15a\sqrt{a\sec^3(x)}}$$

$$= \frac{14\sin(x)}{45a\sqrt{a\sec^3(x)}} + \frac{2\cos^2(x)\sin(x)}{9a\sqrt{a\sec^3(x)}} + \frac{7\int \sqrt{\cos(x)}\,dx}{15a\cos^{\frac{3}{2}}(x)\sqrt{a\sec^3(x)}}$$

$$= \frac{14E\left(\frac{x}{2}\middle|2\right)}{15a\cos^{\frac{3}{2}}(x)\sqrt{a\sec^3(x)}} + \frac{14\sin(x)}{45a\sqrt{a\sec^3(x)}} + \frac{2\cos^2(x)\sin(x)}{9a\sqrt{a\sec^3(x)}}$$

Mathematica [A] time = 0.0940153, size = 43, normalized size = 0.59

$$\frac{33\sin(x) + 5\sin(3x) + \frac{84E\left(\frac{x}{2}\middle|2\right)}{\cos^{\frac{3}{2}}(x)}}{90a\sqrt{a\sec^3(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Sec[x]^3)^(-3/2),x]

[Out] ((84*EllipticE[x/2, 2])/Cos[x]^(3/2) + 33*Sin[x] + 5*Sin[3*x])/(90*a*Sqrt[a
*Sec[x]^3])
```

Maple [C] time = 0.156, size = 198, normalized size = 2.7

$$-\frac{2}{45\,(\cos(x))^5\sin(x)}\left(5\,(\cos(x))^6 - 21\,i\sqrt{(\cos(x)+1)^{-1}}\sqrt{\frac{\cos(x)}{\cos(x)+1}}\mathrm{EllipticF}\left(\frac{i\,(-1+\cos(x))}{\sin(x)},i\right)\cos(x)\sin(x) + \right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*sec(x)^3)^(3/2),x)

[Out] -2/45*(5*cos(x)^6-21*I*(1/(cos(x)+1))^(1/2)*(cos(x)/(cos(x)+1))^(1/2)*Ellip
ticF(I*(-1+cos(x))/sin(x),I)*cos(x)*sin(x)+21*I*cos(x)*sin(x)*(1/(cos(x)+1)
)^(1/2)*(cos(x)/(cos(x)+1))^(1/2)*EllipticE(I*(-1+cos(x))/sin(x),I)-21*I*El
lipticF(I*(-1+cos(x))/sin(x),I)*(1/(cos(x)+1))^(1/2)*(cos(x)/(cos(x)+1))^(1
/2)*sin(x)+21*I*sin(x)*(1/(cos(x)+1))^(1/2)*(cos(x)/(cos(x)+1))^(1/2)*Ellip
ticE(I*(-1+cos(x))/sin(x),I)+2*cos(x)^4+14*cos(x)^2-21*cos(x))/cos(x)^5/sin
(x)/(a/cos(x)^3)^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a\sec(x)^3\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sec(x)^3)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sec(x)^3)^(-3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{a\sec\left(x\right)^{3}}}{a^{2}\sec\left(x\right)^{6}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sec(x)^3)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a*sec(x)^3)/(a^2*sec(x)^6), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\left(a\sec^{3}\left(x\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sec(x)**3)**(3/2),x)
```

```
[Out] Integral((a*sec(x)**3)**(-3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\left(a\sec\left(x\right)^{3}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sec(x)^3)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a*sec(x)^3)^(-3/2), x)
```

3.60 $\int \frac{1}{\left(a\sec^3(x)\right)^{5/2}}\,dx$

Optimal. Leaf size=117

$$\frac{26\text{EllipticF}\left(\frac{x}{2},2\right)}{77a^2\cos^{\frac{3}{2}}(x)\sqrt{a\sec^3(x)}}+\frac{26\tan(x)}{77a^2\sqrt{a\sec^3(x)}}+\frac{2\sin(x)\cos^5(x)}{15a^2\sqrt{a\sec^3(x)}}+\frac{26\sin(x)\cos^3(x)}{165a^2\sqrt{a\sec^3(x)}}+\frac{78\sin(x)\cos(x)}{385a^2\sqrt{a\sec^3(x)}}$$

```
[Out] (26*EllipticF[x/2, 2])/(77*a^2*Cos[x]^(3/2)*Sqrt[a*Sec[x]^3]) + (78*Cos[x]*
Sin[x])/(385*a^2*Sqrt[a*Sec[x]^3]) + (26*Cos[x]^3*Sin[x])/(165*a^2*Sqrt[a*S
ec[x]^3]) + (2*Cos[x]^5*Sin[x])/(15*a^2*Sqrt[a*Sec[x]^3]) + (26*Tan[x])/(77
*a^2*Sqrt[a*Sec[x]^3])
```

Rubi [A] time = 0.0561162, antiderivative size = 117, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {4123, 3769, 3771, 2641}

$$\frac{26\tan(x)}{77a^2\sqrt{a\sec^3(x)}}+\frac{2\sin(x)\cos^5(x)}{15a^2\sqrt{a\sec^3(x)}}+\frac{26\sin(x)\cos^3(x)}{165a^2\sqrt{a\sec^3(x)}}+\frac{26F\left(\frac{x}{2}\middle|2\right)}{77a^2\cos^{\frac{3}{2}}(x)\sqrt{a\sec^3(x)}}+\frac{78\sin(x)\cos(x)}{385a^2\sqrt{a\sec^3(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Sec[x]^3)^(-5/2),x]
```

```
[Out] (26*EllipticF[x/2, 2])/(77*a^2*Cos[x]^(3/2)*Sqrt[a*Sec[x]^3]) + (78*Cos[x]*
Sin[x])/(385*a^2*Sqrt[a*Sec[x]^3]) + (26*Cos[x]^3*Sin[x])/(165*a^2*Sqrt[a*S
ec[x]^3]) + (2*Cos[x]^5*Sin[x])/(15*a^2*Sqrt[a*Sec[x]^3]) + (26*Tan[x])/(77
*a^2*Sqrt[a*Sec[x]^3])
```

Rule 4123

```
Int[((b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> Dist[(b
^IntPart[p]*(b*(c*Sec[e + f*x])^n)^FracPart[p])/(c*Sec[e + f*x])^(n*FracPar
t[p]), Int[(c*Sec[e + f*x])^(n*p), x], x] /; FreeQ[{b, c, e, f, n, p}, x] &
&  !IntegerQ[p]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{1}{\left(a\sec^3(x)\right)^{5/2}}\,dx = \frac{\sec^{\frac{3}{2}}(x)\int \frac{1}{\sec^{\frac{15}{2}}(x)}\,dx}{a^2\sqrt{a\sec^3(x)}}$$

$$= \frac{2\cos^5(x)\sin(x)}{15a^2\sqrt{a\sec^3(x)}} + \frac{\left(13\sec^{\frac{3}{2}}(x)\right)\int \frac{1}{\sec^{\frac{11}{2}}(x)}\,dx}{15a^2\sqrt{a\sec^3(x)}}$$

$$= \frac{26\cos^3(x)\sin(x)}{165a^2\sqrt{a\sec^3(x)}} + \frac{2\cos^5(x)\sin(x)}{15a^2\sqrt{a\sec^3(x)}} + \frac{\left(39\sec^{\frac{3}{2}}(x)\right)\int \frac{1}{\sec^{\frac{7}{2}}(x)}\,dx}{55a^2\sqrt{a\sec^3(x)}}$$

$$= \frac{78\cos(x)\sin(x)}{385a^2\sqrt{a\sec^3(x)}} + \frac{26\cos^3(x)\sin(x)}{165a^2\sqrt{a\sec^3(x)}} + \frac{2\cos^5(x)\sin(x)}{15a^2\sqrt{a\sec^3(x)}} + \frac{\left(39\sec^{\frac{3}{2}}(x)\right)\int \frac{1}{\sec^{\frac{3}{2}}(x)}\,dx}{77a^2\sqrt{a\sec^3(x)}}$$

$$= \frac{78\cos(x)\sin(x)}{385a^2\sqrt{a\sec^3(x)}} + \frac{26\cos^3(x)\sin(x)}{165a^2\sqrt{a\sec^3(x)}} + \frac{2\cos^5(x)\sin(x)}{15a^2\sqrt{a\sec^3(x)}} + \frac{26\tan(x)}{77a^2\sqrt{a\sec^3(x)}} + \frac{\left(13\sec^{\frac{3}{2}}(x)\right)\int \sqrt{\sec}}{77a^2\sqrt{a\sec^3(x}}$$

$$= \frac{78\cos(x)\sin(x)}{385a^2\sqrt{a\sec^3(x)}} + \frac{26\cos^3(x)\sin(x)}{165a^2\sqrt{a\sec^3(x)}} + \frac{2\cos^5(x)\sin(x)}{15a^2\sqrt{a\sec^3(x)}} + \frac{26\tan(x)}{77a^2\sqrt{a\sec^3(x)}} + \frac{13\int \frac{1}{\sqrt{\cos(x)}}\,dx}{77a^2\cos^{\frac{3}{2}}(x)\sqrt{a\sec}}$$

$$= \frac{26F\left(\frac{x}{2}\middle|2\right)}{77a^2\cos^{\frac{3}{2}}(x)\sqrt{a\sec^3(x)}} + \frac{78\cos(x)\sin(x)}{385a^2\sqrt{a\sec^3(x)}} + \frac{26\cos^3(x)\sin(x)}{165a^2\sqrt{a\sec^3(x)}} + \frac{2\cos^5(x)\sin(x)}{15a^2\sqrt{a\sec^3(x)}} + \frac{26\tan(x}{77a^2\sqrt{a\sec}}$$

Mathematica [A] time = 0.096753, size = 59, normalized size = 0.5

$$\frac{\cos(x)\sqrt{a\sec^3(x)}\left(24960\sqrt{\cos(x)}\text{EllipticF}\left(\frac{x}{2},2\right)+19122\sin(2x)+4406\sin(4x)+826\sin(6x)+77\sin(8x)\right)}{73920a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Sec[x]^3)^(-5/2),x]
```

```
[Out] (Cos[x]*Sqrt[a*Sec[x]^3]*(24960*Sqrt[Cos[x]]*EllipticF[x/2, 2] + 19122*Sin[
2*x] + 4406*Sin[4*x] + 826*Sin[6*x] + 77*Sin[8*x]))/(73920*a^3)
```

Maple [C] time = 0.206, size = 114, normalized size = 1.

$$\frac{(-2+2\cos(x))(\cos(x)+1)^2}{1155(\cos(x))^8(\sin(x))^3}\left(77(\cos(x))^8-77(\cos(x))^7+91(\cos(x))^6-91(\cos(x))^5-195\,i\text{EllipticF}\left(\frac{i(-1+\cos}{\sin(x)}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*sec(x)^3)^(5/2),x)
```

```
[Out] 2/1155*(-1+cos(x))*(77*cos(x)^8-77*cos(x)^7+91*cos(x)^6-91*cos(x)^5-195*I*E
llipticF(I*(-1+cos(x))/sin(x),I)*(1/(cos(x)+1))^(1/2)*(cos(x)/(cos(x)+1))^(
1/2)*sin(x)+117*cos(x)^4-117*cos(x)^3+195*cos(x)^2-195*cos(x))*(cos(x)+1)^2
/cos(x)^8/sin(x)^3/(a/cos(x)^3)^(5/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a \sec(x)^3\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sec(x)^3)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sec(x)^3)^(-5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{a \sec(x)^3}}{a^3 \sec(x)^9}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sec(x)^3)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a*sec(x)^3)/(a^3*sec(x)^9), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a \sec^3(x)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sec(x)**3)**(5/2),x)
```

```
[Out] Integral((a*sec(x)**3)**(-5/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a \sec(x)^3\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sec(x)^3)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((a*sec(x)^3)^(-5/2), x)
```

3.61 $\int \left(a\sec^4(x)\right)^{7/2}\,dx$

Optimal. Leaf size=163

$$a^3\sin(x)\cos(x)\sqrt{a\sec^4(x)}+\frac{1}{13}a^3\sin^2(x)\tan^{11}(x)\sqrt{a\sec^4(x)}+\frac{6}{11}a^3\sin^2(x)\tan^9(x)\sqrt{a\sec^4(x)}+\frac{5}{3}a^3\sin^2(x)\tan^7(x)\sqrt{a}$$

```
[Out] a^3*Cos[x]*Sqrt[a*Sec[x]^4]*Sin[x] + 2*a^3*Sqrt[a*Sec[x]^4]*Sin[x]^2*Tan[x]
 + 3*a^3*Sqrt[a*Sec[x]^4]*Sin[x]^2*Tan[x]^3 + (20*a^3*Sqrt[a*Sec[x]^4]*Sin[
x]^2*Tan[x]^5)/7 + (5*a^3*Sqrt[a*Sec[x]^4]*Sin[x]^2*Tan[x]^7)/3 + (6*a^3*Sq
rt[a*Sec[x]^4]*Sin[x]^2*Tan[x]^9)/11 + (a^3*Sqrt[a*Sec[x]^4]*Sin[x]^2*Tan[x
]^11)/13
```

Rubi [A] time = 0.0390895, antiderivative size = 163, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {4123, 3767}

$$a^3\sin(x)\cos(x)\sqrt{a\sec^4(x)}+\frac{1}{13}a^3\sin^2(x)\tan^{11}(x)\sqrt{a\sec^4(x)}+\frac{6}{11}a^3\sin^2(x)\tan^9(x)\sqrt{a\sec^4(x)}+\frac{5}{3}a^3\sin^2(x)\tan^7(x)\sqrt{a}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Sec[x]^4)^(7/2),x]
```



```
[Out] a^3*Cos[x]*Sqrt[a*Sec[x]^4]*Sin[x] + 2*a^3*Sqrt[a*Sec[x]^4]*Sin[x]^2*Tan[x]
 + 3*a^3*Sqrt[a*Sec[x]^4]*Sin[x]^2*Tan[x]^3 + (20*a^3*Sqrt[a*Sec[x]^4]*Sin[
x]^2*Tan[x]^5)/7 + (5*a^3*Sqrt[a*Sec[x]^4]*Sin[x]^2*Tan[x]^7)/3 + (6*a^3*Sq
rt[a*Sec[x]^4]*Sin[x]^2*Tan[x]^9)/11 + (a^3*Sqrt[a*Sec[x]^4]*Sin[x]^2*Tan[x
]^11)/13
```


Rule 4123

```
Int[((b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> Dist[(b
^IntPart[p]*(b*(c*Sec[e + f*x])^n)^FracPart[p])/(c*Sec[e + f*x])^(n*FracPar
t[p]), Int[(c*Sec[e + f*x])^(n*p), x], x] /; FreeQ[{b, c, e, f, n, p}, x] &
&  !IntegerQ[p]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}\int \left(a\sec^4(x)\right)^{7/2}\,dx &= \left(a^3\cos^2(x)\sqrt{a\sec^4(x)}\right)\int \sec^{14}(x)\,dx\\ &= -\left(\left(a^3\cos^2(x)\sqrt{a\sec^4(x)}\right)\text{Subst}\left(\int \left(1+6x^2+15x^4+20x^6+15x^8+6x^{10}+x^{12}\right)dx,x,-\tan(x)\right)\right.\\ &= a^3\cos(x)\sqrt{a\sec^4(x)}\sin(x)+2a^3\sqrt{a\sec^4(x)}\sin^2(x)\tan(x)+3a^3\sqrt{a\sec^4(x)}\sin^2(x)\tan^3(x)+\frac{20}{7}a^3\end{aligned}$$

Mathematica [A] time = 0.168441, size = 54, normalized size = 0.33

$$\frac{\sin(x)\cos(x)(2380\cos(2x)+1093\cos(4x)+378\cos(6x)+92\cos(8x)+14\cos(10x)+\cos(12x)+2048)\left(a\sec^4(x)\right)^{7/2}}{6006}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Sec[x]^4)^(7/2),x]
```

```
[Out] (Cos[x]*(2048 + 2380*Cos[2*x] + 1093*Cos[4*x] + 378*Cos[6*x] + 92*Cos[8*x]
+ 14*Cos[10*x] + Cos[12*x])*(a*Sec[x]^4)^(7/2)*Sin[x])/6006
```

Maple [A] time = 0.233, size = 53, normalized size = 0.3

$$\frac{\left(1024\,(\cos(x))^{12}+512\,(\cos(x))^{10}+384\,(\cos(x))^{8}+320\,(\cos(x))^{6}+280\,(\cos(x))^{4}+252\,(\cos(x))^{2}+231\right)\sin(x)\,c}{3003}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sec(x)^4)^(7/2),x)
```

```
[Out] 1/3003*(1024*cos(x)^12+512*cos(x)^10+384*cos(x)^8+320*cos(x)^6+280*cos(x)^4
+252*cos(x)^2+231)*(a/cos(x)^4)^(7/2)*sin(x)*cos(x)
```

Maxima [A] time = 1.64345, size = 82, normalized size = 0.5

$$\frac{1}{13}\,a^{\frac{7}{2}}\tan(x)^{13}+\frac{6}{11}\,a^{\frac{7}{2}}\tan(x)^{11}+\frac{5}{3}\,a^{\frac{7}{2}}\tan(x)^{9}+\frac{20}{7}\,a^{\frac{7}{2}}\tan(x)^{7}+3\,a^{\frac{7}{2}}\tan(x)^{5}+2\,a^{\frac{7}{2}}\tan(x)^{3}+a^{\frac{7}{2}}\tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sec(x)^4)^(7/2),x, algorithm="maxima")
```

```
[Out] 1/13*a^(7/2)*tan(x)^13 + 6/11*a^(7/2)*tan(x)^11 + 5/3*a^(7/2)*tan(x)^9 + 20
/7*a^(7/2)*tan(x)^7 + 3*a^(7/2)*tan(x)^5 + 2*a^(7/2)*tan(x)^3 + a^(7/2)*tan
(x)
```

Fricas [A] time = 1.58708, size = 228, normalized size = 1.4

$$\frac{\left(1024\,a^{3}\cos(x)^{12}+512\,a^{3}\cos(x)^{10}+384\,a^{3}\cos(x)^{8}+320\,a^{3}\cos(x)^{6}+280\,a^{3}\cos(x)^{4}+252\,a^{3}\cos(x)^{2}+231\,a^{3}\right)\sqrt{}}{3003\,\cos(x)^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sec(x)^4)^(7/2),x, algorithm="fricas")
```

```
[Out] 1/3003*(1024*a^3*cos(x)^12 + 512*a^3*cos(x)^10 + 384*a^3*cos(x)^8 + 320*a^3
*cos(x)^6 + 280*a^3*cos(x)^4 + 252*a^3*cos(x)^2 + 231*a^3)*sqrt(a/cos(x)^4)
*sin(x)/cos(x)^11
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sec(x)**4)**(7/2),x)
```

```
[Out] Timed out
```

<u>Giac</u> [A]   time = 1.32692, size = 90, normalized size = 0.55

$$\frac{1}{3003}\left(231\,a^{3}\tan\left(x\right)^{13}+1638\,a^{3}\tan\left(x\right)^{11}+5005\,a^{3}\tan\left(x\right)^{9}+8580\,a^{3}\tan\left(x\right)^{7}+9009\,a^{3}\tan\left(x\right)^{5}+6006\,a^{3}\tan\left(x\right)^{3}+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sec(x)^4)^(7/2),x, algorithm="giac")
```

```
[Out] 1/3003*(231*a^3*tan(x)^13 + 1638*a^3*tan(x)^11 + 5005*a^3*tan(x)^9 + 8580*a
^3*tan(x)^7 + 9009*a^3*tan(x)^5 + 6006*a^3*tan(x)^3 + 3003*a^3*tan(x))*sqrt
(a)
```

3.62 $\int \left(a \sec^4(x)\right)^{5/2} dx$

Optimal. Leaf size=117

$$a^2 \sin(x)\cos(x)\sqrt{a\sec^4(x)} + \frac{1}{9}a^2\sin^2(x)\tan^7(x)\sqrt{a\sec^4(x)} + \frac{4}{7}a^2\sin^2(x)\tan^5(x)\sqrt{a\sec^4(x)} + \frac{6}{5}a^2\sin^2(x)\tan^3(x)\sqrt{}$$

```
[Out] a^2*Cos[x]*Sqrt[a*Sec[x]^4]*Sin[x] + (4*a^2*Sqrt[a*Sec[x]^4]*Sin[x]^2*Tan[x
])/3 + (6*a^2*Sqrt[a*Sec[x]^4]*Sin[x]^2*Tan[x]^3)/5 + (4*a^2*Sqrt[a*Sec[x]^
4]*Sin[x]^2*Tan[x]^5)/7 + (a^2*Sqrt[a*Sec[x]^4]*Sin[x]^2*Tan[x]^7)/9
```

Rubi [A] time = 0.0303353, antiderivative size = 117, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {4123, 3767}

$$a^2 \sin(x)\cos(x)\sqrt{a\sec^4(x)} + \frac{1}{9}a^2\sin^2(x)\tan^7(x)\sqrt{a\sec^4(x)} + \frac{4}{7}a^2\sin^2(x)\tan^5(x)\sqrt{a\sec^4(x)} + \frac{6}{5}a^2\sin^2(x)\tan^3(x)\sqrt{}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Sec[x]^4)^(5/2),x]
```

```
[Out] a^2*Cos[x]*Sqrt[a*Sec[x]^4]*Sin[x] + (4*a^2*Sqrt[a*Sec[x]^4]*Sin[x]^2*Tan[x
])/3 + (6*a^2*Sqrt[a*Sec[x]^4]*Sin[x]^2*Tan[x]^3)/5 + (4*a^2*Sqrt[a*Sec[x]^
4]*Sin[x]^2*Tan[x]^5)/7 + (a^2*Sqrt[a*Sec[x]^4]*Sin[x]^2*Tan[x]^7)/9
```

Rule 4123

```
Int[((b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> Dist[(b
^IntPart[p]*(b*(c*Sec[e + f*x])^n)^FracPart[p])/(c*Sec[e + f*x])^(n*FracPar
t[p]), Int[(c*Sec[e + f*x])^(n*p), x], x] /; FreeQ[{b, c, e, f, n, p}, x] &
&  !IntegerQ[p]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}
\int \left(a\sec^4(x)\right)^{5/2} dx &= \left(a^2\cos^2(x)\sqrt{a\sec^4(x)}\right)\int \sec^{10}(x)\,dx\\
&= -\left(\left(a^2\cos^2(x)\sqrt{a\sec^4(x)}\right)\text{Subst}\left(\int \left(1+4x^2+6x^4+4x^6+x^8\right) dx, x, -\tan(x)\right)\right)\\
&= a^2\cos(x)\sqrt{a\sec^4(x)}\sin(x) + \frac{4}{3}a^2\sqrt{a\sec^4(x)}\sin^2(x)\tan(x) + \frac{6}{5}a^2\sqrt{a\sec^4(x)}\sin^2(x)\tan^3(x) +
\end{aligned}$$

Mathematica [A] time = 0.0911082, size = 42, normalized size = 0.36

$$\frac{1}{315}\sin(x)\cos(x)(130\cos(2x) + 46\cos(4x) + 10\cos(6x) + \cos(8x) + 128)\left(a\sec^4(x)\right)^{5/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Sec[x]^4)^(5/2),x]
```

```
[Out] (Cos[x]*(128 + 130*Cos[2*x] + 46*Cos[4*x] + 10*Cos[6*x] + Cos[8*x])*(a*Sec[
x]^4)^(5/2)*Sin[x])/315
```

Maple [A] time = 0.113, size = 41, normalized size = 0.4

$$\frac{\left(128 \left(\cos \left(x\right)\right)^{8}+64 \left(\cos \left(x\right)\right)^{6}+48 \left(\cos \left(x\right)\right)^{4}+40 \left(\cos \left(x\right)\right)^{2}+35\right) \cos \left(x\right) \sin \left(x\right)}{315} \left(\frac{a}{\left(\cos \left(x\right)\right)^{4}}\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sec(x)^4)^(5/2),x)
```

```
[Out] 1/315*(128*cos(x)^8+64*cos(x)^6+48*cos(x)^4+40*cos(x)^2+35)*cos(x)*sin(x)*(
a/cos(x)^4)^(5/2)
```

Maxima [A] time = 1.79159, size = 58, normalized size = 0.5

$$\frac{1}{9} a^{\frac{5}{2}} \tan \left(x\right)^{9} + \frac{4}{7} a^{\frac{5}{2}} \tan \left(x\right)^{7} + \frac{6}{5} a^{\frac{5}{2}} \tan \left(x\right)^{5} + \frac{4}{3} a^{\frac{5}{2}} \tan \left(x\right)^{3} + a^{\frac{5}{2}} \tan \left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sec(x)^4)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/9*a^(5/2)*tan(x)^9 + 4/7*a^(5/2)*tan(x)^7 + 6/5*a^(5/2)*tan(x)^5 + 4/3*a^
(5/2)*tan(x)^3 + a^(5/2)*tan(x)
```

Fricas [A] time = 1.4071, size = 165, normalized size = 1.41

$$\frac{\left(128 \, a^{2} \cos \left(x\right)^{8} + 64 \, a^{2} \cos \left(x\right)^{6} + 48 \, a^{2} \cos \left(x\right)^{4} + 40 \, a^{2} \cos \left(x\right)^{2} + 35 \, a^{2}\right) \sqrt{\frac{a}{\cos \left(x\right)^{4}}} \sin \left(x\right)}{315 \, \cos \left(x\right)^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sec(x)^4)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/315*(128*a^2*cos(x)^8 + 64*a^2*cos(x)^6 + 48*a^2*cos(x)^4 + 40*a^2*cos(x)
^2 + 35*a^2)*sqrt(a/cos(x)^4)*sin(x)/cos(x)^7
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sec(x)**4)**(5/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.24393, size = 66, normalized size = 0.56

$$\frac{1}{315}\left(35\,a^{2}\tan\left(x\right)^{9}+180\,a^{2}\tan\left(x\right)^{7}+378\,a^{2}\tan\left(x\right)^{5}+420\,a^{2}\tan\left(x\right)^{3}+315\,a^{2}\tan\left(x\right)\right)\sqrt{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sec(x)^4)^(5/2),x, algorithm="giac")
```

```
[Out] 1/315*(35*a^2*tan(x)^9 + 180*a^2*tan(x)^7 + 378*a^2*tan(x)^5 + 420*a^2*tan(
x)^3 + 315*a^2*tan(x))*sqrt(a)
```

3.63 $\int \left(a \sec^4(x)\right)^{3/2} dx$

Optimal. Leaf size=61

$$a \sin(x)\cos(x)\sqrt{a \sec^4(x)} + \frac{1}{5} a \sin^2(x)\tan^3(x)\sqrt{a \sec^4(x)} + \frac{2}{3} a \sin^2(x)\tan(x)\sqrt{a \sec^4(x)}$$

```
[Out] a*Cos[x]*Sqrt[a*Sec[x]^4]*Sin[x] + (2*a*Sqrt[a*Sec[x]^4]*Sin[x]^2*Tan[x])/3
 + (a*Sqrt[a*Sec[x]^4]*Sin[x]^2*Tan[x]^3)/5
```

Rubi [A] time = 0.0220653, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {4123, 3767}

$$a \sin(x)\cos(x)\sqrt{a \sec^4(x)} + \frac{1}{5} a \sin^2(x)\tan^3(x)\sqrt{a \sec^4(x)} + \frac{2}{3} a \sin^2(x)\tan(x)\sqrt{a \sec^4(x)}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Sec[x]^4)^(3/2),x]
```

```
[Out] a*Cos[x]*Sqrt[a*Sec[x]^4]*Sin[x] + (2*a*Sqrt[a*Sec[x]^4]*Sin[x]^2*Tan[x])/3
 + (a*Sqrt[a*Sec[x]^4]*Sin[x]^2*Tan[x]^3)/5
```

Rule 4123

```
Int[((b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> Dist[(b
^IntPart[p]*(b*(c*Sec[e + f*x])^n)^FracPart[p])/(c*Sec[e + f*x])^(n*FracPar
t[p]), Int[(c*Sec[e + f*x])^(n*p), x], x] /; FreeQ[{b, c, e, f, n, p}, x] &
& !IntegerQ[p]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}
\int \left(a \sec^4(x)\right)^{3/2} dx &= \left(a \cos^2(x)\sqrt{a \sec^4(x)}\right)\int \sec^6(x)\,dx \\
&= -\left(\left(a \cos^2(x)\sqrt{a \sec^4(x)}\right)\text{Subst}\left(\int \left(1+2x^2+x^4\right)dx, x, -\tan(x)\right)\right) \\
&= a\cos(x)\sqrt{a \sec^4(x)}\sin(x) + \frac{2}{3} a\sqrt{a \sec^4(x)}\sin^2(x)\tan(x) + \frac{1}{5} a\sqrt{a \sec^4(x)}\sin^2(x)\tan^3(x)
\end{aligned}$$

Mathematica [A] time = 0.0571263, size = 30, normalized size = 0.49

$$\frac{1}{15}\sin(x)\cos(x)(6\cos(2x)+\cos(4x)+8)\left(a \sec^4(x)\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Sec[x]^4)^(3/2),x]
```

```
[Out] (Cos[x]*(8 + 6*Cos[2*x] + Cos[4*x])*(a*Sec[x]^4)^(3/2)*Sin[x])/15
```

Maple [A] time = 0.067, size = 29, normalized size = 0.5

$$\frac{\left(8\left(\cos\left(x\right)\right)^{4}+4\left(\cos\left(x\right)\right)^{2}+3\right)\cos\left(x\right)\sin\left(x\right)}{15}\left(\frac{a}{\left(\cos\left(x\right)\right)^{4}}\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sec(x)^4)^(3/2),x)
```

```
[Out] 1/15*(8*cos(x)^4+4*cos(x)^2+3)*cos(x)*sin(x)*(a/cos(x)^4)^(3/2)
```

Maxima [A] time = 1.69223, size = 34, normalized size = 0.56

$$\frac{1}{5}a^{\frac{3}{2}}\tan\left(x\right)^{5}+\frac{2}{3}a^{\frac{3}{2}}\tan\left(x\right)^{3}+a^{\frac{3}{2}}\tan\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sec(x)^4)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/5*a^(3/2)*tan(x)^5 + 2/3*a^(3/2)*tan(x)^3 + a^(3/2)*tan(x)
```

Fricas [A] time = 1.4485, size = 101, normalized size = 1.66

$$\frac{\left(8\,a\cos\left(x\right)^{4}+4\,a\cos\left(x\right)^{2}+3\,a\right)\sqrt{\frac{a}{\cos(x)^{4}}}\sin\left(x\right)}{15\,\cos\left(x\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sec(x)^4)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/15*(8*a*cos(x)^4 + 4*a*cos(x)^2 + 3*a)*sqrt(a/cos(x)^4)*sin(x)/cos(x)^3
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\left(a\sec^{4}\left(x\right)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sec(x)**4)**(3/2),x)
```

```
[Out] Integral((a*sec(x)**4)**(3/2), x)
```

Giac [A] time = 1.30526, size = 30, normalized size = 0.49

$$\frac{1}{15}\left(3\,\tan\left(x\right)^{5}+10\,\tan\left(x\right)^{3}+15\,\tan\left(x\right)\right)a^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sec(x)^4)^(3/2),x, algorithm="giac")
```

```
[Out] 1/15*(3*tan(x)^5 + 10*tan(x)^3 + 15*tan(x))*a^(3/2)
```

3.64 $\int \sqrt{a \sec^4(x)}\, dx$

Optimal. Leaf size=15

$$\sin(x)\cos(x)\sqrt{a\sec^4(x)}$$

```
[Out] Cos[x]*Sqrt[a*Sec[x]^4]*Sin[x]
```

Rubi [A] time = 0.0158969, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {4123, 3767, 8}

$$\sin(x)\cos(x)\sqrt{a\sec^4(x)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*Sec[x]^4],x]
```

```
[Out] Cos[x]*Sqrt[a*Sec[x]^4]*Sin[x]
```

Rule 4123

```
Int[((b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> Dist[(b
^IntPart[p]*(b*(c*Sec[e + f*x])^n)^FracPart[p])/(c*Sec[e + f*x])^(n*FracPar
t[p]), Int[(c*Sec[e + f*x])^(n*p), x], x] /; FreeQ[{b, c, e, f, n, p}, x] &
&  !IntegerQ[p]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}
\int \sqrt{a\sec^4(x)}\, dx &= \left(\cos^2(x)\sqrt{a\sec^4(x)}\right)\int \sec^2(x)\, dx\\
&= -\left(\left(\cos^2(x)\sqrt{a\sec^4(x)}\right)\operatorname{Subst}\left(\int 1\, dx, x, -\tan(x)\right)\right)\\
&= \cos(x)\sqrt{a\sec^4(x)}\sin(x)
\end{aligned}$$

Mathematica [A] time = 0.0054477, size = 15, normalized size = 1.

$$\sin(x)\cos(x)\sqrt{a\sec^4(x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*Sec[x]^4],x]
```

```
[Out] Cos[x]*Sqrt[a*Sec[x]^4]*Sin[x]
```

Maple [A] time = 0.063, size = 14, normalized size = 0.9

$$\cos(x)\sin(x)\sqrt{\frac{a}{(\cos(x))^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sec(x)^4)^(1/2),x)
```

```
[Out] cos(x)*sin(x)*(a/cos(x)^4)^(1/2)
```

Maxima [A] time = 1.73949, size = 8, normalized size = 0.53

$$\sqrt{a}\tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sec(x)^4)^(1/2),x, algorithm="maxima")
```

```
[Out] sqrt(a)*tan(x)
```

Fricas [A] time = 1.36385, size = 43, normalized size = 2.87

$$\sqrt{\frac{a}{\cos(x)^4}}\cos(x)\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sec(x)^4)^(1/2),x, algorithm="fricas")
```

```
[Out] sqrt(a/cos(x)^4)*cos(x)*sin(x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a\sec^4(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sec(x)**4)**(1/2),x)
```

```
[Out] Integral(sqrt(a*sec(x)**4), x)
```

Giac [A] time = 1.2576, size = 8, normalized size = 0.53

$$\sqrt{a}\tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sec(x)^4)^(1/2),x, algorithm="giac")
```

```
[Out] sqrt(a)*tan(x)
```

3.65 $\int \frac{1}{\sqrt{a \sec^4(x)}} \, dx$

Optimal. Leaf size=36

$$\frac{x \sec^2(x)}{2\sqrt{a \sec^4(x)}} + \frac{\tan(x)}{2\sqrt{a \sec^4(x)}}$$

```
[Out] (x*Sec[x]^2)/(2*Sqrt[a*Sec[x]^4]) + Tan[x]/(2*Sqrt[a*Sec[x]^4])
```

Rubi [A] time = 0.0146596, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {4123, 2635, 8}

$$\frac{x \sec^2(x)}{2\sqrt{a \sec^4(x)}} + \frac{\tan(x)}{2\sqrt{a \sec^4(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a*Sec[x]^4],x]
```

```
[Out] (x*Sec[x]^2)/(2*Sqrt[a*Sec[x]^4]) + Tan[x]/(2*Sqrt[a*Sec[x]^4])
```

Rule 4123

```
Int[((b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> Dist[(b
^IntPart[p]*(b*(c*Sec[e + f*x])^n)^FracPart[p])/(c*Sec[e + f*x])^(n*FracPar
t[p]), Int[(c*Sec[e + f*x])^(n*p), x], x] /; FreeQ[{b, c, e, f, n, p}, x] &
&  !IntegerQ[p]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt{a \sec^4(x)}} \, dx &= \frac{\sec^2(x) \int \cos^2(x) \, dx}{\sqrt{a \sec^4(x)}} \\
&= \frac{\tan(x)}{2\sqrt{a \sec^4(x)}} + \frac{\sec^2(x) \int 1 \, dx}{2\sqrt{a \sec^4(x)}} \\
&= \frac{x \sec^2(x)}{2\sqrt{a \sec^4(x)}} + \frac{\tan(x)}{2\sqrt{a \sec^4(x)}}
\end{aligned}$$

Mathematica [A] time = 0.0234295, size = 23, normalized size = 0.64

$$\frac{\tan(x) + x \sec^2(x)}{2\sqrt{a \sec^4(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a*Sec[x]^4],x]
```

```
[Out] (x*Sec[x]^2 + Tan[x])/(2*Sqrt[a*Sec[x]^4])
```

Maple [A] time = 0.074, size = 22, normalized size = 0.6

$$\frac{\cos(x)\sin(x)+x}{2(\cos(x))^2}\frac{1}{\sqrt{\frac{a}{(\cos(x))^4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*sec(x)^4)^(1/2),x)
```

```
[Out] 1/2*(cos(x)*sin(x)+x)/cos(x)^2/(a/cos(x)^4)^(1/2)
```

Maxima [A] time = 1.66388, size = 34, normalized size = 0.94

$$\frac{x}{2\sqrt{a}}+\frac{\tan(x)}{2\left(\sqrt{a}\tan(x)^2+\sqrt{a}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sec(x)^4)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*x/sqrt(a) + 1/2*tan(x)/(sqrt(a)*tan(x)^2 + sqrt(a))
```

Fricas [A] time = 1.4198, size = 74, normalized size = 2.06

$$\frac{\left(\cos(x)^3\sin(x)+x\cos(x)^2\right)\sqrt{\frac{a}{\cos(x)^4}}}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sec(x)^4)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*(cos(x)^3*sin(x) + x*cos(x)^2)*sqrt(a/cos(x)^4)/a
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{a\sec^4(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sec(x)**4)**(1/2),x)
```

```
[Out] Integral(1/sqrt(a*sec(x)**4), x)
```

Giac [A] time = 1.31144, size = 53, normalized size = 1.47

$$-\frac{1}{2}\sqrt{a}\left(\frac{\pi\left\lfloor\frac{x}{\pi}+\frac{1}{2}\right\rfloor-x}{a}-\frac{\tan(x)}{\left(\tan(x)^2+1\right)a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sec(x)^4)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(a)*((pi*floor(x/pi + 1/2) - x)/a - tan(x)/((tan(x)^2 + 1)*a))
```

3.66 $\int \frac{1}{\left(a\sec^4(x)\right)^{3/2}}\,dx$

Optimal. Leaf size=86

$$\frac{5x\sec^2(x)}{16a\sqrt{a\sec^4(x)}}+\frac{5\tan(x)}{16a\sqrt{a\sec^4(x)}}+\frac{\sin(x)\cos^3(x)}{6a\sqrt{a\sec^4(x)}}+\frac{5\sin(x)\cos(x)}{24a\sqrt{a\sec^4(x)}}$$

```
[Out] (5*x*Sec[x]^2)/(16*a*Sqrt[a*Sec[x]^4]) + (5*Cos[x]*Sin[x])/(24*a*Sqrt[a*Sec
[x]^4]) + (Cos[x]^3*Sin[x])/(6*a*Sqrt[a*Sec[x]^4]) + (5*Tan[x])/(16*a*Sqrt[
a*Sec[x]^4])
```

Rubi [A] time = 0.0319133, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {4123, 2635, 8}

$$\frac{5x\sec^2(x)}{16a\sqrt{a\sec^4(x)}}+\frac{5\tan(x)}{16a\sqrt{a\sec^4(x)}}+\frac{\sin(x)\cos^3(x)}{6a\sqrt{a\sec^4(x)}}+\frac{5\sin(x)\cos(x)}{24a\sqrt{a\sec^4(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Sec[x]^4)^(-3/2),x]
```

```
[Out] (5*x*Sec[x]^2)/(16*a*Sqrt[a*Sec[x]^4]) + (5*Cos[x]*Sin[x])/(24*a*Sqrt[a*Sec
[x]^4]) + (Cos[x]^3*Sin[x])/(6*a*Sqrt[a*Sec[x]^4]) + (5*Tan[x])/(16*a*Sqrt[
a*Sec[x]^4])
```

Rule 4123

```
Int[((b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> Dist[(b
^IntPart[p]*(b*(c*Sec[e + f*x])^n)^FracPart[p])/(c*Sec[e + f*x])^(n*FracPar
t[p]), Int[(c*Sec[e + f*x])^(n*p), x], x] /; FreeQ[{b, c, e, f, n, p}, x] &
& !IntegerQ[p]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{1}{\left(a\sec^4(x)\right)^{3/2}}\,dx = \frac{\sec^2(x)\int \cos^6(x)\,dx}{a\sqrt{a\sec^4(x)}}$$
$$= \frac{\cos^3(x)\sin(x)}{6a\sqrt{a\sec^4(x)}} + \frac{\left(5\sec^2(x)\right)\int \cos^4(x)\,dx}{6a\sqrt{a\sec^4(x)}}$$
$$= \frac{5\cos(x)\sin(x)}{24a\sqrt{a\sec^4(x)}} + \frac{\cos^3(x)\sin(x)}{6a\sqrt{a\sec^4(x)}} + \frac{\left(5\sec^2(x)\right)\int \cos^2(x)\,dx}{8a\sqrt{a\sec^4(x)}}$$
$$= \frac{5\cos(x)\sin(x)}{24a\sqrt{a\sec^4(x)}} + \frac{\cos^3(x)\sin(x)}{6a\sqrt{a\sec^4(x)}} + \frac{5\tan(x)}{16a\sqrt{a\sec^4(x)}} + \frac{\left(5\sec^2(x)\right)\int 1\,dx}{16a\sqrt{a\sec^4(x)}}$$
$$= \frac{5x\sec^2(x)}{16a\sqrt{a\sec^4(x)}} + \frac{5\cos(x)\sin(x)}{24a\sqrt{a\sec^4(x)}} + \frac{\cos^3(x)\sin(x)}{6a\sqrt{a\sec^4(x)}} + \frac{5\tan(x)}{16a\sqrt{a\sec^4(x)}}$$

Mathematica [A] time = 0.0388284, size = 38, normalized size = 0.44

$$\frac{(60x + 45\sin(2x) + 9\sin(4x) + \sin(6x))\sec^6(x)}{192\left(a\sec^4(x)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Sec[x]^4)^(-3/2),x]
```

```
[Out] (Sec[x]^6*(60*x + 45*Sin[2*x] + 9*Sin[4*x] + Sin[6*x]))/(192*(a*Sec[x]^4)^(
3/2))
```

Maple [A] time = 0.104, size = 41, normalized size = 0.5

$$\frac{8\,(\cos(x))^5\sin(x) + 10\,(\cos(x))^3\sin(x) + 15\,\cos(x)\sin(x) + 15\,x}{48\,(\cos(x))^6}\left(\frac{a}{(\cos(x))^4}\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*sec(x)^4)^(3/2),x)
```

```
[Out] 1/48*(8*cos(x)^5*sin(x)+10*cos(x)^3*sin(x)+15*cos(x)*sin(x)+15*x)/cos(x)^6/
(a/cos(x)^4)^(3/2)
```

Maxima [A] time = 1.64573, size = 78, normalized size = 0.91

$$\frac{15\,\tan(x)^5 + 40\,\tan(x)^3 + 33\,\tan(x)}{48\left(a^{\frac{3}{2}}\tan(x)^6 + 3\,a^{\frac{3}{2}}\tan(x)^4 + 3\,a^{\frac{3}{2}}\tan(x)^2 + a^{\frac{3}{2}}\right)} + \frac{5\,x}{16\,a^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sec(x)^4)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/48*(15*tan(x)^5 + 40*tan(x)^3 + 33*tan(x))/(a^(3/2)*tan(x)^6 + 3*a^(3/2)*
tan(x)^4 + 3*a^(3/2)*tan(x)^2 + a^(3/2)) + 5/16*x/a^(3/2)
```

Fricas [A] time = 1.46067, size = 126, normalized size = 1.47

$$\frac{\left(15\,x\cos\left(x\right)^{2}+\left(8\,\cos\left(x\right)^{7}+10\,\cos\left(x\right)^{5}+15\,\cos\left(x\right)^{3}\right)\sin\left(x\right)\right)\sqrt{\frac{a}{\cos(x)^{4}}}}{48\,a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sec(x)^4)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/48*(15*x*cos(x)^2 + (8*cos(x)^7 + 10*cos(x)^5 + 15*cos(x)^3)*sin(x))*sqrt
(a/cos(x)^4)/a^2
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\left(a\sec^{4}\left(x\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sec(x)**4)**(3/2),x)
```

```
[Out] Integral((a*sec(x)**4)**(-3/2), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sec(x)^4)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.67 $\int \frac{1}{\left(a\sec^4(x)\right)^{5/2}}\,dx$

Optimal. Leaf size=132

$$\frac{63x\sec^2(x)}{256a^2\sqrt{a\sec^4(x)}}+\frac{63\tan(x)}{256a^2\sqrt{a\sec^4(x)}}+\frac{\sin(x)\cos^7(x)}{10a^2\sqrt{a\sec^4(x)}}+\frac{9\sin(x)\cos^5(x)}{80a^2\sqrt{a\sec^4(x)}}+\frac{21\sin(x)\cos^3(x)}{160a^2\sqrt{a\sec^4(x)}}+\frac{21\sin(x)\cos(x)}{128a^2\sqrt{a\sec^4(x)}}$$

```
[Out] (63*x*Sec[x]^2)/(256*a^2*Sqrt[a*Sec[x]^4]) + (21*Cos[x]*Sin[x])/(128*a^2*Sq
rt[a*Sec[x]^4]) + (21*Cos[x]^3*Sin[x])/(160*a^2*Sqrt[a*Sec[x]^4]) + (9*Cos[
x]^5*Sin[x])/(80*a^2*Sqrt[a*Sec[x]^4]) + (Cos[x]^7*Sin[x])/(10*a^2*Sqrt[a*S
ec[x]^4]) + (63*Tan[x])/(256*a^2*Sqrt[a*Sec[x]^4])
```

Rubi [A] time = 0.0508577, antiderivative size = 132, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {4123, 2635, 8}

$$\frac{63x\sec^2(x)}{256a^2\sqrt{a\sec^4(x)}}+\frac{63\tan(x)}{256a^2\sqrt{a\sec^4(x)}}+\frac{\sin(x)\cos^7(x)}{10a^2\sqrt{a\sec^4(x)}}+\frac{9\sin(x)\cos^5(x)}{80a^2\sqrt{a\sec^4(x)}}+\frac{21\sin(x)\cos^3(x)}{160a^2\sqrt{a\sec^4(x)}}+\frac{21\sin(x)\cos(x)}{128a^2\sqrt{a\sec^4(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Sec[x]^4)^(-5/2),x]
```

```
[Out] (63*x*Sec[x]^2)/(256*a^2*Sqrt[a*Sec[x]^4]) + (21*Cos[x]*Sin[x])/(128*a^2*Sq
rt[a*Sec[x]^4]) + (21*Cos[x]^3*Sin[x])/(160*a^2*Sqrt[a*Sec[x]^4]) + (9*Cos[
x]^5*Sin[x])/(80*a^2*Sqrt[a*Sec[x]^4]) + (Cos[x]^7*Sin[x])/(10*a^2*Sqrt[a*S
ec[x]^4]) + (63*Tan[x])/(256*a^2*Sqrt[a*Sec[x]^4])
```

Rule 4123

```
Int[((b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> Dist[(b
^IntPart[p]*(b*(c*Sec[e + f*x])^n)^FracPart[p])/(c*Sec[e + f*x])^(n*FracPar
t[p]), Int[(c*Sec[e + f*x])^(n*p), x], x] /; FreeQ[{b, c, e, f, n, p}, x] &
&  !IntegerQ[p]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{1}{\left(a\sec^4(x)\right)^{5/2}}\,dx = \frac{\sec^2(x)\int \cos^{10}(x)\,dx}{a^2\sqrt{a\sec^4(x)}}$$
$$= \frac{\cos^7(x)\sin(x)}{10a^2\sqrt{a\sec^4(x)}} + \frac{\left(9\sec^2(x)\right)\int \cos^8(x)\,dx}{10a^2\sqrt{a\sec^4(x)}}$$
$$= \frac{9\cos^5(x)\sin(x)}{80a^2\sqrt{a\sec^4(x)}} + \frac{\cos^7(x)\sin(x)}{10a^2\sqrt{a\sec^4(x)}} + \frac{\left(63\sec^2(x)\right)\int \cos^6(x)\,dx}{80a^2\sqrt{a\sec^4(x)}}$$
$$= \frac{21\cos^3(x)\sin(x)}{160a^2\sqrt{a\sec^4(x)}} + \frac{9\cos^5(x)\sin(x)}{80a^2\sqrt{a\sec^4(x)}} + \frac{\cos^7(x)\sin(x)}{10a^2\sqrt{a\sec^4(x)}} + \frac{\left(21\sec^2(x)\right)\int \cos^4(x)\,dx}{32a^2\sqrt{a\sec^4(x)}}$$
$$= \frac{21\cos(x)\sin(x)}{128a^2\sqrt{a\sec^4(x)}} + \frac{21\cos^3(x)\sin(x)}{160a^2\sqrt{a\sec^4(x)}} + \frac{9\cos^5(x)\sin(x)}{80a^2\sqrt{a\sec^4(x)}} + \frac{\cos^7(x)\sin(x)}{10a^2\sqrt{a\sec^4(x)}} + \frac{\left(63\sec^2(x)\right)\int \text{co}}{128a^2\sqrt{a\sec}}$$
$$= \frac{21\cos(x)\sin(x)}{128a^2\sqrt{a\sec^4(x)}} + \frac{21\cos^3(x)\sin(x)}{160a^2\sqrt{a\sec^4(x)}} + \frac{9\cos^5(x)\sin(x)}{80a^2\sqrt{a\sec^4(x)}} + \frac{\cos^7(x)\sin(x)}{10a^2\sqrt{a\sec^4(x)}} + \frac{63\tan(x)}{256a^2\sqrt{a\sec^4(x)}}$$
$$= \frac{63x\sec^2(x)}{256a^2\sqrt{a\sec^4(x)}} + \frac{21\cos(x)\sin(x)}{128a^2\sqrt{a\sec^4(x)}} + \frac{21\cos^3(x)\sin(x)}{160a^2\sqrt{a\sec^4(x)}} + \frac{9\cos^5(x)\sin(x)}{80a^2\sqrt{a\sec^4(x)}} + \frac{\cos^7(x)\sin(x)}{10a^2\sqrt{a\sec^4(x)}}$$

Mathematica [A] time = 0.0824578, size = 55, normalized size = 0.42

$$\frac{(2520x + 2100\sin(2x) + 600\sin(4x) + 150\sin(6x) + 25\sin(8x) + 2\sin(10x))\cos^2(x)\sqrt{a\sec^4(x)}}{10240a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Sec[x]^4)^(-5/2),x]
```

```
[Out] (Cos[x]^2*Sqrt[a*Sec[x]^4]*(2520*x + 2100*Sin[2*x] + 600*Sin[4*x] + 150*Sin
[6*x] + 25*Sin[8*x] + 2*Sin[10*x]))/(10240*a^3)
```

Maple [A] time = 0.221, size = 57, normalized size = 0.4

$$\frac{128\,(\cos(x))^9\sin(x) + 144\,(\cos(x))^7\sin(x) + 168\,(\cos(x))^5\sin(x) + 210\,(\cos(x))^3\sin(x) + 315\,\cos(x)\sin(x) + 31}{1280\,(\cos(x))^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*sec(x)^4)^(5/2),x)
```

```
[Out] 1/1280*(128*cos(x)^9*sin(x)+144*cos(x)^7*sin(x)+168*cos(x)^5*sin(x)+210*cos
(x)^3*sin(x)+315*cos(x)*sin(x)+315*x)/cos(x)^10/(a/cos(x)^4)^(5/2)
```

Maxima [A] time = 1.71142, size = 119, normalized size = 0.9

$$\frac{315\,\tan(x)^9 + 1470\,\tan(x)^7 + 2688\,\tan(x)^5 + 2370\,\tan(x)^3 + 965\,\tan(x)}{1280\left(a^{\frac{5}{2}}\tan(x)^{10} + 5\,a^{\frac{5}{2}}\tan(x)^8 + 10\,a^{\frac{5}{2}}\tan(x)^6 + 10\,a^{\frac{5}{2}}\tan(x)^4 + 5\,a^{\frac{5}{2}}\tan(x)^2 + a^{\frac{5}{2}}\right)} + \frac{63\,x}{256\,a^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sec(x)^4)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/1280*(315*tan(x)^9 + 1470*tan(x)^7 + 2688*tan(x)^5 + 2370*tan(x)^3 + 965*
tan(x))/(a^(5/2)*tan(x)^10 + 5*a^(5/2)*tan(x)^8 + 10*a^(5/2)*tan(x)^6 + 10*
a^(5/2)*tan(x)^4 + 5*a^(5/2)*tan(x)^2 + a^(5/2)) + 63/256*x/a^(5/2)
```

Fricas [A] time = 1.45152, size = 177, normalized size = 1.34

$$\frac{\left(315\, x \cos\left(x\right)^{2} + \left(128\, \cos\left(x\right)^{11} + 144\, \cos\left(x\right)^{9} + 168\, \cos\left(x\right)^{7} + 210\, \cos\left(x\right)^{5} + 315\, \cos\left(x\right)^{3}\right) \sin\left(x\right)\right) \sqrt{\frac{a}{\cos\left(x\right)^{4}}}}{1280\, a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sec(x)^4)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/1280*(315*x*cos(x)^2 + (128*cos(x)^11 + 144*cos(x)^9 + 168*cos(x)^7 + 210
*cos(x)^5 + 315*cos(x)^3)*sin(x))*sqrt(a/cos(x)^4)/a^3
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a \sec^{4}{\left(x \right)}\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sec(x)**4)**(5/2),x)
```

```
[Out] Integral((a*sec(x)**4)**(-5/2), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sec(x)^4)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.68 $\int ((b\sec(c+dx))^p)^n\,dx$

Optimal. Leaf size=81

$$-\frac{\sin(c+dx)\cos(c+dx)\left((b\sec(c+dx))^p\right)^n \text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}(1-np),\frac{1}{2}(3-np),\cos^2(c+dx)\right)}{d(1-np)\sqrt{\sin^2(c+dx)}}$$

```
[Out] -((Cos[c + d*x]*Hypergeometric2F1[1/2, (1 - n*p)/2, (3 - n*p)/2, Cos[c + d*
x]^2]*((b*Sec[c + d*x])^p)^n*Sin[c + d*x])/(d*(1 - n*p)*Sqrt[Sin[c + d*x]^2
]))
```

Rubi [A] time = 0.0532608, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {4123, 3772, 2643}

$$-\frac{\sin(c+dx)\cos(c+dx)\left((b\sec(c+dx))^p\right)^n {}_2F_1\left(\frac{1}{2},\frac{1}{2}(1-np);\frac{1}{2}(3-np);\cos^2(c+dx)\right)}{d(1-np)\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[((b*Sec[c + d*x])^p)^n,x]
```

```
[Out] -((Cos[c + d*x]*Hypergeometric2F1[1/2, (1 - n*p)/2, (3 - n*p)/2, Cos[c + d*
x]^2]*((b*Sec[c + d*x])^p)^n*Sin[c + d*x])/(d*(1 - n*p)*Sqrt[Sin[c + d*x]^2
]))
```

Rule 4123

```
Int[((b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> Dist[(b
^IntPart[p]*(b*(c*Sec[e + f*x])^n)^FracPart[p])/(c*Sec[e + f*x])^(n*FracPar
t[p]), Int[(c*Sec[e + f*x])^(n*p), x], x] /; FreeQ[{b, c, e, f, n, p}, x] &
&  !IntegerQ[p]
```

Rule 3772

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rubi steps

$$\int ((b\sec(c+dx))^p)^n\,dx = \left((b\sec(c+dx))^{-np}((b\sec(c+dx))^p)^n\right)\int (b\sec(c+dx))^{np}\,dx$$
$$= \left(\left(\frac{\cos(c+dx)}{b}\right)^{np}((b\sec(c+dx))^p)^n\right)\int \left(\frac{\cos(c+dx)}{b}\right)^{-np}dx$$
$$= -\frac{\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(1-np);\frac{1}{2}(3-np);\cos^2(c+dx)\right)((b\sec(c+dx))^p)^n\sin(c+dx)}{d(1-np)\sqrt{\sin^2(c+dx)}}$$

Mathematica [A] time = 0.0937918, size = 69, normalized size = 0.85

$$\frac{\sqrt{-\tan^2(c+dx)}\cot(c+dx)((b\sec(c+dx))^p)^n\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{np}{2},\frac{1}{2}(np+2),\sec^2(c+dx)\right)}{dnp}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((b*Sec[c + d*x])^p)^n,x]
```

```
[Out] (Cot[c + d*x]*Hypergeometric2F1[1/2, (n*p)/2, (2 + n*p)/2, Sec[c + d*x]^2]*
((b*Sec[c + d*x])^p)^n*Sqrt[-Tan[c + d*x]^2])/(d*n*p)
```

Maple [F] time = 0.444, size = 0, normalized size = 0.

$$\int \left((b\sec(dx+c))^p\right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*sec(d*x+c))^p)^n,x)
```

```
[Out] int(((b*sec(d*x+c))^p)^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left((b\sec(dx+c))^p\right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*sec(d*x+c))^p)^n,x, algorithm="maxima")
```

```
[Out] integrate(((b*sec(d*x + c))^p)^n, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left((b\sec(dx+c))^p\right)^n, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*sec(d*x+c))^p)^n,x, algorithm="fricas")
```

```
[Out] integral(((b*sec(d*x + c))^p)^n, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left((b\sec (c+dx))^p\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*sec(d*x+c))**p)**n,x)
```

```
[Out] Integral(((b*sec(c + d*x))**p)**n, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left((b\sec (dx+c))^p\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*sec(d*x+c))^p)^n,x, algorithm="giac")
```

```
[Out] integrate(((b*sec(d*x + c))^p)^n, x)
```

3.69 $\int (a(b\sec(c+dx))^p)^n\,dx$

Optimal. Leaf size=83

$$-\frac{\sin(c+dx)\cos(c+dx)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}(1-np),\frac{1}{2}(3-np),\cos^2(c+dx)\right)(a(b\sec(c+dx))^p)^n}{d(1-np)\sqrt{\sin^2(c+dx)}}$$

```
[Out] -((Cos[c + d*x]*Hypergeometric2F1[1/2, (1 - n*p)/2, (3 - n*p)/2, Cos[c + d*
x]^2]*(a*(b*Sec[c + d*x])^p)^n*Sin[c + d*x])/(d*(1 - n*p)*Sqrt[Sin[c + d*x]
^2]))
```

Rubi [A] time = 0.0495264, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {4123, 3772, 2643}

$$-\frac{\sin(c+dx)\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(1-np);\frac{1}{2}(3-np);\cos^2(c+dx)\right)(a(b\sec(c+dx))^p)^n}{d(1-np)\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*(b*Sec[c + d*x])^p)^n,x]
```

```
[Out] -((Cos[c + d*x]*Hypergeometric2F1[1/2, (1 - n*p)/2, (3 - n*p)/2, Cos[c + d*
x]^2]*(a*(b*Sec[c + d*x])^p)^n*Sin[c + d*x])/(d*(1 - n*p)*Sqrt[Sin[c + d*x]
^2]))
```

Rule 4123

```
Int[((b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> Dist[(b
^IntPart[p]*(b*(c*Sec[e + f*x])^n)^FracPart[p])/(c*Sec[e + f*x])^(n*FracPar
t[p]), Int[(c*Sec[e + f*x])^(n*p), x], x] /; FreeQ[{b, c, e, f, n, p}, x] &
& !IntegerQ[p]
```

Rule 3772

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] && !IntegerQ[n]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rubi steps

$$\int (a(b\sec(c+dx))^p)^n\,dx = \left((b\sec(c+dx))^{-np}\left(a(b\sec(c+dx))^p\right)^n\right)\int (b\sec(c+dx))^{np}\,dx$$
$$= \left(\left(\frac{\cos(c+dx)}{b}\right)^{np}(a(b\sec(c+dx))^p)^n\right)\int\left(\frac{\cos(c+dx)}{b}\right)^{-np}dx$$
$$= -\frac{\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(1-np);\frac{1}{2}(3-np);\cos^2(c+dx)\right)(a(b\sec(c+dx))^p)^n\sin(c+dx)}{d(1-np)\sqrt{\sin^2(c+dx)}}$$

Mathematica [A] time = 0.072802, size = 71, normalized size = 0.86

$$\frac{\sqrt{-\tan^2(c+dx)}\cot(c+dx)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{np}{2},\frac{1}{2}(np+2),\sec^2(c+dx)\right)(a(b\sec(c+dx))^p)^n}{dnp}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a*(b*Sec[c + d*x])^p)^n,x]
```

```
[Out] (Cot[c + d*x]*Hypergeometric2F1[1/2, (n*p)/2, (2 + n*p)/2, Sec[c + d*x]^2]*
(a*(b*Sec[c + d*x])^p)^n*Sqrt[-Tan[c + d*x]^2])/(d*n*p)
```

Maple [F] time = 0.197, size = 0, normalized size = 0.

$$\int \left(a\,(b\sec(dx+c))^p\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*(b*sec(d*x+c))^p)^n,x)
```

```
[Out] int((a*(b*sec(d*x+c))^p)^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left((b\sec(dx+c))^p\,a\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*(b*sec(d*x+c))^p)^n,x, algorithm="maxima")
```

```
[Out] integrate(((b*sec(d*x + c))^p*a)^n, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left((b\sec(dx+c))^p\,a\right)^n, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*(b*sec(d*x+c))^p)^n,x, algorithm="fricas")
```

```
[Out] integral(((b*sec(d*x + c))^p*a)^n, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\left(b\sec\left(c+dx\right)\right)^{p}\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*(b*sec(d*x+c))**p)**n,x)
```

```
[Out] Integral((a*(b*sec(c + d*x))**p)**n, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(\left(b\sec\left(dx+c\right)\right)^{p}a\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*(b*sec(d*x+c))^p)^n,x, algorithm="giac")
```

```
[Out] integrate(((b*sec(d*x + c))^p*a)^n, x)
```

3.70 $\int \sec^4(c+dx)\sqrt{b\sec(c+dx)}\,dx$

Optimal. Leaf size=97

$$\frac{10\sqrt{\cos(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{b\sec(c+dx)}}{21d}+\frac{2\sin(c+dx)(b\sec(c+dx))^{7/2}}{7b^3d}+\frac{10\sin(c+dx)(b\sec(c+d}{21bd}$$

```
[Out] (10*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[b*Sec[c + d*x]])/(21*
d) + (10*(b*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(21*b*d) + (2*(b*Sec[c + d*x]
)^(7/2)*Sin[c + d*x])/(7*b^3*d)
```

Rubi [A] time = 0.0604263, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {16, 3768, 3771, 2641}

$$\frac{2\sin(c+dx)(b\sec(c+dx))^{7/2}}{7b^3d}+\frac{10\sin(c+dx)(b\sec(c+dx))^{3/2}}{21bd}+\frac{10\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\sec(c+dx)}}{21d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4*Sqrt[b*Sec[c + d*x]],x]
```

```
[Out] (10*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[b*Sec[c + d*x]])/(21*
d) + (10*(b*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(21*b*d) + (2*(b*Sec[c + d*x]
)^(7/2)*Sin[c + d*x])/(7*b^3*d)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sec^4(c+dx)\sqrt{b\sec(c+dx)}\,dx = \frac{\int (b\sec(c+dx))^{9/2}\,dx}{b^4}$$
$$= \frac{2(b\sec(c+dx))^{7/2}\sin(c+dx)}{7b^3d} + \frac{5\int (b\sec(c+dx))^{5/2}\,dx}{7b^2}$$
$$= \frac{10(b\sec(c+dx))^{3/2}\sin(c+dx)}{21bd} + \frac{2(b\sec(c+dx))^{7/2}\sin(c+dx)}{7b^3d} + \frac{5}{21}\int \sqrt{b\sec(c+$$
$$= \frac{10(b\sec(c+dx))^{3/2}\sin(c+dx)}{21bd} + \frac{2(b\sec(c+dx))^{7/2}\sin(c+dx)}{7b^3d} + \frac{1}{21}\left(5\sqrt{\cos(c+dx}\right.$$
$$= \frac{10\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\sec(c+dx)}}{21d} + \frac{10(b\sec(c+dx))^{3/2}\sin(c+dx)}{21bd} + $$

Mathematica [A] time = 0.21158, size = 69, normalized size = 0.71

$$\frac{\sec^2(c+dx)\sqrt{b\sec(c+dx)}\left(10\cos^{\frac{5}{2}}(c+dx)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+5\sin(2(c+dx))+6\tan(c+dx)\right)}{21d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4*Sqrt[b*Sec[c + d*x]],x]
```

```
[Out] (Sec[c + d*x]^2*Sqrt[b*Sec[c + d*x]]*(10*Cos[c + d*x]^(5/2)*EllipticF[(c +
d*x)/2, 2] + 5*Sin[2*(c + d*x)] + 6*Tan[c + d*x]))/(21*d)
```

Maple [C] time = 0.224, size = 152, normalized size = 1.6

$$-\frac{2\left(\cos\left(dx+c\right)+1\right)^2\left(-1+\cos\left(dx+c\right)\right)}{21\,d\left(\sin\left(dx+c\right)\right)^3\left(\cos\left(dx+c\right)\right)^3}\left(5\,i\left(\cos\left(dx+c\right)\right)^3\sin\left(dx+c\right)\sqrt{\left(\cos\left(dx+c\right)+1\right)^{-1}}\sqrt{\frac{\cos\left(dx+c\right)}{\cos\left(dx+c\right)+1}}\text{Ellipti}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*(b*sec(d*x+c))^(1/2),x)
```

```
[Out] -2/21/d*(cos(d*x+c)+1)^2*(-1+cos(d*x+c))*(5*I*cos(d*x+c)^3*sin(d*x+c)*(1/(c
os(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(-1+cos(d
*x+c))/sin(d*x+c),I)-5*cos(d*x+c)^3+5*cos(d*x+c)^2-3*cos(d*x+c)+3)*(b/cos(d
*x+c))^(1/2)/sin(d*x+c)^3/cos(d*x+c)^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\sec\left(dx+c\right)}\sec\left(dx+c\right)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sec(d*x + c))*sec(d*x + c)^4, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{b\sec(dx+c)}\sec(dx+c)^4, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(d*x + c))*sec(d*x + c)^4, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\sec(c+dx)}\sec^4(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*(b*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(b*sec(c + d*x))*sec(c + d*x)**4, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\sec(dx+c)}\sec(dx+c)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sec(d*x + c))*sec(d*x + c)^4, x)
```

3.71 $\int \sec^3(c+dx)\sqrt{b\sec(c+dx)}\,dx$

Optimal. Leaf size=95

$$\frac{2\sin(c+dx)(b\sec(c+dx))^{5/2}}{5b^2d}+\frac{6\sin(c+dx)\sqrt{b\sec(c+dx)}}{5d}-\frac{6bE\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}$$

```
[Out] (-6*b*EllipticE[(c + d*x)/2, 2])/(5*d*Sqrt[Cos[c + d*x]]*Sqrt[b*Sec[c + d*x
]]) + (6*Sqrt[b*Sec[c + d*x]]*Sin[c + d*x])/(5*d) + (2*(b*Sec[c + d*x])^(5/
2)*Sin[c + d*x])/(5*b^2*d)
```

Rubi [A] time = 0.0606493, antiderivative size = 95, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {16, 3768, 3771, 2639}

$$\frac{2\sin(c+dx)(b\sec(c+dx))^{5/2}}{5b^2d}+\frac{6\sin(c+dx)\sqrt{b\sec(c+dx)}}{5d}-\frac{6bE\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3*Sqrt[b*Sec[c + d*x]],x]
```

```
[Out] (-6*b*EllipticE[(c + d*x)/2, 2])/(5*d*Sqrt[Cos[c + d*x]]*Sqrt[b*Sec[c + d*x
]]) + (6*Sqrt[b*Sec[c + d*x]]*Sin[c + d*x])/(5*d) + (2*(b*Sec[c + d*x])^(5/
2)*Sin[c + d*x])/(5*b^2*d)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sec^3(c+dx)\sqrt{b\sec(c+dx)}\,dx = \frac{\int (b\sec(c+dx))^{7/2}\,dx}{b^3}$$
$$= \frac{2(b\sec(c+dx))^{5/2}\sin(c+dx)}{5b^2d} + \frac{3\int (b\sec(c+dx))^{3/2}\,dx}{5b}$$
$$= \frac{6\sqrt{b\sec(c+dx)}\sin(c+dx)}{5d} + \frac{2(b\sec(c+dx))^{5/2}\sin(c+dx)}{5b^2d} - \frac{1}{5}(3b)\int \frac{1}{\sqrt{b\sec(c}$$
$$= \frac{6\sqrt{b\sec(c+dx)}\sin(c+dx)}{5d} + \frac{2(b\sec(c+dx))^{5/2}\sin(c+dx)}{5b^2d} - \frac{(3b)\int \sqrt{\cos(c}}{5\sqrt{\cos(c+dx)}\sqrt{b}}$$
$$= -\frac{6bE\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}} + \frac{6\sqrt{b\sec(c+dx)}\sin(c+dx)}{5d} + \frac{2(b\sec(c+dx)}{5b}$$

Mathematica [A] time = 0.165085, size = 69, normalized size = 0.73

$$\frac{\sec^2(c+dx)\sqrt{b\sec(c+dx)}\left(7\sin(c+dx)+3\sin(3(c+dx))-12\cos^{\frac{5}{2}}(c+dx)E\left(\frac{1}{2}(c+dx)\middle|2\right)\right)}{10d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3*Sqrt[b*Sec[c + d*x]],x]
```

```
[Out] (Sec[c + d*x]^2*Sqrt[b*Sec[c + d*x]]*(-12*Cos[c + d*x]^(5/2)*EllipticE[(c +
 d*x)/2, 2] + 7*Sin[c + d*x] + 3*Sin[3*(c + d*x)]))/(10*d)
```

Maple [C] time = 0.208, size = 356, normalized size = 3.8

$$\frac{2(\cos(dx+c)+1)^2(-1+\cos(dx+c))^2}{5d(\sin(dx+c))^5(\cos(dx+c))^2}\left(3i\sqrt{(\cos(dx+c)+1)^{-1}}\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}(\cos(dx+c))^3\,\mathrm{EllipticE}\left(\frac{i(-1}{s}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(b*sec(d*x+c))^(1/2),x)
```

```
[Out] 2/5/d*(cos(d*x+c)+1)^2*(-1+cos(d*x+c))^2*(3*I*(1/(cos(d*x+c)+1))^(1/2)*(cos
(d*x+c)/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)^3*EllipticE(I*(-1+cos(d*x+c))/sin(
d*x+c),I)*sin(d*x+c)-3*I*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1
))^(1/2)*EllipticF(I*(-1+cos(d*x+c))/sin(d*x+c),I)*cos(d*x+c)^3*sin(d*x+c)+
3*I*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)^2
*EllipticE(I*(-1+cos(d*x+c))/sin(d*x+c),I)*sin(d*x+c)-3*I*(1/(cos(d*x+c)+1)
)^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(-1+cos(d*x+c))/sin(d
*x+c),I)*cos(d*x+c)^2*sin(d*x+c)-3*cos(d*x+c)^3+2*cos(d*x+c)^2+1)*(b/cos(d*
x+c))^(1/2)/sin(d*x+c)^5/cos(d*x+c)^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\sec(dx+c)}\sec(dx+c)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sec(d*x + c))*sec(d*x + c)^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\sqrt{b\sec\left(dx+c\right)}\sec\left(dx+c\right)^{3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(d*x + c))*sec(d*x + c)^3, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{b\sec\left(c+dx\right)}\sec^{3}\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(b*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(b*sec(c + d*x))*sec(c + d*x)**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{b\sec\left(dx+c\right)}\sec\left(dx+c\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sec(d*x + c))*sec(d*x + c)^3, x)
```

3.72 $\int \sec^2(c+dx)\sqrt{b\sec(c+dx)}\,dx$

Optimal. Leaf size=69

$$\frac{2\sqrt{\cos(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{b\sec(c+dx)}}{3d}+\frac{2\sin(c+dx)(b\sec(c+dx))^{3/2}}{3bd}$$

```
[Out] (2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[b*Sec[c + d*x]])/(3*d)
 + (2*(b*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(3*b*d)
```

Rubi [A] time = 0.0420639, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {16, 3768, 3771, 2641}

$$\frac{2\sin(c+dx)(b\sec(c+dx))^{3/2}}{3bd}+\frac{2\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\sec(c+dx)}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2*Sqrt[b*Sec[c + d*x]],x]
```

```
[Out] (2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[b*Sec[c + d*x]])/(3*d)
 + (2*(b*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(3*b*d)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sec^2(c+dx)\sqrt{b\sec(c+dx)}\,dx = \frac{\int (b\sec(c+dx))^{5/2}\,dx}{b^2}$$
$$= \frac{2(b\sec(c+dx))^{3/2}\sin(c+dx)}{3bd} + \frac{1}{3}\int \sqrt{b\sec(c+dx)}\,dx$$
$$= \frac{2(b\sec(c+dx))^{3/2}\sin(c+dx)}{3bd} + \frac{1}{3}\left(\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx$$
$$= \frac{2\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\sec(c+dx)}}{3d} + \frac{2(b\sec(c+dx))^{3/2}\sin(c+dx)}{3bd}$$

Mathematica [A] time = 0.0836222, size = 51, normalized size = 0.74

$$\frac{2(b\sec(c+dx))^{3/2}\left(\cos^{\frac{3}{2}}(c+dx)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+\sin(c+dx)\right)}{3bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2*Sqrt[b*Sec[c + d*x]],x]
```

```
[Out] (2*(b*Sec[c + d*x])^(3/2)*(Cos[c + d*x]^(3/2)*EllipticF[(c + d*x)/2, 2] + S
in[c + d*x]))/(3*b*d)
```

Maple [C] time = 0.155, size = 130, normalized size = 1.9

$$-\frac{(-2+2\cos(dx+c))(\cos(dx+c)+1)^2}{3d(\sin(dx+c))^3\cos(dx+c)}\sqrt{\frac{b}{\cos(dx+c)}}\left(i\sqrt{(\cos(dx+c)+1)^{-1}}\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\text{EllipticF}\left(\frac{i(-1+\cos}{\sin(d}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(b*sec(d*x+c))^(1/2),x)
```

```
[Out] -2/3/d*(b/cos(d*x+c))^(1/2)*(-1+cos(d*x+c))*(I*(1/(cos(d*x+c)+1))^(1/2)*(co
s(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(-1+cos(d*x+c))/sin(d*x+c),I)*co
s(d*x+c)*sin(d*x+c)-cos(d*x+c)+1)*(cos(d*x+c)+1)^2/sin(d*x+c)^3/cos(d*x+c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\sec(dx+c)}\sec(dx+c)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sec(d*x + c))*sec(d*x + c)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{b\sec(dx+c)}\sec(dx+c)^2,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(d*x + c))*sec(d*x + c)^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\sec(c+dx)}\sec^2(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(b*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(b*sec(c + d*x))*sec(c + d*x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\sec(dx+c)}\sec(dx+c)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sec(d*x + c))*sec(d*x + c)^2, x)
```

3.73 $\int \sec(c+dx)\sqrt{b\sec(c+dx)}\,dx$

Optimal. Leaf size=63

$$\frac{2\sin(c+dx)\sqrt{b\sec(c+dx)}}{d}-\frac{2bE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}$$

```
[Out] (-2*b*EllipticE[(c + d*x)/2, 2])/(d*Sqrt[Cos[c + d*x]]*Sqrt[b*Sec[c + d*x]]
) + (2*Sqrt[b*Sec[c + d*x]]*Sin[c + d*x])/d
```

Rubi [A] time = 0.0407368, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {16, 3768, 3771, 2639}

$$\frac{2\sin(c+dx)\sqrt{b\sec(c+dx)}}{d}-\frac{2bE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*Sqrt[b*Sec[c + d*x]],x]
```

```
[Out] (-2*b*EllipticE[(c + d*x)/2, 2])/(d*Sqrt[Cos[c + d*x]]*Sqrt[b*Sec[c + d*x]]
) + (2*Sqrt[b*Sec[c + d*x]]*Sin[c + d*x])/d
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sec(c+dx)\sqrt{b\sec(c+dx)}\,dx = \frac{\int (b\sec(c+dx))^{3/2}\,dx}{b}$$
$$= \frac{2\sqrt{b\sec(c+dx)}\sin(c+dx)}{d} - b\int \frac{1}{\sqrt{b\sec(c+dx)}}\,dx$$
$$= \frac{2\sqrt{b\sec(c+dx)}\sin(c+dx)}{d} - \frac{b\int \sqrt{\cos(c+dx)}\,dx}{\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}$$
$$= -\frac{2bE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}} + \frac{2\sqrt{b\sec(c+dx)}\sin(c+dx)}{d}$$

Mathematica [A] time = 0.0511457, size = 47, normalized size = 0.75

$$\frac{2\sqrt{b\sec(c+dx)}\left(\sin(c+dx) - \sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]*Sqrt[b*Sec[c + d*x]],x]
```

```
[Out] (2*Sqrt[b*Sec[c + d*x]]*(-(Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]) +
Sin[c + d*x]))/d
```

Maple [C] time = 0.214, size = 316, normalized size = 5.

$$2\,\frac{(\cos(dx+c)+1)^2(-1+\cos(dx+c))^2}{d(\sin(dx+c))^5}\sqrt{\frac{b}{\cos(dx+c)}}\left(i\cos(dx+c)\sin(dx+c)\sqrt{(\cos(dx+c)+1)^{-1}}\sqrt{\frac{\cos(dx+}{\cos(dx+c}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(b*sec(d*x+c))^(1/2),x)
```

```
[Out] 2/d*(b/cos(d*x+c))^(1/2)*(cos(d*x+c)+1)^2*(-1+cos(d*x+c))^2*(I*cos(d*x+c)*s
in(d*x+c)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*Ellipt
icE(I*(-1+cos(d*x+c))/sin(d*x+c),I)-I*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/
(cos(d*x+c)+1))^(1/2)*EllipticF(I*(-1+cos(d*x+c))/sin(d*x+c),I)*cos(d*x+c)*
sin(d*x+c)+I*sin(d*x+c)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1)
)^(1/2)*EllipticE(I*(-1+cos(d*x+c))/sin(d*x+c),I)-I*EllipticF(I*(-1+cos(d*x
+c))/sin(d*x+c),I)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/
2)*sin(d*x+c)-cos(d*x+c)+1)/sin(d*x+c)^5
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\sec(dx+c)}\sec(dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sec(d*x + c))*sec(d*x + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\sqrt{b\sec\left(dx+c\right)}\sec\left(dx+c\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(d*x + c))*sec(d*x + c), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{b\sec\left(c+dx\right)}\sec\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(b*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(b*sec(c + d*x))*sec(c + d*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{b\sec\left(dx+c\right)}\sec\left(dx+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sec(d*x + c))*sec(d*x + c), x)
```

3.74 $\int \sqrt{b \sec(c+dx)}\,dx$

Optimal. Leaf size=38

$$\frac{2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{b\sec(c+dx)}}{d}$$

```
[Out] (2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[b*Sec[c + d*x]])/d
```

Rubi [A] time = 0.0202235, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3771, 2641}

$$\frac{2\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\sec(c+dx)}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*Sec[c + d*x]],x]
```

```
[Out] (2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[b*Sec[c + d*x]])/d
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \sqrt{b\sec(c+dx)}\,dx &= \left(\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx\\ &= \frac{2\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\sec(c+dx)}}{d}\end{aligned}$$

Mathematica [A] time = 0.0202658, size = 38, normalized size = 1.

$$\frac{2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{b\sec(c+dx)}}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*Sec[c + d*x]],x]
```

```
[Out] (2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[b*Sec[c + d*x]])/d
```

Maple [C] time = 0.141, size = 98, normalized size = 2.6

$$\frac{-2\,i\,(-1+\cos(dx+c))\,(\cos(dx+c)+1)^{2}}{d\,(\sin(dx+c))^{2}}\sqrt{\frac{b}{\cos(dx+c)}}\sqrt{(\cos(dx+c)+1)^{-1}}\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\text{EllipticF}\left(\frac{i\,(-1+\cos}{\sin(dx}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sec(d*x+c))^(1/2),x)
```

```
[Out] -2*I/d*(b/cos(d*x+c))^(1/2)*(-1+cos(d*x+c))*(1/(cos(d*x+c)+1))^(1/2)*(cos(d
*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(-1+cos(d*x+c))/sin(d*x+c),I)*(cos(
d*x+c)+1)^2/sin(d*x+c)^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{b\sec(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sec(d*x + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{b\sec(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(d*x + c)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{b\sec(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(b*sec(c + d*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{b\sec(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sec(d*x + c)), x)
```

3.75 $\int \cos(c+dx)\sqrt{b\sec(c+dx)}\,dx$

Optimal. Leaf size=39

$$\frac{2bE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}$$

```
[Out] (2*b*EllipticE[(c + d*x)/2, 2])/(d*Sqrt[Cos[c + d*x]]*Sqrt[b*Sec[c + d*x]])
```

Rubi [A] time = 0.0296558, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {16, 3771, 2639}

$$\frac{2bE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*Sqrt[b*Sec[c + d*x]],x]
```

```
[Out] (2*b*EllipticE[(c + d*x)/2, 2])/(d*Sqrt[Cos[c + d*x]]*Sqrt[b*Sec[c + d*x]])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \cos(c+dx)\sqrt{b\sec(c+dx)}\,dx &= b\int \frac{1}{\sqrt{b\sec(c+dx)}}\,dx\\ &= \frac{b\int \sqrt{\cos(c+dx)}\,dx}{\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}\\ &= \frac{2bE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}\end{aligned}$$

Mathematica [A] time = 0.0348944, size = 39, normalized size = 1.

$$\frac{2bE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*Sqrt[b*Sec[c + d*x]],x]
```

```
[Out] (2*b*EllipticE[(c + d*x)/2, 2])/(d*Sqrt[Cos[c + d*x]]*Sqrt[b*Sec[c + d*x]])
```

Maple [C] time = 0.149, size = 303, normalized size = 7.8

$$2\frac{1}{d\sin(dx+c)}\left(i\sqrt{(\cos(dx+c)+1)^{-1}}\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\text{EllipticF}\left(\frac{i(-1+\cos(dx+c))}{\sin(dx+c)},i\right)\cos(dx+c)\sin(dx+c)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(b*sec(d*x+c))^(1/2),x)
```

```
[Out] 2/d*(I*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticF
(I*(-1+cos(d*x+c))/sin(d*x+c),I)*cos(d*x+c)*sin(d*x+c)-I*EllipticE(I*(-1+co
s(d*x+c))/sin(d*x+c),I)*cos(d*x+c)*sin(d*x+c)*(1/(cos(d*x+c)+1))^(1/2)*(cos
(d*x+c)/(cos(d*x+c)+1))^(1/2)+I*EllipticF(I*(-1+cos(d*x+c))/sin(d*x+c),I)*(
1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-I*Elli
pticE(I*(-1+cos(d*x+c))/sin(d*x+c),I)*sin(d*x+c)*(1/(cos(d*x+c)+1))^(1/2)*(
cos(d*x+c)/(cos(d*x+c)+1))^(1/2)-cos(d*x+c)^2+cos(d*x+c))*(b/cos(d*x+c))^(1
/2)/sin(d*x+c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{b\sec(dx+c)}\cos(dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sec(d*x + c))*cos(d*x + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{b\sec(dx+c)}\cos(dx+c),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(d*x + c))*cos(d*x + c), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{b\sec(c+dx)}\cos(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(b*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(b*sec(c + d*x))*cos(c + d*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\sec(dx+c)}\cos(dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sec(d*x + c))*cos(d*x + c), x)
```

3.76 $\int \cos^2(c+dx)\sqrt{b\sec(c+dx)}\,dx$

Optimal. Leaf size=67

$$\frac{2\sqrt{\cos(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{b\sec(c+dx)}}{3d}+\frac{2b\sin(c+dx)}{3d\sqrt{b\sec(c+dx)}}$$

```
[Out] (2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[b*Sec[c + d*x]])/(3*d)
 + (2*b*Sin[c + d*x])/(3*d*Sqrt[b*Sec[c + d*x]])
```

Rubi [A] time = 0.0516895, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {16, 3769, 3771, 2641}

$$\frac{2b\sin(c+dx)}{3d\sqrt{b\sec(c+dx)}}+\frac{2\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\sec(c+dx)}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*Sqrt[b*Sec[c + d*x]],x]
```

```
[Out] (2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[b*Sec[c + d*x]])/(3*d)
 + (2*b*Sin[c + d*x])/(3*d*Sqrt[b*Sec[c + d*x]])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cos^2(c+dx)\sqrt{b\sec(c+dx)}\,dx = b^2\int \frac{1}{(b\sec(c+dx))^{3/2}}\,dx$$
$$= \frac{2b\sin(c+dx)}{3d\sqrt{b\sec(c+dx)}} + \frac{1}{3}\int \sqrt{b\sec(c+dx)}\,dx$$
$$= \frac{2b\sin(c+dx)}{3d\sqrt{b\sec(c+dx)}} + \frac{1}{3}\left(\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx$$
$$= \frac{2\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\sec(c+dx)}}{3d} + \frac{2b\sin(c+dx)}{3d\sqrt{b\sec(c+dx)}}$$

Mathematica [A] time = 0.0513573, size = 51, normalized size = 0.76

$$\frac{\sqrt{b\sec(c+dx)}\left(2\sqrt{\cos(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+\sin(2(c+dx))\right)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*Sqrt[b*Sec[c + d*x]],x]
```

```
[Out] (Sqrt[b*Sec[c + d*x]]*(2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + Sin
[2*(c + d*x)]))/(3*d)
```

Maple [C] time = 0.164, size = 123, normalized size = 1.8

$$-\frac{(-2+2\cos(dx+c))(\cos(dx+c)+1)^2}{3d(\sin(dx+c))^3}\left(i\text{EllipticF}\left(\frac{i(-1+\cos(dx+c))}{\sin(dx+c)},i\right)\sqrt{(\cos(dx+c)+1)^{-1}}\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,s\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(b*sec(d*x+c))^(1/2),x)
```

```
[Out] -2/3/d*(-1+cos(d*x+c))*(I*EllipticF(I*(-1+cos(d*x+c))/sin(d*x+c),I)*(1/(cos
(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-cos(d*x+c)^2
+cos(d*x+c))*(cos(d*x+c)+1)^2*(b/cos(d*x+c))^(1/2)/sin(d*x+c)^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\sec(dx+c)}\cos(dx+c)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sec(d*x + c))*cos(d*x + c)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{b\sec(dx+c)}\cos(dx+c)^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(d*x + c))*cos(d*x + c)^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\sec\left(c+dx\right)}\cos^2\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(b*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(b*sec(c + d*x))*cos(c + d*x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\sec\left(dx+c\right)}\cos\left(dx+c\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sec(d*x + c))*cos(d*x + c)^2, x)
```

3.77 $\int \cos^3(c+dx)\sqrt{b\sec(c+dx)}\,dx$

Optimal. Leaf size=70

$$\frac{2b^2\sin(c+dx)}{5d(b\sec(c+dx))^{3/2}}+\frac{6bE\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}$$

```
[Out] (6*b*EllipticE[(c + d*x)/2, 2])/(5*d*Sqrt[Cos[c + d*x]]*Sqrt[b*Sec[c + d*x]
]) + (2*b^2*Sin[c + d*x])/(5*d*(b*Sec[c + d*x])^(3/2))
```

Rubi [A] time = 0.0514325, antiderivative size = 70, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {16, 3769, 3771, 2639}

$$\frac{2b^2\sin(c+dx)}{5d(b\sec(c+dx))^{3/2}}+\frac{6bE\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*Sqrt[b*Sec[c + d*x]],x]
```

```
[Out] (6*b*EllipticE[(c + d*x)/2, 2])/(5*d*Sqrt[Cos[c + d*x]]*Sqrt[b*Sec[c + d*x]
]) + (2*b^2*Sin[c + d*x])/(5*d*(b*Sec[c + d*x])^(3/2))
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \cos^3(c+dx)\sqrt{b\sec(c+dx)}\,dx &= b^3\int \frac{1}{(b\sec(c+dx))^{5/2}}\,dx\\ &= \frac{2b^2\sin(c+dx)}{5d(b\sec(c+dx))^{3/2}}+\frac{1}{5}(3b)\int \frac{1}{\sqrt{b\sec(c+dx)}}\,dx\\ &= \frac{2b^2\sin(c+dx)}{5d(b\sec(c+dx))^{3/2}}+\frac{(3b)\int \sqrt{\cos(c+dx)}\,dx}{5\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}\\ &= \frac{6bE\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}+\frac{2b^2\sin(c+dx)}{5d(b\sec(c+dx))^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.0594578, size = 57, normalized size = 0.81

$$\frac{\sqrt{b\sec(c+dx)}\left(\sin(c+dx)+\sin(3(c+dx))+12\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\right)}{10d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*Sqrt[b*Sec[c + d*x]],x]
```

```
[Out] (Sqrt[b*Sec[c + d*x]]*(12*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + Si
n[c + d*x] + Sin[3*(c + d*x)]))/(10*d)
```

Maple [C] time = 0.208, size = 315, normalized size = 4.5

$$\frac{2}{5\,d\sin(dx+c)}\sqrt{\frac{b}{\cos(dx+c)}}\left(3\,i\sqrt{(\cos(dx+c)+1)^{-1}}\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\mathrm{EllipticF}\left(\frac{i(-1+\cos(dx+c))}{\sin(dx+c)},i\right)\cos(dx\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(b*sec(d*x+c))^(1/2),x)
```

```
[Out] 2/5/d*(b/cos(d*x+c))^(1/2)*(3*I*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d
*x+c)+1))^(1/2)*EllipticF(I*(-1+cos(d*x+c))/sin(d*x+c),I)*cos(d*x+c)*sin(d*
x+c)-3*I*cos(d*x+c)*sin(d*x+c)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*
x+c)+1))^(1/2)*EllipticE(I*(-1+cos(d*x+c))/sin(d*x+c),I)+3*I*EllipticF(I*(-
1+cos(d*x+c))/sin(d*x+c),I)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c
)+1))^(1/2)*sin(d*x+c)-3*I*sin(d*x+c)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/
(cos(d*x+c)+1))^(1/2)*EllipticE(I*(-1+cos(d*x+c))/sin(d*x+c),I)-cos(d*x+c)^
4-2*cos(d*x+c)^2+3*cos(d*x+c))/sin(d*x+c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\sec(dx+c)}\cos(dx+c)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sec(d*x + c))*cos(d*x + c)^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{b\sec\left(dx+c\right)}\cos\left(dx+c\right)^{3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(d*x + c))*cos(d*x + c)^3, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(b*sec(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\sec\left(dx+c\right)}\cos\left(dx+c\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sec(d*x + c))*cos(d*x + c)^3, x)
```

3.78 $\int \cos^4(c+dx)\sqrt{b\sec(c+dx)}\,dx$

Optimal. Leaf size=95

$$\frac{10\sqrt{\cos(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{b\sec(c+dx)}}{21d}+\frac{2b^3\sin(c+dx)}{7d(b\sec(c+dx))^{5/2}}+\frac{10b\sin(c+dx)}{21d\sqrt{b\sec(c+dx)}}$$

```
[Out] (10*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[b*Sec[c + d*x]])/(21*
d) + (2*b^3*Sin[c + d*x])/(7*d*(b*Sec[c + d*x])^(5/2)) + (10*b*Sin[c + d*x]
)/(21*d*Sqrt[b*Sec[c + d*x]])
```

Rubi [A] time = 0.0701063, antiderivative size = 95, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {16, 3769, 3771, 2641}

$$\frac{2b^3\sin(c+dx)}{7d(b\sec(c+dx))^{5/2}}+\frac{10b\sin(c+dx)}{21d\sqrt{b\sec(c+dx)}}+\frac{10\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\sec(c+dx)}}{21d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*Sqrt[b*Sec[c + d*x]],x]
```

```
[Out] (10*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[b*Sec[c + d*x]])/(21*
d) + (2*b^3*Sin[c + d*x])/(7*d*(b*Sec[c + d*x])^(5/2)) + (10*b*Sin[c + d*x]
)/(21*d*Sqrt[b*Sec[c + d*x]])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cos^4(c+dx)\sqrt{b\sec(c+dx)}\,dx = b^4\int \frac{1}{(b\sec(c+dx))^{7/2}}\,dx$$
$$= \frac{2b^3\sin(c+dx)}{7d(b\sec(c+dx))^{5/2}} + \frac{1}{7}\left(5b^2\right)\int \frac{1}{(b\sec(c+dx))^{3/2}}\,dx$$
$$= \frac{2b^3\sin(c+dx)}{7d(b\sec(c+dx))^{5/2}} + \frac{10b\sin(c+dx)}{21d\sqrt{b\sec(c+dx)}} + \frac{5}{21}\int \sqrt{b\sec(c+dx)}\,dx$$
$$= \frac{2b^3\sin(c+dx)}{7d(b\sec(c+dx))^{5/2}} + \frac{10b\sin(c+dx)}{21d\sqrt{b\sec(c+dx)}} + \frac{1}{21}\left(5\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}\right)\int \frac{}{\sqrt{}}$$
$$= \frac{10\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\sec(c+dx)}}{21d} + \frac{2b^3\sin(c+dx)}{7d(b\sec(c+dx))^{5/2}} + \frac{10b\sin(c}{21d\sqrt{b\sec(}}$$

Mathematica [A] time = 0.089581, size = 63, normalized size = 0.66

$$\frac{\sqrt{b\sec(c+dx)}\left(40\sqrt{\cos(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+26\sin(2(c+dx))+3\sin(4(c+dx))\right)}{84d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*Sqrt[b*Sec[c + d*x]],x]
```

```
[Out] (Sqrt[b*Sec[c + d*x]]*(40*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + 26
*Sin[2*(c + d*x)] + 3*Sin[4*(c + d*x)]))/(84*d)
```

Maple [C] time = 0.188, size = 145, normalized size = 1.5

$$-\frac{(-2+2\cos(dx+c))(\cos(dx+c)+1)^2}{21\,d\,(\sin(dx+c))^3}\left(5\,i\text{EllipticF}\left(\frac{i(-1+\cos(dx+c))}{\sin(dx+c)},i\right)\sqrt{(\cos(dx+c)+1)^{-1}}\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*(b*sec(d*x+c))^(1/2),x)
```

```
[Out] -2/21/d*(-1+cos(d*x+c))*(5*I*EllipticF(I*(-1+cos(d*x+c))/sin(d*x+c),I)*(1/(
cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-3*cos(d*x
+c)^4+3*cos(d*x+c)^3-5*cos(d*x+c)^2+5*cos(d*x+c))*(cos(d*x+c)+1)^2*(b/cos(d
*x+c))^(1/2)/sin(d*x+c)^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\sec(dx+c)}\cos(dx+c)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sec(d*x + c))*cos(d*x + c)^4, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{b\sec\left(dx+c\right)}\cos\left(dx+c\right)^{4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(d*x + c))*cos(d*x + c)^4, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*(b*sec(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.79 $\int \cos^5(c+dx)\sqrt{b\sec(c+dx)}\,dx$

Optimal. Leaf size=98

$$\frac{2b^4\sin(c+dx)}{9d(b\sec(c+dx))^{7/2}}+\frac{14b^2\sin(c+dx)}{45d(b\sec(c+dx))^{3/2}}+\frac{14bE\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}$$

```
[Out] (14*b*EllipticE[(c + d*x)/2, 2])/(15*d*Sqrt[Cos[c + d*x]]*Sqrt[b*Sec[c + d*
x]]) + (2*b^4*Sin[c + d*x])/(9*d*(b*Sec[c + d*x])^(7/2)) + (14*b^2*Sin[c +
d*x])/(45*d*(b*Sec[c + d*x])^(3/2))
```

Rubi [A] time = 0.0693045, antiderivative size = 98, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {16, 3769, 3771, 2639}

$$\frac{2b^4\sin(c+dx)}{9d(b\sec(c+dx))^{7/2}}+\frac{14b^2\sin(c+dx)}{45d(b\sec(c+dx))^{3/2}}+\frac{14bE\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5*Sqrt[b*Sec[c + d*x]],x]
```

```
[Out] (14*b*EllipticE[(c + d*x)/2, 2])/(15*d*Sqrt[Cos[c + d*x]]*Sqrt[b*Sec[c + d*
x]]) + (2*b^4*Sin[c + d*x])/(9*d*(b*Sec[c + d*x])^(7/2)) + (14*b^2*Sin[c +
d*x])/(45*d*(b*Sec[c + d*x])^(3/2))
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cos^5(c+dx)\sqrt{b\sec(c+dx)}\,dx = b^5\int \frac{1}{(b\sec(c+dx))^{9/2}}\,dx$$
$$= \frac{2b^4\sin(c+dx)}{9d(b\sec(c+dx))^{7/2}} + \frac{1}{9}\left(7b^3\right)\int \frac{1}{(b\sec(c+dx))^{5/2}}\,dx$$
$$= \frac{2b^4\sin(c+dx)}{9d(b\sec(c+dx))^{7/2}} + \frac{14b^2\sin(c+dx)}{45d(b\sec(c+dx))^{3/2}} + \frac{1}{15}(7b)\int \frac{1}{\sqrt{b\sec(c+dx)}}\,dx$$
$$= \frac{2b^4\sin(c+dx)}{9d(b\sec(c+dx))^{7/2}} + \frac{14b^2\sin(c+dx)}{45d(b\sec(c+dx))^{3/2}} + \frac{(7b)\int \sqrt{\cos(c+dx)}\,dx}{15\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}$$
$$= \frac{14bE\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}} + \frac{2b^4\sin(c+dx)}{9d(b\sec(c+dx))^{7/2}} + \frac{14b^2\sin(c+dx)}{45d(b\sec(c+dx))^{3/2}}$$

Mathematica [A] time = 0.243606, size = 71, normalized size = 0.72

$$\frac{\sqrt{b\sec(c+dx)}\left((33\sin(c+dx)+5\sin(3(c+dx)))\cos^2(c+dx)+84\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\right)}{90d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5*Sqrt[b*Sec[c + d*x]],x]
```

```
[Out] (Sqrt[b*Sec[c + d*x]]*(84*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + Co
s[c + d*x]^2*(33*Sin[c + d*x] + 5*Sin[3*(c + d*x)])))/(90*d)
```

Maple [C] time = 0.181, size = 325, normalized size = 3.3

$$-\frac{2}{45\,d\sin(dx+c)}\sqrt{\frac{b}{\cos(dx+c)}}\left(21\,i\cos(dx+c)\sin(dx+c)\sqrt{(\cos(dx+c)+1)^{-1}}\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\mathrm{EllipticE}\left(\frac{i(-\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*(b*sec(d*x+c))^(1/2),x)
```

```
[Out] -2/45/d*(b/cos(d*x+c))^(1/2)*(21*I*cos(d*x+c)*sin(d*x+c)*(1/(cos(d*x+c)+1))
^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticE(I*(-1+cos(d*x+c))/sin(d*
x+c),I)-21*I*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*Ell
ipticF(I*(-1+cos(d*x+c))/sin(d*x+c),I)*cos(d*x+c)*sin(d*x+c)+5*cos(d*x+c)^6
+21*I*sin(d*x+c)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)
*EllipticE(I*(-1+cos(d*x+c))/sin(d*x+c),I)-21*I*EllipticF(I*(-1+cos(d*x+c))
/sin(d*x+c),I)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*s
in(d*x+c)+2*cos(d*x+c)^4+14*cos(d*x+c)^2-21*cos(d*x+c))/sin(d*x+c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\sec(dx+c)}\cos(dx+c)^5\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sec(d*x + c))*cos(d*x + c)^5, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{b\sec\left(dx+c\right)}\cos\left(dx+c\right)^{5},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(d*x + c))*cos(d*x + c)^5, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*(b*sec(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{b\sec\left(dx+c\right)}\cos\left(dx+c\right)^{5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sec(d*x + c))*cos(d*x + c)^5, x)
```

3.80 $\int \sec^3(c+dx)(b\sec(c+dx))^{3/2}\,dx$

Optimal. Leaf size=95

$$\frac{10b\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{b\sec(c+dx)}}{21d}+\frac{2\sin(c+dx)(b\sec(c+dx))^{7/2}}{7b^2d}+\frac{10\sin(c+dx)(b\sec(c+d$$

```
[Out] (10*b*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[b*Sec[c + d*x]])/(2
1*d) + (10*(b*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(21*d) + (2*(b*Sec[c + d*x]
)^(7/2)*Sin[c + d*x])/(7*b^2*d)
```

Rubi [A] time = 0.0594839, antiderivative size = 95, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {16, 3768, 3771, 2641}

$$\frac{2\sin(c+dx)(b\sec(c+dx))^{7/2}}{7b^2d}+\frac{10\sin(c+dx)(b\sec(c+dx))^{3/2}}{21d}+\frac{10b\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\sec(c+dx)}}{21d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3*(b*Sec[c + d*x])^(3/2),x]
```

```
[Out] (10*b*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[b*Sec[c + d*x]])/(2
1*d) + (10*(b*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(21*d) + (2*(b*Sec[c + d*x]
)^(7/2)*Sin[c + d*x])/(7*b^2*d)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sec^3(c+dx)(b\sec(c+dx))^{3/2}\,dx = \frac{\int (b\sec(c+dx))^{9/2}\,dx}{b^3}$$
$$= \frac{2(b\sec(c+dx))^{7/2}\sin(c+dx)}{7b^2d} + \frac{5\int (b\sec(c+dx))^{5/2}\,dx}{7b}$$
$$= \frac{10(b\sec(c+dx))^{3/2}\sin(c+dx)}{21d} + \frac{2(b\sec(c+dx))^{7/2}\sin(c+dx)}{7b^2d} + \frac{1}{21}(5b)\int \sqrt{b\,\mathrm{se}$$
$$= \frac{10(b\sec(c+dx))^{3/2}\sin(c+dx)}{21d} + \frac{2(b\sec(c+dx))^{7/2}\sin(c+dx)}{7b^2d} + \frac{1}{21}\left(5b\sqrt{\cos(c+\right.$$
$$= \frac{10b\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\sec(c+dx)}}{21d} + \frac{10(b\sec(c+dx))^{3/2}\sin(c+dx)}{21d}$$

Mathematica [A] time = 0.183115, size = 64, normalized size = 0.67

$$\frac{(b\sec(c+dx))^{5/2}\left(10\cos^{\frac{5}{2}}(c+dx)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+5\sin(2(c+dx))+6\tan(c+dx)\right)}{21bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3*(b*Sec[c + d*x])^(3/2),x]
```

```
[Out] ((b*Sec[c + d*x])^(5/2)*(10*Cos[c + d*x]^(5/2)*EllipticF[(c + d*x)/2, 2] +
5*Sin[2*(c + d*x)] + 6*Tan[c + d*x]))/(21*b*d)
```

Maple [C] time = 0.195, size = 152, normalized size = 1.6

$$-\frac{2\left(\cos(dx+c)+1\right)^2\left(-1+\cos(dx+c)\right)}{21\,d\left(\cos(dx+c)\right)^2\left(\sin(dx+c)\right)^3}\left(5\,i\left(\cos(dx+c)\right)^3\sin(dx+c)\sqrt{\left(\cos(dx+c)+1\right)^{-1}}\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\text{Ellipti}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(b*sec(d*x+c))^(3/2),x)
```

```
[Out] -2/21/d*(cos(d*x+c)+1)^2*(-1+cos(d*x+c))*(5*I*cos(d*x+c)^3*sin(d*x+c)*(1/(c
os(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(-1+cos(d
*x+c))/sin(d*x+c),I)-5*cos(d*x+c)^3+5*cos(d*x+c)^2-3*cos(d*x+c)+3)*(b/cos(d
*x+c))^(3/2)/cos(d*x+c)^2/sin(d*x+c)^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b\sec(dx+c))^{\frac{3}{2}}\sec(dx+c)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c))^(3/2)*sec(d*x + c)^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\sqrt{b\sec(dx+c)}b\sec(dx+c)^4\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(d*x + c))*b*sec(d*x + c)^4, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(b*sec(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b\sec(dx+c))^{\frac{3}{2}}\sec(dx+c)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c))^(3/2)*sec(d*x + c)^3, x)
```

3.81 $\int \sec^2(c+dx)(b\sec(c+dx))^{3/2}\,dx$

Optimal. Leaf size=98

$$-\frac{6b^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}+\frac{2\sin(c+dx)(b\sec(c+dx))^{5/2}}{5bd}+\frac{6b\sin(c+dx)\sqrt{b\sec(c+dx)}}{5d}$$

```
[Out] (-6*b^2*EllipticE[(c + d*x)/2, 2])/(5*d*Sqrt[Cos[c + d*x]]*Sqrt[b*Sec[c + d
*x]]) + (6*b*Sqrt[b*Sec[c + d*x]]*Sin[c + d*x])/(5*d) + (2*(b*Sec[c + d*x])
^(5/2)*Sin[c + d*x])/(5*b*d)
```

Rubi [A] time = 0.0591151, antiderivative size = 98, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {16, 3768, 3771, 2639}

$$-\frac{6b^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}+\frac{2\sin(c+dx)(b\sec(c+dx))^{5/2}}{5bd}+\frac{6b\sin(c+dx)\sqrt{b\sec(c+dx)}}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2*(b*Sec[c + d*x])^(3/2),x]
```

```
[Out] (-6*b^2*EllipticE[(c + d*x)/2, 2])/(5*d*Sqrt[Cos[c + d*x]]*Sqrt[b*Sec[c + d
*x]]) + (6*b*Sqrt[b*Sec[c + d*x]]*Sin[c + d*x])/(5*d) + (2*(b*Sec[c + d*x])
^(5/2)*Sin[c + d*x])/(5*b*d)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sec^2(c+dx)(b\sec(c+dx))^{3/2}\,dx = \frac{\int (b\sec(c+dx))^{7/2}\,dx}{b^2}$$
$$= \frac{2(b\sec(c+dx))^{5/2}\sin(c+dx)}{5bd} + \frac{3}{5}\int (b\sec(c+dx))^{3/2}\,dx$$
$$= \frac{6b\sqrt{b\sec(c+dx)}\sin(c+dx)}{5d} + \frac{2(b\sec(c+dx))^{5/2}\sin(c+dx)}{5bd} - \frac{1}{5}\left(3b^2\right)\int \frac{}{\sqrt{b}}$$
$$= \frac{6b\sqrt{b\sec(c+dx)}\sin(c+dx)}{5d} + \frac{2(b\sec(c+dx))^{5/2}\sin(c+dx)}{5bd} - \frac{\left(3b^2\right)\int \sqrt{\mathrm{co}}}{5\sqrt{\cos(c+dx)}}$$
$$= -\frac{6b^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}} + \frac{6b\sqrt{b\sec(c+dx)}\sin(c+dx)}{5d} + \frac{2(b\sec(c+}{}$$

Mathematica [A] time = 0.174518, size = 64, normalized size = 0.65

$$\frac{(b\sec(c+dx))^{5/2}\left(7\sin(c+dx)+3\sin(3(c+dx))-12\cos^{\frac{5}{2}}(c+dx)E\left(\frac{1}{2}(c+dx)\middle|2\right)\right)}{10bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2*(b*Sec[c + d*x])^(3/2),x]
```

```
[Out] ((b*Sec[c + d*x])^(5/2)*(-12*Cos[c + d*x]^(5/2)*EllipticE[(c + d*x)/2, 2] +
 7*Sin[c + d*x] + 3*Sin[3*(c + d*x)]))/(10*b*d)
```

Maple [C] time = 0.19, size = 356, normalized size = 3.6

$$\frac{2\left(\cos(dx+c)+1\right)^2\left(-1+\cos(dx+c)\right)^2}{5d\left(\sin(dx+c)\right)^5\cos(dx+c)}\left(3i\sqrt{\left(\cos(dx+c)+1\right)^{-1}}\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\left(\cos(dx+c)\right)^3\mathrm{EllipticE}\left(\frac{i(-1}{\mathrm{s}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(b*sec(d*x+c))^(3/2),x)
```

```
[Out] 2/5/d*(cos(d*x+c)+1)^2*(-1+cos(d*x+c))^2*(3*I*(1/(cos(d*x+c)+1))^(1/2)*(cos
(d*x+c)/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)^3*EllipticE(I*(-1+cos(d*x+c))/sin(
d*x+c),I)*sin(d*x+c)-3*I*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1
))^(1/2)*EllipticF(I*(-1+cos(d*x+c))/sin(d*x+c),I)*cos(d*x+c)^3*sin(d*x+c)+
3*I*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)^2
*EllipticE(I*(-1+cos(d*x+c))/sin(d*x+c),I)*sin(d*x+c)-3*I*(1/(cos(d*x+c)+1)
)^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(-1+cos(d*x+c))/sin(d
*x+c),I)*cos(d*x+c)^2*sin(d*x+c)-3*cos(d*x+c)^3+2*cos(d*x+c)^2+1)*(b/cos(d*
x+c))^(3/2)/sin(d*x+c)^5/cos(d*x+c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b\sec(dx+c))^{\frac{3}{2}}\sec(dx+c)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c))^(3/2)*sec(d*x + c)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{b\sec(dx+c)}b\sec(dx+c)^3, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(d*x + c))*b*sec(d*x + c)^3, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (b\sec(c+dx))^{\frac{3}{2}}\sec^2(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(b*sec(d*x+c))**(3/2),x)
```

```
[Out] Integral((b*sec(c + d*x))**(3/2)*sec(c + d*x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b\sec(dx+c))^{\frac{3}{2}}\sec(dx+c)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c))^(3/2)*sec(d*x + c)^2, x)
```

3.82 $\int \sec(c+dx)(b\sec(c+dx))^{3/2}\,dx$

Optimal. Leaf size=67

$$\frac{2b\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{b\sec(c+dx)}}{3d}+\frac{2\sin(c+dx)(b\sec(c+dx))^{3/2}}{3d}$$

```
[Out] (2*b*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[b*Sec[c + d*x]])/(3*
d) + (2*(b*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(3*d)
```

Rubi [A] time = 0.0407184, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {16, 3768, 3771, 2641}

$$\frac{2\sin(c+dx)(b\sec(c+dx))^{3/2}}{3d}+\frac{2b\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\sec(c+dx)}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*(b*Sec[c + d*x])^(3/2),x]
```

```
[Out] (2*b*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[b*Sec[c + d*x]])/(3*
d) + (2*(b*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(3*d)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\int \sec(c+dx)(b\sec(c+dx))^{3/2}\,dx &= \frac{\int (b\sec(c+dx))^{5/2}\,dx}{b} \\
&= \frac{2(b\sec(c+dx))^{3/2}\sin(c+dx)}{3d} + \frac{1}{3}b\int \sqrt{b\sec(c+dx)}\,dx \\
&= \frac{2(b\sec(c+dx))^{3/2}\sin(c+dx)}{3d} + \frac{1}{3}\left(b\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}} \\
&= \frac{2b\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\sec(c+dx)}}{3d} + \frac{2(b\sec(c+dx))^{3/2}\sin(c+dx)}{3d}
\end{aligned}
$$

Mathematica [A] time = 0.0721231, size = 49, normalized size = 0.73

$$
\frac{2b\sqrt{b\sec(c+dx)}\left(\sqrt{\cos(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+\tan(c+dx)\right)}{3d}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]*(b*Sec[c + d*x])^(3/2),x]
```

```
[Out] (2*b*Sqrt[b*Sec[c + d*x]]*(Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + T
an[c + d*x]))/(3*d)
```

Maple [C] time = 0.146, size = 122, normalized size = 1.8

$$
-\frac{2\left(\cos(dx+c)+1\right)^{2}\left(-1+\cos(dx+c)\right)}{3\,d\left(\sin(dx+c)\right)^{3}}\left(i\sqrt{\left(\cos(dx+c)+1\right)^{-1}}\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\text{EllipticF}\left(\frac{i\left(-1+\cos(dx+c)\right)}{\sin(dx+c)},i\right)\text{c}\right.
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(b*sec(d*x+c))^(3/2),x)
```

```
[Out] -2/3/d*(cos(d*x+c)+1)^2*(-1+cos(d*x+c))*(I*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*
x+c)/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(-1+cos(d*x+c))/sin(d*x+c),I)*cos(d*
x+c)*sin(d*x+c)-cos(d*x+c)+1)*(b/cos(d*x+c))^(3/2)/sin(d*x+c)^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$
\int \left(b\sec(dx+c)\right)^{\frac{3}{2}}\sec(dx+c)\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c))^(3/2)*sec(d*x + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$
\text{integral}\left(\sqrt{b\sec(dx+c)}b\sec(dx+c)^{2},x\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(d*x + c))*b*sec(d*x + c)^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (b\sec(c+dx))^{\frac{3}{2}}\sec(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(b*sec(d*x+c))**(3/2),x)
```

```
[Out] Integral((b*sec(c + d*x))**(3/2)*sec(c + d*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b\sec(dx+c))^{\frac{3}{2}}\sec(dx+c)\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c))^(3/2)*sec(d*x + c), x)
```

3.83 $\int (b\sec(c+dx))^{3/2}\,dx$

Optimal. Leaf size=66

$$\frac{2b\sin(c+dx)\sqrt{b\sec(c+dx)}}{d}-\frac{2b^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}$$

```
[Out] (-2*b^2*EllipticE[(c + d*x)/2, 2])/(d*Sqrt[Cos[c + d*x]]*Sqrt[b*Sec[c + d*x
]]) + (2*b*Sqrt[b*Sec[c + d*x]]*Sin[c + d*x])/d
```

Rubi [A] time = 0.0335228, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {3768, 3771, 2639}

$$\frac{2b\sin(c+dx)\sqrt{b\sec(c+dx)}}{d}-\frac{2b^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Sec[c + d*x])^(3/2),x]
```

```
[Out] (-2*b^2*EllipticE[(c + d*x)/2, 2])/(d*Sqrt[Cos[c + d*x]]*Sqrt[b*Sec[c + d*x
]]) + (2*b*Sqrt[b*Sec[c + d*x]]*Sin[c + d*x])/d
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int (b\sec(c+dx))^{3/2}\,dx &= \frac{2b\sqrt{b\sec(c+dx)}\sin(c+dx)}{d}-b^2\int \frac{1}{\sqrt{b\sec(c+dx)}}\,dx\\
&= \frac{2b\sqrt{b\sec(c+dx)}\sin(c+dx)}{d}-\frac{b^2\int \sqrt{\cos(c+dx)}\,dx}{\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}\\
&= -\frac{2b^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}+\frac{2b\sqrt{b\sec(c+dx)}\sin(c+dx)}{d}
\end{aligned}$$

Mathematica [A] time = 0.0370968, size = 48, normalized size = 0.73

$$\frac{2b\sqrt{b\sec(c+dx)}\left(\sin(c+dx)-\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Sec[c + d*x])^(3/2),x]
```

```
[Out] (2*b*Sqrt[b*Sec[c + d*x]]*(-(Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])
+ Sin[c + d*x]))/d
```

Maple [C] time = 0.199, size = 320, normalized size = 4.9

$$-2\frac{(\cos(dx+c)+1)^2(-1+\cos(dx+c))^2\cos(dx+c)}{d(\sin(dx+c))^5}\left(i\sqrt{(\cos(dx+c)+1)^{-1}}\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\mathrm{EllipticF}\left(\frac{i(-1+c}{\sin(}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sec(d*x+c))^(3/2),x)
```

```
[Out] -2/d*(cos(d*x+c)+1)^2*(-1+cos(d*x+c))^2*(I*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*
x+c)/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(-1+cos(d*x+c))/sin(d*x+c),I)*cos(d*
x+c)*sin(d*x+c)-I*EllipticE(I*(-1+cos(d*x+c))/sin(d*x+c),I)*cos(d*x+c)*sin(
d*x+c)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+I*Ellipti
cF(I*(-1+cos(d*x+c))/sin(d*x+c),I)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(co
s(d*x+c)+1))^(1/2)*sin(d*x+c)-I*EllipticE(I*(-1+cos(d*x+c))/sin(d*x+c),I)*s
in(d*x+c)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+cos(d*
x+c)-1)*cos(d*x+c)*(b/cos(d*x+c))^(3/2)/sin(d*x+c)^5
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b\sec(dx+c))^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c))^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\sqrt{b\sec(dx+c)}b\sec(dx+c),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(d*x + c))*b*sec(d*x + c), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (b\sec(c+dx))^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))**(3/2),x)
```

```
[Out] Integral((b*sec(c + d*x))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b\sec(dx+c))^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c))^(3/2), x)
```

3.84 $\int \cos(c+dx)(b\sec(c+dx))^{3/2}\,dx$

Optimal. Leaf size=39

$$\frac{2b\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{b\sec(c+dx)}}{d}$$

```
[Out] (2*b*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[b*Sec[c + d*x]])/d
```

Rubi [A] time = 0.030127, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {16, 3771, 2641}

$$\frac{2b\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\sec(c+dx)}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(b*Sec[c + d*x])^(3/2),x]
```

```
[Out] (2*b*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[b*Sec[c + d*x]])/d
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \cos(c+dx)(b\sec(c+dx))^{3/2}\,dx &= b\int \sqrt{b\sec(c+dx)}\,dx\\
&= \left(b\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx\\
&= \frac{2b\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\sec(c+dx)}}{d}
\end{aligned}$$

Mathematica [A] time = 0.0193133, size = 39, normalized size = 1.

$$\frac{2b\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{b\sec(c+dx)}}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(b*Sec[c + d*x])^(3/2),x]
```

```
[Out] (2*b*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[b*Sec[c + d*x]])/d
```

Maple [C] time = 0.136, size = 98, normalized size = 2.5

$$\frac{-2i(-1+\cos(dx+c))(\cos(dx+c)+1)^3}{d(\sin(dx+c))^2}\left(\frac{b}{\cos(dx+c)}\right)^{\frac{3}{2}}\left(\frac{\cos(dx+c)}{\cos(dx+c)+1}\right)^{\frac{3}{2}}\text{EllipticF}\left(\frac{i(-1+\cos(dx+c))}{\sin(dx+c)},i\right)\sqrt{(\cos$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(b*sec(d*x+c))^(3/2),x)
```

```
[Out] -2*I/d*(b/cos(d*x+c))^(3/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(3/2)*(-1+cos(d*x+c
))*EllipticF(I*(-1+cos(d*x+c))/sin(d*x+c),I)*(cos(d*x+c)+1)^3*(1/(cos(d*x+c
)+1))^(1/2)/sin(d*x+c)^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b\sec(dx+c))^{\frac{3}{2}}\cos(dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c))^(3/2)*cos(d*x + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{b\sec(dx+c)}b\cos(dx+c)\sec(dx+c),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(d*x + c))*b*cos(d*x + c)*sec(d*x + c), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(b*sec(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b \sec (dx+c))^{\frac{3}{2}} \cos (dx+c) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c))^(3/2)*cos(d*x + c), x)
```

3.85 $\int \cos^2(c+dx)(b\sec(c+dx))^{3/2}\,dx$

Optimal. Leaf size=41

$$\frac{2b^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}$$

```
[Out] (2*b^2*EllipticE[(c + d*x)/2, 2])/(d*Sqrt[Cos[c + d*x]]*Sqrt[b*Sec[c + d*x]
])
```

Rubi [A] time = 0.0378043, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {16, 3771, 2639}

$$\frac{2b^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(b*Sec[c + d*x])^(3/2),x]
```

```
[Out] (2*b^2*EllipticE[(c + d*x)/2, 2])/(d*Sqrt[Cos[c + d*x]]*Sqrt[b*Sec[c + d*x]
])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \cos^2(c+dx)(b\sec(c+dx))^{3/2}\,dx &= b^2\int \frac{1}{\sqrt{b\sec(c+dx)}}\,dx\\ &= \frac{b^2\int \sqrt{\cos(c+dx)}\,dx}{\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}\\ &= \frac{2b^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}\end{aligned}$$

Mathematica [A] time = 0.0235292, size = 41, normalized size = 1.

$$\frac{2b^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*(b*Sec[c + d*x])^(3/2),x]
```

```
[Out] (2*b^2*EllipticE[(c + d*x)/2, 2])/(d*Sqrt[Cos[c + d*x]]*Sqrt[b*Sec[c + d*x]
])
```

Maple [C] time = 0.147, size = 309, normalized size = 7.5

$$2\frac{\cos(dx+c)}{d\sin(dx+c)}\left(i\sqrt{(\cos(dx+c)+1)^{-1}}\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\mathrm{EllipticF}\left(\frac{i(-1+\cos(dx+c))}{\sin(dx+c)},i\right)\cos(dx+c)\sin(dx+c)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(b*sec(d*x+c))^(3/2),x)
```

```
[Out] 2/d*(I*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticF
(I*(-1+cos(d*x+c))/sin(d*x+c),I)*cos(d*x+c)*sin(d*x+c)-I*EllipticE(I*(-1+co
s(d*x+c))/sin(d*x+c),I)*cos(d*x+c)*sin(d*x+c)*(1/(cos(d*x+c)+1))^(1/2)*(cos
(d*x+c)/(cos(d*x+c)+1))^(1/2)+I*EllipticF(I*(-1+cos(d*x+c))/sin(d*x+c),I)*(
1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-I*Elli
pticE(I*(-1+cos(d*x+c))/sin(d*x+c),I)*sin(d*x+c)*(1/(cos(d*x+c)+1))^(1/2)*(
cos(d*x+c)/(cos(d*x+c)+1))^(1/2)-cos(d*x+c)^2+cos(d*x+c))*cos(d*x+c)*(b/cos
(d*x+c))^(3/2)/sin(d*x+c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int(b\sec(dx+c))^{\frac{3}{2}}\cos(dx+c)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c))^(3/2)*cos(d*x + c)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\sqrt{b\sec(dx+c)}b\cos(dx+c)^2\sec(dx+c),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(d*x + c))*b*cos(d*x + c)^2*sec(d*x + c), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(b*sec(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b\sec(dx+c))^{\frac{3}{2}}\cos(dx+c)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c))^(3/2)*cos(d*x + c)^2, x)
```

3.86 $\int \cos^3(c+dx)(b\sec(c+dx))^{3/2}\,dx$

Optimal. Leaf size=70

$$\frac{2b\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{b\sec(c+dx)}}{3d}+\frac{2b^2\sin(c+dx)}{3d\sqrt{b\sec(c+dx)}}$$

```
[Out] (2*b*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[b*Sec[c + d*x]])/(3*
d) + (2*b^2*Sin[c + d*x])/(3*d*Sqrt[b*Sec[c + d*x]])
```

Rubi [A] time = 0.0542509, antiderivative size = 70, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {16, 3769, 3771, 2641}

$$\frac{2b^2\sin(c+dx)}{3d\sqrt{b\sec(c+dx)}}+\frac{2b\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\sec(c+dx)}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*(b*Sec[c + d*x])^(3/2),x]
```

```
[Out] (2*b*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[b*Sec[c + d*x]])/(3*
d) + (2*b^2*Sin[c + d*x])/(3*d*Sqrt[b*Sec[c + d*x]])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \cos^3(c+dx)(b\sec(c+dx))^{3/2}\,dx &= b^3\int \frac{1}{(b\sec(c+dx))^{3/2}}\,dx\\ &= \frac{2b^2\sin(c+dx)}{3d\sqrt{b\sec(c+dx)}}+\frac{1}{3}b\int \sqrt{b\sec(c+dx)}\,dx\\ &= \frac{2b^2\sin(c+dx)}{3d\sqrt{b\sec(c+dx)}}+\frac{1}{3}\left(b\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx\\ &= \frac{2b\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\sec(c+dx)}}{3d}+\frac{2b^2\sin(c+dx)}{3d\sqrt{b\sec(c+dx)}}\end{aligned}$$

Mathematica [A] time = 0.0493521, size = 52, normalized size = 0.74

$$\frac{b\sqrt{b\sec(c+dx)}\left(2\sqrt{\cos(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+\sin(2(c+dx))\right)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*(b*Sec[c + d*x])^(3/2),x]
```

```
[Out] (b*Sqrt[b*Sec[c + d*x]]*(2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + S
in[2*(c + d*x)]))/(3*d)
```

Maple [C] time = 0.16, size = 129, normalized size = 1.8

$$-\frac{2\left(\cos(dx+c)+1\right)^2\left(-1+\cos(dx+c)\right)\cos(dx+c)}{3d\left(\sin(dx+c)\right)^3}\left(\frac{b}{\cos(dx+c)}\right)^{\frac{3}{2}}\left(i\text{EllipticF}\left(\frac{i\left(-1+\cos(dx+c)\right)}{\sin(dx+c)},i\right)\sqrt{(\cos(dx+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(b*sec(d*x+c))^(3/2),x)
```

```
[Out] -2/3/d*(cos(d*x+c)+1)^2*(b/cos(d*x+c))^(3/2)*(-1+cos(d*x+c))*cos(d*x+c)*(I*
EllipticF(I*(-1+cos(d*x+c))/sin(d*x+c),I)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x
+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-cos(d*x+c)^2+cos(d*x+c))/sin(d*x+c)^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b\sec(dx+c))^{\frac{3}{2}}\cos(dx+c)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c))^(3/2)*cos(d*x + c)^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{b\sec(dx+c)}b\cos(dx+c)^3\sec(dx+c),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(d*x + c))*b*cos(d*x + c)^3*sec(d*x + c), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(b*sec(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b\sec(dx+c))^{\frac{3}{2}}\cos(dx+c)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c))^(3/2)*cos(d*x + c)^3, x)
```

3.87 $\int \cos^4(c+dx)(b\sec(c+dx))^{3/2}\,dx$

Optimal. Leaf size=72

$$\frac{2b^3\sin(c+dx)}{5d(b\sec(c+dx))^{3/2}}+\frac{6b^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}$$

```
[Out] (6*b^2*EllipticE[(c + d*x)/2, 2])/(5*d*Sqrt[Cos[c + d*x]]*Sqrt[b*Sec[c + d*
x]]) + (2*b^3*Sin[c + d*x])/(5*d*(b*Sec[c + d*x])^(3/2))
```

Rubi [A] time = 0.0541705, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {16, 3769, 3771, 2639}

$$\frac{2b^3\sin(c+dx)}{5d(b\sec(c+dx))^{3/2}}+\frac{6b^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*(b*Sec[c + d*x])^(3/2),x]
```


```
[Out] (6*b^2*EllipticE[(c + d*x)/2, 2])/(5*d*Sqrt[Cos[c + d*x]]*Sqrt[b*Sec[c + d*
x]]) + (2*b^3*Sin[c + d*x])/(5*d*(b*Sec[c + d*x])^(3/2))
```


Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cos^4(c+dx)(b\sec(c+dx))^{3/2}\,dx = b^4\int \frac{1}{(b\sec(c+dx))^{5/2}}\,dx$$
$$= \frac{2b^3\sin(c+dx)}{5d(b\sec(c+dx))^{3/2}} + \frac{1}{5}\left(3b^2\right)\int \frac{1}{\sqrt{b\sec(c+dx)}}\,dx$$
$$= \frac{2b^3\sin(c+dx)}{5d(b\sec(c+dx))^{3/2}} + \frac{\left(3b^2\right)\int \sqrt{\cos(c+dx)}\,dx}{5\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}$$
$$= \frac{6b^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}} + \frac{2b^3\sin(c+dx)}{5d(b\sec(c+dx))^{3/2}}$$

Mathematica [A] time = 0.0491233, size = 58, normalized size = 0.81

$$\frac{b\sqrt{b\sec(c+dx)}\left(\sin(c+dx)+\sin(3(c+dx))+12\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\right)}{10d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*(b*Sec[c + d*x])^(3/2),x]
```

```
[Out] (b*Sqrt[b*Sec[c + d*x]]*(12*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] +
Sin[c + d*x] + Sin[3*(c + d*x)]))/(10*d)
```

Maple [C] time = 0.196, size = 321, normalized size = 4.5

$$\frac{2\cos(dx+c)}{5d\sin(dx+c)}\left(3i\sqrt{(\cos(dx+c)+1)^{-1}}\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\mathrm{EllipticF}\left(\frac{i(-1+\cos(dx+c))}{\sin(dx+c)},i\right)\cos(dx+c)\sin(dx+c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*(b*sec(d*x+c))^(3/2),x)
```

```
[Out] 2/5/d*(3*I*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*Ellip
ticF(I*(-1+cos(d*x+c))/sin(d*x+c),I)*cos(d*x+c)*sin(d*x+c)-3*I*cos(d*x+c)*s
in(d*x+c)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*Ellipt
icE(I*(-1+cos(d*x+c))/sin(d*x+c),I)+3*I*EllipticF(I*(-1+cos(d*x+c))/sin(d*x
+c),I)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c
)-3*I*sin(d*x+c)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)
*EllipticE(I*(-1+cos(d*x+c))/sin(d*x+c),I)-cos(d*x+c)^4-2*cos(d*x+c)^2+3*co
s(d*x+c))*cos(d*x+c)*(b/cos(d*x+c))^(3/2)/sin(d*x+c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b\sec(dx+c))^{\frac{3}{2}}\cos(dx+c)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c))^(3/2)*cos(d*x + c)^4, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{b\sec\left(dx+c\right)}b\cos\left(dx+c\right)^4\sec\left(dx+c\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(d*x + c))*b*cos(d*x + c)^4*sec(d*x + c), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*(b*sec(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(b\sec\left(dx+c\right)\right)^{\frac{3}{2}}\cos\left(dx+c\right)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c))^(3/2)*cos(d*x + c)^4, x)
```

3.88 $\int \cos^5(c+dx)(b\sec(c+dx))^{3/2}\,dx$

Optimal. Leaf size=98

$$\frac{10b\sqrt{\cos(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{b\sec(c+dx)}}{21d}+\frac{2b^4\sin(c+dx)}{7d(b\sec(c+dx))^{5/2}}+\frac{10b^2\sin(c+dx)}{21d\sqrt{b\sec(c+dx)}}$$

```
[Out] (10*b*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[b*Sec[c + d*x]])/(2
1*d) + (2*b^4*Sin[c + d*x])/(7*d*(b*Sec[c + d*x])^(5/2)) + (10*b^2*Sin[c +
d*x])/(21*d*Sqrt[b*Sec[c + d*x]])
```

Rubi [A] time = 0.0730224, antiderivative size = 98, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {16, 3769, 3771, 2641}

$$\frac{2b^4\sin(c+dx)}{7d(b\sec(c+dx))^{5/2}}+\frac{10b^2\sin(c+dx)}{21d\sqrt{b\sec(c+dx)}}+\frac{10b\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\sec(c+dx)}}{21d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5*(b*Sec[c + d*x])^(3/2),x]
```

```
[Out] (10*b*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[b*Sec[c + d*x]])/(2
1*d) + (2*b^4*Sin[c + d*x])/(7*d*(b*Sec[c + d*x])^(5/2)) + (10*b^2*Sin[c +
d*x])/(21*d*Sqrt[b*Sec[c + d*x]])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cos^5(c+dx)(b\sec(c+dx))^{3/2}\,dx = b^5\int \frac{1}{(b\sec(c+dx))^{7/2}}\,dx$$
$$= \frac{2b^4\sin(c+dx)}{7d(b\sec(c+dx))^{5/2}} + \frac{1}{7}\left(5b^3\right)\int \frac{1}{(b\sec(c+dx))^{3/2}}\,dx$$
$$= \frac{2b^4\sin(c+dx)}{7d(b\sec(c+dx))^{5/2}} + \frac{10b^2\sin(c+dx)}{21d\sqrt{b\sec(c+dx)}} + \frac{1}{21}(5b)\int \sqrt{b\sec(c+dx)}\,dx$$
$$= \frac{2b^4\sin(c+dx)}{7d(b\sec(c+dx))^{5/2}} + \frac{10b^2\sin(c+dx)}{21d\sqrt{b\sec(c+dx)}} + \frac{1}{21}\left(5b\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}\right)\int$$
$$= \frac{10b\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\sec(c+dx)}}{21d} + \frac{2b^4\sin(c+dx)}{7d(b\sec(c+dx))^{5/2}} + \frac{10b^2\sin}{21d\sqrt{b\,s}}$$

Mathematica [A] time = 0.0779546, size = 64, normalized size = 0.65

$$\frac{b\sqrt{b\sec(c+dx)}\left(40\sqrt{\cos(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+26\sin(2(c+dx))+3\sin(4(c+dx))\right)}{84d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5*(b*Sec[c + d*x])^(3/2),x]
```

```
[Out] (b*Sqrt[b*Sec[c + d*x]]*(40*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] +
26*Sin[2*(c + d*x)] + 3*Sin[4*(c + d*x)]))/(84*d)
```

Maple [C] time = 0.184, size = 151, normalized size = 1.5

$$-\frac{2\left(\cos(dx+c)+1\right)^2\left(-1+\cos(dx+c)\right)\cos(dx+c)}{21\,d\left(\sin(dx+c)\right)^3}\left(\frac{b}{\cos(dx+c)}\right)^{\frac{3}{2}}\left(5\,i\text{EllipticF}\left(\frac{i\left(-1+\cos(dx+c)\right)}{\sin(dx+c)},i\right)\sqrt{(\cos(dx}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*(b*sec(d*x+c))^(3/2),x)
```

```
[Out] -2/21/d*(cos(d*x+c)+1)^2*(b/cos(d*x+c))^(3/2)*(-1+cos(d*x+c))*cos(d*x+c)*(5
*I*EllipticF(I*(-1+cos(d*x+c))/sin(d*x+c),I)*(1/(cos(d*x+c)+1))^(1/2)*(cos(
d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-3*cos(d*x+c)^4+3*cos(d*x+c)^3-5*cos
(d*x+c)^2+5*cos(d*x+c))/sin(d*x+c)^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b\sec(dx+c))^{\frac{3}{2}}\cos(dx+c)^5\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c))^(3/2)*cos(d*x + c)^5, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{b\sec\left(dx+c\right)}b\cos\left(dx+c\right)^{5}\sec\left(dx+c\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(d*x + c))*b*cos(d*x + c)^5*sec(d*x + c), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*(b*sec(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.89 $\int \cos^6(c+dx)(b\sec(c+dx))^{3/2}\,dx$

Optimal. Leaf size=100

$$\frac{2b^5\sin(c+dx)}{9d(b\sec(c+dx))^{7/2}}+\frac{14b^3\sin(c+dx)}{45d(b\sec(c+dx))^{3/2}}+\frac{14b^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}$$

```
[Out] (14*b^2*EllipticE[(c + d*x)/2, 2])/(15*d*Sqrt[Cos[c + d*x]]*Sqrt[b*Sec[c +
d*x]]) + (2*b^5*Sin[c + d*x])/(9*d*(b*Sec[c + d*x])^(7/2)) + (14*b^3*Sin[c
+ d*x])/(45*d*(b*Sec[c + d*x])^(3/2))
```

Rubi [A] time = 0.0747851, antiderivative size = 100, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {16, 3769, 3771, 2639}

$$\frac{2b^5\sin(c+dx)}{9d(b\sec(c+dx))^{7/2}}+\frac{14b^3\sin(c+dx)}{45d(b\sec(c+dx))^{3/2}}+\frac{14b^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^6*(b*Sec[c + d*x])^(3/2),x]
```

```
[Out] (14*b^2*EllipticE[(c + d*x)/2, 2])/(15*d*Sqrt[Cos[c + d*x]]*Sqrt[b*Sec[c +
d*x]]) + (2*b^5*Sin[c + d*x])/(9*d*(b*Sec[c + d*x])^(7/2)) + (14*b^3*Sin[c
+ d*x])/(45*d*(b*Sec[c + d*x])^(3/2))
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cos^6(c+dx)(b\sec(c+dx))^{3/2}\,dx = b^6\int \frac{1}{(b\sec(c+dx))^{9/2}}\,dx$$
$$= \frac{2b^5\sin(c+dx)}{9d(b\sec(c+dx))^{7/2}} + \frac{1}{9}\left(7b^4\right)\int \frac{1}{(b\sec(c+dx))^{5/2}}\,dx$$
$$= \frac{2b^5\sin(c+dx)}{9d(b\sec(c+dx))^{7/2}} + \frac{14b^3\sin(c+dx)}{45d(b\sec(c+dx))^{3/2}} + \frac{1}{15}\left(7b^2\right)\int \frac{1}{\sqrt{b\sec(c+dx)}}\,dx$$
$$= \frac{2b^5\sin(c+dx)}{9d(b\sec(c+dx))^{7/2}} + \frac{14b^3\sin(c+dx)}{45d(b\sec(c+dx))^{3/2}} + \frac{\left(7b^2\right)\int \sqrt{\cos(c+dx)}\,dx}{15\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}$$
$$= \frac{14b^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}} + \frac{2b^5\sin(c+dx)}{9d(b\sec(c+dx))^{7/2}} + \frac{14b^3\sin(c+dx)}{45d(b\sec(c+dx))^{3/2}}$$

Mathematica [A] time = 0.160356, size = 72, normalized size = 0.72

$$\frac{b\sqrt{b\sec(c+dx)}\left((33\sin(c+dx)+5\sin(3(c+dx)))\cos^2(c+dx)+84\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\right)}{90d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^6*(b*Sec[c + d*x])^(3/2),x]
```

```
[Out] (b*Sqrt[b*Sec[c + d*x]]*(84*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] +
Cos[c + d*x]^2*(33*Sin[c + d*x] + 5*Sin[3*(c + d*x)])))/(90*d)
```

Maple [C] time = 0.182, size = 331, normalized size = 3.3

$$-\frac{2\cos(dx+c)}{45d\sin(dx+c)}\left(21i\cos(dx+c)\sin(dx+c)\sqrt{(\cos(dx+c)+1)^{-1}}\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\mathrm{EllipticE}\left(\frac{i(-1+\cos(dx+c)}{\sin(dx+c)}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*(b*sec(d*x+c))^(3/2),x)
```

```
[Out] -2/45/d*(21*I*cos(d*x+c)*sin(d*x+c)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(c
os(d*x+c)+1))^(1/2)*EllipticE(I*(-1+cos(d*x+c))/sin(d*x+c),I)-21*I*(1/(cos(
d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(-1+cos(d*x+
c))/sin(d*x+c),I)*cos(d*x+c)*sin(d*x+c)+5*cos(d*x+c)^6+21*I*sin(d*x+c)*(1/(
cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticE(I*(-1+cos(
d*x+c))/sin(d*x+c),I)-21*I*EllipticF(I*(-1+cos(d*x+c))/sin(d*x+c),I)*(1/(co
s(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+2*cos(d*x+c
)^4+14*cos(d*x+c)^2-21*cos(d*x+c))*cos(d*x+c)*(b/cos(d*x+c))^(3/2)/sin(d*x+
c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b\sec(dx+c))^{\frac{3}{2}}\cos(dx+c)^6\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*(b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c))^(3/2)*cos(d*x + c)^6, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{b\sec\left(dx+c\right)}b\cos\left(dx+c\right)^{6}\sec\left(dx+c\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*(b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(d*x + c))*b*cos(d*x + c)^6*sec(d*x + c), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*(b*sec(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(b\sec\left(dx+c\right)\right)^{\frac{3}{2}}\cos\left(dx+c\right)^{6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*(b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c))^(3/2)*cos(d*x + c)^6, x)
```

3.90 $\int \sec^2(c+dx)(b\sec(c+dx))^{5/2}\,dx$

Optimal. Leaf size=98

$$\frac{10b^2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{b\sec(c+dx)}}{21d}+\frac{2\sin(c+dx)(b\sec(c+dx))^{7/2}}{7bd}+\frac{10b\sin(c+dx)(b\sec(c+$$

```
[Out] (10*b^2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[b*Sec[c + d*x]])/
(21*d) + (10*b*(b*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(21*d) + (2*(b*Sec[c +
d*x])^(7/2)*Sin[c + d*x])/(7*b*d)
```

Rubi [A] time = 0.0588525, antiderivative size = 98, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {16, 3768, 3771, 2641}

$$\frac{10b^2\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\sec(c+dx)}}{21d}+\frac{2\sin(c+dx)(b\sec(c+dx))^{7/2}}{7bd}+\frac{10b\sin(c+dx)(b\sec(c+dx))^{3/2}}{21d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2*(b*Sec[c + d*x])^(5/2),x]
```

```
[Out] (10*b^2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[b*Sec[c + d*x]])/
(21*d) + (10*b*(b*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(21*d) + (2*(b*Sec[c +
d*x])^(7/2)*Sin[c + d*x])/(7*b*d)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sec^2(c+dx)(b\sec(c+dx))^{5/2}\,dx = \frac{\int (b\sec(c+dx))^{9/2}\,dx}{b^2}$$
$$= \frac{2(b\sec(c+dx))^{7/2}\sin(c+dx)}{7bd} + \frac{5}{7}\int (b\sec(c+dx))^{5/2}\,dx$$
$$= \frac{10b(b\sec(c+dx))^{3/2}\sin(c+dx)}{21d} + \frac{2(b\sec(c+dx))^{7/2}\sin(c+dx)}{7bd} + \frac{1}{21}\left(5b^2\right)\int \sqrt{\text{}}$$
$$= \frac{10b(b\sec(c+dx))^{3/2}\sin(c+dx)}{21d} + \frac{2(b\sec(c+dx))^{7/2}\sin(c+dx)}{7bd} + \frac{1}{21}\left(5b^2\sqrt{\cos(}\right.$$
$$= \frac{10b^2\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\sec(c+dx)}}{21d} + \frac{10b(b\sec(c+dx))^{3/2}\sin(c+d}{21d}$$

Mathematica [A] time = 0.179666, size = 61, normalized size = 0.62

$$\frac{(b\sec(c+dx))^{5/2}\left(10\cos^{\frac{5}{2}}(c+dx)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+5\sin(2(c+dx))+6\tan(c+dx)\right)}{21d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2*(b*Sec[c + d*x])^(5/2),x]
```

```
[Out] ((b*Sec[c + d*x])^(5/2)*(10*Cos[c + d*x]^(5/2)*EllipticF[(c + d*x)/2, 2] +
5*Sin[2*(c + d*x)] + 6*Tan[c + d*x]))/(21*d)
```

Maple [C] time = 0.191, size = 152, normalized size = 1.6

$$-\frac{2\left(\cos(dx+c)+1\right)^2\left(-1+\cos(dx+c)\right)}{21\,d\cos(dx+c)\left(\sin(dx+c)\right)^3}\left(5\,i\left(\cos(dx+c)\right)^3\sin(dx+c)\sqrt{\left(\cos(dx+c)+1\right)^{-1}}\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\text{Ellipti}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(b*sec(d*x+c))^(5/2),x)
```

```
[Out] -2/21/d*(cos(d*x+c)+1)^2*(-1+cos(d*x+c))*(5*I*cos(d*x+c)^3*sin(d*x+c)*(1/(c
os(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(-1+cos(d
*x+c))/sin(d*x+c),I)-5*cos(d*x+c)^3+5*cos(d*x+c)^2-3*cos(d*x+c)+3)*(b/cos(d
*x+c))^(5/2)/cos(d*x+c)/sin(d*x+c)^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sec(dx+c)\right)^{\frac{5}{2}}\sec(dx+c)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(b*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c))^(5/2)*sec(d*x + c)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{b\sec(dx+c)}b^2\sec(dx+c)^4,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(d*x + c))*b^2*sec(d*x + c)^4, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(b*sec(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b\sec(dx+c))^{\frac{5}{2}}\sec(dx+c)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c))^(5/2)*sec(d*x + c)^2, x)
```

3.91 $\int \sec(c+dx)(b\sec(c+dx))^{5/2}\,dx$

Optimal. Leaf size=97

$$\frac{6b^2\sin(c+dx)\sqrt{b\sec(c+dx)}}{5d}-\frac{6b^3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}+\frac{2\sin(c+dx)(b\sec(c+dx))^{5/2}}{5d}$$

```
[Out] (-6*b^3*EllipticE[(c + d*x)/2, 2])/(5*d*Sqrt[Cos[c + d*x]]*Sqrt[b*Sec[c + d
*x]]) + (6*b^2*Sqrt[b*Sec[c + d*x]]*Sin[c + d*x])/(5*d) + (2*(b*Sec[c + d*x
])^(5/2)*Sin[c + d*x])/(5*d)
```

Rubi [A] time = 0.0567826, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {16, 3768, 3771, 2639}

$$\frac{6b^2\sin(c+dx)\sqrt{b\sec(c+dx)}}{5d}-\frac{6b^3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}+\frac{2\sin(c+dx)(b\sec(c+dx))^{5/2}}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*(b*Sec[c + d*x])^(5/2),x]
```


```
[Out] (-6*b^3*EllipticE[(c + d*x)/2, 2])/(5*d*Sqrt[Cos[c + d*x]]*Sqrt[b*Sec[c + d
*x]]) + (6*b^2*Sqrt[b*Sec[c + d*x]]*Sin[c + d*x])/(5*d) + (2*(b*Sec[c + d*x
])^(5/2)*Sin[c + d*x])/(5*d)
```


Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sec(c+dx)(b\sec(c+dx))^{5/2}\,dx = \frac{\int (b\sec(c+dx))^{7/2}\,dx}{b}$$
$$= \frac{2(b\sec(c+dx))^{5/2}\sin(c+dx)}{5d} + \frac{1}{5}(3b)\int (b\sec(c+dx))^{3/2}\,dx$$
$$= \frac{6b^2\sqrt{b\sec(c+dx)}\sin(c+dx)}{5d} + \frac{2(b\sec(c+dx))^{5/2}\sin(c+dx)}{5d} - \frac{1}{5}\left(3b^3\right)\int \frac{}{\sqrt{b}}$$
$$= \frac{6b^2\sqrt{b\sec(c+dx)}\sin(c+dx)}{5d} + \frac{2(b\sec(c+dx))^{5/2}\sin(c+dx)}{5d} - \frac{\left(3b^3\right)\int \sqrt{\cos}}{5\sqrt{\cos(c+dx)}}$$
$$= -\frac{6b^3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}} + \frac{6b^2\sqrt{b\sec(c+dx)}\sin(c+dx)}{5d} + \frac{2(b\sec(c+}{}$$

Mathematica [A] time = 0.164128, size = 61, normalized size = 0.63

$$\frac{(b\sec(c+dx))^{5/2}\left(7\sin(c+dx)+3\sin(3(c+dx))-12\cos^{\frac{5}{2}}(c+dx)E\left(\frac{1}{2}(c+dx)\middle|2\right)\right)}{10d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]*(b*Sec[c + d*x])^(5/2),x]
```

```
[Out] ((b*Sec[c + d*x])^(5/2)*(-12*Cos[c + d*x]^(5/2)*EllipticE[(c + d*x)/2, 2] +
 7*Sin[c + d*x] + 3*Sin[3*(c + d*x)]))/(10*d)
```

Maple [C] time = 0.204, size = 348, normalized size = 3.6

$$\frac{2\,(\cos(dx+c)+1)^2\,(-1+\cos(dx+c))^2}{5\,d\,(\sin(dx+c))^5}\left(3\,i\sqrt{(\cos(dx+c)+1)^{-1}}\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,(\cos(dx+c))^3\,\mathrm{EllipticE}\left(\frac{i\,(-1}{s}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(b*sec(d*x+c))^(5/2),x)
```

```
[Out] 2/5/d*(cos(d*x+c)+1)^2*(-1+cos(d*x+c))^2*(3*I*(1/(cos(d*x+c)+1))^(1/2)*(cos
(d*x+c)/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)^3*EllipticE(I*(-1+cos(d*x+c))/sin(
d*x+c),I)*sin(d*x+c)-3*I*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1
))^(1/2)*EllipticF(I*(-1+cos(d*x+c))/sin(d*x+c),I)*cos(d*x+c)^3*sin(d*x+c)+
3*I*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)^2
*EllipticE(I*(-1+cos(d*x+c))/sin(d*x+c),I)*sin(d*x+c)-3*I*(1/(cos(d*x+c)+1)
)^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(-1+cos(d*x+c))/sin(d
*x+c),I)*cos(d*x+c)^2*sin(d*x+c)-3*cos(d*x+c)^3+2*cos(d*x+c)^2+1)*(b/cos(d*
x+c))^(5/2)/sin(d*x+c)^5
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b\sec(dx+c))^{\frac{5}{2}}\sec(dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(b*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c))^(5/2)*sec(d*x + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{b\sec(dx+c)}b^2\sec(dx+c)^3, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(d*x + c))*b^2*sec(d*x + c)^3, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(b*sec(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b\sec(dx+c))^{\frac{5}{2}}\sec(dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c))^(5/2)*sec(d*x + c), x)
```

3.92 $\int (b\sec(c+dx))^{5/2}\,dx$

Optimal. Leaf size=70

$$\frac{2b^2\sqrt{\cos(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{b\sec(c+dx)}}{3d}+\frac{2b\sin(c+dx)(b\sec(c+dx))^{3/2}}{3d}$$

```
[Out] (2*b^2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[b*Sec[c + d*x]])/(
3*d) + (2*b*(b*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(3*d)
```

Rubi [A] time = 0.0323627, antiderivative size = 70, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {3768, 3771, 2641}

$$\frac{2b^2\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\sec(c+dx)}}{3d}+\frac{2b\sin(c+dx)(b\sec(c+dx))^{3/2}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Sec[c + d*x])^(5/2),x]
```

```
[Out] (2*b^2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[b*Sec[c + d*x]])/(
3*d) + (2*b*(b*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(3*d)
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int (b\sec(c+dx))^{5/2}\,dx &= \frac{2b(b\sec(c+dx))^{3/2}\sin(c+dx)}{3d}+\frac{1}{3}b^2\int \sqrt{b\sec(c+dx)}\,dx\\ &= \frac{2b(b\sec(c+dx))^{3/2}\sin(c+dx)}{3d}+\frac{1}{3}\left(b^2\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx\\ &= \frac{2b^2\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\sec(c+dx)}}{3d}+\frac{2b(b\sec(c+dx))^{3/2}\sin(c+dx)}{3d}\end{aligned}$$

Mathematica [A] time = 0.0169936, size = 51, normalized size = 0.73

$$\frac{2b^2\sqrt{b\sec(c+dx)}\left(\sqrt{\cos(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+\tan(c+dx)\right)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Sec[c + d*x])^(5/2),x]
```

```
[Out] (2*b^2*Sqrt[b*Sec[c + d*x]]*(Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] +
 Tan[c + d*x]))/(3*d)
```

Maple [C] time = 0.159, size = 128, normalized size = 1.8

$$-\frac{(-2+2\cos(dx+c))\cos(dx+c)(\cos(dx+c)+1)^2}{3d(\sin(dx+c))^3}\left(i\sqrt{(\cos(dx+c)+1)^{-1}}\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\text{EllipticF}\left(\frac{i(-1+\cos(\right.\right.}{\sin(dx+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sec(d*x+c))^(5/2),x)
```

```
[Out] -2/3/d*(-1+cos(d*x+c))*(I*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+
1))^(1/2)*EllipticF(I*(-1+cos(d*x+c))/sin(d*x+c),I)*cos(d*x+c)*sin(d*x+c)-c
os(d*x+c)+1)*cos(d*x+c)*(cos(d*x+c)+1)^2*(b/cos(d*x+c))^(5/2)/sin(d*x+c)^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b\sec(dx+c))^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c))^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{b\sec(dx+c)}b^2\sec(dx+c)^2,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(d*x + c))*b^2*sec(d*x + c)^2, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F]   time = 0., size = 0, normalized size = 0.

$$\int (b \sec (dx + c))^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c))^(5/2), x)
```

3.93 $\int \cos(c+dx)(b\sec(c+dx))^{5/2}\,dx$

Optimal. Leaf size=68

$$\frac{2b^2\sin(c+dx)\sqrt{b\sec(c+dx)}}{d}-\frac{2b^3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}$$

```
[Out] (-2*b^3*EllipticE[(c + d*x)/2, 2])/(d*Sqrt[Cos[c + d*x]]*Sqrt[b*Sec[c + d*x
]]) + (2*b^2*Sqrt[b*Sec[c + d*x]]*Sin[c + d*x])/d
```

Rubi [A] time = 0.0445958, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {16, 3768, 3771, 2639}

$$\frac{2b^2\sin(c+dx)\sqrt{b\sec(c+dx)}}{d}-\frac{2b^3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(b*Sec[c + d*x])^(5/2),x]
```

```
[Out] (-2*b^3*EllipticE[(c + d*x)/2, 2])/(d*Sqrt[Cos[c + d*x]]*Sqrt[b*Sec[c + d*x
]]) + (2*b^2*Sqrt[b*Sec[c + d*x]]*Sin[c + d*x])/d
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \cos(c+dx)(b\sec(c+dx))^{5/2}\,dx &= b\int (b\sec(c+dx))^{3/2}\,dx\\ &= \frac{2b^2\sqrt{b\sec(c+dx)}\sin(c+dx)}{d} - b^3\int \frac{1}{\sqrt{b\sec(c+dx)}}\,dx\\ &= \frac{2b^2\sqrt{b\sec(c+dx)}\sin(c+dx)}{d} - \frac{b^3\int \sqrt{\cos(c+dx)}\,dx}{\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}\\ &= -\frac{2b^3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}} + \frac{2b^2\sqrt{b\sec(c+dx)}\sin(c+dx)}{d}\end{aligned}$$

Mathematica [A] time = 0.0312653, size = 50, normalized size = 0.74

$$\frac{2b^2\sqrt{b\sec(c+dx)}\left(\sin(c+dx)-\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(b*Sec[c + d*x])^(5/2),x]
```

```
[Out] (2*b^2*Sqrt[b*Sec[c + d*x]]*(-(Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]
) + Sin[c + d*x]))/d
```

Maple [C] time = 0.218, size = 324, normalized size = 4.8

$$2\,\frac{(\cos(dx+c)+1)^2(-1+\cos(dx+c))^2(\cos(dx+c))^2}{d(\sin(dx+c))^5}\left(i\cos(dx+c)\sin(dx+c)\sqrt{(\cos(dx+c)+1)^{-1}}\sqrt{\frac{\cos(dx+}{\cos(dx+c}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(b*sec(d*x+c))^(5/2),x)
```

```
[Out] 2/d*(cos(d*x+c)+1)^2*(-1+cos(d*x+c))^2*(I*cos(d*x+c)*sin(d*x+c)*(1/(cos(d*x
+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticE(I*(-1+cos(d*x+c))
/sin(d*x+c),I)-I*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)
*EllipticF(I*(-1+cos(d*x+c))/sin(d*x+c),I)*cos(d*x+c)*sin(d*x+c)+I*sin(d*x+
c)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticE(I*(
-1+cos(d*x+c))/sin(d*x+c),I)-I*EllipticF(I*(-1+cos(d*x+c))/sin(d*x+c),I)*(1
/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-cos(d*x
+c)+1)*cos(d*x+c)^2*(b/cos(d*x+c))^(5/2)/sin(d*x+c)^5
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b\sec(dx+c))^{\frac{5}{2}}\cos(dx+c)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(b*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c))^(5/2)*cos(d*x + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{b\sec\left(dx+c\right)}b^2\cos\left(dx+c\right)\sec\left(dx+c\right)^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(d*x + c))*b^2*cos(d*x + c)*sec(d*x + c)^2, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(b*sec(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sec\left(dx+c\right)\right)^{\frac{5}{2}}\cos\left(dx+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c))^(5/2)*cos(d*x + c), x)
```

3.94 $\int \cos^2(c+dx)(b\sec(c+dx))^{5/2}\,dx$

Optimal. Leaf size=41

$$\frac{2b^2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{b\sec(c+dx)}}{d}$$

```
[Out] (2*b^2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[b*Sec[c + d*x]])/d
```

Rubi [A] time = 0.0379056, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {16, 3771, 2641}

$$\frac{2b^2\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\sec(c+dx)}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(b*Sec[c + d*x])^(5/2),x]
```

```
[Out] (2*b^2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[b*Sec[c + d*x]])/d
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \cos^2(c+dx)(b\sec(c+dx))^{5/2}\,dx &= b^2\int \sqrt{b\sec(c+dx)}\,dx\\ &= \left(b^2\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx\\ &= \frac{2b^2\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\sec(c+dx)}}{d}\end{aligned}$$

Mathematica [A] time = 0.0159645, size = 41, normalized size = 1.

$$\frac{2b^2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{b\sec(c+dx)}}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*(b*Sec[c + d*x])^(5/2),x]
```

```
[Out] (2*b^2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[b*Sec[c + d*x]])/d
```

Maple [C] time = 0.153, size = 98, normalized size = 2.4

$$\frac{-2i(-1+\cos(dx+c))(\cos(dx+c)+1)^5}{d(\sin(dx+c))^2}\left(\frac{b}{\cos(dx+c)}\right)^{\frac{5}{2}}\left(\frac{\cos(dx+c)}{\cos(dx+c)+1}\right)^{\frac{5}{2}}\mathrm{EllipticF}\left(\frac{i(-1+\cos(dx+c))}{\sin(dx+c)},i\right)((\cos$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(b*sec(d*x+c))^(5/2),x)
```

```
[Out] -2*I/d*(b/cos(d*x+c))^(5/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(5/2)*(-1+cos(d*x+c
))*EllipticF(I*(-1+cos(d*x+c))/sin(d*x+c),I)*(cos(d*x+c)+1)^5*(1/(cos(d*x+c
)+1))^(3/2)/sin(d*x+c)^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b\sec(dx+c))^{\frac{5}{2}}\cos(dx+c)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(b*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c))^(5/2)*cos(d*x + c)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\sqrt{b\sec(dx+c)}b^2\cos(dx+c)^2\sec(dx+c)^2,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(d*x + c))*b^2*cos(d*x + c)^2*sec(d*x + c)^2, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(b*sec(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b\sec(dx+c))^{\frac{5}{2}}\cos(dx+c)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c))^(5/2)*cos(d*x + c)^2, x)
```

3.95 $\int \cos^3(c+dx)(b\sec(c+dx))^{5/2}\,dx$

Optimal. Leaf size=41

$$\frac{2b^3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}$$

```
[Out] (2*b^3*EllipticE[(c + d*x)/2, 2])/(d*Sqrt[Cos[c + d*x]]*Sqrt[b*Sec[c + d*x]
])
```

Rubi [A] time = 0.0367042, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {16, 3771, 2639}

$$\frac{2b^3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*(b*Sec[c + d*x])^(5/2),x]
```

```
[Out] (2*b^3*EllipticE[(c + d*x)/2, 2])/(d*Sqrt[Cos[c + d*x]]*Sqrt[b*Sec[c + d*x]
])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \cos^3(c+dx)(b\sec(c+dx))^{5/2}\,dx &= b^3\int \frac{1}{\sqrt{b\sec(c+dx)}}\,dx\\ &= \frac{b^3\int \sqrt{\cos(c+dx)}\,dx}{\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}\\ &= \frac{2b^3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}\end{aligned}$$

Mathematica [A] time = 0.0433608, size = 38, normalized size = 0.93

$$\frac{2\cos^{\frac{5}{2}}(c+dx)E\left(\frac{1}{2}(c+dx)\middle|2\right)(b\sec(c+dx))^{5/2}}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*(b*Sec[c + d*x])^(5/2),x]
```

```
[Out] (2*Cos[c + d*x]^(5/2)*EllipticE[(c + d*x)/2, 2]*(b*Sec[c + d*x])^(5/2))/d
```

Maple [C] time = 0.158, size = 311, normalized size = 7.6

$$2\,\frac{(\cos(dx+c))^2}{d\sin(dx+c)}\left(i\sqrt{(\cos(dx+c)+1)^{-1}}\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\mathrm{EllipticF}\left(\frac{i(-1+\cos(dx+c))}{\sin(dx+c)},i\right)\cos(dx+c)\sin(dx+c\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(b*sec(d*x+c))^(5/2),x)
```

```
[Out] 2/d*(I*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticF
(I*(-1+cos(d*x+c))/sin(d*x+c),I)*cos(d*x+c)*sin(d*x+c)-I*EllipticE(I*(-1+co
s(d*x+c))/sin(d*x+c),I)*cos(d*x+c)*sin(d*x+c)*(1/(cos(d*x+c)+1))^(1/2)*(cos
(d*x+c)/(cos(d*x+c)+1))^(1/2)+I*EllipticF(I*(-1+cos(d*x+c))/sin(d*x+c),I)*(
1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-I*Elli
pticE(I*(-1+cos(d*x+c))/sin(d*x+c),I)*sin(d*x+c)*(1/(cos(d*x+c)+1))^(1/2)*(
cos(d*x+c)/(cos(d*x+c)+1))^(1/2)-cos(d*x+c)^2+cos(d*x+c))*cos(d*x+c)^2*(b/c
os(d*x+c))^(5/2)/sin(d*x+c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b\sec(dx+c))^{\frac{5}{2}}\cos(dx+c)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(b*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c))^(5/2)*cos(d*x + c)^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\sqrt{b\sec(dx+c)}b^2\cos(dx+c)^3\sec(dx+c)^2,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(d*x + c))*b^2*cos(d*x + c)^3*sec(d*x + c)^2, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(b*sec(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b\sec(dx+c))^{\frac{5}{2}} \cos(dx+c)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c))^(5/2)*cos(d*x + c)^3, x)
```

3.96 $\int \cos^4(c+dx)(b\sec(c+dx))^{5/2}\,dx$

Optimal. Leaf size=72

$$\frac{2b^2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{b\sec(c+dx)}}{3d}+\frac{2b^3\sin(c+dx)}{3d\sqrt{b\sec(c+dx)}}$$

```
[Out] (2*b^2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[b*Sec[c + d*x]])/(
3*d) + (2*b^3*Sin[c + d*x])/(3*d*Sqrt[b*Sec[c + d*x]])
```

Rubi [A] time = 0.0546094, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {16, 3769, 3771, 2641}

$$\frac{2b^3\sin(c+dx)}{3d\sqrt{b\sec(c+dx)}}+\frac{2b^2\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\sec(c+dx)}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*(b*Sec[c + d*x])^(5/2),x]
```

```
[Out] (2*b^2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[b*Sec[c + d*x]])/(
3*d) + (2*b^3*Sin[c + d*x])/(3*d*Sqrt[b*Sec[c + d*x]])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cos^4(c+dx)(b\sec(c+dx))^{5/2}\,dx = b^4\int \frac{1}{(b\sec(c+dx))^{3/2}}\,dx$$
$$= \frac{2b^3\sin(c+dx)}{3d\sqrt{b\sec(c+dx)}} + \frac{1}{3}b^2\int \sqrt{b\sec(c+dx)}\,dx$$
$$= \frac{2b^3\sin(c+dx)}{3d\sqrt{b\sec(c+dx)}} + \frac{1}{3}\left(b^2\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx$$
$$= \frac{2b^2\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\sec(c+dx)}}{3d} + \frac{2b^3\sin(c+dx)}{3d\sqrt{b\sec(c+dx)}}$$

Mathematica [A] time = 0.0494816, size = 54, normalized size = 0.75

$$\frac{b^2\sqrt{b\sec(c+dx)}\left(2\sqrt{\cos(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+\sin(2(c+dx))\right)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*(b*Sec[c + d*x])^(5/2),x]
```

```
[Out] (b^2*Sqrt[b*Sec[c + d*x]]*(2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] +
 Sin[2*(c + d*x)]))/(3*d)
```

Maple [C] time = 0.167, size = 131, normalized size = 1.8

$$-\frac{2\left(\cos(dx+c)+1\right)^2\left(-1+\cos(dx+c)\right)\left(\cos(dx+c)\right)^2}{3d\left(\sin(dx+c)\right)^3}\left(\frac{b}{\cos(dx+c)}\right)^{\frac{5}{2}}\left(i\text{EllipticF}\left(\frac{i\left(-1+\cos(dx+c)\right)}{\sin(dx+c)},i\right)\sqrt{(\cos(dx\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*(b*sec(d*x+c))^(5/2),x)
```

```
[Out] -2/3/d*(cos(d*x+c)+1)^2*(b/cos(d*x+c))^(5/2)*(-1+cos(d*x+c))*cos(d*x+c)^2*(
I*EllipticF(I*(-1+cos(d*x+c))/sin(d*x+c),I)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d
*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-cos(d*x+c)^2+cos(d*x+c))/sin(d*x+c)^
3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b\sec(dx+c))^{\frac{5}{2}}\cos(dx+c)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(b*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c))^(5/2)*cos(d*x + c)^4, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{b\sec\left(dx+c\right)}b^2\cos\left(dx+c\right)^4\sec\left(dx+c\right)^2,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(d*x + c))*b^2*cos(d*x + c)^4*sec(d*x + c)^2, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*(b*sec(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(b\sec\left(dx+c\right)\right)^{\frac{5}{2}}\cos\left(dx+c\right)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c))^(5/2)*cos(d*x + c)^4, x)
```

3.97 $\int \cos^5(c+dx)(b\sec(c+dx))^{5/2}\,dx$

Optimal. Leaf size=72

$$\frac{2b^4\sin(c+dx)}{5d(b\sec(c+dx))^{3/2}}+\frac{6b^3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}$$

```
[Out] (6*b^3*EllipticE[(c + d*x)/2, 2])/(5*d*Sqrt[Cos[c + d*x]]*Sqrt[b*Sec[c + d*
x]]) + (2*b^4*Sin[c + d*x])/(5*d*(b*Sec[c + d*x])^(3/2))
```

Rubi [A] time = 0.0537524, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {16, 3769, 3771, 2639}

$$\frac{2b^4\sin(c+dx)}{5d(b\sec(c+dx))^{3/2}}+\frac{6b^3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5*(b*Sec[c + d*x])^(5/2),x]
```

```
[Out] (6*b^3*EllipticE[(c + d*x)/2, 2])/(5*d*Sqrt[Cos[c + d*x]]*Sqrt[b*Sec[c + d*
x]]) + (2*b^4*Sin[c + d*x])/(5*d*(b*Sec[c + d*x])^(3/2))
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \cos^5(c+dx)(b\sec(c+dx))^{5/2}\,dx &= b^5\int \frac{1}{(b\sec(c+dx))^{5/2}}\,dx\\ &= \frac{2b^4\sin(c+dx)}{5d(b\sec(c+dx))^{3/2}}+\frac{1}{5}\left(3b^3\right)\int \frac{1}{\sqrt{b\sec(c+dx)}}\,dx\\ &= \frac{2b^4\sin(c+dx)}{5d(b\sec(c+dx))^{3/2}}+\frac{\left(3b^3\right)\int \sqrt{\cos(c+dx)}\,dx}{5\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}\\ &= \frac{6b^3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}+\frac{2b^4\sin(c+dx)}{5d(b\sec(c+dx))^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.0513129, size = 60, normalized size = 0.83

$$\frac{b^2\sqrt{b\sec(c+dx)}\left(\sin(c+dx)+\sin(3(c+dx))+12\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\right)}{10d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5*(b*Sec[c + d*x])^(5/2),x]
```

```
[Out] (b^2*Sqrt[b*Sec[c + d*x]]*(12*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]
+ Sin[c + d*x] + Sin[3*(c + d*x)]))/(10*d)
```

Maple [C] time = 0.201, size = 321, normalized size = 4.5

$$-\frac{2\,(\cos(dx+c))^2}{5\,d\sin(dx+c)}\left(\frac{b}{\cos(dx+c)}\right)^{\frac{5}{2}}\left(3\,i\cos(dx+c)\sin(dx+c)\sqrt{(\cos(dx+c)+1)^{-1}}\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\mathrm{EllipticE}\left(\frac{i(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*(b*sec(d*x+c))^(5/2),x)
```

```
[Out] -2/5/d*(b/cos(d*x+c))^(5/2)*cos(d*x+c)^2*(3*I*cos(d*x+c)*sin(d*x+c)*(1/(cos
(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticE(I*(-1+cos(d*x
+c))/sin(d*x+c),I)-3*I*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))
^(1/2)*EllipticF(I*(-1+cos(d*x+c))/sin(d*x+c),I)*sin(d*x+c)*cos(d*x+c)+3*I*
sin(d*x+c)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*Ellip
ticE(I*(-1+cos(d*x+c))/sin(d*x+c),I)-3*I*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+
c)/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(-1+cos(d*x+c))/sin(d*x+c),I)*sin(d*x+
c)+cos(d*x+c)^4+2*cos(d*x+c)^2-3*cos(d*x+c))/sin(d*x+c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b\sec(dx+c))^{\frac{5}{2}}\cos(dx+c)^5\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(b*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c))^(5/2)*cos(d*x + c)^5, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{b\sec\left(dx+c\right)}b^2\cos\left(dx+c\right)^5\sec\left(dx+c\right)^2,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(d*x + c))*b^2*cos(d*x + c)^5*sec(d*x + c)^2, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*(b*sec(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sec\left(dx+c\right)\right)^{\frac{5}{2}}\cos\left(dx+c\right)^5\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c))^(5/2)*cos(d*x + c)^5, x)
```

3.98 $\int \cos^6(c+dx)(b\sec(c+dx))^{5/2}\,dx$

Optimal. Leaf size=100

$$\frac{10b^2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{b\sec(c+dx)}}{21d}+\frac{2b^5\sin(c+dx)}{7d(b\sec(c+dx))^{5/2}}+\frac{10b^3\sin(c+dx)}{21d\sqrt{b\sec(c+dx)}}$$

```
[Out] (10*b^2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[b*Sec[c + d*x]])/
(21*d) + (2*b^5*Sin[c + d*x])/(7*d*(b*Sec[c + d*x])^(5/2)) + (10*b^3*Sin[c
+ d*x])/(21*d*Sqrt[b*Sec[c + d*x]])
```

Rubi [A] time = 0.0714372, antiderivative size = 100, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {16, 3769, 3771, 2641}

$$\frac{2b^5\sin(c+dx)}{7d(b\sec(c+dx))^{5/2}}+\frac{10b^3\sin(c+dx)}{21d\sqrt{b\sec(c+dx)}}+\frac{10b^2\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\sec(c+dx)}}{21d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^6*(b*Sec[c + d*x])^(5/2),x]
```

```
[Out] (10*b^2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[b*Sec[c + d*x]])/
(21*d) + (2*b^5*Sin[c + d*x])/(7*d*(b*Sec[c + d*x])^(5/2)) + (10*b^3*Sin[c
+ d*x])/(21*d*Sqrt[b*Sec[c + d*x]])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cos^6(c+dx)(b\sec(c+dx))^{5/2}\,dx = b^6\int \frac{1}{(b\sec(c+dx))^{7/2}}\,dx$$
$$= \frac{2b^5\sin(c+dx)}{7d(b\sec(c+dx))^{5/2}} + \frac{1}{7}\left(5b^4\right)\int \frac{1}{(b\sec(c+dx))^{3/2}}\,dx$$
$$= \frac{2b^5\sin(c+dx)}{7d(b\sec(c+dx))^{5/2}} + \frac{10b^3\sin(c+dx)}{21d\sqrt{b\sec(c+dx)}} + \frac{1}{21}\left(5b^2\right)\int \sqrt{b\sec(c+dx)}\,dx$$
$$= \frac{2b^5\sin(c+dx)}{7d(b\sec(c+dx))^{5/2}} + \frac{10b^3\sin(c+dx)}{21d\sqrt{b\sec(c+dx)}} + \frac{1}{21}\left(5b^2\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}\right)$$
$$= \frac{10b^2\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\sec(c+dx)}}{21d} + \frac{2b^5\sin(c+dx)}{7d(b\sec(c+dx))^{5/2}} + \frac{10b^3\sin}{21d\sqrt{b}}$$

Mathematica [A] time = 0.0732741, size = 66, normalized size = 0.66

$$\frac{b^2\sqrt{b\sec(c+dx)}\left(40\sqrt{\cos(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+26\sin(2(c+dx))+3\sin(4(c+dx))\right)}{84d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^6*(b*Sec[c + d*x])^(5/2),x]
```

```
[Out] (b^2*Sqrt[b*Sec[c + d*x]]*(40*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]
+ 26*Sin[2*(c + d*x)] + 3*Sin[4*(c + d*x)]))/(84*d)
```

Maple [C] time = 0.188, size = 153, normalized size = 1.5

$$\frac{2\,(\cos(dx+c)+1)^2\,(-1+\cos(dx+c))\,(\cos(dx+c))^2}{21\,d\,(\sin(dx+c))^3}\left(\frac{b}{\cos(dx+c)}\right)^{\frac{5}{2}}\left(-5\,i\sqrt{(\cos(dx+c)+1)^{-1}}\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\text{Ell}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*(b*sec(d*x+c))^(5/2),x)
```

```
[Out] 2/21/d*(cos(d*x+c)+1)^2*(b/cos(d*x+c))^(5/2)*(-1+cos(d*x+c))*cos(d*x+c)^2*(
-5*I*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticF(I
*(-1+cos(d*x+c))/sin(d*x+c),I)*sin(d*x+c)+3*cos(d*x+c)^4-3*cos(d*x+c)^3+5*c
os(d*x+c)^2-5*cos(d*x+c))/sin(d*x+c)^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b\sec(dx+c))^{\frac{5}{2}}\cos(dx+c)^6\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*(b*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c))^(5/2)*cos(d*x + c)^6, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{b\sec(dx+c)}b^2\cos(dx+c)^6\sec(dx+c)^2,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*(b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(d*x + c))*b^2*cos(d*x + c)^6*sec(d*x + c)^2, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*(b*sec(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b\sec(dx+c))^{\frac{5}{2}}\cos(dx+c)^6\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*(b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c))^(5/2)*cos(d*x + c)^6, x)
```

3.99 $\int \cos^7(c+dx)(b\sec(c+dx))^{5/2}\,dx$

Optimal. Leaf size=100

$$\frac{2b^6\sin(c+dx)}{9d(b\sec(c+dx))^{7/2}}+\frac{14b^4\sin(c+dx)}{45d(b\sec(c+dx))^{3/2}}+\frac{14b^3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}$$

```
[Out] (14*b^3*EllipticE[(c + d*x)/2, 2])/(15*d*Sqrt[Cos[c + d*x]]*Sqrt[b*Sec[c +
d*x]]) + (2*b^6*Sin[c + d*x])/(9*d*(b*Sec[c + d*x])^(7/2)) + (14*b^4*Sin[c
+ d*x])/(45*d*(b*Sec[c + d*x])^(3/2))
```

Rubi [A] time = 0.0740449, antiderivative size = 100, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {16, 3769, 3771, 2639}

$$\frac{2b^6\sin(c+dx)}{9d(b\sec(c+dx))^{7/2}}+\frac{14b^4\sin(c+dx)}{45d(b\sec(c+dx))^{3/2}}+\frac{14b^3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^7*(b*Sec[c + d*x])^(5/2),x]
```

```
[Out] (14*b^3*EllipticE[(c + d*x)/2, 2])/(15*d*Sqrt[Cos[c + d*x]]*Sqrt[b*Sec[c +
d*x]]) + (2*b^6*Sin[c + d*x])/(9*d*(b*Sec[c + d*x])^(7/2)) + (14*b^4*Sin[c
+ d*x])/(45*d*(b*Sec[c + d*x])^(3/2))
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\int \cos^7(c+dx)(b\sec(c+dx))^{5/2}\,dx &= b^7\int \frac{1}{(b\sec(c+dx))^{9/2}}\,dx \\
&= \frac{2b^6\sin(c+dx)}{9d(b\sec(c+dx))^{7/2}}+\frac{1}{9}\left(7b^5\right)\int \frac{1}{(b\sec(c+dx))^{5/2}}\,dx \\
&= \frac{2b^6\sin(c+dx)}{9d(b\sec(c+dx))^{7/2}}+\frac{14b^4\sin(c+dx)}{45d(b\sec(c+dx))^{3/2}}+\frac{1}{15}\left(7b^3\right)\int \frac{1}{\sqrt{b\sec(c+dx)}}\,dx \\
&= \frac{2b^6\sin(c+dx)}{9d(b\sec(c+dx))^{7/2}}+\frac{14b^4\sin(c+dx)}{45d(b\sec(c+dx))^{3/2}}+\frac{\left(7b^3\right)\int \sqrt{\cos(c+dx)}\,dx}{15\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}} \\
&= \frac{14b^3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}+\frac{2b^6\sin(c+dx)}{9d(b\sec(c+dx))^{7/2}}+\frac{14b^4\sin(c+dx)}{45d(b\sec(c+dx))^{3/2}}
\end{aligned}
$$

Mathematica [A] time = 0.166259, size = 74, normalized size = 0.74

$$
\frac{b^2\sqrt{b\sec(c+dx)}\left((33\sin(c+dx)+5\sin(3(c+dx)))\cos^2(c+dx)+84\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\right)}{90d}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^7*(b*Sec[c + d*x])^(5/2),x]
```

```
[Out] (b^2*Sqrt[b*Sec[c + d*x]]*(84*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]
+ Cos[c + d*x]^2*(33*Sin[c + d*x] + 5*Sin[3*(c + d*x)])))/(90*d)
```

Maple [C] time = 0.186, size = 333, normalized size = 3.3

$$
\frac{2\,(\cos(dx+c))^2}{45\,d\sin(dx+c)}\left(\frac{b}{\cos(dx+c)}\right)^{\frac{5}{2}}\left(21\,i\sqrt{(\cos(dx+c)+1)^{-1}}\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\mathrm{EllipticF}\left(\frac{i(-1+\cos(dx+c))}{\sin(dx+c)},i\right)\sin\right.
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^7*(b*sec(d*x+c))^(5/2),x)
```

```
[Out] 2/45/d*(b/cos(d*x+c))^(5/2)*cos(d*x+c)^2*(21*I*(1/(cos(d*x+c)+1))^(1/2)*(co
s(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(-1+cos(d*x+c))/sin(d*x+c),I)*si
n(d*x+c)*cos(d*x+c)-21*I*cos(d*x+c)*sin(d*x+c)*(1/(cos(d*x+c)+1))^(1/2)*(co
s(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticE(I*(-1+cos(d*x+c))/sin(d*x+c),I)-5*
cos(d*x+c)^6+21*I*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2
)*EllipticF(I*(-1+cos(d*x+c))/sin(d*x+c),I)*sin(d*x+c)-21*I*sin(d*x+c)*(1/(
cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticE(I*(-1+cos(
d*x+c))/sin(d*x+c),I)-2*cos(d*x+c)^4-14*cos(d*x+c)^2+21*cos(d*x+c))/sin(d*x
+c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$
\int (b\sec(dx+c))^{\frac{5}{2}}\cos(dx+c)^7\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*(b*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c))^(5/2)*cos(d*x + c)^7, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{b\sec\left(dx+c\right)}b^{2}\cos\left(dx+c\right)^{7}\sec\left(dx+c\right)^{2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*(b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(d*x + c))*b^2*cos(d*x + c)^7*sec(d*x + c)^2, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**7*(b*sec(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sec\left(dx+c\right)\right)^{\frac{5}{2}}\cos\left(dx+c\right)^{7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*(b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c))^(5/2)*cos(d*x + c)^7, x)
```

3.100 $\int (b\sec(c+dx))^{7/2}\,dx$

Optimal. Leaf size=98

$$\frac{6b^3\sin(c+dx)\sqrt{b\sec(c+dx)}}{5d}-\frac{6b^4E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}+\frac{2b\sin(c+dx)(b\sec(c+dx))^{5/2}}{5d}$$

```
[Out] (-6*b^4*EllipticE[(c + d*x)/2, 2])/(5*d*Sqrt[Cos[c + d*x]]*Sqrt[b*Sec[c + d
*x]]) + (6*b^3*Sqrt[b*Sec[c + d*x]]*Sin[c + d*x])/(5*d) + (2*b*(b*Sec[c + d
*x])^(5/2)*Sin[c + d*x])/(5*d)
```

Rubi [A] time = 0.0484491, antiderivative size = 98, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {3768, 3771, 2639}

$$\frac{6b^3\sin(c+dx)\sqrt{b\sec(c+dx)}}{5d}-\frac{6b^4E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}+\frac{2b\sin(c+dx)(b\sec(c+dx))^{5/2}}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Sec[c + d*x])^(7/2),x]
```

```
[Out] (-6*b^4*EllipticE[(c + d*x)/2, 2])/(5*d*Sqrt[Cos[c + d*x]]*Sqrt[b*Sec[c + d
*x]]) + (6*b^3*Sqrt[b*Sec[c + d*x]]*Sin[c + d*x])/(5*d) + (2*b*(b*Sec[c + d
*x])^(5/2)*Sin[c + d*x])/(5*d)
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (b\sec(c+dx))^{7/2}\,dx = \frac{2b(b\sec(c+dx))^{5/2}\sin(c+dx)}{5d} + \frac{1}{5}\left(3b^2\right)\int (b\sec(c+dx))^{3/2}\,dx$$
$$= \frac{6b^3\sqrt{b\sec(c+dx)}\sin(c+dx)}{5d} + \frac{2b(b\sec(c+dx))^{5/2}\sin(c+dx)}{5d} - \frac{1}{5}\left(3b^4\right)\int \frac{1}{\sqrt{b\sec(c+dx)}}\,dx$$
$$= \frac{6b^3\sqrt{b\sec(c+dx)}\sin(c+dx)}{5d} + \frac{2b(b\sec(c+dx))^{5/2}\sin(c+dx)}{5d} - \frac{\left(3b^4\right)\int \sqrt{\cos(c+dx)}\,dx}{5\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}$$
$$= -\frac{6b^4E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}} + \frac{6b^3\sqrt{b\sec(c+dx)}\sin(c+dx)}{5d} + \frac{2b(b\sec(c+dx))^{5/2}\sin(c+dx)}{5d}$$

Mathematica [A] time = 0.0847728, size = 62, normalized size = 0.63

$$\frac{b(b\sec(c+dx))^{5/2}\left(7\sin(c+dx)+3\sin(3(c+dx))-12\cos^{\frac{5}{2}}(c+dx)E\left(\frac{1}{2}(c+dx)\middle|2\right)\right)}{10d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Sec[c + d*x])^(7/2),x]
```

```
[Out] (b*(b*Sec[c + d*x])^(5/2)*(-12*Cos[c + d*x]^(5/2)*EllipticE[(c + d*x)/2, 2]
 + 7*Sin[c + d*x] + 3*Sin[3*(c + d*x)]))/(10*d)
```

Maple [C] time = 0.222, size = 354, normalized size = 3.6

$$-\frac{2\,(-1+\cos(dx+c))^2\cos(dx+c)\,(\cos(dx+c)+1)^2}{5\,d\,(\sin(dx+c))^5}\left(3\,i\mathrm{EllipticF}\left(\frac{i\,(-1+\cos(dx+c))}{\sin(dx+c)},i\right)(\cos(dx+c))^3\sqrt{(\cos(dx+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sec(d*x+c))^(7/2),x)
```

```
[Out] -2/5/d*(-1+cos(d*x+c))^2*(3*I*EllipticF(I*(-1+cos(d*x+c))/sin(d*x+c),I)*cos
(d*x+c)^3*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*
x+c)-3*I*EllipticE(I*(-1+cos(d*x+c))/sin(d*x+c),I)*cos(d*x+c)^3*(1/(cos(d*x
+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+3*I*EllipticF(I*
(-1+cos(d*x+c))/sin(d*x+c),I)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x
+c)+1))^(1/2)*cos(d*x+c)^2*sin(d*x+c)-3*I*EllipticE(I*(-1+cos(d*x+c))/sin(d
*x+c),I)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*cos(d*x
+c)^2*sin(d*x+c)+3*cos(d*x+c)^3-2*cos(d*x+c)^2-1)*cos(d*x+c)*(cos(d*x+c)+1)
^2*(b/cos(d*x+c))^(7/2)/sin(d*x+c)^5
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b\sec(dx+c))^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c))^(7/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{b\sec\left(dx+c\right)}b^3\sec\left(dx+c\right)^3,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(d*x + c))*b^3*sec(d*x + c)^3, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(b\sec\left(dx+c\right)\right)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c))^(7/2), x)
```

3.101 $\int \frac{\sec^5(c+dx)}{\sqrt{b\sec(c+dx)}}\,dx$

Optimal. Leaf size=100

$$\frac{10\sqrt{\cos(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{b\sec(c+dx)}}{21bd}+\frac{2\sin(c+dx)(b\sec(c+dx))^{7/2}}{7b^4d}+\frac{10\sin(c+dx)(b\sec(c+dx))^{3/2}}{21b^2d}$$

```
[Out] (10*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[b*Sec[c + d*x]])/(21*
b*d) + (10*(b*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(21*b^2*d) + (2*(b*Sec[c +
d*x])^(7/2)*Sin[c + d*x])/(7*b^4*d)
```

Rubi [A] time = 0.0574047, antiderivative size = 100, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {16, 3768, 3771, 2641}

$$\frac{2\sin(c+dx)(b\sec(c+dx))^{7/2}}{7b^4d}+\frac{10\sin(c+dx)(b\sec(c+dx))^{3/2}}{21b^2d}+\frac{10\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\sec(c+dx)}}{21bd}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^5/Sqrt[b*Sec[c + d*x]],x]
```

```
[Out] (10*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[b*Sec[c + d*x]])/(21*
b*d) + (10*(b*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(21*b^2*d) + (2*(b*Sec[c +
d*x])^(7/2)*Sin[c + d*x])/(7*b^4*d)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sec^5(c+dx)}{\sqrt{b\sec(c+dx)}}\,dx = \frac{\int (b\sec(c+dx))^{9/2}\,dx}{b^5}$$
$$= \frac{2(b\sec(c+dx))^{7/2}\sin(c+dx)}{7b^4d} + \frac{5\int (b\sec(c+dx))^{5/2}\,dx}{7b^3}$$
$$= \frac{10(b\sec(c+dx))^{3/2}\sin(c+dx)}{21b^2d} + \frac{2(b\sec(c+dx))^{7/2}\sin(c+dx)}{7b^4d} + \frac{5\int \sqrt{b\sec(c+dx)}\,dx}{21b}$$
$$= \frac{10(b\sec(c+dx))^{3/2}\sin(c+dx)}{21b^2d} + \frac{2(b\sec(c+dx))^{7/2}\sin(c+dx)}{7b^4d} + \frac{\left(5\sqrt{\cos(c+dx)}\sqrt{b\sec(c+\right.}{21b}$$
$$= \frac{10\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\sec(c+dx)}}{21bd} + \frac{10(b\sec(c+dx))^{3/2}\sin(c+dx)}{21b^2d} + \frac{2(b\sec(c}{}$$

Mathematica [A] time = 0.0827893, size = 69, normalized size = 0.69

$$\frac{\sec^3(c+dx)\left(10\cos^{\frac{5}{2}}(c+dx)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+5\sin(2(c+dx))+6\tan(c+dx)\right)}{21d\sqrt{b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^5/Sqrt[b*Sec[c + d*x]],x]
```

```
[Out] (Sec[c + d*x]^3*(10*Cos[c + d*x]^(5/2)*EllipticF[(c + d*x)/2, 2] + 5*Sin[2*
(c + d*x)] + 6*Tan[c + d*x]))/(21*d*Sqrt[b*Sec[c + d*x]])
```

Maple [C] time = 0.23, size = 152, normalized size = 1.5

$$-\frac{(-2+2\cos(dx+c))(\cos(dx+c)+1)^2}{21\,d(\sin(dx+c))^3(\cos(dx+c))^4}\left(5\,i(\cos(dx+c))^3\sin(dx+c)\sqrt{(\cos(dx+c)+1)^{-1}}\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\text{Elli}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5/(b*sec(d*x+c))^(1/2),x)
```

```
[Out] -2/21/d*(-1+cos(d*x+c))*(5*I*cos(d*x+c)^3*sin(d*x+c)*(1/(cos(d*x+c)+1))^(1/
2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(-1+cos(d*x+c))/sin(d*x+c)
,I)-5*cos(d*x+c)^3+5*cos(d*x+c)^2-3*cos(d*x+c)+3)*(cos(d*x+c)+1)^2/sin(d*x+
c)^3/cos(d*x+c)^4/(b/cos(d*x+c))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)^5}{\sqrt{b\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)^5/sqrt(b*sec(d*x + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{b\sec\left(dx+c\right)}\sec\left(dx+c\right)^{4}}{b},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(d*x + c))*sec(d*x + c)^4/b, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sec^{5}\left(c+dx\right)}{\sqrt{b\sec\left(c+dx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5/(b*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral(sec(c + d*x)**5/sqrt(b*sec(c + d*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sec\left(dx+c\right)^{5}}{\sqrt{b\sec\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^5/sqrt(b*sec(d*x + c)), x)
```

3.102 $\int \frac{\sec^4(c+dx)}{\sqrt{b\sec(c+dx)}}\,dx$

Optimal. Leaf size=97

$$\frac{2\sin(c+dx)(b\sec(c+dx))^{5/2}}{5b^3d}+\frac{6\sin(c+dx)\sqrt{b\sec(c+dx)}}{5bd}-\frac{6E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}$$

```
[Out] (-6*EllipticE[(c + d*x)/2, 2])/(5*d*Sqrt[Cos[c + d*x]]*Sqrt[b*Sec[c + d*x]]
) + (6*Sqrt[b*Sec[c + d*x]]*Sin[c + d*x])/(5*b*d) + (2*(b*Sec[c + d*x])^(5/
2)*Sin[c + d*x])/(5*b^3*d)
```

Rubi [A] time = 0.0586433, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {16, 3768, 3771, 2639}

$$\frac{2\sin(c+dx)(b\sec(c+dx))^{5/2}}{5b^3d}+\frac{6\sin(c+dx)\sqrt{b\sec(c+dx)}}{5bd}-\frac{6E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4/Sqrt[b*Sec[c + d*x]],x]
```


```
[Out] (-6*EllipticE[(c + d*x)/2, 2])/(5*d*Sqrt[Cos[c + d*x]]*Sqrt[b*Sec[c + d*x]]
) + (6*Sqrt[b*Sec[c + d*x]]*Sin[c + d*x])/(5*b*d) + (2*(b*Sec[c + d*x])^(5/
2)*Sin[c + d*x])/(5*b^3*d)
```


Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sec^4(c+dx)}{\sqrt{b\sec(c+dx)}}\,dx = \frac{\int (b\sec(c+dx))^{7/2}\,dx}{b^4}$$
$$= \frac{2(b\sec(c+dx))^{5/2}\sin(c+dx)}{5b^3d} + \frac{3\int (b\sec(c+dx))^{3/2}\,dx}{5b^2}$$
$$= \frac{6\sqrt{b\sec(c+dx)}\sin(c+dx)}{5bd} + \frac{2(b\sec(c+dx))^{5/2}\sin(c+dx)}{5b^3d} - \frac{3}{5}\int \frac{1}{\sqrt{b\sec(c+dx)}}\,dx$$
$$= \frac{6\sqrt{b\sec(c+dx)}\sin(c+dx)}{5bd} + \frac{2(b\sec(c+dx))^{5/2}\sin(c+dx)}{5b^3d} - \frac{3\int \sqrt{\cos(c+dx)}\,dx}{5\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}$$
$$= -\frac{6E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}} + \frac{6\sqrt{b\sec(c+dx)}\sin(c+dx)}{5bd} + \frac{2(b\sec(c+dx))^{5/2}\sin(c+dx)}{5b^3d}$$

Mathematica [A] time = 0.225734, size = 61, normalized size = 0.63

$$\frac{2\tan(c+dx)\left(\sec^2(c+dx)+3\right) - \frac{6E\left(\frac{1}{2}(c+dx)\middle|2\right)}{\sqrt{\cos(c+dx)}}}{5d\sqrt{b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4/Sqrt[b*Sec[c + d*x]],x]
```

```
[Out] ((-6*EllipticE[(c + d*x)/2, 2])/Sqrt[Cos[c + d*x]] + 2*(3 + Sec[c + d*x]^2)
*Tan[c + d*x])/(5*d*Sqrt[b*Sec[c + d*x]])
```

Maple [C] time = 0.236, size = 356, normalized size = 3.7

$$\frac{2(-1+\cos(dx+c))^2(\cos(dx+c)+1)^2}{5d(\cos(dx+c))^3(\sin(dx+c))^5}\left(3i\sqrt{(\cos(dx+c)+1)^{-1}}\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}(\cos(dx+c))^3\,\mathrm{EllipticE}\left(\frac{i(-1+c}{\sin}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4/(b*sec(d*x+c))^(1/2),x)
```

```
[Out] 2/5/d*(-1+cos(d*x+c))^2*(3*I*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+
c)+1))^(1/2)*cos(d*x+c)^3*EllipticE(I*(-1+cos(d*x+c))/sin(d*x+c),I)*sin(d*x
+c)-3*I*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*Elliptic
F(I*(-1+cos(d*x+c))/sin(d*x+c),I)*cos(d*x+c)^3*sin(d*x+c)+3*I*(1/(cos(d*x+c
)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)^2*EllipticE(I*(-1+
cos(d*x+c))/sin(d*x+c),I)*sin(d*x+c)-3*I*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+
c)/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(-1+cos(d*x+c))/sin(d*x+c),I)*cos(d*x+
c)^2*sin(d*x+c)-3*cos(d*x+c)^3+2*cos(d*x+c)^2+1)*(cos(d*x+c)+1)^2/cos(d*x+c
)^3/(b/cos(d*x+c))^(1/2)/sin(d*x+c)^5
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)^4}{\sqrt{b\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)^4/sqrt(b*sec(d*x + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{b\sec\left(dx+c\right)}\sec\left(dx+c\right)^{3}}{b},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(d*x + c))*sec(d*x + c)^3/b, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sec^{4}\left(c+dx\right)}{\sqrt{b\sec\left(c+dx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4/(b*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral(sec(c + d*x)**4/sqrt(b*sec(c + d*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sec\left(dx+c\right)^{4}}{\sqrt{b\sec\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^4/sqrt(b*sec(d*x + c)), x)
```

3.103 $\int \frac{\sec^3(c+dx)}{\sqrt{b\sec(c+dx)}}\,dx$

Optimal. Leaf size=72

$$\frac{2\sqrt{\cos(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{b\sec(c+dx)}}{3bd}+\frac{2\sin(c+dx)(b\sec(c+dx))^{3/2}}{3b^2d}$$

```
[Out] (2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[b*Sec[c + d*x]])/(3*b*
d) + (2*(b*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(3*b^2*d)
```

Rubi [A] time = 0.0394617, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {16, 3768, 3771, 2641}

$$\frac{2\sin(c+dx)(b\sec(c+dx))^{3/2}}{3b^2d}+\frac{2\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\sec(c+dx)}}{3bd}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3/Sqrt[b*Sec[c + d*x]],x]
```

```
[Out] (2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[b*Sec[c + d*x]])/(3*b*
d) + (2*(b*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(3*b^2*d)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sec^3(c+dx)}{\sqrt{b\sec(c+dx)}}\,dx = \frac{\int (b\sec(c+dx))^{5/2}\,dx}{b^3}$$
$$= \frac{2(b\sec(c+dx))^{3/2}\sin(c+dx)}{3b^2d} + \frac{\int \sqrt{b\sec(c+dx)}\,dx}{3b}$$
$$= \frac{2(b\sec(c+dx))^{3/2}\sin(c+dx)}{3b^2d} + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx}{3b}$$
$$= \frac{2\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\sec(c+dx)}}{3bd} + \frac{2(b\sec(c+dx))^{3/2}\sin(c+dx)}{3b^2d}$$

Mathematica [A] time = 0.0744825, size = 51, normalized size = 0.71

$$\frac{2\sqrt{b\sec(c+dx)}\left(\sqrt{\cos(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+\tan(c+dx)\right)}{3bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3/Sqrt[b*Sec[c + d*x]],x]
```

```
[Out] (2*Sqrt[b*Sec[c + d*x]]*(Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + Tan
[c + d*x]))/(3*b*d)
```

Maple [C] time = 0.181, size = 130, normalized size = 1.8

$$-\frac{(-2+2\cos(dx+c))(\cos(dx+c)+1)^2}{3d(\cos(dx+c))^2(\sin(dx+c))^3}\left(i\sqrt{(\cos(dx+c)+1)^{-1}}\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\text{EllipticF}\left(\frac{i(-1+\cos(dx+c))}{\sin(dx+c)},\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3/(b*sec(d*x+c))^(1/2),x)
```

```
[Out] -2/3/d*(-1+cos(d*x+c))*(I*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+
1))^(1/2)*EllipticF(I*(-1+cos(d*x+c))/sin(d*x+c),I)*cos(d*x+c)*sin(d*x+c)-c
os(d*x+c)+1)*(cos(d*x+c)+1)^2/cos(d*x+c)^2/(b/cos(d*x+c))^(1/2)/sin(d*x+c)^
3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)^3}{\sqrt{b\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)^3/sqrt(b*sec(d*x + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{b\sec(dx+c)}\sec(dx+c)^2}{b}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(d*x + c))*sec(d*x + c)^2/b, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec^3(c+dx)}{\sqrt{b\sec(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3/(b*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral(sec(c + d*x)**3/sqrt(b*sec(c + d*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)^3}{\sqrt{b\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^3/sqrt(b*sec(d*x + c)), x)
```

3.104 $\int \frac{\sec^2(c+dx)}{\sqrt{b\sec(c+dx)}}\,dx$

Optimal. Leaf size=65

$$\frac{2\sin(c+dx)\sqrt{b\sec(c+dx)}}{bd}-\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}$$

```
[Out] (-2*EllipticE[(c + d*x)/2, 2])/(d*Sqrt[Cos[c + d*x]]*Sqrt[b*Sec[c + d*x]])
+ (2*Sqrt[b*Sec[c + d*x]]*Sin[c + d*x])/(b*d)
```

Rubi [A] time = 0.0401477, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {16, 3768, 3771, 2639}

$$\frac{2\sin(c+dx)\sqrt{b\sec(c+dx)}}{bd}-\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2/Sqrt[b*Sec[c + d*x]],x]
```

```
[Out] (-2*EllipticE[(c + d*x)/2, 2])/(d*Sqrt[Cos[c + d*x]]*Sqrt[b*Sec[c + d*x]])
+ (2*Sqrt[b*Sec[c + d*x]]*Sin[c + d*x])/(b*d)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sec^2(c+dx)}{\sqrt{b\sec(c+dx)}}\,dx = \frac{\int (b\sec(c+dx))^{3/2}\,dx}{b^2}$$
$$= \frac{2\sqrt{b\sec(c+dx)}\sin(c+dx)}{bd} - \int \frac{1}{\sqrt{b\sec(c+dx)}}\,dx$$
$$= \frac{2\sqrt{b\sec(c+dx)}\sin(c+dx)}{bd} - \frac{\int \sqrt{\cos(c+dx)}\,dx}{\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}$$
$$= -\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}} + \frac{2\sqrt{b\sec(c+dx)}\sin(c+dx)}{bd}$$

Mathematica [A] time = 0.0782271, size = 48, normalized size = 0.74

$$\frac{2\tan(c+dx) - \frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{\sqrt{\cos(c+dx)}}}{d\sqrt{b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2/Sqrt[b*Sec[c + d*x]],x]
```

```
[Out] ((-2*EllipticE[(c + d*x)/2, 2])/Sqrt[Cos[c + d*x]] + 2*Tan[c + d*x])/(d*Sqr
t[b*Sec[c + d*x]])
```

Maple [C] time = 0.218, size = 319, normalized size = 4.9

$$2\,\frac{(\cos(dx+c)+1)^2(-1+\cos(dx+c))^2}{db(\sin(dx+c))^5}\left(i\cos(dx+c)\sin(dx+c)\sqrt{(\cos(dx+c)+1)^{-1}}\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\mathrm{EllipticE}\left(\frac{i}{-}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2/(b*sec(d*x+c))^(1/2),x)
```

```
[Out] 2/d*(cos(d*x+c)+1)^2*(-1+cos(d*x+c))^2*(I*cos(d*x+c)*sin(d*x+c)*(1/(cos(d*x
+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticE(I*(-1+cos(d*x+c))
/sin(d*x+c),I)-I*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)
*EllipticF(I*(-1+cos(d*x+c))/sin(d*x+c),I)*cos(d*x+c)*sin(d*x+c)+I*sin(d*x+
c)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticE(I*(
-1+cos(d*x+c))/sin(d*x+c),I)-I*EllipticF(I*(-1+cos(d*x+c))/sin(d*x+c),I)*(1
/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-cos(d*x
+c)+1)*(b/cos(d*x+c))^(1/2)/b/sin(d*x+c)^5
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)^2}{\sqrt{b\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)^2/sqrt(b*sec(d*x + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{b\sec\left(dx+c\right)}\sec\left(dx+c\right)}{b},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(d*x + c))*sec(d*x + c)/b, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sec^2\left(c+dx\right)}{\sqrt{b\sec\left(c+dx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2/(b*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral(sec(c + d*x)**2/sqrt(b*sec(c + d*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sec\left(dx+c\right)^2}{\sqrt{b\sec\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^2/sqrt(b*sec(d*x + c)), x)
```

3.105 $\int \frac{\sec(c+dx)}{\sqrt{b\sec(c+dx)}}\,dx$

Optimal. Leaf size=41

$$\frac{2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{b\sec(c+dx)}}{bd}$$

```
[Out] (2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[b*Sec[c + d*x]])/(b*d)
```

Rubi [A] time = 0.0212964, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {16, 3771, 2641}

$$\frac{2\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\sec(c+dx)}}{bd}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]/Sqrt[b*Sec[c + d*x]],x]
```

```
[Out] (2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[b*Sec[c + d*x]])/(b*d)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec(c+dx)}{\sqrt{b\sec(c+dx)}}\,dx &= \frac{\int \sqrt{b\sec(c+dx)}\,dx}{b}\\ &= \frac{\left(\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx}{b}\\ &= \frac{2\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\sec(c+dx)}}{bd}\end{aligned}$$

Mathematica [A] time = 0.0163518, size = 41, normalized size = 1.

$$\frac{2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{b\sec(c+dx)}}{bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]/Sqrt[b*Sec[c + d*x]],x]
```

```
[Out] (2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[b*Sec[c + d*x]])/(b*d)
```

Maple [C] time = 0.158, size = 98, normalized size = 2.4

$$\frac{-2i(-1+\cos(dx+c))(\cos(dx+c)+1)^2}{d(\sin(dx+c))^2}\left((\cos(dx+c)+1)^{-1}\right)^{\frac{3}{2}}\text{EllipticF}\left(\frac{i(-1+\cos(dx+c))}{\sin(dx+c)},i\right)\frac{1}{\sqrt{\frac{b}{\cos(dx+c)}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)/(b*sec(d*x+c))^(1/2),x)
```

```
[Out] -2*I/d*(1/(cos(d*x+c)+1))^(3/2)*(-1+cos(d*x+c))*EllipticF(I*(-1+cos(d*x+c))
/sin(d*x+c),I)*(cos(d*x+c)+1)^2/(b/cos(d*x+c))^(1/2)/(cos(d*x+c)/(cos(d*x+c
)+1))^(1/2)/sin(d*x+c)^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)}{\sqrt{b\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)/sqrt(b*sec(d*x + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{b\sec(dx+c)}}{b},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(d*x + c))/b, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(c+dx)}{\sqrt{b\sec(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(b*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral(sec(c + d*x)/sqrt(b*sec(c + d*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)}{\sqrt{b\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)/sqrt(b*sec(d*x + c)), x)
```

3.106 $\int \frac{1}{\sqrt{b\sec(c+dx)}}\,dx$

Optimal. Leaf size=38

$$\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}$$

```
[Out] (2*EllipticE[(c + d*x)/2, 2])/(d*Sqrt[Cos[c + d*x]]*Sqrt[b*Sec[c + d*x]])
```

Rubi [A] time = 0.0190712, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3771, 2639}

$$\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[b*Sec[c + d*x]],x]
```

```
[Out] (2*EllipticE[(c + d*x)/2, 2])/(d*Sqrt[Cos[c + d*x]]*Sqrt[b*Sec[c + d*x]])
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{b\sec(c+dx)}}\,dx &= \frac{\int \sqrt{\cos(c+dx)}\,dx}{\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}\\ &= \frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}\end{aligned}$$

Mathematica [A] time = 0.014566, size = 38, normalized size = 1.

$$\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[b*Sec[c + d*x]],x]
```

```
[Out] (2*EllipticE[(c + d*x)/2, 2])/(d*Sqrt[Cos[c + d*x]]*Sqrt[b*Sec[c + d*x]])
```

Maple [C] time = 0.157, size = 306, normalized size = 8.1

$$2\frac{1}{d\sin(dx+c)b}\left(i\sqrt{(\cos(dx+c)+1)^{-1}}\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\mathrm{EllipticF}\left(\frac{i(-1+\cos(dx+c))}{\sin(dx+c)},i\right)\cos(dx+c)\sin(dx+c)-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*sec(d*x+c))^(1/2),x)
```

```
[Out] 2/d*(I*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticF
(I*(-1+cos(d*x+c))/sin(d*x+c),I)*cos(d*x+c)*sin(d*x+c)-I*EllipticE(I*(-1+co
s(d*x+c))/sin(d*x+c),I)*cos(d*x+c)*sin(d*x+c)*(1/(cos(d*x+c)+1))^(1/2)*(cos
(d*x+c)/(cos(d*x+c)+1))^(1/2)+I*EllipticF(I*(-1+cos(d*x+c))/sin(d*x+c),I)*(
1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-I*Elli
pticE(I*(-1+cos(d*x+c))/sin(d*x+c),I)*sin(d*x+c)*(1/(cos(d*x+c)+1))^(1/2)*(
cos(d*x+c)/(cos(d*x+c)+1))^(1/2)-cos(d*x+c)^2+cos(d*x+c))*(b/cos(d*x+c))^(1
/2)/sin(d*x+c)/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{b\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(b*sec(d*x + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{b\sec(dx+c)}}{b\sec(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(d*x + c))/(b*sec(d*x + c)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{b\sec(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral(1/sqrt(b*sec(c + d*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{b \sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(b*sec(d*x + c)), x)
```

3.107 $\int \frac{\cos(c+dx)}{\sqrt{b\sec(c+dx)}}\,dx$

Optimal. Leaf size=69

$$\frac{2\sqrt{\cos(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{b\sec(c+dx)}}{3bd}+\frac{2\sin(c+dx)}{3d\sqrt{b\sec(c+dx)}}$$

```
[Out] (2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[b*Sec[c + d*x]])/(3*b*
d) + (2*Sin[c + d*x])/(3*d*Sqrt[b*Sec[c + d*x]])
```

Rubi [A] time = 0.0434153, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {16, 3769, 3771, 2641}

$$\frac{2\sin(c+dx)}{3d\sqrt{b\sec(c+dx)}}+\frac{2\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\sec(c+dx)}}{3bd}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]/Sqrt[b*Sec[c + d*x]],x]
```

```
[Out] (2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[b*Sec[c + d*x]])/(3*b*
d) + (2*Sin[c + d*x])/(3*d*Sqrt[b*Sec[c + d*x]])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cos(c+dx)}{\sqrt{b\sec(c+dx)}}\,dx &= b\int \frac{1}{(b\sec(c+dx))^{3/2}}\,dx \\
&= \frac{2\sin(c+dx)}{3d\sqrt{b\sec(c+dx)}} + \frac{\int \sqrt{b\sec(c+dx)}\,dx}{3b} \\
&= \frac{2\sin(c+dx)}{3d\sqrt{b\sec(c+dx)}} + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx}{3b} \\
&= \frac{2\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\sec(c+dx)}}{3bd} + \frac{2\sin(c+dx)}{3d\sqrt{b\sec(c+dx)}}
\end{aligned}
$$

Mathematica [A] time = 0.0675756, size = 60, normalized size = 0.87

$$
\frac{b\sec^2(c+dx)\left(2\sqrt{\cos(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+\sin(2(c+dx))\right)}{3d(b\sec(c+dx))^{3/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]/Sqrt[b*Sec[c + d*x]],x]
```

```
[Out] (b*Sec[c + d*x]^2*(2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + Sin[2*(
c + d*x)]))/(3*d*(b*Sec[c + d*x])^(3/2))
```

Maple [C] time = 0.16, size = 126, normalized size = 1.8

$$
-\frac{(-2+2\cos(dx+c))(\cos(dx+c)+1)^2}{3db(\sin(dx+c))^3}\left(i\text{EllipticF}\left(\frac{i(-1+\cos(dx+c))}{\sin(dx+c)},i\right)\sqrt{(\cos(dx+c)+1)^{-1}}\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+}}\right.
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)/(b*sec(d*x+c))^(1/2),x)
```

```
[Out] -2/3/d*(-1+cos(d*x+c))*(I*EllipticF(I*(-1+cos(d*x+c))/sin(d*x+c),I)*(1/(cos
(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-cos(d*x+c)^2
+cos(d*x+c))*(cos(d*x+c)+1)^2*(b/cos(d*x+c))^(1/2)/b/sin(d*x+c)^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$
\int \frac{\cos(dx+c)}{\sqrt{b\sec(dx+c)}}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)/sqrt(b*sec(d*x + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{b\sec\left(dx+c\right)}\cos\left(dx+c\right)}{b\sec\left(dx+c\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(d*x + c))*cos(d*x + c)/(b*sec(d*x + c)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\cos\left(c+dx\right)}{\sqrt{b\sec\left(c+dx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(b*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral(cos(c + d*x)/sqrt(b*sec(c + d*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\cos\left(dx+c\right)}{\sqrt{b\sec\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)/sqrt(b*sec(d*x + c)), x)
```

3.108 $\int \frac{\cos^2(c+dx)}{\sqrt{b\sec(c+dx)}}\,dx$

Optimal. Leaf size=67

$$\frac{2b\sin(c+dx)}{5d(b\sec(c+dx))^{3/2}}+\frac{6E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}$$

```
[Out] (6*EllipticE[(c + d*x)/2, 2])/(5*d*Sqrt[Cos[c + d*x]]*Sqrt[b*Sec[c + d*x]])
 + (2*b*Sin[c + d*x])/(5*d*(b*Sec[c + d*x])^(3/2))
```

Rubi [A] time = 0.0507569, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {16, 3769, 3771, 2639}

$$\frac{2b\sin(c+dx)}{5d(b\sec(c+dx))^{3/2}}+\frac{6E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2/Sqrt[b*Sec[c + d*x]],x]
```

```
[Out] (6*EllipticE[(c + d*x)/2, 2])/(5*d*Sqrt[Cos[c + d*x]]*Sqrt[b*Sec[c + d*x]])
 + (2*b*Sin[c + d*x])/(5*d*(b*Sec[c + d*x])^(3/2))
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^2(c+dx)}{\sqrt{b\sec(c+dx)}}\,dx &= b^2\int \frac{1}{(b\sec(c+dx))^{5/2}}\,dx\\ &= \frac{2b\sin(c+dx)}{5d(b\sec(c+dx))^{3/2}}+\frac{3}{5}\int \frac{1}{\sqrt{b\sec(c+dx)}}\,dx\\ &= \frac{2b\sin(c+dx)}{5d(b\sec(c+dx))^{3/2}}+\frac{3\int \sqrt{\cos(c+dx)}\,dx}{5\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}\\ &= \frac{6E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}+\frac{2b\sin(c+dx)}{5d(b\sec(c+dx))^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.0817869, size = 60, normalized size = 0.9

$$\frac{\sqrt{b\sec(c+dx)}\left(\sin(c+dx)+\sin(3(c+dx))+12\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\right)}{10bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2/Sqrt[b*Sec[c + d*x]],x]
```

```
[Out] (Sqrt[b*Sec[c + d*x]]*(12*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + Si
n[c + d*x] + Sin[3*(c + d*x)]))/(10*b*d)
```

Maple [C] time = 0.215, size = 316, normalized size = 4.7

$$-\frac{2}{5\,d\sin(dx+c)\,b}\left(3\,i\cos(dx+c)\sin(dx+c)\sqrt{(\cos(dx+c)+1)^{-1}}\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\mathrm{EllipticE}\left(\frac{i(-1+\cos(dx+c))}{\sin(dx+c)},i\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2/(b*sec(d*x+c))^(1/2),x)
```

```
[Out] -2/5/d*(3*I*cos(d*x+c)*sin(d*x+c)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos
(d*x+c)+1))^(1/2)*EllipticE(I*(-1+cos(d*x+c))/sin(d*x+c),I)-3*I*(1/(cos(d*x
+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(-1+cos(d*x+c))
/sin(d*x+c),I)*sin(d*x+c)*cos(d*x+c)+3*I*sin(d*x+c)*(1/(cos(d*x+c)+1))^(1/2
)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticE(I*(-1+cos(d*x+c))/sin(d*x+c),
I)-3*I*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticF
(I*(-1+cos(d*x+c))/sin(d*x+c),I)*sin(d*x+c)+cos(d*x+c)^4+2*cos(d*x+c)^2-3*c
os(d*x+c))*(b/cos(d*x+c))^(1/2)/sin(d*x+c)/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)^2}{\sqrt{b\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)^2/sqrt(b*sec(d*x + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{b\sec\left(dx+c\right)}\cos\left(dx+c\right)^{2}}{b\sec\left(dx+c\right)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(d*x + c))*cos(d*x + c)^2/(b*sec(d*x + c)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos^{2}\left(c+dx\right)}{\sqrt{b\sec\left(c+dx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2/(b*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral(cos(c + d*x)**2/sqrt(b*sec(c + d*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos\left(dx+c\right)^{2}}{\sqrt{b\sec\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^2/sqrt(b*sec(d*x + c)), x)
```

3.109 $\int \frac{\cos^3(c+dx)}{\sqrt{b\sec(c+dx)}}\,dx$

Optimal. Leaf size=97

$$\frac{10\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{b\sec(c+dx)}}{21bd}+\frac{2b^2\sin(c+dx)}{7d(b\sec(c+dx))^{5/2}}+\frac{10\sin(c+dx)}{21d\sqrt{b\sec(c+dx)}}$$

```
[Out] (10*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[b*Sec[c + d*x]])/(21*
b*d) + (2*b^2*Sin[c + d*x])/(7*d*(b*Sec[c + d*x])^(5/2)) + (10*Sin[c + d*x]
)/(21*d*Sqrt[b*Sec[c + d*x]])
```

Rubi [A] time = 0.0690577, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {16, 3769, 3771, 2641}

$$\frac{2b^2\sin(c+dx)}{7d(b\sec(c+dx))^{5/2}}+\frac{10\sin(c+dx)}{21d\sqrt{b\sec(c+dx)}}+\frac{10\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\sec(c+dx)}}{21bd}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3/Sqrt[b*Sec[c + d*x]],x]
```


```
[Out] (10*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[b*Sec[c + d*x]])/(21*
b*d) + (2*b^2*Sin[c + d*x])/(7*d*(b*Sec[c + d*x])^(5/2)) + (10*Sin[c + d*x]
)/(21*d*Sqrt[b*Sec[c + d*x]])
```


Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cos^3(c+dx)}{\sqrt{b\sec(c+dx)}}\,dx &= b^3\int \frac{1}{(b\sec(c+dx))^{7/2}}\,dx \\
&= \frac{2b^2\sin(c+dx)}{7d(b\sec(c+dx))^{5/2}} + \frac{1}{7}(5b)\int \frac{1}{(b\sec(c+dx))^{3/2}}\,dx \\
&= \frac{2b^2\sin(c+dx)}{7d(b\sec(c+dx))^{5/2}} + \frac{10\sin(c+dx)}{21d\sqrt{b\sec(c+dx)}} + \frac{5\int \sqrt{b\sec(c+dx)}\,dx}{21b} \\
&= \frac{2b^2\sin(c+dx)}{7d(b\sec(c+dx))^{5/2}} + \frac{10\sin(c+dx)}{21d\sqrt{b\sec(c+dx)}} + \frac{\left(5\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx}{21b} \\
&= \frac{10\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\sec(c+dx)}}{21bd} + \frac{2b^2\sin(c+dx)}{7d(b\sec(c+dx))^{5/2}} + \frac{10\sin(c+dx)}{21d\sqrt{b\sec(c+dx)}}
\end{aligned}
$$

Mathematica [A] time = 0.0748665, size = 66, normalized size = 0.68

$$
\frac{\sqrt{b\sec(c+dx)}\left(40\sqrt{\cos(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+26\sin(2(c+dx))+3\sin(4(c+dx))\right)}{84bd}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3/Sqrt[b*Sec[c + d*x]],x]
```

```
[Out] (Sqrt[b*Sec[c + d*x]]*(40*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + 26
*Sin[2*(c + d*x)] + 3*Sin[4*(c + d*x)]))/(84*b*d)
```

Maple [C] time = 0.204, size = 148, normalized size = 1.5

$$
-\frac{2\left(\cos(dx+c)+1\right)^2\left(-1+\cos(dx+c)\right)}{21\,db\left(\sin(dx+c)\right)^3}\left(5\,i\text{EllipticF}\left(\frac{i\left(-1+\cos(dx+c)\right)}{\sin(dx+c)},i\right)\sqrt{\left(\cos(dx+c)+1\right)^{-1}}\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)}}\right.
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3/(b*sec(d*x+c))^(1/2),x)
```

```
[Out] -2/21/d*(cos(d*x+c)+1)^2*(-1+cos(d*x+c))*(5*I*EllipticF(I*(-1+cos(d*x+c))/s
in(d*x+c),I)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin
(d*x+c)-3*cos(d*x+c)^4+3*cos(d*x+c)^3-5*cos(d*x+c)^2+5*cos(d*x+c))*(b/cos(d
*x+c))^(1/2)/b/sin(d*x+c)^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$
\int \frac{\cos(dx+c)^3}{\sqrt{b\sec(dx+c)}}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)^3/sqrt(b*sec(d*x + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{b\sec\left(dx+c\right)}\cos\left(dx+c\right)^{3}}{b\sec\left(dx+c\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(b*sec(d*x+c))^(1/2),x, algorithm="fricas")

[Out] integral(sqrt(b*sec(d*x + c))*cos(d*x + c)^3/(b*sec(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3/(b*sec(d*x+c))**(1/2),x)

[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\cos\left(dx+c\right)^{3}}{\sqrt{b\sec\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(b*sec(d*x+c))^(1/2),x, algorithm="giac")

[Out] integrate(cos(d*x + c)^3/sqrt(b*sec(d*x + c)), x)
```

3.110 $\int \frac{\cos^4(c+dx)}{\sqrt{b\sec(c+dx)}}\,dx$

Optimal. Leaf size=95

$$\frac{2b^3\sin(c+dx)}{9d(b\sec(c+dx))^{7/2}}+\frac{14b\sin(c+dx)}{45d(b\sec(c+dx))^{3/2}}+\frac{14E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}$$

```
[Out] (14*EllipticE[(c + d*x)/2, 2])/(15*d*Sqrt[Cos[c + d*x]]*Sqrt[b*Sec[c + d*x]
]) + (2*b^3*Sin[c + d*x])/(9*d*(b*Sec[c + d*x])^(7/2)) + (14*b*Sin[c + d*x]
)/(45*d*(b*Sec[c + d*x])^(3/2))
```

Rubi [A] time = 0.069369, antiderivative size = 95, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {16, 3769, 3771, 2639}

$$\frac{2b^3\sin(c+dx)}{9d(b\sec(c+dx))^{7/2}}+\frac{14b\sin(c+dx)}{45d(b\sec(c+dx))^{3/2}}+\frac{14E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4/Sqrt[b*Sec[c + d*x]],x]
```

```
[Out] (14*EllipticE[(c + d*x)/2, 2])/(15*d*Sqrt[Cos[c + d*x]]*Sqrt[b*Sec[c + d*x]
]) + (2*b^3*Sin[c + d*x])/(9*d*(b*Sec[c + d*x])^(7/2)) + (14*b*Sin[c + d*x]
)/(45*d*(b*Sec[c + d*x])^(3/2))
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cos^4(c+dx)}{\sqrt{b\sec(c+dx)}}\,dx &= b^4\int \frac{1}{(b\sec(c+dx))^{9/2}}\,dx \\
&= \frac{2b^3\sin(c+dx)}{9d(b\sec(c+dx))^{7/2}} + \frac{1}{9}\left(7b^2\right)\int \frac{1}{(b\sec(c+dx))^{5/2}}\,dx \\
&= \frac{2b^3\sin(c+dx)}{9d(b\sec(c+dx))^{7/2}} + \frac{14b\sin(c+dx)}{45d(b\sec(c+dx))^{3/2}} + \frac{7}{15}\int \frac{1}{\sqrt{b\sec(c+dx)}}\,dx \\
&= \frac{2b^3\sin(c+dx)}{9d(b\sec(c+dx))^{7/2}} + \frac{14b\sin(c+dx)}{45d(b\sec(c+dx))^{3/2}} + \frac{7\int \sqrt{\cos(c+dx)}\,dx}{15\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}} \\
&= \frac{14E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}} + \frac{2b^3\sin(c+dx)}{9d(b\sec(c+dx))^{7/2}} + \frac{14b\sin(c+dx)}{45d(b\sec(c+dx))^{3/2}}
\end{aligned}
$$

Mathematica [A] time = 0.189403, size = 70, normalized size = 0.74

$$
\frac{4(33\sin(c+dx)+5\sin(3(c+dx)))\cos(c+dx)+\frac{336E\left(\frac{1}{2}(c+dx)\middle|2\right)}{\sqrt{\cos(c+dx)}}}{360d\sqrt{b\sec(c+dx)}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4/Sqrt[b*Sec[c + d*x]],x]
```

```
[Out] ((336*EllipticE[(c + d*x)/2, 2])/Sqrt[Cos[c + d*x]] + 4*Cos[c + d*x]*(33*Si
n[c + d*x] + 5*Sin[3*(c + d*x)]))/(360*d*Sqrt[b*Sec[c + d*x]])
```

Maple [C] time = 0.202, size = 328, normalized size = 3.5

$$
\frac{2}{45\,d\sin(dx+c)\,b}\left(21\,i\sqrt{(\cos(dx+c)+1)^{-1}}\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\mathrm{EllipticF}\left(\frac{i(-1+\cos(dx+c))}{\sin(dx+c)},i\right)\sin(dx+c)\cos(dx+
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4/(b*sec(d*x+c))^(1/2),x)
```

```
[Out] 2/45/d*(21*I*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*Ell
ipticF(I*(-1+cos(d*x+c))/sin(d*x+c),I)*sin(d*x+c)*cos(d*x+c)-21*I*cos(d*x+c
)*sin(d*x+c)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*Ell
ipticE(I*(-1+cos(d*x+c))/sin(d*x+c),I)-5*cos(d*x+c)^6+21*I*(1/(cos(d*x+c)+1
))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(-1+cos(d*x+c))/sin(
d*x+c),I)*sin(d*x+c)-21*I*sin(d*x+c)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(
cos(d*x+c)+1))^(1/2)*EllipticE(I*(-1+cos(d*x+c))/sin(d*x+c),I)-2*cos(d*x+c)
^4-14*cos(d*x+c)^2+21*cos(d*x+c))*(b/cos(d*x+c))^(1/2)/sin(d*x+c)/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$
\int \frac{\cos(dx+c)^4}{\sqrt{b\sec(dx+c)}}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(b*sec(d*x+c))^(1/2),x, algorithm="maxima")

[Out] integrate(cos(d*x + c)^4/sqrt(b*sec(d*x + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{b\sec\left(dx+c\right)}\cos\left(dx+c\right)^{4}}{b\sec\left(dx+c\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(b*sec(d*x+c))^(1/2),x, algorithm="fricas")

[Out] integral(sqrt(b*sec(d*x + c))*cos(d*x + c)^4/(b*sec(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4/(b*sec(d*x+c))**(1/2),x)

[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\cos\left(dx+c\right)^{4}}{\sqrt{b\sec\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(b*sec(d*x+c))^(1/2),x, algorithm="giac")

[Out] integrate(cos(d*x + c)^4/sqrt(b*sec(d*x + c)), x)
```

3.111 $\int \frac{\sec^6(c+dx)}{(b\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=100

$$\frac{10\sqrt{\cos(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{b\sec(c+dx)}}{21b^2d}+\frac{2\sin(c+dx)(b\sec(c+dx))^{7/2}}{7b^5d}+\frac{10\sin(c+dx)(b\sec(c+dx))^{3/2}}{21b^3d}$$

```
[Out] (10*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[b*Sec[c + d*x]])/(21*
b^2*d) + (10*(b*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(21*b^3*d) + (2*(b*Sec[c
+ d*x])^(7/2)*Sin[c + d*x])/(7*b^5*d)
```

Rubi [A] time = 0.0586986, antiderivative size = 100, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {16, 3768, 3771, 2641}

$$\frac{2\sin(c+dx)(b\sec(c+dx))^{7/2}}{7b^5d}+\frac{10\sin(c+dx)(b\sec(c+dx))^{3/2}}{21b^3d}+\frac{10\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\sec(c+dx)}}{21b^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^6/(b*Sec[c + d*x])^(3/2),x]
```

```
[Out] (10*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[b*Sec[c + d*x]])/(21*
b^2*d) + (10*(b*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(21*b^3*d) + (2*(b*Sec[c
+ d*x])^(7/2)*Sin[c + d*x])/(7*b^5*d)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sec^6(c+dx)}{(b\sec(c+dx))^{3/2}}\,dx = \frac{\int (b\sec(c+dx))^{9/2}\,dx}{b^6}$$
$$= \frac{2(b\sec(c+dx))^{7/2}\sin(c+dx)}{7b^5d} + \frac{5\int (b\sec(c+dx))^{5/2}\,dx}{7b^4}$$
$$= \frac{10(b\sec(c+dx))^{3/2}\sin(c+dx)}{21b^3d} + \frac{2(b\sec(c+dx))^{7/2}\sin(c+dx)}{7b^5d} + \frac{5\int \sqrt{b\sec(c+dx)}\,dx}{21b^2}$$
$$= \frac{10(b\sec(c+dx))^{3/2}\sin(c+dx)}{21b^3d} + \frac{2(b\sec(c+dx))^{7/2}\sin(c+dx)}{7b^5d} + \frac{\left(5\sqrt{\cos(c+dx)}\sqrt{b\sec(c\right.}{21l}$$
$$= \frac{10\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\sec(c+dx)}}{21b^2d} + \frac{10(b\sec(c+dx))^{3/2}\sin(c+dx)}{21b^3d} + 2(b\sec$$

Mathematica [A] time = 0.166195, size = 69, normalized size = 0.69

$$\frac{\sec^4(c+dx)\left(10\cos^{\frac{5}{2}}(c+dx)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+5\sin(2(c+dx))+6\tan(c+dx)\right)}{21d(b\sec(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^6/(b*Sec[c + d*x])^(3/2),x]
```

```
[Out] (Sec[c + d*x]^4*(10*Cos[c + d*x]^(5/2)*EllipticF[(c + d*x)/2, 2] + 5*Sin[2*
(c + d*x)] + 6*Tan[c + d*x]))/(21*d*(b*Sec[c + d*x])^(3/2))
```

Maple [C] time = 0.219, size = 152, normalized size = 1.5

$$-\frac{2\,(\cos(dx+c)+1)^2\,(-1+\cos(dx+c))}{21\,d\,(\cos(dx+c))^5\,(\sin(dx+c))^3}\left(5\,i\,(\cos(dx+c))^3\sin(dx+c)\sqrt{(\cos(dx+c)+1)^{-1}}\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\text{Elli}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^6/(b*sec(d*x+c))^(3/2),x)
```

```
[Out] -2/21/d*(cos(d*x+c)+1)^2*(-1+cos(d*x+c))*(5*I*cos(d*x+c)^3*sin(d*x+c)*(1/(c
os(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(-1+cos(d
*x+c))/sin(d*x+c),I)-5*cos(d*x+c)^3+5*cos(d*x+c)^2-3*cos(d*x+c)+3)/cos(d*x+
c)^5/(b/cos(d*x+c))^(3/2)/sin(d*x+c)^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)^6}{(b\sec(dx+c))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6/(b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)^6/(b*sec(d*x + c))^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{b\sec\left(dx+c\right)}\sec\left(dx+c\right)^{4}}{b^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6/(b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(d*x + c))*sec(d*x + c)^4/b^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec^{6}\left(c+dx\right)}{\left(b\sec\left(c+dx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**6/(b*sec(d*x+c))**(3/2),x)
```

```
[Out] Integral(sec(c + d*x)**6/(b*sec(c + d*x))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec\left(dx+c\right)^{6}}{\left(b\sec\left(dx+c\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6/(b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^6/(b*sec(d*x + c))^(3/2), x)
```

3.112 $\int \frac{\sec^5(c+dx)}{(b\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=100

$$\frac{2\sin(c+dx)(b\sec(c+dx))^{5/2}}{5b^4d}+\frac{6\sin(c+dx)\sqrt{b\sec(c+dx)}}{5b^2d}-\frac{6E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5bd\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}$$

```
[Out] (-6*EllipticE[(c + d*x)/2, 2])/(5*b*d*Sqrt[Cos[c + d*x]]*Sqrt[b*Sec[c + d*x
]]) + (6*Sqrt[b*Sec[c + d*x]]*Sin[c + d*x])/(5*b^2*d) + (2*(b*Sec[c + d*x])
^(5/2)*Sin[c + d*x])/(5*b^4*d)
```

Rubi [A] time = 0.0576487, antiderivative size = 100, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {16, 3768, 3771, 2639}

$$\frac{2\sin(c+dx)(b\sec(c+dx))^{5/2}}{5b^4d}+\frac{6\sin(c+dx)\sqrt{b\sec(c+dx)}}{5b^2d}-\frac{6E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5bd\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^5/(b*Sec[c + d*x])^(3/2),x]
```


```
[Out] (-6*EllipticE[(c + d*x)/2, 2])/(5*b*d*Sqrt[Cos[c + d*x]]*Sqrt[b*Sec[c + d*x
]]) + (6*Sqrt[b*Sec[c + d*x]]*Sin[c + d*x])/(5*b^2*d) + (2*(b*Sec[c + d*x])
^(5/2)*Sin[c + d*x])/(5*b^4*d)
```


Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sec^5(c+dx)}{(b\sec(c+dx))^{3/2}}\,dx = \frac{\int (b\sec(c+dx))^{7/2}\,dx}{b^5}$$
$$= \frac{2(b\sec(c+dx))^{5/2}\sin(c+dx)}{5b^4d} + \frac{3\int (b\sec(c+dx))^{3/2}\,dx}{5b^3}$$
$$= \frac{6\sqrt{b\sec(c+dx)}\sin(c+dx)}{5b^2d} + \frac{2(b\sec(c+dx))^{5/2}\sin(c+dx)}{5b^4d} - \frac{3\int \frac{1}{\sqrt{b\sec(c+dx)}}\,dx}{5b}$$
$$= \frac{6\sqrt{b\sec(c+dx)}\sin(c+dx)}{5b^2d} + \frac{2(b\sec(c+dx))^{5/2}\sin(c+dx)}{5b^4d} - \frac{3\int \sqrt{\cos(c+dx)}\,dx}{5b\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}$$
$$= -\frac{6E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5bd\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}} + \frac{6\sqrt{b\sec(c+dx)}\sin(c+dx)}{5b^2d} + \frac{2(b\sec(c+dx))^{5/2}\sin(c+dx)}{5b^4d}$$

Mathematica [A] time = 0.0534337, size = 64, normalized size = 0.64

$$\frac{2\tan(c+dx)\left(\sec^2(c+dx)+3\right) - \frac{6E\left(\frac{1}{2}(c+dx)\middle|2\right)}{\sqrt{\cos(c+dx)}}}{5bd\sqrt{b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^5/(b*Sec[c + d*x])^(3/2),x]
```

```
[Out] ((-6*EllipticE[(c + d*x)/2, 2])/Sqrt[Cos[c + d*x]] + 2*(3 + Sec[c + d*x]^2)
*Tan[c + d*x])/(5*b*d*Sqrt[b*Sec[c + d*x]])
```

Maple [C] time = 0.223, size = 356, normalized size = 3.6

$$\frac{2\,(\cos(dx+c)+1)^2\,(-1+\cos(dx+c))^2}{5\,d\,(\sin(dx+c))^5\,(\cos(dx+c))^4}\left(3\,i\sqrt{(\cos(dx+c)+1)^{-1}}\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,(\cos(dx+c))^3\,\mathrm{EllipticE}\left(\frac{i(-1+c}{\sin}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5/(b*sec(d*x+c))^(3/2),x)
```

```
[Out] 2/5/d*(cos(d*x+c)+1)^2*(-1+cos(d*x+c))^2*(3*I*(1/(cos(d*x+c)+1))^(1/2)*(cos
(d*x+c)/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)^3*EllipticE(I*(-1+cos(d*x+c))/sin(
d*x+c),I)*sin(d*x+c)-3*I*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1
))^(1/2)*EllipticF(I*(-1+cos(d*x+c))/sin(d*x+c),I)*cos(d*x+c)^3*sin(d*x+c)+
3*I*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)^2
*EllipticE(I*(-1+cos(d*x+c))/sin(d*x+c),I)*sin(d*x+c)-3*I*(1/(cos(d*x+c)+1)
)^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(-1+cos(d*x+c))/sin(d
*x+c),I)*cos(d*x+c)^2*sin(d*x+c)-3*cos(d*x+c)^3+2*cos(d*x+c)^2+1)/sin(d*x+c
)^5/cos(d*x+c)^4/(b/cos(d*x+c))^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)^5}{(b\sec(dx+c))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)^5/(b*sec(d*x + c))^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{b\sec(dx+c)}\sec(dx+c)^3}{b^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(d*x + c))*sec(d*x + c)^3/b^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec^5(c+dx)}{(b\sec(c+dx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5/(b*sec(d*x+c))**(3/2),x)
```

```
[Out] Integral(sec(c + d*x)**5/(b*sec(c + d*x))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)^5}{(b\sec(dx+c))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^5/(b*sec(d*x + c))^(3/2), x)
```

3.113 $\int \frac{\sec^4(c+dx)}{(b\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=72

$$\frac{2\sqrt{\cos(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{b\sec(c+dx)}}{3b^2d}+\frac{2\sin(c+dx)(b\sec(c+dx))^{3/2}}{3b^3d}$$

```
[Out] (2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[b*Sec[c + d*x]])/(3*b^
2*d) + (2*(b*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(3*b^3*d)
```

Rubi [A] time = 0.0377128, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {16, 3768, 3771, 2641}

$$\frac{2\sin(c+dx)(b\sec(c+dx))^{3/2}}{3b^3d}+\frac{2\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\sec(c+dx)}}{3b^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4/(b*Sec[c + d*x])^(3/2),x]
```



```
[Out] (2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[b*Sec[c + d*x]])/(3*b^
2*d) + (2*(b*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(3*b^3*d)
```


Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sec^4(c+dx)}{(b\sec(c+dx))^{3/2}}\,dx &= \frac{\int (b\sec(c+dx))^{5/2}\,dx}{b^4} \\
&= \frac{2(b\sec(c+dx))^{3/2}\sin(c+dx)}{3b^3d} + \frac{\int \sqrt{b\sec(c+dx)}\,dx}{3b^2} \\
&= \frac{2(b\sec(c+dx))^{3/2}\sin(c+dx)}{3b^3d} + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx}{3b^2} \\
&= \frac{2\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\sec(c+dx)}}{3b^2d} + \frac{2(b\sec(c+dx))^{3/2}\sin(c+dx)}{3b^3d}
\end{aligned}
$$

Mathematica [A] time = 0.0968243, size = 56, normalized size = 0.78

$$
\frac{2\sec^3(c+dx)\left(\cos^{\frac{3}{2}}(c+dx)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+\sin(c+dx)\right)}{3d(b\sec(c+dx))^{3/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4/(b*Sec[c + d*x])^(3/2),x]
```

```
[Out] (2*Sec[c + d*x]^3*(Cos[c + d*x]^(3/2)*EllipticF[(c + d*x)/2, 2] + Sin[c + d
*x]))/(3*d*(b*Sec[c + d*x])^(3/2))
```

Maple [C] time = 0.17, size = 125, normalized size = 1.7

$$
-\frac{(-2+2\cos(dx+c))(\cos(dx+c)+1)^2}{3\,db^3(\sin(dx+c))^3}\left(i\sqrt{(\cos(dx+c)+1)^{-1}}\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\text{EllipticF}\left(\frac{i(-1+\cos(dx+c))}{\sin(dx+c)},\right.\right.
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4/(b*sec(d*x+c))^(3/2),x)
```

```
[Out] -2/3/d*(-1+cos(d*x+c))*(I*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+
1))^(1/2)*EllipticF(I*(-1+cos(d*x+c))/sin(d*x+c),I)*cos(d*x+c)*sin(d*x+c)-c
os(d*x+c)+1)*(cos(d*x+c)+1)^2*(b/cos(d*x+c))^(3/2)/b^3/sin(d*x+c)^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$
\int \frac{\sec(dx+c)^4}{(b\sec(dx+c))^{\frac{3}{2}}}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)^4/(b*sec(d*x + c))^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{b\sec\left(dx+c\right)}\sec\left(dx+c\right)^{2}}{b^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(d*x + c))*sec(d*x + c)^2/b^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sec^{4}\left(c+dx\right)}{\left(b\sec\left(c+dx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4/(b*sec(d*x+c))**(3/2),x)
```

```
[Out] Integral(sec(c + d*x)**4/(b*sec(c + d*x))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sec\left(dx+c\right)^{4}}{\left(b\sec\left(dx+c\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^4/(b*sec(d*x + c))^(3/2), x)
```

3.114 $\int \frac{\sec^3(c+dx)}{(b\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=68

$$\frac{2\sin(c+dx)\sqrt{b\sec(c+dx)}}{b^2d}-\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}$$

```
[Out] (-2*EllipticE[(c + d*x)/2, 2])/(b*d*Sqrt[Cos[c + d*x]]*Sqrt[b*Sec[c + d*x]]
) + (2*Sqrt[b*Sec[c + d*x]]*Sin[c + d*x])/(b^2*d)
```

Rubi [A] time = 0.0395441, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {16, 3768, 3771, 2639}

$$\frac{2\sin(c+dx)\sqrt{b\sec(c+dx)}}{b^2d}-\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3/(b*Sec[c + d*x])^(3/2),x]
```

```
[Out] (-2*EllipticE[(c + d*x)/2, 2])/(b*d*Sqrt[Cos[c + d*x]]*Sqrt[b*Sec[c + d*x]]
) + (2*Sqrt[b*Sec[c + d*x]]*Sin[c + d*x])/(b^2*d)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sec^3(c+dx)}{(b\sec(c+dx))^{3/2}}\,dx = \frac{\int (b\sec(c+dx))^{3/2}\,dx}{b^3}$$
$$= \frac{2\sqrt{b\sec(c+dx)}\sin(c+dx)}{b^2d} - \frac{\int \frac{1}{\sqrt{b\sec(c+dx)}}\,dx}{b}$$
$$= \frac{2\sqrt{b\sec(c+dx)}\sin(c+dx)}{b^2d} - \frac{\int \sqrt{\cos(c+dx)}\,dx}{b\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}$$
$$= -\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}} + \frac{2\sqrt{b\sec(c+dx)}\sin(c+dx)}{b^2d}$$

Mathematica [A] time = 0.0570341, size = 51, normalized size = 0.75

$$\frac{2\tan(c+dx) - \frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{\sqrt{\cos(c+dx)}}}{bd\sqrt{b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3/(b*Sec[c + d*x])^(3/2),x]
```

```
[Out] ((-2*EllipticE[(c + d*x)/2, 2])/Sqrt[Cos[c + d*x]] + 2*Tan[c + d*x])/(b*d*S
qrt[b*Sec[c + d*x]])
```

Maple [C] time = 0.224, size = 322, normalized size = 4.7

$$-2\,\frac{(\cos(dx+c)+1)^2(-1+\cos(dx+c))^2}{d(\cos(dx+c))^2(\sin(dx+c))^5}\left(i\sqrt{(\cos(dx+c)+1)^{-1}}\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\mathrm{EllipticF}\left(\frac{i(-1+\cos(dx+c))}{\sin(dx+c)},i\right)c\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3/(b*sec(d*x+c))^(3/2),x)
```

```
[Out] -2/d*(cos(d*x+c)+1)^2*(-1+cos(d*x+c))^2*(I*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*
x+c)/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(-1+cos(d*x+c))/sin(d*x+c),I)*cos(d*
x+c)*sin(d*x+c)-I*EllipticE(I*(-1+cos(d*x+c))/sin(d*x+c),I)*cos(d*x+c)*sin(
d*x+c)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+I*Ellipti
cF(I*(-1+cos(d*x+c))/sin(d*x+c),I)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(co
s(d*x+c)+1))^(1/2)*sin(d*x+c)-I*EllipticE(I*(-1+cos(d*x+c))/sin(d*x+c),I)*s
in(d*x+c)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+cos(d*
x+c)-1)/cos(d*x+c)^2/sin(d*x+c)^5/(b/cos(d*x+c))^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)^3}{(b\sec(dx+c))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)^3/(b*sec(d*x + c))^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{b\sec\left(dx+c\right)}\sec\left(dx+c\right)}{b^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(d*x + c))*sec(d*x + c)/b^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sec^3\left(c+dx\right)}{\left(b\sec\left(c+dx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3/(b*sec(d*x+c))**(3/2),x)
```

```
[Out] Integral(sec(c + d*x)**3/(b*sec(c + d*x))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sec\left(dx+c\right)^3}{\left(b\sec\left(dx+c\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^3/(b*sec(d*x + c))^(3/2), x)
```

3.115 $\int \frac{\sec^2(c+dx)}{(b\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=41

$$\frac{2\sqrt{\cos(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{b\sec(c+dx)}}{b^2d}$$

```
[Out] (2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[b*Sec[c + d*x]])/(b^2*
d)
```

Rubi [A] time = 0.023263, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {16, 3771, 2641}

$$\frac{2\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\sec(c+dx)}}{b^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2/(b*Sec[c + d*x])^(3/2),x]
```

```
[Out] (2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[b*Sec[c + d*x]])/(b^2*
d)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec^2(c+dx)}{(b\sec(c+dx))^{3/2}}\,dx &= \frac{\int \sqrt{b\sec(c+dx)}\,dx}{b^2}\\ &= \frac{\left(\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx}{b^2}\\ &= \frac{2\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\sec(c+dx)}}{b^2d}\end{aligned}$$

Mathematica [A] time = 0.0216751, size = 41, normalized size = 1.

$$\frac{2\sqrt{\cos(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{b\sec(c+dx)}}{b^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2/(b*Sec[c + d*x])^(3/2),x]
```

```
[Out] (2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[b*Sec[c + d*x]])/(b^2*
d)
```

Maple [C] time = 0.135, size = 98, normalized size = 2.4

$$\frac{-2\,i\,(-1+\cos(dx+c))\,(\cos(dx+c)+1)^2}{d\,(\sin(dx+c))^2}\left((\cos(dx+c)+1)^{-1}\right)^{\frac{5}{2}}\text{EllipticF}\left(\frac{i\,(-1+\cos(dx+c))}{\sin(dx+c)},i\right)\left(\frac{b}{\cos(dx+c)}\right)^{-}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2/(b*sec(d*x+c))^(3/2),x)
```

```
[Out] -2*I/d*(1/(cos(d*x+c)+1))^(5/2)*(-1+cos(d*x+c))*EllipticF(I*(-1+cos(d*x+c))
/sin(d*x+c),I)*(cos(d*x+c)+1)^2/(b/cos(d*x+c))^(3/2)/(cos(d*x+c)/(cos(d*x+c
)+1))^(3/2)/sin(d*x+c)^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)^2}{(b\sec(dx+c))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)^2/(b*sec(d*x + c))^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{b\sec(dx+c)}}{b^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(d*x + c))/b^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec^2(c+dx)}{(b\sec(c+dx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2/(b*sec(d*x+c))**(3/2),x)
```

```
[Out] Integral(sec(c + d*x)**2/(b*sec(c + d*x))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)^2}{(b\sec(dx+c))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^2/(b*sec(d*x + c))^(3/2), x)
```

3.116 $\int \frac{\sec(c+dx)}{(b\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=41

$$\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}$$

```
[Out] (2*EllipticE[(c + d*x)/2, 2])/(b*d*Sqrt[Cos[c + d*x]]*Sqrt[b*Sec[c + d*x]])
```

Rubi [A] time = 0.021754, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {16, 3771, 2639}

$$\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]/(b*Sec[c + d*x])^(3/2),x]

[Out] (2*EllipticE[(c + d*x)/2, 2])/(b*d*Sqrt[Cos[c + d*x]]*Sqrt[b*Sec[c + d*x]])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec(c+dx)}{(b\sec(c+dx))^{3/2}}\,dx &= \frac{\int \frac{1}{\sqrt{b\sec(c+dx)}}\,dx}{b}\\ &= \frac{\int \sqrt{\cos(c+dx)}\,dx}{b\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}\\ &= \frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}\end{aligned}$$

Mathematica [A] time = 0.0312372, size = 41, normalized size = 1.

$$\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]/(b*Sec[c + d*x])^(3/2),x]
```

```
[Out] (2*EllipticE[(c + d*x)/2, 2])/(b*d*Sqrt[Cos[c + d*x]]*Sqrt[b*Sec[c + d*x]])
```

Maple [C] time = 0.144, size = 311, normalized size = 7.6

$$2\frac{1}{d(\cos(dx+c))^2\sin(dx+c)}\left(i\sqrt{(\cos(dx+c)+1)^{-1}}\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\text{EllipticF}\left(\frac{i(-1+\cos(dx+c))}{\sin(dx+c)},i\right)\cos(dx+c)\text{s}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)/(b*sec(d*x+c))^(3/2),x)
```

```
[Out] 2/d*(I*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticF
(I*(-1+cos(d*x+c))/sin(d*x+c),I)*cos(d*x+c)*sin(d*x+c)-I*EllipticE(I*(-1+co
s(d*x+c))/sin(d*x+c),I)*cos(d*x+c)*sin(d*x+c)*(1/(cos(d*x+c)+1))^(1/2)*(cos
(d*x+c)/(cos(d*x+c)+1))^(1/2)+I*EllipticF(I*(-1+cos(d*x+c))/sin(d*x+c),I)*(
1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-I*Elli
pticE(I*(-1+cos(d*x+c))/sin(d*x+c),I)*sin(d*x+c)*(1/(cos(d*x+c)+1))^(1/2)*(
cos(d*x+c)/(cos(d*x+c)+1))^(1/2)-cos(d*x+c)^2+cos(d*x+c))/cos(d*x+c)^2/(b/c
os(d*x+c))^(3/2)/sin(d*x+c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)}{(b\sec(dx+c))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)/(b*sec(d*x + c))^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{b\sec(dx+c)}}{b^2\sec(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(d*x + c))/(b^2*sec(d*x + c)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(c+dx)}{(b\sec(c+dx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(b*sec(d*x+c))**(3/2),x)
```

```
[Out] Integral(sec(c + d*x)/(b*sec(c + d*x))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx + c)}{(b\sec(dx + c))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)/(b*sec(d*x + c))^(3/2), x)
```

3.117 $\int \frac{1}{(b\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=72

$$\frac{2\sqrt{\cos(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{b\sec(c+dx)}}{3b^2d}+\frac{2\sin(c+dx)}{3bd\sqrt{b\sec(c+dx)}}$$

```
[Out] (2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[b*Sec[c + d*x]])/(3*b^
2*d) + (2*Sin[c + d*x])/(3*b*d*Sqrt[b*Sec[c + d*x]])
```

Rubi [A] time = 0.0329195, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {3769, 3771, 2641}

$$\frac{2\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\sec(c+dx)}}{3b^2d}+\frac{2\sin(c+dx)}{3bd\sqrt{b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Sec[c + d*x])^(-3/2),x]
```

```
[Out] (2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[b*Sec[c + d*x]])/(3*b^
2*d) + (2*Sin[c + d*x])/(3*b*d*Sqrt[b*Sec[c + d*x]])
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(b\sec(c+dx))^{3/2}}\,dx &= \frac{2\sin(c+dx)}{3bd\sqrt{b\sec(c+dx)}}+\frac{\int \sqrt{b\sec(c+dx)}\,dx}{3b^2}\\ &= \frac{2\sin(c+dx)}{3bd\sqrt{b\sec(c+dx)}}+\frac{\left(\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx}{3b^2}\\ &= \frac{2\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\sec(c+dx)}}{3b^2d}+\frac{2\sin(c+dx)}{3bd\sqrt{b\sec(c+dx)}}\end{aligned}$$

Mathematica [A] time = 0.0492983, size = 59, normalized size = 0.82

$$\frac{\sec^2(c+dx)\left(2\sqrt{\cos(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+\sin(2(c+dx))\right)}{3d(b\sec(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Sec[c + d*x])^(-3/2),x]
```

```
[Out] (Sec[c + d*x]^2*(2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + Sin[2*(c
+ d*x)]))/(3*d*(b*Sec[c + d*x])^(3/2))
```

Maple [C] time = 0.155, size = 131, normalized size = 1.8

$$-\frac{2\,(\cos(dx+c)+1)^2\,(-1+\cos(dx+c))}{3\,d\,(\cos(dx+c))^2\,(\sin(dx+c))^3}\left(i\text{EllipticF}\left(\frac{i\,(-1+\cos(dx+c))}{\sin(dx+c)},i\right)\sqrt{(\cos(dx+c)+1)^{-1}}\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*sec(d*x+c))^(3/2),x)
```

```
[Out] -2/3/d*(cos(d*x+c)+1)^2*(-1+cos(d*x+c))*(I*EllipticF(I*(-1+cos(d*x+c))/sin(
d*x+c),I)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*
x+c)-cos(d*x+c)^2+cos(d*x+c))/cos(d*x+c)^2/sin(d*x+c)^3/(b/cos(d*x+c))^(3/2
)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b\sec(dx+c))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c))^(-3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{b\sec(dx+c)}}{b^2\sec(dx+c)^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(d*x + c))/(b^2*sec(d*x + c)^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b \sec \left(c + dx\right)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*sec(d*x+c))**(3/2),x)
```

```
[Out] Integral((b*sec(c + d*x))**(-3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b \sec \left(dx + c\right)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c))^(-3/2), x)
```

3.118 $\int \frac{\cos(c+dx)}{(b\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=69

$$\frac{2\sin(c+dx)}{5d(b\sec(c+dx))^{3/2}}+\frac{6E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5bd\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}$$

```
[Out] (6*EllipticE[(c + d*x)/2, 2])/(5*b*d*Sqrt[Cos[c + d*x]]*Sqrt[b*Sec[c + d*x]
]) + (2*Sin[c + d*x])/(5*d*(b*Sec[c + d*x])^(3/2))
```

Rubi [A] time = 0.0448313, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {16, 3769, 3771, 2639}

$$\frac{2\sin(c+dx)}{5d(b\sec(c+dx))^{3/2}}+\frac{6E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5bd\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]/(b*Sec[c + d*x])^(3/2),x]
```

```
[Out] (6*EllipticE[(c + d*x)/2, 2])/(5*b*d*Sqrt[Cos[c + d*x]]*Sqrt[b*Sec[c + d*x]
]) + (2*Sin[c + d*x])/(5*d*(b*Sec[c + d*x])^(3/2))
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cos(c+dx)}{(b\sec(c+dx))^{3/2}}\,dx &= b\int \frac{1}{(b\sec(c+dx))^{5/2}}\,dx \\
&= \frac{2\sin(c+dx)}{5d(b\sec(c+dx))^{3/2}} + \frac{3\int \frac{1}{\sqrt{b\sec(c+dx)}}\,dx}{5b} \\
&= \frac{2\sin(c+dx)}{5d(b\sec(c+dx))^{3/2}} + \frac{3\int \sqrt{\cos(c+dx)}\,dx}{5b\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}} \\
&= \frac{6E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5bd\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}} + \frac{2\sin(c+dx)}{5d(b\sec(c+dx))^{3/2}}
\end{aligned}
$$

Mathematica [A] time = 0.0322174, size = 60, normalized size = 0.87

$$
\frac{\sqrt{b\sec(c+dx)}\left(\sin(c+dx)+\sin(3(c+dx))+12\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\right)}{10b^2d}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]/(b*Sec[c + d*x])^(3/2),x]
```

```
[Out] (Sqrt[b*Sec[c + d*x]]*(12*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + Si
n[c + d*x] + Sin[3*(c + d*x)]))/(10*b^2*d)
```

Maple [C] time = 0.197, size = 321, normalized size = 4.7

$$
-\frac{2}{5\,d\,(\cos(dx+c))^2\sin(dx+c)}\left(3\,i\cos(dx+c)\sin(dx+c)\sqrt{(\cos(dx+c)+1)^{-1}}\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\mathrm{EllipticE}\left(\frac{i(-1+c}{\sin(}\right.\right.
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)/(b*sec(d*x+c))^(3/2),x)
```

```
[Out] -2/5/d*(3*I*cos(d*x+c)*sin(d*x+c)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos
(d*x+c)+1))^(1/2)*EllipticE(I*(-1+cos(d*x+c))/sin(d*x+c),I)-3*I*(1/(cos(d*x
+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(-1+cos(d*x+c))
/sin(d*x+c),I)*sin(d*x+c)*cos(d*x+c)+3*I*sin(d*x+c)*(1/(cos(d*x+c)+1))^(1/2
)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticE(I*(-1+cos(d*x+c))/sin(d*x+c),
I)-3*I*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticF
(I*(-1+cos(d*x+c))/sin(d*x+c),I)*sin(d*x+c)+cos(d*x+c)^4+2*cos(d*x+c)^2-3*c
os(d*x+c))/cos(d*x+c)^2/(b/cos(d*x+c))^(3/2)/sin(d*x+c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$
\int \frac{\cos(dx+c)}{(b\sec(dx+c))^{\frac{3}{2}}}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)/(b*sec(d*x + c))^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{b\sec\left(dx+c\right)}\cos\left(dx+c\right)}{b^2\sec\left(dx+c\right)^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(d*x + c))*cos(d*x + c)/(b^2*sec(d*x + c)^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\cos\left(c+dx\right)}{\left(b\sec\left(c+dx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(b*sec(d*x+c))**(3/2),x)
```

```
[Out] Integral(cos(c + d*x)/(b*sec(c + d*x))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\cos\left(dx+c\right)}{\left(b\sec\left(dx+c\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)/(b*sec(d*x + c))^(3/2), x)
```

3.119 $\int \frac{\cos^2(c+dx)}{(b\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=98

$$\frac{10\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{b\sec(c+dx)}}{21b^2d}+\frac{10\sin(c+dx)}{21bd\sqrt{b\sec(c+dx)}}+\frac{2b\sin(c+dx)}{7d(b\sec(c+dx))^{5/2}}$$

```
[Out] (10*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[b*Sec[c + d*x]])/(21*
b^2*d) + (2*b*Sin[c + d*x])/(7*d*(b*Sec[c + d*x])^(5/2)) + (10*Sin[c + d*x]
)/(21*b*d*Sqrt[b*Sec[c + d*x]])
```

Rubi [A] time = 0.0721163, antiderivative size = 98, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {16, 3769, 3771, 2641}

$$\frac{10\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\sec(c+dx)}}{21b^2d}+\frac{10\sin(c+dx)}{21bd\sqrt{b\sec(c+dx)}}+\frac{2b\sin(c+dx)}{7d(b\sec(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2/(b*Sec[c + d*x])^(3/2),x]
```



```
[Out] (10*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[b*Sec[c + d*x]])/(21*
b^2*d) + (2*b*Sin[c + d*x])/(7*d*(b*Sec[c + d*x])^(5/2)) + (10*Sin[c + d*x]
)/(21*b*d*Sqrt[b*Sec[c + d*x]])
```


Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cos^2(c+dx)}{(b\sec(c+dx))^{3/2}}\,dx &= b^2\int \frac{1}{(b\sec(c+dx))^{7/2}}\,dx \\
&= \frac{2b\sin(c+dx)}{7d(b\sec(c+dx))^{5/2}} + \frac{5}{7}\int \frac{1}{(b\sec(c+dx))^{3/2}}\,dx \\
&= \frac{2b\sin(c+dx)}{7d(b\sec(c+dx))^{5/2}} + \frac{10\sin(c+dx)}{21bd\sqrt{b\sec(c+dx)}} + \frac{5\int\sqrt{b\sec(c+dx)}\,dx}{21b^2} \\
&= \frac{2b\sin(c+dx)}{7d(b\sec(c+dx))^{5/2}} + \frac{10\sin(c+dx)}{21bd\sqrt{b\sec(c+dx)}} + \frac{\left(5\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}\right)\int\frac{1}{\sqrt{\cos(c+dx)}}\,dx}{21b^2} \\
&= \frac{10\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\sec(c+dx)}}{21b^2d} + \frac{2b\sin(c+dx)}{7d(b\sec(c+dx))^{5/2}} + \frac{10\sin(c+dx)}{21bd\sqrt{b\sec(c+dx)}}
\end{aligned}
$$

Mathematica [A] time = 0.0905225, size = 66, normalized size = 0.67

$$
\frac{\sqrt{b\sec(c+dx)}\left(40\sqrt{\cos(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+26\sin(2(c+dx))+3\sin(4(c+dx))\right)}{84b^2d}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2/(b*Sec[c + d*x])^(3/2),x]
```

```
[Out] (Sqrt[b*Sec[c + d*x]]*(40*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + 26
*Sin[2*(c + d*x)] + 3*Sin[4*(c + d*x)]))/(84*b^2*d)
```

Maple [C] time = 0.184, size = 153, normalized size = 1.6

$$
-\frac{2\left(\cos(dx+c)+1\right)^2\left(-1+\cos(dx+c)\right)}{21\,d\left(\sin(dx+c)\right)^3\left(\cos(dx+c)\right)^2}\left(5\,i\text{EllipticF}\left(\frac{i\left(-1+\cos(dx+c)\right)}{\sin(dx+c)},i\right)\sqrt{\left(\cos(dx+c)+1\right)^{-1}}\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)}}\right.
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2/(b*sec(d*x+c))^(3/2),x)
```

```
[Out] -2/21/d*(cos(d*x+c)+1)^2*(-1+cos(d*x+c))*(5*I*EllipticF(I*(-1+cos(d*x+c))/s
in(d*x+c),I)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin
(d*x+c)-3*cos(d*x+c)^4+3*cos(d*x+c)^3-5*cos(d*x+c)^2+5*cos(d*x+c))/sin(d*x+
c)^3/cos(d*x+c)^2/(b/cos(d*x+c))^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$
\int \frac{\cos(dx+c)^2}{(b\sec(dx+c))^{\frac{3}{2}}}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)^2/(b*sec(d*x + c))^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{b\sec\left(dx+c\right)}\cos\left(dx+c\right)^{2}}{b^{2}\sec\left(dx+c\right)^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(d*x + c))*cos(d*x + c)^2/(b^2*sec(d*x + c)^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\cos^{2}\left(c+dx\right)}{\left(b\sec\left(c+dx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2/(b*sec(d*x+c))**(3/2),x)
```

```
[Out] Integral(cos(c + d*x)**2/(b*sec(c + d*x))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\cos\left(dx+c\right)^{2}}{\left(b\sec\left(dx+c\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^2/(b*sec(d*x + c))^(3/2), x)
```

3.120 $\int \frac{\cos^3(c+dx)}{(b\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=97

$$\frac{2b^2\sin(c+dx)}{9d(b\sec(c+dx))^{7/2}}+\frac{14\sin(c+dx)}{45d(b\sec(c+dx))^{3/2}}+\frac{14E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15bd\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}$$

```
[Out] (14*EllipticE[(c + d*x)/2, 2])/(15*b*d*Sqrt[Cos[c + d*x]]*Sqrt[b*Sec[c + d*
x]]) + (2*b^2*Sin[c + d*x])/(9*d*(b*Sec[c + d*x])^(7/2)) + (14*Sin[c + d*x]
)/(45*d*(b*Sec[c + d*x])^(3/2))
```

Rubi [A] time = 0.0712229, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {16, 3769, 3771, 2639}

$$\frac{2b^2\sin(c+dx)}{9d(b\sec(c+dx))^{7/2}}+\frac{14\sin(c+dx)}{45d(b\sec(c+dx))^{3/2}}+\frac{14E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15bd\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3/(b*Sec[c + d*x])^(3/2),x]
```

```
[Out] (14*EllipticE[(c + d*x)/2, 2])/(15*b*d*Sqrt[Cos[c + d*x]]*Sqrt[b*Sec[c + d*
x]]) + (2*b^2*Sin[c + d*x])/(9*d*(b*Sec[c + d*x])^(7/2)) + (14*Sin[c + d*x]
)/(45*d*(b*Sec[c + d*x])^(3/2))
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cos^3(c+dx)}{(b\sec(c+dx))^{3/2}}\,dx &= b^3\int \frac{1}{(b\sec(c+dx))^{9/2}}\,dx \\
&= \frac{2b^2\sin(c+dx)}{9d(b\sec(c+dx))^{7/2}} + \frac{1}{9}(7b)\int \frac{1}{(b\sec(c+dx))^{5/2}}\,dx \\
&= \frac{2b^2\sin(c+dx)}{9d(b\sec(c+dx))^{7/2}} + \frac{14\sin(c+dx)}{45d(b\sec(c+dx))^{3/2}} + \frac{7\int \frac{1}{\sqrt{b\sec(c+dx)}}\,dx}{15b} \\
&= \frac{2b^2\sin(c+dx)}{9d(b\sec(c+dx))^{7/2}} + \frac{14\sin(c+dx)}{45d(b\sec(c+dx))^{3/2}} + \frac{7\int \sqrt{\cos(c+dx)}\,dx}{15b\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}} \\
&= \frac{14E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15bd\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}} + \frac{2b^2\sin(c+dx)}{9d(b\sec(c+dx))^{7/2}} + \frac{14\sin(c+dx)}{45d(b\sec(c+dx))^{3/2}}
\end{aligned}
$$

Mathematica [A] time = 0.0770183, size = 73, normalized size = 0.75

$$
\frac{84E\left(\frac{1}{2}(c+dx)\middle|2\right) + (33\sin(c+dx) + 5\sin(3(c+dx)))\cos^{\frac{3}{2}}(c+dx)}{90d\cos^{\frac{3}{2}}(c+dx)(b\sec(c+dx))^{3/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3/(b*Sec[c + d*x])^(3/2),x]
```

```
[Out] (84*EllipticE[(c + d*x)/2, 2] + Cos[c + d*x]^(3/2)*(33*Sin[c + d*x] + 5*Sin
[3*(c + d*x)]))/(90*d*Cos[c + d*x]^(3/2)*(b*Sec[c + d*x])^(3/2))
```

Maple [C] time = 0.19, size = 333, normalized size = 3.4

$$
\frac{2}{45\,d\sin(dx+c)(\cos(dx+c))^2}\left(21\,i\sqrt{(\cos(dx+c)+1)^{-1}}\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\mathrm{EllipticF}\left(\frac{i(-1+\cos(dx+c))}{\sin(dx+c)},i\right)\sin(dx+
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3/(b*sec(d*x+c))^(3/2),x)
```

```
[Out] 2/45/d*(21*I*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*Ell
ipticF(I*(-1+cos(d*x+c))/sin(d*x+c),I)*sin(d*x+c)*cos(d*x+c)-21*I*cos(d*x+c
)*sin(d*x+c)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*Ell
ipticE(I*(-1+cos(d*x+c))/sin(d*x+c),I)-5*cos(d*x+c)^6+21*I*(1/(cos(d*x+c)+1
))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(-1+cos(d*x+c))/sin(
d*x+c),I)*sin(d*x+c)-21*I*sin(d*x+c)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(
cos(d*x+c)+1))^(1/2)*EllipticE(I*(-1+cos(d*x+c))/sin(d*x+c),I)-2*cos(d*x+c)
^4-14*cos(d*x+c)^2+21*cos(d*x+c))/sin(d*x+c)/cos(d*x+c)^2/(b/cos(d*x+c))^(3
/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$
\int \frac{\cos(dx+c)^3}{(b\sec(dx+c))^{\frac{3}{2}}}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)^3/(b*sec(d*x + c))^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{b\sec\left(dx+c\right)}\cos\left(dx+c\right)^{3}}{b^{2}\sec\left(dx+c\right)^{2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(d*x + c))*cos(d*x + c)^3/(b^2*sec(d*x + c)^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3/(b*sec(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos\left(dx+c\right)^{3}}{\left(b\sec\left(dx+c\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^3/(b*sec(d*x + c))^(3/2), x)
```

3.121 $\int \frac{\sec^7(c+dx)}{(b\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=100

$$\frac{10\sqrt{\cos(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{b\sec(c+dx)}}{21b^3d}+\frac{2\sin(c+dx)(b\sec(c+dx))^{7/2}}{7b^6d}+\frac{10\sin(c+dx)(b\sec(c+dx))^{3/2}}{21b^4d}$$

```
[Out] (10*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[b*Sec[c + d*x]])/(21*
b^3*d) + (10*(b*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(21*b^4*d) + (2*(b*Sec[c
+ d*x])^(7/2)*Sin[c + d*x])/(7*b^6*d)
```

Rubi [A] time = 0.0571708, antiderivative size = 100, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {16, 3768, 3771, 2641}

$$\frac{2\sin(c+dx)(b\sec(c+dx))^{7/2}}{7b^6d}+\frac{10\sin(c+dx)(b\sec(c+dx))^{3/2}}{21b^4d}+\frac{10\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\sec(c+dx)}}{21b^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^7/(b*Sec[c + d*x])^(5/2),x]
```

```
[Out] (10*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[b*Sec[c + d*x]])/(21*
b^3*d) + (10*(b*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(21*b^4*d) + (2*(b*Sec[c
+ d*x])^(7/2)*Sin[c + d*x])/(7*b^6*d)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sec^7(c+dx)}{(b\sec(c+dx))^{5/2}}\,dx = \frac{\int (b\sec(c+dx))^{9/2}\,dx}{b^7}$$
$$= \frac{2(b\sec(c+dx))^{7/2}\sin(c+dx)}{7b^6d} + \frac{5\int (b\sec(c+dx))^{5/2}\,dx}{7b^5}$$
$$= \frac{10(b\sec(c+dx))^{3/2}\sin(c+dx)}{21b^4d} + \frac{2(b\sec(c+dx))^{7/2}\sin(c+dx)}{7b^6d} + \frac{5\int \sqrt{b\sec(c+dx)}\,dx}{21b^3}$$
$$= \frac{10(b\sec(c+dx))^{3/2}\sin(c+dx)}{21b^4d} + \frac{2(b\sec(c+dx))^{7/2}\sin(c+dx)}{7b^6d} + \frac{\left(5\sqrt{\cos(c+dx)}\sqrt{b\sec(c\right.}{21l}$$
$$= \frac{10\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\sec(c+dx)}}{21b^3d} + \frac{10(b\sec(c+dx))^{3/2}\sin(c+dx)}{21b^4d} + \frac{2(b\sec}{}$$

Mathematica [A] time = 0.132118, size = 64, normalized size = 0.64

$$\frac{(b\sec(c+dx))^{5/2}\left(10\cos^{\frac{5}{2}}(c+dx)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+5\sin(2(c+dx))+6\tan(c+dx)\right)}{21b^5d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^7/(b*Sec[c + d*x])^(5/2),x]
```

```
[Out] ((b*Sec[c + d*x])^(5/2)*(10*Cos[c + d*x]^(5/2)*EllipticF[(c + d*x)/2, 2] +
5*Sin[2*(c + d*x)] + 6*Tan[c + d*x]))/(21*b^5*d)
```

Maple [C] time = 0.217, size = 152, normalized size = 1.5

$$-\frac{2\left(\cos(dx+c)+1\right)^2\left(-1+\cos(dx+c)\right)}{21\,d\left(\sin(dx+c)\right)^3\left(\cos(dx+c)\right)^6}\left(5\,i\left(\cos(dx+c)\right)^3\sin(dx+c)\sqrt{\left(\cos(dx+c)+1\right)^{-1}}\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\text{Elli}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^7/(b*sec(d*x+c))^(5/2),x)
```

```
[Out] -2/21/d*(cos(d*x+c)+1)^2*(-1+cos(d*x+c))*(5*I*cos(d*x+c)^3*sin(d*x+c)*(1/(c
os(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(-1+cos(d
*x+c))/sin(d*x+c),I)-5*cos(d*x+c)^3+5*cos(d*x+c)^2-3*cos(d*x+c)+3)/sin(d*x+
c)^3/cos(d*x+c)^6/(b/cos(d*x+c))^(5/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)^7}{(b\sec(dx+c))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7/(b*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)^7/(b*sec(d*x + c))^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{b\sec\left(dx+c\right)}\sec\left(dx+c\right)^{4}}{b^{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7/(b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(d*x + c))*sec(d*x + c)^4/b^3, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**7/(b*sec(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec\left(dx+c\right)^{7}}{\left(b\sec\left(dx+c\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7/(b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^7/(b*sec(d*x + c))^(5/2), x)
```

3.122 $\int \frac{\sec^6(c+dx)}{(b\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=100

$$\frac{2\sin(c+dx)(b\sec(c+dx))^{5/2}}{5b^5d}+\frac{6\sin(c+dx)\sqrt{b\sec(c+dx)}}{5b^3d}-\frac{6E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5b^2d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}$$

```
[Out] (-6*EllipticE[(c + d*x)/2, 2])/(5*b^2*d*Sqrt[Cos[c + d*x]]*Sqrt[b*Sec[c + d
*x]]) + (6*Sqrt[b*Sec[c + d*x]]*Sin[c + d*x])/(5*b^3*d) + (2*(b*Sec[c + d*x
])^(5/2)*Sin[c + d*x])/(5*b^5*d)
```

Rubi [A] time = 0.058089, antiderivative size = 100, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {16, 3768, 3771, 2639}

$$\frac{2\sin(c+dx)(b\sec(c+dx))^{5/2}}{5b^5d}+\frac{6\sin(c+dx)\sqrt{b\sec(c+dx)}}{5b^3d}-\frac{6E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5b^2d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^6/(b*Sec[c + d*x])^(5/2),x]
```

```
[Out] (-6*EllipticE[(c + d*x)/2, 2])/(5*b^2*d*Sqrt[Cos[c + d*x]]*Sqrt[b*Sec[c + d
*x]]) + (6*Sqrt[b*Sec[c + d*x]]*Sin[c + d*x])/(5*b^3*d) + (2*(b*Sec[c + d*x
])^(5/2)*Sin[c + d*x])/(5*b^5*d)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sec^6(c+dx)}{(b\sec(c+dx))^{5/2}}\,dx = \frac{\int (b\sec(c+dx))^{7/2}\,dx}{b^6}$$
$$= \frac{2(b\sec(c+dx))^{5/2}\sin(c+dx)}{5b^5d} + \frac{3\int (b\sec(c+dx))^{3/2}\,dx}{5b^4}$$
$$= \frac{6\sqrt{b\sec(c+dx)}\sin(c+dx)}{5b^3d} + \frac{2(b\sec(c+dx))^{5/2}\sin(c+dx)}{5b^5d} - \frac{3\int \frac{1}{\sqrt{b\sec(c+dx)}}\,dx}{5b^2}$$
$$= \frac{6\sqrt{b\sec(c+dx)}\sin(c+dx)}{5b^3d} + \frac{2(b\sec(c+dx))^{5/2}\sin(c+dx)}{5b^5d} - \frac{3\int \sqrt{\cos(c+dx)}\,dx}{5b^2\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx}}$$
$$= -\frac{6E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5b^2d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}} + \frac{6\sqrt{b\sec(c+dx)}\sin(c+dx)}{5b^3d} + \frac{2(b\sec(c+dx))^{5/2}\sin(c+}{5b^5d}$$

Mathematica [A] time = 0.113415, size = 64, normalized size = 0.64

$$\frac{2\tan(c+dx)\left(\sec^2(c+dx)+3\right) - \frac{6E\left(\frac{1}{2}(c+dx)\middle|2\right)}{\sqrt{\cos(c+dx)}}}{5b^2d\sqrt{b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^6/(b*Sec[c + d*x])^(5/2),x]
```

```
[Out] ((-6*EllipticE[(c + d*x)/2, 2])/Sqrt[Cos[c + d*x]] + 2*(3 + Sec[c + d*x]^2)
*Tan[c + d*x])/(5*b^2*d*Sqrt[b*Sec[c + d*x]])
```

Maple [C] time = 0.222, size = 351, normalized size = 3.5

$$-\frac{2\,(\cos(dx+c)+1)^2(-1+\cos(dx+c))^2}{5\,db^5(\sin(dx+c))^5}\left(3\,i\mathrm{EllipticF}\left(\frac{i(-1+\cos(dx+c))}{\sin(dx+c)},i\right)(\cos(dx+c))^3\sqrt{(\cos(dx+c)+1)^{-1}}\sqrt{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^6/(b*sec(d*x+c))^(5/2),x)
```

```
[Out] -2/5/d*(cos(d*x+c)+1)^2*(-1+cos(d*x+c))^2*(3*I*EllipticF(I*(-1+cos(d*x+c))/
sin(d*x+c),I)*cos(d*x+c)^3*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)
+1))^(1/2)*sin(d*x+c)-3*I*EllipticE(I*(-1+cos(d*x+c))/sin(d*x+c),I)*cos(d*x
+c)^3*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)
+3*I*EllipticF(I*(-1+cos(d*x+c))/sin(d*x+c),I)*(1/(cos(d*x+c)+1))^(1/2)*(co
s(d*x+c)/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)^2*sin(d*x+c)-3*I*EllipticE(I*(-1+
cos(d*x+c))/sin(d*x+c),I)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+
1))^(1/2)*cos(d*x+c)^2*sin(d*x+c)+3*cos(d*x+c)^3-2*cos(d*x+c)^2-1)*(b/cos(d
*x+c))^(5/2)/b^5/sin(d*x+c)^5
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)^6}{(b\sec(dx+c))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6/(b*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)^6/(b*sec(d*x + c))^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{b\sec\left(dx+c\right)}\sec\left(dx+c\right)^{3}}{b^{3}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6/(b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(d*x + c))*sec(d*x + c)^3/b^3, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec^{6}\left(c+dx\right)}{\left(b\sec\left(c+dx\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**6/(b*sec(d*x+c))**(5/2),x)
```

```
[Out] Integral(sec(c + d*x)**6/(b*sec(c + d*x))**(5/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec\left(dx+c\right)^{6}}{\left(b\sec\left(dx+c\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6/(b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^6/(b*sec(d*x + c))^(5/2), x)
```

3.123 $\int \frac{\sec^5(c+dx)}{(b\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=72

$$\frac{2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{b\sec(c+dx)}}{3b^3d}+\frac{2\sin(c+dx)(b\sec(c+dx))^{3/2}}{3b^4d}$$

```
[Out] (2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[b*Sec[c + d*x]])/(3*b^
3*d) + (2*(b*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(3*b^4*d)
```

Rubi [A] time = 0.0383166, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {16, 3768, 3771, 2641}

$$\frac{2\sin(c+dx)(b\sec(c+dx))^{3/2}}{3b^4d}+\frac{2\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\sec(c+dx)}}{3b^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^5/(b*Sec[c + d*x])^(5/2),x]
```

```
[Out] (2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[b*Sec[c + d*x]])/(3*b^
3*d) + (2*(b*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(3*b^4*d)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sec^5(c+dx)}{(b\sec(c+dx))^{5/2}}\,dx = \frac{\int (b\sec(c+dx))^{5/2}\,dx}{b^5}$$
$$= \frac{2(b\sec(c+dx))^{3/2}\sin(c+dx)}{3b^4d} + \frac{\int \sqrt{b\sec(c+dx)}\,dx}{3b^3}$$
$$= \frac{2(b\sec(c+dx))^{3/2}\sin(c+dx)}{3b^4d} + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx}{3b^3}$$
$$= \frac{2\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\sec(c+dx)}}{3b^3d} + \frac{2(b\sec(c+dx))^{3/2}\sin(c+dx)}{3b^4d}$$

Mathematica [A] time = 0.065955, size = 51, normalized size = 0.71

$$\frac{2\sqrt{b\sec(c+dx)}\left(\sqrt{\cos(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+\tan(c+dx)\right)}{3b^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^5/(b*Sec[c + d*x])^(5/2),x]
```

```
[Out] (2*Sqrt[b*Sec[c + d*x]]*(Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + Tan
[c + d*x]))/(3*b^3*d)
```

Maple [C] time = 0.174, size = 130, normalized size = 1.8

$$-\frac{2\,(\cos(dx+c)+1)^2(-1+\cos(dx+c))}{3\,d\,(\cos(dx+c))^4(\sin(dx+c))^3}\left(i\sqrt{(\cos(dx+c)+1)^{-1}}\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\text{EllipticF}\left(\frac{i(-1+\cos(dx+c))}{\sin(dx+c)},\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5/(b*sec(d*x+c))^(5/2),x)
```

```
[Out] -2/3/d*(cos(d*x+c)+1)^2*(-1+cos(d*x+c))*(I*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*
x+c)/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(-1+cos(d*x+c))/sin(d*x+c),I)*cos(d*
x+c)*sin(d*x+c)-cos(d*x+c)+1)/cos(d*x+c)^4/(b/cos(d*x+c))^(5/2)/sin(d*x+c)^
3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)^5}{(b\sec(dx+c))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(b*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)^5/(b*sec(d*x + c))^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{b\sec\left(dx+c\right)}\sec\left(dx+c\right)^{2}}{b^{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(d*x + c))*sec(d*x + c)^2/b^3, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec^{5}\left(c+dx\right)}{\left(b\sec\left(c+dx\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5/(b*sec(d*x+c))**(5/2),x)
```

```
[Out] Integral(sec(c + d*x)**5/(b*sec(c + d*x))**(5/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec\left(dx+c\right)^{5}}{\left(b\sec\left(dx+c\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^5/(b*sec(d*x + c))^(5/2), x)
```

3.124 $\int \frac{\sec^4(c+dx)}{(b\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=68

$$\frac{2\sin(c+dx)\sqrt{b\sec(c+dx)}}{b^3d}-\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^2d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}$$

```
[Out] (-2*EllipticE[(c + d*x)/2, 2])/(b^2*d*Sqrt[Cos[c + d*x]]*Sqrt[b*Sec[c + d*x
]]) + (2*Sqrt[b*Sec[c + d*x]]*Sin[c + d*x])/(b^3*d)
```

Rubi [A] time = 0.0383202, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {16, 3768, 3771, 2639}

$$\frac{2\sin(c+dx)\sqrt{b\sec(c+dx)}}{b^3d}-\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^2d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4/(b*Sec[c + d*x])^(5/2),x]
```

```
[Out] (-2*EllipticE[(c + d*x)/2, 2])/(b^2*d*Sqrt[Cos[c + d*x]]*Sqrt[b*Sec[c + d*x
]]) + (2*Sqrt[b*Sec[c + d*x]]*Sin[c + d*x])/(b^3*d)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sec^4(c+dx)}{(b\sec(c+dx))^{5/2}}\,dx = \frac{\int (b\sec(c+dx))^{3/2}\,dx}{b^4}$$
$$= \frac{2\sqrt{b\sec(c+dx)}\sin(c+dx)}{b^3d} - \frac{\int \frac{1}{\sqrt{b\sec(c+dx)}}\,dx}{b^2}$$
$$= \frac{2\sqrt{b\sec(c+dx)}\sin(c+dx)}{b^3d} - \frac{\int \sqrt{\cos(c+dx)}\,dx}{b^2\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}$$
$$= -\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^2d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}} + \frac{2\sqrt{b\sec(c+dx)}\sin(c+dx)}{b^3d}$$

Mathematica [A] time = 0.0525358, size = 51, normalized size = 0.75

$$\frac{2\tan(c+dx) - \frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{\sqrt{\cos(c+dx)}}}{b^2d\sqrt{b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4/(b*Sec[c + d*x])^(5/2),x]
```

```
[Out] ((-2*EllipticE[(c + d*x)/2, 2])/Sqrt[Cos[c + d*x]] + 2*Tan[c + d*x])/(b^2*d
*Sqrt[b*Sec[c + d*x]])
```

Maple [C] time = 0.234, size = 322, normalized size = 4.7

$$-2\,\frac{(\cos(dx+c)+1)^2(-1+\cos(dx+c))^2}{d(\sin(dx+c))^5(\cos(dx+c))^3}\left(i\sqrt{(\cos(dx+c)+1)^{-1}}\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\mathrm{EllipticF}\left(\frac{i(-1+\cos(dx+c))}{\sin(dx+c)},i\right)c\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4/(b*sec(d*x+c))^(5/2),x)
```

```
[Out] -2/d*(cos(d*x+c)+1)^2*(-1+cos(d*x+c))^2*(I*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*
x+c)/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(-1+cos(d*x+c))/sin(d*x+c),I)*cos(d*
x+c)*sin(d*x+c)-I*EllipticE(I*(-1+cos(d*x+c))/sin(d*x+c),I)*cos(d*x+c)*sin(
d*x+c)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+I*Ellipti
cF(I*(-1+cos(d*x+c))/sin(d*x+c),I)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(co
s(d*x+c)+1))^(1/2)*sin(d*x+c)-I*EllipticE(I*(-1+cos(d*x+c))/sin(d*x+c),I)*s
in(d*x+c)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+cos(d*
x+c)-1)/sin(d*x+c)^5/cos(d*x+c)^3/(b/cos(d*x+c))^(5/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)^4}{(b\sec(dx+c))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(b*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)^4/(b*sec(d*x + c))^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\sqrt{b\sec\left(dx+c\right)}\sec\left(dx+c\right)}{b^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(d*x + c))*sec(d*x + c)/b^3, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec^4\left(c+dx\right)}{\left(b\sec\left(c+dx\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4/(b*sec(d*x+c))**(5/2),x)
```

```
[Out] Integral(sec(c + d*x)**4/(b*sec(c + d*x))**(5/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec\left(dx+c\right)^4}{\left(b\sec\left(dx+c\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^4/(b*sec(d*x + c))^(5/2), x)
```

3.125 $\int \frac{\sec^3(c+dx)}{(b\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=41

$$\frac{2\sqrt{\cos(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{b\sec(c+dx)}}{b^3d}$$

```
[Out] (2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[b*Sec[c + d*x]])/(b^3*
d)
```

Rubi [A] time = 0.0218402, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {16, 3771, 2641}

$$\frac{2\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\sec(c+dx)}}{b^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3/(b*Sec[c + d*x])^(5/2),x]
```

```
[Out] (2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[b*Sec[c + d*x]])/(b^3*
d)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec^3(c+dx)}{(b\sec(c+dx))^{5/2}}\,dx &= \frac{\int \sqrt{b\sec(c+dx)}\,dx}{b^3}\\ &= \frac{\left(\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx}{b^3}\\ &= \frac{2\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\sec(c+dx)}}{b^3d}\end{aligned}$$

Mathematica [A] time = 0.0156618, size = 41, normalized size = 1.

$$\frac{2\sqrt{\cos(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{b\sec(c+dx)}}{b^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3/(b*Sec[c + d*x])^(5/2),x]
```

```
[Out] (2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[b*Sec[c + d*x]])/(b^3*
d)
```

Maple [C] time = 0.14, size = 98, normalized size = 2.4

$$\frac{-2\,i\,(-1+\cos(dx+c))\,(\cos(dx+c)+1)^2}{d\,(\sin(dx+c))^2}\left((\cos(dx+c)+1)^{-1}\right)^{\frac{7}{2}}\text{EllipticF}\left(\frac{i\,(-1+\cos(dx+c))}{\sin(dx+c)},i\right)\left(\frac{b}{\cos(dx+c)}\right)^{-}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3/(b*sec(d*x+c))^(5/2),x)
```

```
[Out] -2*I/d*(1/(cos(d*x+c)+1))^(7/2)*(-1+cos(d*x+c))*EllipticF(I*(-1+cos(d*x+c))
/sin(d*x+c),I)*(cos(d*x+c)+1)^2/(b/cos(d*x+c))^(5/2)/(cos(d*x+c)/(cos(d*x+c
)+1))^(5/2)/sin(d*x+c)^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)^3}{(b\sec(dx+c))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(b*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)^3/(b*sec(d*x + c))^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{b\sec(dx+c)}}{b^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(d*x + c))/b^3, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec^3(c+dx)}{(b\sec(c+dx))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3/(b*sec(d*x+c))**(5/2),x)
```

```
[Out] Integral(sec(c + d*x)**3/(b*sec(c + d*x))**(5/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)^3}{(b\sec(dx+c))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^3/(b*sec(d*x + c))^(5/2), x)
```

3.126 $\int \frac{\sec^2(c+dx)}{(b\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=41

$$\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^2d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}$$

```
[Out] (2*EllipticE[(c + d*x)/2, 2])/(b^2*d*Sqrt[Cos[c + d*x]]*Sqrt[b*Sec[c + d*x]
])
```

Rubi [A] time = 0.0226226, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {16, 3771, 2639}

$$\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^2d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2/(b*Sec[c + d*x])^(5/2),x]
```

```
[Out] (2*EllipticE[(c + d*x)/2, 2])/(b^2*d*Sqrt[Cos[c + d*x]]*Sqrt[b*Sec[c + d*x]
])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec^2(c+dx)}{(b\sec(c+dx))^{5/2}}\,dx &= \frac{\int \frac{1}{\sqrt{b\sec(c+dx)}}\,dx}{b^2}\\ &= \frac{\int \sqrt{\cos(c+dx)}\,dx}{b^2\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}\\ &= \frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^2d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}\end{aligned}$$

Mathematica [A] time = 0.0364556, size = 38, normalized size = 0.93

$$\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\cos^{\frac{5}{2}}(c+dx)(b\sec(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2/(b*Sec[c + d*x])^(5/2),x]
```

```
[Out] (2*EllipticE[(c + d*x)/2, 2])/(d*Cos[c + d*x]^(5/2)*(b*Sec[c + d*x])^(5/2))
```

Maple [C] time = 0.143, size = 311, normalized size = 7.6

$$2\frac{1}{d(\cos(dx+c))^3\sin(dx+c)}\left(i\sqrt{(\cos(dx+c)+1)^{-1}}\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\text{EllipticF}\left(\frac{i(-1+\cos(dx+c))}{\sin(dx+c)},i\right)\cos(dx+c)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2/(b*sec(d*x+c))^(5/2),x)
```

```
[Out] 2/d*(I*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticF
(I*(-1+cos(d*x+c))/sin(d*x+c),I)*cos(d*x+c)*sin(d*x+c)-I*EllipticE(I*(-1+co
s(d*x+c))/sin(d*x+c),I)*cos(d*x+c)*sin(d*x+c)*(1/(cos(d*x+c)+1))^(1/2)*(cos
(d*x+c)/(cos(d*x+c)+1))^(1/2)+I*EllipticF(I*(-1+cos(d*x+c))/sin(d*x+c),I)*(
1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-I*Elli
pticE(I*(-1+cos(d*x+c))/sin(d*x+c),I)*sin(d*x+c)*(1/(cos(d*x+c)+1))^(1/2)*(
cos(d*x+c)/(cos(d*x+c)+1))^(1/2)-cos(d*x+c)^2+cos(d*x+c))/cos(d*x+c)^3/(b/c
os(d*x+c))^(5/2)/sin(d*x+c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sec(dx+c)^2}{(b\sec(dx+c))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(b*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)^2/(b*sec(d*x + c))^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{b\sec(dx+c)}}{b^3\sec(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(d*x + c))/(b^3*sec(d*x + c)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec^2(c+dx)}{(b\sec(c+dx))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2/(b*sec(d*x+c))**(5/2),x)
```

```
[Out] Integral(sec(c + d*x)**2/(b*sec(c + d*x))**(5/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)^2}{(b\sec(dx+c))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^2/(b*sec(d*x + c))^(5/2), x)
```

3.127 $\int \frac{\sec(c+dx)}{(b\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=72

$$\frac{2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{b\sec(c+dx)}}{3b^3d}+\frac{2\sin(c+dx)}{3b^2d\sqrt{b\sec(c+dx)}}$$

```
[Out] (2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[b*Sec[c + d*x]])/(3*b^
3*d) + (2*Sin[c + d*x])/(3*b^2*d*Sqrt[b*Sec[c + d*x]])
```

Rubi [A] time = 0.0400432, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {16, 3769, 3771, 2641}

$$\frac{2\sin(c+dx)}{3b^2d\sqrt{b\sec(c+dx)}}+\frac{2\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\sec(c+dx)}}{3b^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]/(b*Sec[c + d*x])^(5/2),x]
```

```
[Out] (2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[b*Sec[c + d*x]])/(3*b^
3*d) + (2*Sin[c + d*x])/(3*b^2*d*Sqrt[b*Sec[c + d*x]])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sec(c+dx)}{(b\sec(c+dx))^{5/2}}\,dx = \frac{\int \frac{1}{(b\sec(c+dx))^{3/2}}\,dx}{b}$$
$$= \frac{2\sin(c+dx)}{3b^2d\sqrt{b\sec(c+dx)}} + \frac{\int \sqrt{b\sec(c+dx)}\,dx}{3b^3}$$
$$= \frac{2\sin(c+dx)}{3b^2d\sqrt{b\sec(c+dx)}} + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx}{3b^3}$$
$$= \frac{2\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\sec(c+dx)}}{3b^3d} + \frac{2\sin(c+dx)}{3b^2d\sqrt{b\sec(c+dx)}}$$

Mathematica [A] time = 0.0306355, size = 62, normalized size = 0.86

$$\frac{\sec^2(c+dx)\left(2\sqrt{\cos(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+\sin(2(c+dx))\right)}{3bd(b\sec(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]/(b*Sec[c + d*x])^(5/2),x]
```

```
[Out] (Sec[c + d*x]^2*(2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + Sin[2*(c
+ d*x)]))/(3*b*d*(b*Sec[c + d*x])^(3/2))
```

Maple [C] time = 0.147, size = 131, normalized size = 1.8

$$-\frac{(-2+2\cos(dx+c))(\cos(dx+c)+1)^2}{3d(\sin(dx+c))^3(\cos(dx+c))^3}\left(i\text{EllipticF}\left(\frac{i(-1+\cos(dx+c))}{\sin(dx+c)},i\right)\sqrt{(\cos(dx+c)+1)^{-1}}\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)/(b*sec(d*x+c))^(5/2),x)
```

```
[Out] -2/3/d*(-1+cos(d*x+c))*(I*EllipticF(I*(-1+cos(d*x+c))/sin(d*x+c),I)*(1/(cos
(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-cos(d*x+c)^2
+cos(d*x+c))*(cos(d*x+c)+1)^2/sin(d*x+c)^3/cos(d*x+c)^3/(b/cos(d*x+c))^(5/2
)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)}{(b\sec(dx+c))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(b*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)/(b*sec(d*x + c))^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{b\sec\left(dx+c\right)}}{b^3\sec\left(dx+c\right)^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(d*x + c))/(b^3*sec(d*x + c)^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sec\left(c+dx\right)}{\left(b\sec\left(c+dx\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(b*sec(d*x+c))**(5/2),x)
```

```
[Out] Integral(sec(c + d*x)/(b*sec(c + d*x))**(5/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sec\left(dx+c\right)}{\left(b\sec\left(dx+c\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)/(b*sec(d*x + c))^(5/2), x)
```

3.128 $\int \frac{1}{(b\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=72

$$\frac{6E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5b^2d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}+\frac{2\sin(c+dx)}{5bd(b\sec(c+dx))^{3/2}}$$

```
[Out] (6*EllipticE[(c + d*x)/2, 2])/(5*b^2*d*Sqrt[Cos[c + d*x]]*Sqrt[b*Sec[c + d*
x]]) + (2*Sin[c + d*x])/(5*b*d*(b*Sec[c + d*x])^(3/2))
```

Rubi [A] time = 0.0337027, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {3769, 3771, 2639}

$$\frac{6E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5b^2d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}+\frac{2\sin(c+dx)}{5bd(b\sec(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Sec[c + d*x])^(-5/2),x]
```

```
[Out] (6*EllipticE[(c + d*x)/2, 2])/(5*b^2*d*Sqrt[Cos[c + d*x]]*Sqrt[b*Sec[c + d*
x]]) + (2*Sin[c + d*x])/(5*b*d*(b*Sec[c + d*x])^(3/2))
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(b\sec(c+dx))^{5/2}}\,dx &= \frac{2\sin(c+dx)}{5bd(b\sec(c+dx))^{3/2}}+\frac{3\int \frac{1}{\sqrt{b\sec(c+dx)}}\,dx}{5b^2}\\ &= \frac{2\sin(c+dx)}{5bd(b\sec(c+dx))^{3/2}}+\frac{3\int \sqrt{\cos(c+dx)}\,dx}{5b^2\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}\\ &= \frac{6E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5b^2d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}+\frac{2\sin(c+dx)}{5bd(b\sec(c+dx))^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.0180601, size = 60, normalized size = 0.83

$$\frac{\sqrt{b\sec(c+dx)}\left(\sin(c+dx)+\sin(3(c+dx))+12\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\right)}{10b^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Sec[c + d*x])^(-5/2),x]
```

```
[Out] (Sqrt[b*Sec[c + d*x]]*(12*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + Si
n[c + d*x] + Sin[3*(c + d*x)]))/(10*b^3*d)
```

Maple [C] time = 0.199, size = 321, normalized size = 4.5

$$-\frac{2}{5\,d\,(\cos(dx+c))^3\sin(dx+c)}\left(3\,i\cos(dx+c)\sin(dx+c)\sqrt{(\cos(dx+c)+1)^{-1}}\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\mathrm{EllipticE}\left(\frac{i(-1+c}{\sin(}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*sec(d*x+c))^(5/2),x)
```

```
[Out] -2/5/d*(3*I*cos(d*x+c)*sin(d*x+c)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos
(d*x+c)+1))^(1/2)*EllipticE(I*(-1+cos(d*x+c))/sin(d*x+c),I)-3*I*(1/(cos(d*x
+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(-1+cos(d*x+c))
/sin(d*x+c),I)*sin(d*x+c)*cos(d*x+c)+3*I*sin(d*x+c)*(1/(cos(d*x+c)+1))^(1/2
)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticE(I*(-1+cos(d*x+c))/sin(d*x+c),
I)-3*I*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticF
(I*(-1+cos(d*x+c))/sin(d*x+c),I)*sin(d*x+c)+cos(d*x+c)^4+2*cos(d*x+c)^2-3*c
os(d*x+c))/cos(d*x+c)^3/sin(d*x+c)/(b/cos(d*x+c))^(5/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b\sec(dx+c))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c))^(-5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{b\sec(dx+c)}}{b^3\sec(dx+c)^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(d*x + c))/(b^3*sec(d*x + c)^3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b\sec(c+dx))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*sec(d*x+c))**(5/2),x)
```

```
[Out] Integral((b*sec(c + d*x))**(-5/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b\sec(dx+c))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c))^(-5/2), x)
```

3.129 $\int \frac{\cos(c+dx)}{(b\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=97

$$\frac{10\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{b\sec(c+dx)}}{21b^3d}+\frac{10\sin(c+dx)}{21b^2d\sqrt{b\sec(c+dx)}}+\frac{2\sin(c+dx)}{7d(b\sec(c+dx))^{5/2}}$$

```
[Out] (10*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[b*Sec[c + d*x]])/(21*
b^3*d) + (2*Sin[c + d*x])/(7*d*(b*Sec[c + d*x])^(5/2)) + (10*Sin[c + d*x])/
(21*b^2*d*Sqrt[b*Sec[c + d*x]])
```

Rubi [A] time = 0.0634044, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {16, 3769, 3771, 2641}

$$\frac{10\sin(c+dx)}{21b^2d\sqrt{b\sec(c+dx)}}+\frac{10\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\sec(c+dx)}}{21b^3d}+\frac{2\sin(c+dx)}{7d(b\sec(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]/(b*Sec[c + d*x])^(5/2),x]
```

```
[Out] (10*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[b*Sec[c + d*x]])/(21*
b^3*d) + (2*Sin[c + d*x])/(7*d*(b*Sec[c + d*x])^(5/2)) + (10*Sin[c + d*x])/
(21*b^2*d*Sqrt[b*Sec[c + d*x]])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cos(c+dx)}{(b\sec(c+dx))^{5/2}}\,dx = b\int \frac{1}{(b\sec(c+dx))^{7/2}}\,dx$$
$$= \frac{2\sin(c+dx)}{7d(b\sec(c+dx))^{5/2}} + \frac{5\int \frac{1}{(b\sec(c+dx))^{3/2}}\,dx}{7b}$$
$$= \frac{2\sin(c+dx)}{7d(b\sec(c+dx))^{5/2}} + \frac{10\sin(c+dx)}{21b^2d\sqrt{b\sec(c+dx)}} + \frac{5\int \sqrt{b\sec(c+dx)}\,dx}{21b^3}$$
$$= \frac{2\sin(c+dx)}{7d(b\sec(c+dx))^{5/2}} + \frac{10\sin(c+dx)}{21b^2d\sqrt{b\sec(c+dx)}} + \frac{\left(5\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}}\,d}{21b^3}$$
$$= \frac{10\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\sec(c+dx)}}{21b^3d} + \frac{2\sin(c+dx)}{7d(b\sec(c+dx))^{5/2}} + \frac{10\sin(c+dx)}{21b^2d\sqrt{b\sec(c+d}}$$

Mathematica [A] time = 0.0689545, size = 66, normalized size = 0.68

$$\frac{\sqrt{b\sec(c+dx)}\left(40\sqrt{\cos(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+26\sin(2(c+dx))+3\sin(4(c+dx))\right)}{84b^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]/(b*Sec[c + d*x])^(5/2),x]
```

```
[Out] (Sqrt[b*Sec[c + d*x]]*(40*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + 26
*Sin[2*(c + d*x)] + 3*Sin[4*(c + d*x)]))/(84*b^3*d)
```

Maple [C] time = 0.178, size = 153, normalized size = 1.6

$$-\frac{2\,(\cos(dx+c)+1)^2(-1+\cos(dx+c))}{21\,d\,(\sin(dx+c))^3(\cos(dx+c))^3}\left(5\,i\text{EllipticF}\left(\frac{i(-1+\cos(dx+c))}{\sin(dx+c)},i\right)\sqrt{(\cos(dx+c)+1)^{-1}}\sqrt{\frac{\cos(dx+c}{\cos(dx+c)}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)/(b*sec(d*x+c))^(5/2),x)
```

```
[Out] -2/21/d*(cos(d*x+c)+1)^2*(-1+cos(d*x+c))*(5*I*EllipticF(I*(-1+cos(d*x+c))/s
in(d*x+c),I)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin
(d*x+c)-3*cos(d*x+c)^4+3*cos(d*x+c)^3-5*cos(d*x+c)^2+5*cos(d*x+c))/sin(d*x+
c)^3/cos(d*x+c)^3/(b/cos(d*x+c))^(5/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)}{(b\sec(dx+c))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(b*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)/(b*sec(d*x + c))^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{b\sec\left(dx+c\right)}\cos\left(dx+c\right)}{b^3\sec\left(dx+c\right)^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(d*x + c))*cos(d*x + c)/(b^3*sec(d*x + c)^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(b*sec(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos\left(dx+c\right)}{\left(b\sec\left(dx+c\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)/(b*sec(d*x + c))^(5/2), x)
```

3.130 $\int \frac{\cos^2(c+dx)}{(b\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=98

$$\frac{14E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15b^2d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}+\frac{14\sin(c+dx)}{45bd(b\sec(c+dx))^{3/2}}+\frac{2b\sin(c+dx)}{9d(b\sec(c+dx))^{7/2}}$$

```
[Out] (14*EllipticE[(c + d*x)/2, 2])/(15*b^2*d*Sqrt[Cos[c + d*x]]*Sqrt[b*Sec[c +
d*x]]) + (2*b*Sin[c + d*x])/(9*d*(b*Sec[c + d*x])^(7/2)) + (14*Sin[c + d*x]
)/(45*b*d*(b*Sec[c + d*x])^(3/2))
```

Rubi [A] time = 0.0723242, antiderivative size = 98, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {16, 3769, 3771, 2639}

$$\frac{14E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15b^2d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}+\frac{14\sin(c+dx)}{45bd(b\sec(c+dx))^{3/2}}+\frac{2b\sin(c+dx)}{9d(b\sec(c+dx))^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2/(b*Sec[c + d*x])^(5/2),x]
```

```
[Out] (14*EllipticE[(c + d*x)/2, 2])/(15*b^2*d*Sqrt[Cos[c + d*x]]*Sqrt[b*Sec[c +
d*x]]) + (2*b*Sin[c + d*x])/(9*d*(b*Sec[c + d*x])^(7/2)) + (14*Sin[c + d*x]
)/(45*b*d*(b*Sec[c + d*x])^(3/2))
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)}{(b\sec(c+dx))^{5/2}}\,dx = b^2\int \frac{1}{(b\sec(c+dx))^{9/2}}\,dx$$
$$= \frac{2b\sin(c+dx)}{9d(b\sec(c+dx))^{7/2}} + \frac{7}{9}\int \frac{1}{(b\sec(c+dx))^{5/2}}\,dx$$
$$= \frac{2b\sin(c+dx)}{9d(b\sec(c+dx))^{7/2}} + \frac{14\sin(c+dx)}{45bd(b\sec(c+dx))^{3/2}} + \frac{7\int \frac{1}{\sqrt{b\sec(c+dx)}}\,dx}{15b^2}$$
$$= \frac{2b\sin(c+dx)}{9d(b\sec(c+dx))^{7/2}} + \frac{14\sin(c+dx)}{45bd(b\sec(c+dx))^{3/2}} + \frac{7\int \sqrt{\cos(c+dx)}\,dx}{15b^2\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}$$
$$= \frac{14E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15b^2d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}} + \frac{2b\sin(c+dx)}{9d(b\sec(c+dx))^{7/2}} + \frac{14\sin(c+dx)}{45bd(b\sec(c+dx))^{3/2}}$$

Mathematica [A] time = 0.120966, size = 73, normalized size = 0.74

$$\frac{4(33\sin(c+dx)+5\sin(3(c+dx)))\cos(c+dx)+\frac{336E\left(\frac{1}{2}(c+dx)\middle|2\right)}{\sqrt{\cos(c+dx)}}}{360b^2d\sqrt{b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2/(b*Sec[c + d*x])^(5/2),x]
```

```
[Out] ((336*EllipticE[(c + d*x)/2, 2])/Sqrt[Cos[c + d*x]] + 4*Cos[c + d*x]*(33*Si
n[c + d*x] + 5*Sin[3*(c + d*x)]))/(360*b^2*d*Sqrt[b*Sec[c + d*x]])
```

Maple [C] time = 0.183, size = 333, normalized size = 3.4

$$-\frac{2}{45\,d\sin(dx+c)(\cos(dx+c))^3}\left(21\,i\cos(dx+c)\sin(dx+c)\sqrt{(\cos(dx+c)+1)^{-1}}\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\mathrm{EllipticE}\left(\frac{i(-1+}{\mathrm{si}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2/(b*sec(d*x+c))^(5/2),x)
```

```
[Out] -2/45/d*(21*I*cos(d*x+c)*sin(d*x+c)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(c
os(d*x+c)+1))^(1/2)*EllipticE(I*(-1+cos(d*x+c))/sin(d*x+c),I)-21*I*(1/(cos(
d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(-1+cos(d*x+
c))/sin(d*x+c),I)*cos(d*x+c)*sin(d*x+c)+5*cos(d*x+c)^6+21*I*sin(d*x+c)*(1/(
cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticE(I*(-1+cos(
d*x+c))/sin(d*x+c),I)-21*I*EllipticF(I*(-1+cos(d*x+c))/sin(d*x+c),I)*(1/(co
s(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+2*cos(d*x+c
)^4+14*cos(d*x+c)^2-21*cos(d*x+c))/sin(d*x+c)/cos(d*x+c)^3/(b/cos(d*x+c))^(
5/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)^2}{(b\sec(dx+c))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(b*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)^2/(b*sec(d*x + c))^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{b\sec\left(dx+c\right)}\cos\left(dx+c\right)^{2}}{b^{3}\sec\left(dx+c\right)^{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(d*x + c))*cos(d*x + c)^2/(b^3*sec(d*x + c)^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2/(b*sec(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\cos\left(dx+c\right)^{2}}{\left(b\sec\left(dx+c\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^2/(b*sec(d*x + c))^(5/2), x)
```

3.131 $\int \frac{1}{(b\sec(c+dx))^{7/2}}\,dx$

Optimal. Leaf size=100

$$\frac{10\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{b\sec(c+dx)}}{21b^4d}+\frac{10\sin(c+dx)}{21b^3d\sqrt{b\sec(c+dx)}}+\frac{2\sin(c+dx)}{7bd(b\sec(c+dx))^{5/2}}$$

```
[Out] (10*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[b*Sec[c + d*x]])/(21*
b^4*d) + (2*Sin[c + d*x])/(7*b*d*(b*Sec[c + d*x])^(5/2)) + (10*Sin[c + d*x]
)/(21*b^3*d*Sqrt[b*Sec[c + d*x]])
```

Rubi [A] time = 0.0512744, antiderivative size = 100, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {3769, 3771, 2641}

$$\frac{10\sin(c+dx)}{21b^3d\sqrt{b\sec(c+dx)}}+\frac{10\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\sec(c+dx)}}{21b^4d}+\frac{2\sin(c+dx)}{7bd(b\sec(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Sec[c + d*x])^(-7/2),x]
```

```
[Out] (10*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[b*Sec[c + d*x]])/(21*
b^4*d) + (2*Sin[c + d*x])/(7*b*d*(b*Sec[c + d*x])^(5/2)) + (10*Sin[c + d*x]
)/(21*b^3*d*Sqrt[b*Sec[c + d*x]])
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{1}{(b\sec(c+dx))^{7/2}}\,dx = \frac{2\sin(c+dx)}{7bd(b\sec(c+dx))^{5/2}} + \frac{5\int \frac{1}{(b\sec(c+dx))^{3/2}}\,dx}{7b^2}$$
$$= \frac{2\sin(c+dx)}{7bd(b\sec(c+dx))^{5/2}} + \frac{10\sin(c+dx)}{21b^3d\sqrt{b\sec(c+dx)}} + \frac{5\int \sqrt{b\sec(c+dx)}\,dx}{21b^4}$$
$$= \frac{2\sin(c+dx)}{7bd(b\sec(c+dx))^{5/2}} + \frac{10\sin(c+dx)}{21b^3d\sqrt{b\sec(c+dx)}} + \frac{\left(5\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}}}{21b^4}$$
$$= \frac{10\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\sec(c+dx)}}{21b^4d} + \frac{2\sin(c+dx)}{7bd(b\sec(c+dx))^{5/2}} + \frac{10\sin(c+dx)}{21b^3d\sqrt{b\sec(c+}}$$

Mathematica [A] time = 0.0204968, size = 66, normalized size = 0.66

$$\frac{\sqrt{b\sec(c+dx)}\left(40\sqrt{\cos(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+26\sin(2(c+dx))+3\sin(4(c+dx))\right)}{84b^4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Sec[c + d*x])^(-7/2),x]
```

```
[Out] (Sqrt[b*Sec[c + d*x]]*(40*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + 26
*Sin[2*(c + d*x)] + 3*Sin[4*(c + d*x)]))/(84*b^4*d)
```

Maple [C] time = 0.187, size = 153, normalized size = 1.5

$$-\frac{2\,(\cos(dx+c)+1)^2(-1+\cos(dx+c))}{21\,d\,(\cos(dx+c))^4(\sin(dx+c))^3}\left(5\,i\text{EllipticF}\left(\frac{i(-1+\cos(dx+c))}{\sin(dx+c)},i\right)\sqrt{(\cos(dx+c)+1)^{-1}}\sqrt{\frac{\cos(dx+c}{\cos(dx+c)}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*sec(d*x+c))^(7/2),x)
```

```
[Out] -2/21/d*(cos(d*x+c)+1)^2*(-1+cos(d*x+c))*(5*I*EllipticF(I*(-1+cos(d*x+c))/s
in(d*x+c),I)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin
(d*x+c)-3*cos(d*x+c)^4+3*cos(d*x+c)^3-5*cos(d*x+c)^2+5*cos(d*x+c))/(b/cos(d
*x+c))^(7/2)/cos(d*x+c)^4/sin(d*x+c)^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b\sec(dx+c))^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*sec(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c))^(-7/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{b\sec(dx+c)}}{b^4\sec(dx+c)^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*sec(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(d*x + c))/(b^4*sec(d*x + c)^4), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*sec(d*x+c))**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b\sec(dx+c))^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*sec(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c))^(-7/2), x)
```

3.132 $\int \sec^{\frac{9}{2}}(c+dx)\sqrt{b\sec(c+dx)}\,dx$

Optimal. Leaf size=107

$$\frac{\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)\sqrt{b\sec(c+dx)}}{4d}+\frac{3\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\sqrt{b\sec(c+dx)}}{8d}+\frac{3\sqrt{b\sec(c+dx)}\tanh^{-1}(\sin(c+}{8d\sqrt{\sec(c+dx)}}$$

```
[Out] (3*ArcTanh[Sin[c + d*x]]*Sqrt[b*Sec[c + d*x]])/(8*d*Sqrt[Sec[c + d*x]]) + (
3*Sec[c + d*x]^(3/2)*Sqrt[b*Sec[c + d*x]]*Sin[c + d*x])/(8*d) + (Sec[c + d*
x]^(7/2)*Sqrt[b*Sec[c + d*x]]*Sin[c + d*x])/(4*d)
```

Rubi [A] time = 0.0332845, antiderivative size = 107, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {17, 3768, 3770}

$$\frac{\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)\sqrt{b\sec(c+dx)}}{4d}+\frac{3\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\sqrt{b\sec(c+dx)}}{8d}+\frac{3\sqrt{b\sec(c+dx)}\tanh^{-1}(\sin(c+}{8d\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(9/2)*Sqrt[b*Sec[c + d*x]],x]
```

```
[Out] (3*ArcTanh[Sin[c + d*x]]*Sqrt[b*Sec[c + d*x]])/(8*d*Sqrt[Sec[c + d*x]]) + (
3*Sec[c + d*x]^(3/2)*Sqrt[b*Sec[c + d*x]]*Sin[c + d*x])/(8*d) + (Sec[c + d*
x]^(7/2)*Sqrt[b*Sec[c + d*x]]*Sin[c + d*x])/(4*d)
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(a^(m + 1/
2)*b^(n - 1/2)*Sqrt[b*v])/Sqrt[a*v], Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sec^{\frac{9}{2}}(c+dx)\sqrt{b\sec(c+dx)}\,dx = \frac{\sqrt{b\sec(c+dx)}\int \sec^5(c+dx)\,dx}{\sqrt{\sec(c+dx)}}$$
$$= \frac{\sec^{\frac{7}{2}}(c+dx)\sqrt{b\sec(c+dx)}\sin(c+dx)}{4d} + \frac{\left(3\sqrt{b\sec(c+dx)}\right)\int \sec^3(c+dx)\,dx}{4\sqrt{\sec(c+dx)}}$$
$$= \frac{3\sec^{\frac{3}{2}}(c+dx)\sqrt{b\sec(c+dx)}\sin(c+dx)}{8d} + \frac{\sec^{\frac{7}{2}}(c+dx)\sqrt{b\sec(c+dx)}\sin(c+dx)}{4d} +$$
$$= \frac{3\tanh^{-1}(\sin(c+dx))\sqrt{b\sec(c+dx)}}{8d\sqrt{\sec(c+dx)}} + \frac{3\sec^{\frac{3}{2}}(c+dx)\sqrt{b\sec(c+dx)}\sin(c+dx)}{8d} +$$

Mathematica [A] time = 0.141481, size = 64, normalized size = 0.6

$$\frac{\sqrt{b\sec(c+dx)}\left(3\tanh^{-1}(\sin(c+dx))+\tan(c+dx)\sec(c+dx)\left(2\sec^2(c+dx)+3\right)\right)}{8d\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^(9/2)*Sqrt[b*Sec[c + d*x]],x]
```

```
[Out] (Sqrt[b*Sec[c + d*x]]*(3*ArcTanh[Sin[c + d*x]] + Sec[c + d*x]*(3 + 2*Sec[c
+ d*x]^2)*Tan[c + d*x]))/(8*d*Sqrt[Sec[c + d*x]])
```

Maple [A] time = 0.178, size = 131, normalized size = 1.2

$$\frac{\cos(dx+c)}{8d}\left(3\left(\cos(dx+c)\right)^4\ln\left(-\frac{-1+\cos(dx+c)-\sin(dx+c)}{\sin(dx+c)}\right)-3\left(\cos(dx+c)\right)^4\ln\left(-\frac{-1+\cos(dx+c)+\sin(dx}{\sin(dx+c)}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(9/2)*(b*sec(d*x+c))^(1/2),x)
```

```
[Out] 1/8/d*(3*cos(d*x+c)^4*ln(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))-3*cos(d*x+
c)^4*ln(-(-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))+3*cos(d*x+c)^2*sin(d*x+c)+2
*sin(d*x+c))*cos(d*x+c)*(1/cos(d*x+c))^(9/2)*(b/cos(d*x+c))^(1/2)
```

Maxima [B] time = 2.48916, size = 2236, normalized size = 20.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(9/2)*(b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] -1/16*(12*(sin(8*d*x + 8*c) + 4*sin(6*d*x + 6*c) + 6*sin(4*d*x + 4*c) + 4*s
in(2*d*x + 2*c))*cos(7/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 44*
(sin(8*d*x + 8*c) + 4*sin(6*d*x + 6*c) + 6*sin(4*d*x + 4*c) + 4*sin(2*d*x +
 2*c))*cos(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 44*(sin(8*d*x
 + 8*c) + 4*sin(6*d*x + 6*c) + 6*sin(4*d*x + 4*c) + 4*sin(2*d*x + 2*c))*cos
(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 12*(sin(8*d*x + 8*c) +
```

```
4*sin(6*d*x + 6*c) + 6*sin(4*d*x + 4*c) + 4*sin(2*d*x + 2*c))*cos(1/2*arcta
n2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 3*(2*(4*cos(6*d*x + 6*c) + 6*cos(
4*d*x + 4*c) + 4*cos(2*d*x + 2*c) + 1)*cos(8*d*x + 8*c) + cos(8*d*x + 8*c)^
2 + 8*(6*cos(4*d*x + 4*c) + 4*cos(2*d*x + 2*c) + 1)*cos(6*d*x + 6*c) + 16*c
os(6*d*x + 6*c)^2 + 12*(4*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + 36*cos(4
*d*x + 4*c)^2 + 16*cos(2*d*x + 2*c)^2 + 4*(2*sin(6*d*x + 6*c) + 3*sin(4*d*x
 + 4*c) + 2*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + sin(8*d*x + 8*c)^2 + 16*(3
*sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + 16*sin(6*d*x + 6
*c)^2 + 36*sin(4*d*x + 4*c)^2 + 48*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 16*s
in(2*d*x + 2*c)^2 + 8*cos(2*d*x + 2*c) + 1)*log(cos(1/2*arctan2(sin(2*d*x +
 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
 2*c)))^2 + 2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) + 3
*(2*(4*cos(6*d*x + 6*c) + 6*cos(4*d*x + 4*c) + 4*cos(2*d*x + 2*c) + 1)*cos(
8*d*x + 8*c) + cos(8*d*x + 8*c)^2 + 8*(6*cos(4*d*x + 4*c) + 4*cos(2*d*x + 2
*c) + 1)*cos(6*d*x + 6*c) + 16*cos(6*d*x + 6*c)^2 + 12*(4*cos(2*d*x + 2*c)
+ 1)*cos(4*d*x + 4*c) + 36*cos(4*d*x + 4*c)^2 + 16*cos(2*d*x + 2*c)^2 + 4*(
2*sin(6*d*x + 6*c) + 3*sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*sin(8*d*x + 8
*c) + sin(8*d*x + 8*c)^2 + 16*(3*sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*sin
(6*d*x + 6*c) + 16*sin(6*d*x + 6*c)^2 + 36*sin(4*d*x + 4*c)^2 + 48*sin(4*d*
x + 4*c)*sin(2*d*x + 2*c) + 16*sin(2*d*x + 2*c)^2 + 8*cos(2*d*x + 2*c) + 1)
*log(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arcta
n2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sin(1/2*arctan2(sin(2*d*x + 2
*c), cos(2*d*x + 2*c))) + 1) - 12*(cos(8*d*x + 8*c) + 4*cos(6*d*x + 6*c) +
6*cos(4*d*x + 4*c) + 4*cos(2*d*x + 2*c) + 1)*sin(7/2*arctan2(sin(2*d*x + 2*
c), cos(2*d*x + 2*c))) - 44*(cos(8*d*x + 8*c) + 4*cos(6*d*x + 6*c) + 6*cos(
4*d*x + 4*c) + 4*cos(2*d*x + 2*c) + 1)*sin(5/2*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c))) + 44*(cos(8*d*x + 8*c) + 4*cos(6*d*x + 6*c) + 6*cos(4*d*x
+ 4*c) + 4*cos(2*d*x + 2*c) + 1)*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*
x + 2*c))) + 12*(cos(8*d*x + 8*c) + 4*cos(6*d*x + 6*c) + 6*cos(4*d*x + 4*c)
 + 4*cos(2*d*x + 2*c) + 1)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*
c))))*sqrt(b)/((2*(4*cos(6*d*x + 6*c) + 6*cos(4*d*x + 4*c) + 4*cos(2*d*x +
2*c) + 1)*cos(8*d*x + 8*c) + cos(8*d*x + 8*c)^2 + 8*(6*cos(4*d*x + 4*c) + 4
*cos(2*d*x + 2*c) + 1)*cos(6*d*x + 6*c) + 16*cos(6*d*x + 6*c)^2 + 12*(4*cos
(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + 36*cos(4*d*x + 4*c)^2 + 16*cos(2*d*x
+ 2*c)^2 + 4*(2*sin(6*d*x + 6*c) + 3*sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))
*sin(8*d*x + 8*c) + sin(8*d*x + 8*c)^2 + 16*(3*sin(4*d*x + 4*c) + 2*sin(2*d
*x + 2*c))*sin(6*d*x + 6*c) + 16*sin(6*d*x + 6*c)^2 + 36*sin(4*d*x + 4*c)^2
 + 48*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 16*sin(2*d*x + 2*c)^2 + 8*cos(2*d
*x + 2*c) + 1)*d)
```

Fricas [A] time = 1.60803, size = 608, normalized size = 5.68

$$\left[\frac{3\sqrt{b}\cos\left(dx+c\right)^{3}\log\left(-\frac{b\cos(dx+c)^{2}-2\sqrt{b}\sqrt{\frac{b}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)-2b}{\cos(dx+c)^{2}}\right)+\frac{2\left(3\cos(dx+c)^{2}+2\right)\sqrt{\frac{b}{\cos(dx+c)}}\sin(dx+c)}{\sqrt{\cos(dx+c)}}}{16\,d\cos\left(dx+c\right)^{3}},\ -\frac{3\sqrt{-b}\,\mathrm{ar}\cdots}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(9/2)*(b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/16*(3*sqrt(b)*cos(d*x + c)^3*log(-(b*cos(d*x + c)^2 - 2*sqrt(b)*sqrt(b/c
os(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) - 2*b)/cos(d*x + c)^2) + 2*(3*
cos(d*x + c)^2 + 2)*sqrt(b/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(
d*cos(d*x + c)^3), -1/8*(3*sqrt(-b)*arctan(sqrt(-b)*sqrt(b/cos(d*x + c))*sq
```

```
rt(cos(d*x + c))*sin(d*x + c)/b)*cos(d*x + c)^3 - (3*cos(d*x + c)^2 + 2)*sq
rt(b/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x + c)^3)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(9/2)*(b*sec(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b \sec (dx + c)} \sec (dx + c)^{\frac{9}{2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(9/2)*(b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sec(d*x + c))*sec(d*x + c)^(9/2), x)
```

3.133 $\int \sec^{\frac{7}{2}}(c+dx)\sqrt{b\sec(c+dx)}\,dx$

Optimal. Leaf size=70

$$\frac{\sin^3(c+dx)\sec^{\frac{5}{2}}(c+dx)\sqrt{b\sec(c+dx)}}{3d}+\frac{\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{b\sec(c+dx)}}{d}$$

```
[Out] (Sqrt[Sec[c + d*x]]*Sqrt[b*Sec[c + d*x]]*Sin[c + d*x])/d + (Sec[c + d*x]^(5
/2)*Sqrt[b*Sec[c + d*x]]*Sin[c + d*x]^3)/(3*d)
```

Rubi [A] time = 0.0165725, antiderivative size = 70, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {17, 3767}

$$\frac{\sin^3(c+dx)\sec^{\frac{5}{2}}(c+dx)\sqrt{b\sec(c+dx)}}{3d}+\frac{\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{b\sec(c+dx)}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(7/2)*Sqrt[b*Sec[c + d*x]],x]
```

```
[Out] (Sqrt[Sec[c + d*x]]*Sqrt[b*Sec[c + d*x]]*Sin[c + d*x])/d + (Sec[c + d*x]^(5
/2)*Sqrt[b*Sec[c + d*x]]*Sin[c + d*x]^3)/(3*d)
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(a^(m + 1/
2)*b^(n - 1/2)*Sqrt[b*v])/Sqrt[a*v], Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] && !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}\int \sec^{\frac{7}{2}}(c+dx)\sqrt{b\sec(c+dx)}\,dx &= \frac{\sqrt{b\sec(c+dx)}\int \sec^4(c+dx)\,dx}{\sqrt{\sec(c+dx)}}\\ &= -\frac{\sqrt{b\sec(c+dx)}\,\text{Subst}\left(\int \left(1+x^2\right)\,dx, x, -\tan(c+dx)\right)}{d\sqrt{\sec(c+dx)}}\\ &= \frac{\sqrt{\sec(c+dx)}\sqrt{b\sec(c+dx)}\sin(c+dx)}{d}+\frac{\sec^{\frac{5}{2}}(c+dx)\sqrt{b\sec(c+dx)}\sin^3(c+d}{3d}\end{aligned}$$

Mathematica [A] time = 0.0867656, size = 45, normalized size = 0.64

$$\frac{\left(\frac{1}{3}\tan^3(c+dx)+\tan(c+dx)\right)\sqrt{b\sec(c+dx)}}{d\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^(7/2)*Sqrt[b*Sec[c + d*x]],x]
```

```
[Out] (Sqrt[b*Sec[c + d*x]]*(Tan[c + d*x] + Tan[c + d*x]^3/3))/(d*Sqrt[Sec[c + d*
x]])
```

Maple [A] time = 0.129, size = 52, normalized size = 0.7

$$\frac{\left(2\left(\cos\left(dx+c\right)\right)^{2}+1\right)\cos\left(dx+c\right)\sin\left(dx+c\right)}{3d}\left(\left(\cos\left(dx+c\right)\right)^{-1}\right)^{\frac{7}{2}}\sqrt{\frac{b}{\cos\left(dx+c\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(7/2)*(b*sec(d*x+c))^(1/2),x)
```

```
[Out] 1/3/d*(2*cos(d*x+c)^2+1)*(1/cos(d*x+c))^(7/2)*(b/cos(d*x+c))^(1/2)*cos(d*x+
c)*sin(d*x+c)
```

Maxima [B] time = 2.07997, size = 397, normalized size = 5.67

$$\frac{4\left(\left(3\cos\left(2dx+2c\right)+1\right)\mathrm{si}\right.}{3\left(2\left(3\cos\left(4dx+4c\right)+3\cos\left(2dx+2c\right)+1\right)\cos\left(6dx+6c\right)+\cos\left(6dx+6c\right)^{2}+6\left(3\cos\left(2dx+2c\right)+1\right)\cos\left(4dx+\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(7/2)*(b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] 4/3*((3*cos(2*d*x + 2*c) + 1)*sin(6*d*x + 6*c) + 3*(3*cos(2*d*x + 2*c) + 1)
*sin(4*d*x + 4*c) - 3*cos(6*d*x + 6*c)*sin(2*d*x + 2*c) - 9*cos(4*d*x + 4*c
)*sin(2*d*x + 2*c))*sqrt(b)/((2*(3*cos(4*d*x + 4*c) + 3*cos(2*d*x + 2*c) +
1)*cos(6*d*x + 6*c) + cos(6*d*x + 6*c)^2 + 6*(3*cos(2*d*x + 2*c) + 1)*cos(4
*d*x + 4*c) + 9*cos(4*d*x + 4*c)^2 + 9*cos(2*d*x + 2*c)^2 + 6*(sin(4*d*x +
4*c) + sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + sin(6*d*x + 6*c)^2 + 9*sin(4*d*
x + 4*c)^2 + 18*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 9*sin(2*d*x + 2*c)^2 +
6*cos(2*d*x + 2*c) + 1)*d)
```

Fricas [A] time = 1.41191, size = 115, normalized size = 1.64

$$\frac{\left(2\cos\left(dx+c\right)^{2}+1\right)\sqrt{\frac{b}{\cos(dx+c)}}\sin\left(dx+c\right)}{3d\cos\left(dx+c\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(7/2)*(b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3*(2*cos(d*x + c)^2 + 1)*sqrt(b/cos(d*x + c))*sin(d*x + c)/(d*cos(d*x + c
)^(5/2))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(7/2)*(b*sec(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b \sec (dx + c)} \sec (dx + c)^{\frac{7}{2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(7/2)*(b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sec(d*x + c))*sec(d*x + c)^(7/2), x)
```

3.134 $\int \sec^{\frac{5}{2}}(c+dx)\sqrt{b\sec(c+dx)}\,dx$

Optimal. Leaf size=72

$$\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\sqrt{b\sec(c+dx)}}{2d}+\frac{\sqrt{b\sec(c+dx)}\tanh^{-1}(\sin(c+dx))}{2d\sqrt{\sec(c+dx)}}$$

```
[Out] (ArcTanh[Sin[c + d*x]]*Sqrt[b*Sec[c + d*x]])/(2*d*Sqrt[Sec[c + d*x]]) + (Se
c[c + d*x]^(3/2)*Sqrt[b*Sec[c + d*x]]*Sin[c + d*x])/(2*d)
```

Rubi [A] time = 0.0192353, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {17, 3768, 3770}

$$\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\sqrt{b\sec(c+dx)}}{2d}+\frac{\sqrt{b\sec(c+dx)}\tanh^{-1}(\sin(c+dx))}{2d\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(5/2)*Sqrt[b*Sec[c + d*x]],x]
```

```
[Out] (ArcTanh[Sin[c + d*x]]*Sqrt[b*Sec[c + d*x]])/(2*d*Sqrt[Sec[c + d*x]]) + (Se
c[c + d*x]^(3/2)*Sqrt[b*Sec[c + d*x]]*Sin[c + d*x])/(2*d)
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(a^(m + 1/
2)*b^(n - 1/2)*Sqrt[b*v])/Sqrt[a*v], Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] && !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \sec^{\frac{5}{2}}(c+dx)\sqrt{b\sec(c+dx)}\,dx &= \frac{\sqrt{b\sec(c+dx)}\int \sec^3(c+dx)\,dx}{\sqrt{\sec(c+dx)}}\\ &= \frac{\sec^{\frac{3}{2}}(c+dx)\sqrt{b\sec(c+dx)}\sin(c+dx)}{2d}+\frac{\sqrt{b\sec(c+dx)}\int \sec(c+dx)\,dx}{2\sqrt{\sec(c+dx)}}\\ &= \frac{\tanh^{-1}(\sin(c+dx))\sqrt{b\sec(c+dx)}}{2d\sqrt{\sec(c+dx)}}+\frac{\sec^{\frac{3}{2}}(c+dx)\sqrt{b\sec(c+dx)}\sin(c+dx)}{2d}\end{aligned}$$

Mathematica [A] time = 0.0517154, size = 50, normalized size = 0.69

$$\frac{\sqrt{b\sec(c+dx)}\left(\tanh^{-1}(\sin(c+dx))+\tan(c+dx)\sec(c+dx)\right)}{2d\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^(5/2)*Sqrt[b*Sec[c + d*x]],x]
```

```
[Out] (Sqrt[b*Sec[c + d*x]]*(ArcTanh[Sin[c + d*x]] + Sec[c + d*x]*Tan[c + d*x]))/
(2*d*Sqrt[Sec[c + d*x]])
```

Maple [A] time = 0.135, size = 112, normalized size = 1.6

$$\frac{\cos(dx+c)}{2d}\left(\ln\left(-\frac{-1+\cos(dx+c)-\sin(dx+c)}{\sin(dx+c)}\right)(\cos(dx+c))^2-\ln\left(-\frac{-1+\cos(dx+c)+\sin(dx+c)}{\sin(dx+c)}\right)(\cos(dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(5/2)*(b*sec(d*x+c))^(1/2),x)
```

```
[Out] 1/2/d*(ln(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))*cos(d*x+c)^2-ln(-(-1+cos(
d*x+c)+sin(d*x+c))/sin(d*x+c))*cos(d*x+c)^2+sin(d*x+c))*cos(d*x+c)*(1/cos(d
*x+c))^(5/2)*(b/cos(d*x+c))^(1/2)
```

Maxima [B] time = 2.11265, size = 892, normalized size = 12.39

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)*(b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] -1/4*(4*(sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*cos(3/2*arctan2(sin(2*d*x +
 2*c), cos(2*d*x + 2*c))) - 4*(sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*cos(1
/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - (2*(2*cos(2*d*x + 2*c) +
1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*x
 + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*
cos(2*d*x + 2*c) + 1)*log(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c
)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/2*
arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) + (2*(2*cos(2*d*x + 2*c)
+ 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d
*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 +
4*cos(2*d*x + 2*c) + 1)*log(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2
*c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sin(1/
2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) - 4*(cos(4*d*x + 4*c) +
 2*cos(2*d*x + 2*c) + 1)*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
)) + 4*(cos(4*d*x + 4*c) + 2*cos(2*d*x + 2*c) + 1)*sin(1/2*arctan2(sin(2*d*
x + 2*c), cos(2*d*x + 2*c))))*sqrt(b)/((2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*
x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2 +
 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x +
 2*c) + 1)*d)
```

Fricas [A] time = 1.5485, size = 528, normalized size = 7.33

$$\left[\frac{\sqrt{b}\cos\left(dx+c\right)\log\left(-\frac{b\cos(dx+c)^2-2\sqrt{b}\sqrt{\frac{b}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)-2\,b}{\cos(dx+c)^2}\right)+\frac{2\sqrt{\frac{b}{\cos(dx+c)}}\sin(dx+c)}{\sqrt{\cos(dx+c)}}}{4\,d\cos\left(dx+c\right)},-\frac{\sqrt{-b}\arctan\left(\frac{\sqrt{-b}\sqrt{\frac{b}{\cos(dx+c)}}\sqrt{c\cdots}}{b\cdots}\right)\cdots}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)*(b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(b)*cos(d*x + c)*log(-(b*cos(d*x + c)^2 - 2*sqrt(b)*sqrt(b/cos(d*
x + c))*sqrt(cos(d*x + c))*sin(d*x + c) - 2*b)/cos(d*x + c)^2) + 2*sqrt(b/c
os(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x + c)), -1/2*(sqrt(
-b)*arctan(sqrt(-b)*sqrt(b/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/b)
*cos(d*x + c) - sqrt(b/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*co
s(d*x + c))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(5/2)*(b*sec(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\sec\left(dx+c\right)}\sec\left(dx+c\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)*(b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sec(d*x + c))*sec(d*x + c)^(5/2), x)
```

3.135 $\int \sec^{\frac{3}{2}}(c+dx)\sqrt{b\sec(c+dx)}\,dx$

Optimal. Leaf size=32

$$\frac{\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{b\sec(c+dx)}}{d}$$

```
[Out] (Sqrt[Sec[c + d*x]]*Sqrt[b*Sec[c + d*x]]*Sin[c + d*x])/d
```

Rubi [A] time = 0.0117126, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {17, 3767, 8}

$$\frac{\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{b\sec(c+dx)}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(3/2)*Sqrt[b*Sec[c + d*x]],x]
```

```
[Out] (Sqrt[Sec[c + d*x]]*Sqrt[b*Sec[c + d*x]]*Sin[c + d*x])/d
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(a^(m + 1/
2)*b^(n - 1/2)*Sqrt[b*v])/Sqrt[a*v], Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] && !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \sec^{\frac{3}{2}}(c+dx)\sqrt{b\sec(c+dx)}\,dx &= \frac{\sqrt{b\sec(c+dx)}\int \sec^2(c+dx)\,dx}{\sqrt{\sec(c+dx)}}\\ &= -\frac{\sqrt{b\sec(c+dx)}\,\text{Subst}\left(\int 1\,dx,x,-\tan(c+dx)\right)}{d\sqrt{\sec(c+dx)}}\\ &= \frac{\sqrt{\sec(c+dx)}\sqrt{b\sec(c+dx)}\sin(c+dx)}{d}\end{aligned}$$

Mathematica [A] time = 0.0176707, size = 32, normalized size = 1.

$$\frac{\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{b\sec(c+dx)}}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^(3/2)*Sqrt[b*Sec[c + d*x]],x]
```

```
[Out] (Sqrt[Sec[c + d*x]]*Sqrt[b*Sec[c + d*x]]*Sin[c + d*x])/d
```

Maple [A] time = 0.13, size = 39, normalized size = 1.2

$$\frac{\cos(dx+c)\sin(dx+c)}{d}\left((\cos(dx+c))^{-1}\right)^{\frac{3}{2}}\sqrt{\frac{b}{\cos(dx+c)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(3/2)*(b*sec(d*x+c))^(1/2),x)
```

```
[Out] 1/d*cos(d*x+c)*sin(d*x+c)*(1/cos(d*x+c))^(3/2)*(b/cos(d*x+c))^(1/2)
```

Maxima [A] time = 2.14705, size = 73, normalized size = 2.28

$$\frac{2\sqrt{b}\sin(2dx+2c)}{\left(\cos(2dx+2c)^2+\sin(2dx+2c)^2+2\cos(2dx+2c)+1\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] 2*sqrt(b)*sin(2*d*x + 2*c)/((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*co
s(2*d*x + 2*c) + 1)*d)
```

Fricas [A] time = 1.36651, size = 78, normalized size = 2.44

$$\frac{\sqrt{\frac{b}{\cos(dx+c)}}\sin(dx+c)}{d\sqrt{\cos(dx+c)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] sqrt(b/cos(d*x + c))*sin(d*x + c)/(d*sqrt(cos(d*x + c)))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(3/2)*(b*sec(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\sec(dx+c)}\sec(dx+c)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sec(d*x + c))*sec(d*x + c)^(3/2), x)
```

3.136 $\int \sqrt{\sec(c+dx)}\sqrt{b\sec(c+dx)}\,dx$

Optimal. Leaf size=33

$$\frac{\sqrt{b\sec(c+dx)}\tanh^{-1}(\sin(c+dx))}{d\sqrt{\sec(c+dx)}}$$

```
[Out] (ArcTanh[Sin[c + d*x]]*Sqrt[b*Sec[c + d*x]])/(d*Sqrt[Sec[c + d*x]])
```

Rubi [A] time = 0.006833, antiderivative size = 33, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {17, 3770}

$$\frac{\sqrt{b\sec(c+dx)}\tanh^{-1}(\sin(c+dx))}{d\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Sec[c + d*x]]*Sqrt[b*Sec[c + d*x]],x]
```

```
[Out] (ArcTanh[Sin[c + d*x]]*Sqrt[b*Sec[c + d*x]])/(d*Sqrt[Sec[c + d*x]])
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(a^(m + 1/
2)*b^(n - 1/2)*Sqrt[b*v])/Sqrt[a*v], Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \sqrt{\sec(c+dx)}\sqrt{b\sec(c+dx)}\,dx &= \frac{\sqrt{b\sec(c+dx)}\int \sec(c+dx)\,dx}{\sqrt{\sec(c+dx)}}\\ &= \frac{\tanh^{-1}(\sin(c+dx))\sqrt{b\sec(c+dx)}}{d\sqrt{\sec(c+dx)}}\end{aligned}$$

Mathematica [A] time = 0.0113776, size = 33, normalized size = 1.

$$\frac{\sqrt{b\sec(c+dx)}\tanh^{-1}(\sin(c+dx))}{d\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Sec[c + d*x]]*Sqrt[b*Sec[c + d*x]],x]
```

```
[Out] (ArcTanh[Sin[c + d*x]]*Sqrt[b*Sec[c + d*x]])/(d*Sqrt[Sec[c + d*x]])
```

Maple [A] time = 0.116, size = 52, normalized size = 1.6

$$-2\,\frac{\cos\left(dx+c\right)\sqrt{\left(\cos\left(dx+c\right)\right)^{-1}}}{d}\mathrm{Artanh}\left(\frac{-1+\cos\left(dx+c\right)}{\sin\left(dx+c\right)}\right)\sqrt{\frac{b}{\cos\left(dx+c\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(1/2)*(b*sec(d*x+c))^(1/2),x)
```

```
[Out] -2/d*arctanh((-1+cos(d*x+c))/sin(d*x+c))*cos(d*x+c)*(1/cos(d*x+c))^(1/2)*(b
/cos(d*x+c))^(1/2)
```

Maxima [B] time = 2.06356, size = 88, normalized size = 2.67

$$\frac{\sqrt{b}\left(\log\left(\cos\left(dx+c\right)^2+\sin\left(dx+c\right)^2+2\,\sin\left(dx+c\right)+1\right)-\log\left(\cos\left(dx+c\right)^2+\sin\left(dx+c\right)^2-2\,\sin\left(dx+c\right)+1\right)\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)*(b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(b)*(log(cos(d*x + c)^2 + sin(d*x + c)^2 + 2*sin(d*x + c) + 1) - lo
g(cos(d*x + c)^2 + sin(d*x + c)^2 - 2*sin(d*x + c) + 1))/d
```

Fricas [A] time = 1.49578, size = 290, normalized size = 8.79

$$\left[\frac{\sqrt{b}\log\left(-\frac{b\cos(dx+c)^2-2\sqrt{b}\sqrt{\frac{b}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)-2\,b}{\cos(dx+c)^2}\right)}{2\,d},\ -\frac{\sqrt{-b}\arctan\left(\frac{\sqrt{-b}\sqrt{\frac{b}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)}{b}\right)}{d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)*(b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*sqrt(b)*log(-(b*cos(d*x + c)^2 - 2*sqrt(b)*sqrt(b/cos(d*x + c))*sqrt(c
os(d*x + c))*sin(d*x + c) - 2*b)/cos(d*x + c)^2)/d, -sqrt(-b)*arctan(sqrt(-
b)*sqrt(b/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/b)/d]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{b\sec\left(c+dx\right)}\sqrt{\sec\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(1/2)*(b*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(b*sec(c + d*x))*sqrt(sec(c + d*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\sec(dx+c)}\sqrt{\sec(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)*(b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sec(d*x + c))*sqrt(sec(d*x + c)), x)
```

3.137 $\int \frac{\sqrt{b\sec(c+dx)}}{\sqrt{\sec(c+dx)}} dx$

Optimal. Leaf size=24

$$\frac{x\sqrt{b\sec(c+dx)}}{\sqrt{\sec(c+dx)}}$$

```
[Out] (x*Sqrt[b*Sec[c + d*x]])/Sqrt[Sec[c + d*x]]
```

Rubi [A] time = 0.0022498, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {17, 8}

$$\frac{x\sqrt{b\sec(c+dx)}}{\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*Sec[c + d*x]]/Sqrt[Sec[c + d*x]],x]
```

```
[Out] (x*Sqrt[b*Sec[c + d*x]])/Sqrt[Sec[c + d*x]]
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(a^(m + 1/
2)*b^(n - 1/2)*Sqrt[b*v])/Sqrt[a*v], Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] && !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{b\sec(c+dx)}}{\sqrt{\sec(c+dx)}} dx &= \frac{\sqrt{b\sec(c+dx)} \int 1 \, dx}{\sqrt{\sec(c+dx)}}\\ &= \frac{x\sqrt{b\sec(c+dx)}}{\sqrt{\sec(c+dx)}}\end{aligned}$$

Mathematica [A] time = 0.0144343, size = 24, normalized size = 1.

$$\frac{x\sqrt{b\sec(c+dx)}}{\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*Sec[c + d*x]]/Sqrt[Sec[c + d*x]],x]
```

```
[Out] (x*Sqrt[b*Sec[c + d*x]])/Sqrt[Sec[c + d*x]]
```

Maple [A] time = 0.108, size = 32, normalized size = 1.3

$$\frac{dx+c}{d}\sqrt{\frac{b}{\cos(dx+c)}}\frac{1}{\sqrt{(\cos(dx+c))^{-1}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sec(d*x+c))^(1/2)/sec(d*x+c)^(1/2),x)
```

```
[Out] 1/d*(d*x+c)*(b/cos(d*x+c))^(1/2)/(1/cos(d*x+c))^(1/2)
```

Maxima [A] time = 1.7862, size = 35, normalized size = 1.46

$$\frac{2\sqrt{b}\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))^(1/2)/sec(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] 2*sqrt(b)*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/d
```

Fricas [A] time = 1.63846, size = 266, normalized size = 11.08

$$\left[\frac{\sqrt{-b}\log\left(-2\sqrt{-b}\sqrt{\frac{b}{\cos(dx+c)}}\cos(dx+c)^{\frac{3}{2}}\sin(dx+c)+2b\cos(dx+c)^2-b\right)}{2d}, \frac{\sqrt{b}\arctan\left(\frac{\sqrt{\frac{b}{\cos(dx+c)}}\sin(dx+c)}{\sqrt{b}\sqrt{\cos(dx+c)}}\right)}{d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))^(1/2)/sec(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*sqrt(-b)*log(-2*sqrt(-b)*sqrt(b/cos(d*x + c))*cos(d*x + c)^(3/2)*sin(d
*x + c) + 2*b*cos(d*x + c)^2 - b)/d, sqrt(b)*arctan(sqrt(b/cos(d*x + c))*si
n(d*x + c)/(sqrt(b)*sqrt(cos(d*x + c))))/d]
```

Sympy [A] time = 1.55017, size = 5, normalized size = 0.21

$$\sqrt{b}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))**(1/2)/sec(d*x+c)**(1/2),x)
```

```
[Out] sqrt(b)*x
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{b\sec(dx+c)}}{\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))^(1/2)/sec(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sec(d*x + c))/sqrt(sec(d*x + c)), x)
```

3.138 $\int \frac{\sqrt{b\sec(c+dx)}}{\sec^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=32

$$\frac{\sin(c+dx)\sqrt{b\sec(c+dx)}}{d\sqrt{\sec(c+dx)}}$$

```
[Out] (Sqrt[b*Sec[c + d*x]]*Sin[c + d*x])/(d*Sqrt[Sec[c + d*x]])
```

Rubi [A] time = 0.0066801, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {17, 2637}

$$\frac{\sin(c+dx)\sqrt{b\sec(c+dx)}}{d\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*Sec[c + d*x]]/Sec[c + d*x]^(3/2),x]
```

```
[Out] (Sqrt[b*Sec[c + d*x]]*Sin[c + d*x])/(d*Sqrt[Sec[c + d*x]])
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(a^(m + 1/
2)*b^(n - 1/2)*Sqrt[b*v])/Sqrt[a*v], Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] && !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{b\sec(c+dx)}}{\sec^{\frac{3}{2}}(c+dx)}\,dx &= \frac{\sqrt{b\sec(c+dx)}\int \cos(c+dx)\,dx}{\sqrt{\sec(c+dx)}}\\ &= \frac{\sqrt{b\sec(c+dx)}\sin(c+dx)}{d\sqrt{\sec(c+dx)}}\end{aligned}$$

Mathematica [A] time = 0.0355135, size = 32, normalized size = 1.

$$\frac{\sin(c+dx)\sqrt{b\sec(c+dx)}}{d\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*Sec[c + d*x]]/Sec[c + d*x]^(3/2),x]
```

```
[Out] (Sqrt[b*Sec[c + d*x]]*Sin[c + d*x])/(d*Sqrt[Sec[c + d*x]])
```

Maple [A] time = 0.127, size = 41, normalized size = 1.3

$$\frac{\sin(dx+c)}{d\cos(dx+c)}\sqrt{\frac{b}{\cos(dx+c)}}\left((\cos(dx+c))^{-1}\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sec(d*x+c))^(1/2)/sec(d*x+c)^(3/2),x)
```

```
[Out] 1/d*sin(d*x+c)*(b/cos(d*x+c))^(1/2)/(1/cos(d*x+c))^(3/2)/cos(d*x+c)
```

Maxima [A] time = 2.05672, size = 18, normalized size = 0.56

$$\frac{\sqrt{b}\sin(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))^(1/2)/sec(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] sqrt(b)*sin(d*x + c)/d
```

Fricas [A] time = 1.40192, size = 76, normalized size = 2.38

$$\frac{\sqrt{\frac{b}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))^(1/2)/sec(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] sqrt(b/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/d
```

Sympy [A] time = 20.9001, size = 36, normalized size = 1.12

$$\begin{cases} \frac{\sqrt{b}\tan(c+dx)}{d\sec(c+dx)} & \text{for}\ d \neq 0 \\ \frac{x\sqrt{b\sec(c)}}{\sec^{\frac{3}{2}}(c)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))**(1/2)/sec(d*x+c)**(3/2),x)
```

```
[Out] Piecewise((sqrt(b)*tan(c + d*x)/(d*sec(c + d*x)), Ne(d, 0)), (x*sqrt(b*sec(
c))/sec(c)**(3/2), True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{b \sec(dx+c)}}{\sec(dx+c)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))^(1/2)/sec(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sec(d*x + c))/sec(d*x + c)^(3/2), x)
```

3.139 $\int \frac{\sqrt{b\sec(c+dx)}}{\sec^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=63

$$\frac{x\sqrt{b\sec(c+dx)}}{2\sqrt{\sec(c+dx)}}+\frac{\sin(c+dx)\sqrt{b\sec(c+dx)}}{2d\sec^{\frac{3}{2}}(c+dx)}$$

```
[Out] (x*Sqrt[b*Sec[c + d*x]])/(2*Sqrt[Sec[c + d*x]]) + (Sqrt[b*Sec[c + d*x]]*Sin
[c + d*x])/(2*d*Sec[c + d*x]^(3/2))
```

Rubi [A] time = 0.0137245, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {17, 2635, 8}

$$\frac{x\sqrt{b\sec(c+dx)}}{2\sqrt{\sec(c+dx)}}+\frac{\sin(c+dx)\sqrt{b\sec(c+dx)}}{2d\sec^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*Sec[c + d*x]]/Sec[c + d*x]^(5/2),x]
```

```
[Out] (x*Sqrt[b*Sec[c + d*x]])/(2*Sqrt[Sec[c + d*x]]) + (Sqrt[b*Sec[c + d*x]]*Sin
[c + d*x])/(2*d*Sec[c + d*x]^(3/2))
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(a^(m + 1/
2)*b^(n - 1/2)*Sqrt[b*v])/Sqrt[a*v], Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{b\sec(c+dx)}}{\sec^{\frac{5}{2}}(c+dx)}\,dx &= \frac{\sqrt{b\sec(c+dx)}\int \cos^2(c+dx)\,dx}{\sqrt{\sec(c+dx)}}\\ &= \frac{\sqrt{b\sec(c+dx)}\sin(c+dx)}{2d\sec^{\frac{3}{2}}(c+dx)}+\frac{\sqrt{b\sec(c+dx)}\int 1\,dx}{2\sqrt{\sec(c+dx)}}\\ &= \frac{x\sqrt{b\sec(c+dx)}}{2\sqrt{\sec(c+dx)}}+\frac{\sqrt{b\sec(c+dx)}\sin(c+dx)}{2d\sec^{\frac{3}{2}}(c+dx)}\end{aligned}$$

Mathematica [A] time = 0.0639679, size = 45, normalized size = 0.71

$$\frac{(2(c+dx)+\sin(2(c+dx)))\sqrt{b\sec(c+dx)}}{4d\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*Sec[c + d*x]]/Sec[c + d*x]^(5/2),x]
```

```
[Out] (Sqrt[b*Sec[c + d*x]]*(2*(c + d*x) + Sin[2*(c + d*x)]))/(4*d*Sqrt[Sec[c + d
*x]])
```

Maple [A] time = 0.14, size = 54, normalized size = 0.9

$$\frac{\cos(dx+c)\sin(dx+c)+dx+c}{2d(\cos(dx+c))^2}\sqrt{\frac{b}{\cos(dx+c)}}\left((\cos(dx+c))^{-1}\right)^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sec(d*x+c))^(1/2)/sec(d*x+c)^(5/2),x)
```

```
[Out] 1/2/d*(cos(d*x+c)*sin(d*x+c)+d*x+c)*(b/cos(d*x+c))^(1/2)/cos(d*x+c)^2/(1/co
s(d*x+c))^(5/2)
```

Maxima [A] time = 2.01943, size = 34, normalized size = 0.54

$$\frac{(2dx+2c+\sin(2dx+2c))\sqrt{b}}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))^(1/2)/sec(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/4*(2*d*x + 2*c + sin(2*d*x + 2*c))*sqrt(b)/d
```

Fricas [A] time = 1.6486, size = 428, normalized size = 6.79

$$\left[\frac{2\sqrt{\frac{b}{\cos(dx+c)}}\cos(dx+c)^{\frac{3}{2}}\sin(dx+c)+\sqrt{-b}\log\left(-2\sqrt{-b}\sqrt{\frac{b}{\cos(dx+c)}}\cos(dx+c)^{\frac{3}{2}}\sin(dx+c)+2b\cos(dx+c)^2-b\right)}{4d},\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))^(1/2)/sec(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/4*(2*sqrt(b/cos(d*x + c))*cos(d*x + c)^(3/2)*sin(d*x + c) + sqrt(-b)*log
(-2*sqrt(-b)*sqrt(b/cos(d*x + c))*cos(d*x + c)^(3/2)*sin(d*x + c) + 2*b*cos
(d*x + c)^2 - b))/d, 1/2*(sqrt(b/cos(d*x + c))*cos(d*x + c)^(3/2)*sin(d*x +
```

```
 c) + sqrt(b)*arctan(sqrt(b/cos(d*x + c))*sin(d*x + c)/(sqrt(b)*sqrt(cos(d*
x + c)))))/d]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))**(1/2)/sec(d*x+c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{b\sec(dx+c)}}{\sec(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))^(1/2)/sec(d*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sec(d*x + c))/sec(d*x + c)^(5/2), x)
```

3.140 $\int \frac{\sqrt{b \sec(c+dx)}}{\sec^{\frac{7}{2}}(c+dx)}\,dx$

Optimal. Leaf size=70

$$\frac{\sin(c+dx)\sqrt{b\sec(c+dx)}}{d\sqrt{\sec(c+dx)}}-\frac{\sin^3(c+dx)\sqrt{b\sec(c+dx)}}{3d\sqrt{\sec(c+dx)}}$$

```
[Out] (Sqrt[b*Sec[c + d*x]]*Sin[c + d*x])/(d*Sqrt[Sec[c + d*x]]) - (Sqrt[b*Sec[c
+ d*x]]*Sin[c + d*x]^3)/(3*d*Sqrt[Sec[c + d*x]])
```

Rubi [A] time = 0.0163579, antiderivative size = 70, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {17, 2633}

$$\frac{\sin(c+dx)\sqrt{b\sec(c+dx)}}{d\sqrt{\sec(c+dx)}}-\frac{\sin^3(c+dx)\sqrt{b\sec(c+dx)}}{3d\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*Sec[c + d*x]]/Sec[c + d*x]^(7/2),x]
```

```
[Out] (Sqrt[b*Sec[c + d*x]]*Sin[c + d*x])/(d*Sqrt[Sec[c + d*x]]) - (Sqrt[b*Sec[c
+ d*x]]*Sin[c + d*x]^3)/(3*d*Sqrt[Sec[c + d*x]])
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(a^(m + 1/
2)*b^(n - 1/2)*Sqrt[b*v])/Sqrt[a*v], Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] && !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{b\sec(c+dx)}}{\sec^{\frac{7}{2}}(c+dx)}\,dx &= \frac{\sqrt{b\sec(c+dx)}\int \cos^3(c+dx)\,dx}{\sqrt{\sec(c+dx)}}\\
&= -\frac{\sqrt{b\sec(c+dx)}\,\text{Subst}\left(\int \left(1-x^2\right)\,dx, x, -\sin(c+dx)\right)}{d\sqrt{\sec(c+dx)}}\\
&= \frac{\sqrt{b\sec(c+dx)}\sin(c+dx)}{d\sqrt{\sec(c+dx)}}-\frac{\sqrt{b\sec(c+dx)}\sin^3(c+dx)}{3d\sqrt{\sec(c+dx)}}
\end{aligned}$$

Mathematica [A] time = 0.108713, size = 45, normalized size = 0.64

$$\frac{\sin(c+dx)(\cos(2(c+dx))+5)\sqrt{b\sec(c+dx)}}{6d\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*Sec[c + d*x]]/Sec[c + d*x]^(7/2),x]
```

```
[Out] ((5 + Cos[2*(c + d*x)])*Sqrt[b*Sec[c + d*x]]*Sin[c + d*x])/(6*d*Sqrt[Sec[c
+ d*x]])
```

Maple [A] time = 0.135, size = 52, normalized size = 0.7

$$\frac{\left((\cos(dx+c))^2+2\right)\sin(dx+c)}{3\,d\,(\cos(dx+c))^3}\sqrt{\frac{b}{\cos(dx+c)}}\left((\cos(dx+c))^{-1}\right)^{-\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sec(d*x+c))^(1/2)/sec(d*x+c)^(7/2),x)
```

```
[Out] 1/3/d*(cos(d*x+c)^2+2)*(b/cos(d*x+c))^(1/2)*sin(d*x+c)/(1/cos(d*x+c))^(7/2)
/cos(d*x+c)^3
```

Maxima [A] time = 2.07164, size = 57, normalized size = 0.81

$$\frac{\sqrt{b}\left(\sin(3\,dx+3\,c)+9\,\sin\left(\frac{1}{3}\,\arctan(\sin(3\,dx+3\,c),\cos(3\,dx+3\,c))\right)\right)}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))^(1/2)/sec(d*x+c)^(7/2),x, algorithm="maxima")
```

```
[Out] 1/12*sqrt(b)*(sin(3*d*x + 3*c) + 9*sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*
d*x + 3*c))))/d
```

Fricas [A] time = 1.38295, size = 130, normalized size = 1.86

$$\frac{\left(\cos(dx+c)^3+2\,\cos(dx+c)\right)\sqrt{\frac{b}{\cos(dx+c)}}\,\sin(dx+c)}{3\,d\sqrt{\cos(dx+c)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))^(1/2)/sec(d*x+c)^(7/2),x, algorithm="fricas")
```

```
[Out] 1/3*(cos(d*x + c)^3 + 2*cos(d*x + c))*sqrt(b/cos(d*x + c))*sin(d*x + c)/(d*
sqrt(cos(d*x + c)))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))**(1/2)/sec(d*x+c)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{b \sec(dx+c)}}{\sec(dx+c)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))^(1/2)/sec(d*x+c)^(7/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sec(d*x + c))/sec(d*x + c)^(7/2), x)
```

3.141 $\int \frac{\sqrt{b\sec(c+dx)}}{\sec^{\frac{9}{2}}(c+dx)}\,dx$

Optimal. Leaf size=98

$$\frac{3x\sqrt{b\sec(c+dx)}}{8\sqrt{\sec(c+dx)}}+\frac{3\sin(c+dx)\sqrt{b\sec(c+dx)}}{8d\sec^{\frac{3}{2}}(c+dx)}+\frac{\sin(c+dx)\sqrt{b\sec(c+dx)}}{4d\sec^{\frac{7}{2}}(c+dx)}$$

```
[Out] (3*x*Sqrt[b*Sec[c + d*x]])/(8*Sqrt[Sec[c + d*x]]) + (Sqrt[b*Sec[c + d*x]]*S
in[c + d*x])/(4*d*Sec[c + d*x]^(7/2)) + (3*Sqrt[b*Sec[c + d*x]]*Sin[c + d*x
])/(8*d*Sec[c + d*x]^(3/2))
```

Rubi [A] time = 0.0251071, antiderivative size = 98, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {17, 2635, 8}

$$\frac{3x\sqrt{b\sec(c+dx)}}{8\sqrt{\sec(c+dx)}}+\frac{3\sin(c+dx)\sqrt{b\sec(c+dx)}}{8d\sec^{\frac{3}{2}}(c+dx)}+\frac{\sin(c+dx)\sqrt{b\sec(c+dx)}}{4d\sec^{\frac{7}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*Sec[c + d*x]]/Sec[c + d*x]^(9/2),x]
```


```
[Out] (3*x*Sqrt[b*Sec[c + d*x]])/(8*Sqrt[Sec[c + d*x]]) + (Sqrt[b*Sec[c + d*x]]*S
in[c + d*x])/(4*d*Sec[c + d*x]^(7/2)) + (3*Sqrt[b*Sec[c + d*x]]*Sin[c + d*x
])/(8*d*Sec[c + d*x]^(3/2))
```


Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(a^(m + 1/
2)*b^(n - 1/2)*Sqrt[b*v])/Sqrt[a*v], Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] && !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{\sqrt{b\sec(c+dx)}}{\sec^{\frac{9}{2}}(c+dx)}\,dx = \frac{\sqrt{b\sec(c+dx)}\int \cos^4(c+dx)\,dx}{\sqrt{\sec(c+dx)}}$$
$$= \frac{\sqrt{b\sec(c+dx)}\sin(c+dx)}{4d\sec^{\frac{7}{2}}(c+dx)} + \frac{\left(3\sqrt{b\sec(c+dx)}\right)\int \cos^2(c+dx)\,dx}{4\sqrt{\sec(c+dx)}}$$
$$= \frac{\sqrt{b\sec(c+dx)}\sin(c+dx)}{4d\sec^{\frac{7}{2}}(c+dx)} + \frac{3\sqrt{b\sec(c+dx)}\sin(c+dx)}{8d\sec^{\frac{3}{2}}(c+dx)} + \frac{\left(3\sqrt{b\sec(c+dx)}\right)\int 1\,dx}{8\sqrt{\sec(c+dx)}}$$
$$= \frac{3x\sqrt{b\sec(c+dx)}}{8\sqrt{\sec(c+dx)}} + \frac{\sqrt{b\sec(c+dx)}\sin(c+dx)}{4d\sec^{\frac{7}{2}}(c+dx)} + \frac{3\sqrt{b\sec(c+dx)}\sin(c+dx)}{8d\sec^{\frac{3}{2}}(c+dx)}$$

Mathematica [A] time = 0.116334, size = 55, normalized size = 0.56

$$\frac{(12(c+dx)+8\sin(2(c+dx))+\sin(4(c+dx)))\sqrt{b\sec(c+dx)}}{32d\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*Sec[c + d*x]]/Sec[c + d*x]^(9/2),x]
```

```
[Out] (Sqrt[b*Sec[c + d*x]]*(12*(c + d*x) + 8*Sin[2*(c + d*x)] + Sin[4*(c + d*x)]
))/(32*d*Sqrt[Sec[c + d*x]])
```

Maple [A] time = 0.157, size = 74, normalized size = 0.8

$$\frac{2\,(\cos(dx+c))^3\sin(dx+c)+3\,\cos(dx+c)\sin(dx+c)+3\,dx+3\,c}{8\,d\,(\cos(dx+c))^4}\sqrt{\frac{b}{\cos(dx+c)}}\left((\cos(dx+c))^{-1}\right)^{-\frac{9}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sec(d*x+c))^(1/2)/sec(d*x+c)^(9/2),x)
```

```
[Out] 1/8/d*(2*cos(d*x+c)^3*sin(d*x+c)+3*cos(d*x+c)*sin(d*x+c)+3*d*x+3*c)*(b/cos(
d*x+c))^(1/2)/cos(d*x+c)^4/(1/cos(d*x+c))^(9/2)
```

Maxima [A] time = 2.13244, size = 66, normalized size = 0.67

$$\frac{\left(12\,dx+12\,c+\sin(4\,dx+4\,c)+8\,\sin\left(\frac{1}{2}\,\arctan(\sin(4\,dx+4\,c),\cos(4\,dx+4\,c))\right)\right)\sqrt{b}}{32\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))^(1/2)/sec(d*x+c)^(9/2),x, algorithm="maxima")
```

```
[Out] 1/32*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(1/2*arctan2(sin(4*d*x + 4*c)
, cos(4*d*x + 4*c))))*sqrt(b)/d
```

Fricas [A] time = 1.78951, size = 537, normalized size = 5.48

$$\left[\frac{\frac{2\left(2\cos(dx+c)^4+3\cos(dx+c)^2\right)\sqrt{\frac{b}{\cos(dx+c)}}\sin(dx+c)}{\sqrt{\cos(dx+c)}}+3\sqrt{-b}\log\left(-2\sqrt{-b}\sqrt{\frac{b}{\cos(dx+c)}}\cos(dx+c)^{\frac{3}{2}}\sin(dx+c)+2b\cos(dx+c\right.}{16d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))^(1/2)/sec(d*x+c)^(9/2),x, algorithm="fricas")
```

```
[Out] [1/16*(2*(2*cos(d*x + c)^4 + 3*cos(d*x + c)^2)*sqrt(b/cos(d*x + c))*sin(d*x
 + c)/sqrt(cos(d*x + c)) + 3*sqrt(-b)*log(-2*sqrt(-b)*sqrt(b/cos(d*x + c))*
cos(d*x + c)^(3/2)*sin(d*x + c) + 2*b*cos(d*x + c)^2 - b))/d, 1/8*((2*cos(d
*x + c)^4 + 3*cos(d*x + c)^2)*sqrt(b/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*
x + c)) + 3*sqrt(b)*arctan(sqrt(b/cos(d*x + c))*sin(d*x + c)/(sqrt(b)*sqrt(
cos(d*x + c)))))/d]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))**(1/2)/sec(d*x+c)**(9/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{b\sec(dx+c)}}{\sec(dx+c)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))^(1/2)/sec(d*x+c)^(9/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sec(d*x + c))/sec(d*x + c)^(9/2), x)
```

3.142 $\int \sec^{\frac{7}{2}}(c+dx)(b\sec(c+dx))^{3/2}\,dx$

Optimal. Leaf size=110

$$\frac{b\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)\sqrt{b\sec(c+dx)}}{4d}+\frac{3b\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\sqrt{b\sec(c+dx)}}{8d}+\frac{3b\sqrt{b\sec(c+dx)}\tanh^{-1}(\sin(c}{8d\sqrt{\sec(c+dx)}}$$

```
[Out] (3*b*ArcTanh[Sin[c + d*x]]*Sqrt[b*Sec[c + d*x]])/(8*d*Sqrt[Sec[c + d*x]]) +
 (3*b*Sec[c + d*x]^(3/2)*Sqrt[b*Sec[c + d*x]]*Sin[c + d*x])/(8*d) + (b*Sec[
c + d*x]^(7/2)*Sqrt[b*Sec[c + d*x]]*Sin[c + d*x])/(4*d)
```

Rubi [A] time = 0.0348635, antiderivative size = 110, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {17, 3768, 3770}

$$\frac{b\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)\sqrt{b\sec(c+dx)}}{4d}+\frac{3b\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\sqrt{b\sec(c+dx)}}{8d}+\frac{3b\sqrt{b\sec(c+dx)}\tanh^{-1}(\sin(c}{8d\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(7/2)*(b*Sec[c + d*x])^(3/2),x]
```


```
[Out] (3*b*ArcTanh[Sin[c + d*x]]*Sqrt[b*Sec[c + d*x]])/(8*d*Sqrt[Sec[c + d*x]]) +
 (3*b*Sec[c + d*x]^(3/2)*Sqrt[b*Sec[c + d*x]]*Sin[c + d*x])/(8*d) + (b*Sec[
c + d*x]^(7/2)*Sqrt[b*Sec[c + d*x]]*Sin[c + d*x])/(4*d)
```


Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(a^(m + 1/
2)*b^(n - 1/2)*Sqrt[b*v])/Sqrt[a*v], Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sec^{\frac{7}{2}}(c+dx)(b\sec(c+dx))^{3/2}\,dx = \frac{\left(b\sqrt{b\sec(c+dx)}\right)\int \sec^5(c+dx)\,dx}{\sqrt{\sec(c+dx)}}$$
$$= \frac{b\sec^{\frac{7}{2}}(c+dx)\sqrt{b\sec(c+dx)}\sin(c+dx)}{4d} + \frac{\left(3b\sqrt{b\sec(c+dx)}\right)\int \sec^3(c+dx)\,d}{4\sqrt{\sec(c+dx)}}$$
$$= \frac{3b\sec^{\frac{3}{2}}(c+dx)\sqrt{b\sec(c+dx)}\sin(c+dx)}{8d} + \frac{b\sec^{\frac{7}{2}}(c+dx)\sqrt{b\sec(c+dx)}\sin(}{4d}$$
$$= \frac{3b\tanh^{-1}(\sin(c+dx))\sqrt{b\sec(c+dx)}}{8d\sqrt{\sec(c+dx)}} + \frac{3b\sec^{\frac{3}{2}}(c+dx)\sqrt{b\sec(c+dx)}\sin(c+}{8d}$$

Mathematica [A] time = 0.11093, size = 64, normalized size = 0.58

$$\frac{(b\sec(c+dx))^{3/2}\left(3\tanh^{-1}(\sin(c+dx)) + \tan(c+dx)\sec(c+dx)\left(2\sec^2(c+dx)+3\right)\right)}{8d\sec^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^(7/2)*(b*Sec[c + d*x])^(3/2),x]
```

```
[Out] ((b*Sec[c + d*x])^(3/2)*(3*ArcTanh[Sin[c + d*x]] + Sec[c + d*x]*(3 + 2*Sec[
c + d*x]^2)*Tan[c + d*x]))/(8*d*Sec[c + d*x]^(3/2))
```

Maple [A] time = 0.139, size = 131, normalized size = 1.2

$$-\frac{\cos(dx+c)}{8d}\left(3(\cos(dx+c))^4\ln\left(-\frac{-1+\cos(dx+c)+\sin(dx+c)}{\sin(dx+c)}\right) - 3(\cos(dx+c))^4\ln\left(-\frac{-1+\cos(dx+c)-\mathrm{si}}{\sin(dx+c)}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(7/2)*(b*sec(d*x+c))^(3/2),x)
```

```
[Out] -1/8/d*(3*cos(d*x+c)^4*ln(-(-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))-3*cos(d*x
+c)^4*ln(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))-3*cos(d*x+c)^2*sin(d*x+c)-
2*sin(d*x+c))*cos(d*x+c)*(1/cos(d*x+c))^(7/2)*(b/cos(d*x+c))^(3/2)
```

Maxima [B] time = 2.63137, size = 2352, normalized size = 21.38

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(7/2)*(b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] -1/16*(12*(b*sin(8*d*x + 8*c) + 4*b*sin(6*d*x + 6*c) + 6*b*sin(4*d*x + 4*c)
 + 4*b*sin(2*d*x + 2*c))*cos(7/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
)) + 44*(b*sin(8*d*x + 8*c) + 4*b*sin(6*d*x + 6*c) + 6*b*sin(4*d*x + 4*c) +
 4*b*sin(2*d*x + 2*c))*cos(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))
 - 44*(b*sin(8*d*x + 8*c) + 4*b*sin(6*d*x + 6*c) + 6*b*sin(4*d*x + 4*c) + 4
```

```
*b*sin(2*d*x + 2*c))*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) -
 12*(b*sin(8*d*x + 8*c) + 4*b*sin(6*d*x + 6*c) + 6*b*sin(4*d*x + 4*c) + 4*b
*sin(2*d*x + 2*c))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 3
*(b*cos(8*d*x + 8*c)^2 + 16*b*cos(6*d*x + 6*c)^2 + 36*b*cos(4*d*x + 4*c)^2
+ 16*b*cos(2*d*x + 2*c)^2 + b*sin(8*d*x + 8*c)^2 + 16*b*sin(6*d*x + 6*c)^2
+ 36*b*sin(4*d*x + 4*c)^2 + 48*b*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 16*b*s
in(2*d*x + 2*c)^2 + 2*(4*b*cos(6*d*x + 6*c) + 6*b*cos(4*d*x + 4*c) + 4*b*co
s(2*d*x + 2*c) + b)*cos(8*d*x + 8*c) + 8*(6*b*cos(4*d*x + 4*c) + 4*b*cos(2*
d*x + 2*c) + b)*cos(6*d*x + 6*c) + 12*(4*b*cos(2*d*x + 2*c) + b)*cos(4*d*x
+ 4*c) + 8*b*cos(2*d*x + 2*c) + 4*(2*b*sin(6*d*x + 6*c) + 3*b*sin(4*d*x + 4
*c) + 2*b*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + 16*(3*b*sin(4*d*x + 4*c) + 2
*b*sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + b)*log(cos(1/2*arctan2(sin(2*d*x +
2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
2*c)))^2 + 2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) + 3*
(b*cos(8*d*x + 8*c)^2 + 16*b*cos(6*d*x + 6*c)^2 + 36*b*cos(4*d*x + 4*c)^2 +
 16*b*cos(2*d*x + 2*c)^2 + b*sin(8*d*x + 8*c)^2 + 16*b*sin(6*d*x + 6*c)^2 +
 36*b*sin(4*d*x + 4*c)^2 + 48*b*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 16*b*si
n(2*d*x + 2*c)^2 + 2*(4*b*cos(6*d*x + 6*c) + 6*b*cos(4*d*x + 4*c) + 4*b*cos
(2*d*x + 2*c) + b)*cos(8*d*x + 8*c) + 8*(6*b*cos(4*d*x + 4*c) + 4*b*cos(2*d
*x + 2*c) + b)*cos(6*d*x + 6*c) + 12*(4*b*cos(2*d*x + 2*c) + b)*cos(4*d*x +
 4*c) + 8*b*cos(2*d*x + 2*c) + 4*(2*b*sin(6*d*x + 6*c) + 3*b*sin(4*d*x + 4*
c) + 2*b*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + 16*(3*b*sin(4*d*x + 4*c) + 2*
b*sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + b)*log(cos(1/2*arctan2(sin(2*d*x + 2
*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2
*c)))^2 - 2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) - 12*
(b*cos(8*d*x + 8*c) + 4*b*cos(6*d*x + 6*c) + 6*b*cos(4*d*x + 4*c) + 4*b*cos
(2*d*x + 2*c) + b)*sin(7/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 4
4*(b*cos(8*d*x + 8*c) + 4*b*cos(6*d*x + 6*c) + 6*b*cos(4*d*x + 4*c) + 4*b*c
os(2*d*x + 2*c) + b)*sin(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) +
 44*(b*cos(8*d*x + 8*c) + 4*b*cos(6*d*x + 6*c) + 6*b*cos(4*d*x + 4*c) + 4*b
*cos(2*d*x + 2*c) + b)*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))
 + 12*(b*cos(8*d*x + 8*c) + 4*b*cos(6*d*x + 6*c) + 6*b*cos(4*d*x + 4*c) + 4
*b*cos(2*d*x + 2*c) + b)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
)))*sqrt(b)/((2*(4*cos(6*d*x + 6*c) + 6*cos(4*d*x + 4*c) + 4*cos(2*d*x + 2*
c) + 1)*cos(8*d*x + 8*c) + cos(8*d*x + 8*c)^2 + 8*(6*cos(4*d*x + 4*c) + 4*c
os(2*d*x + 2*c) + 1)*cos(6*d*x + 6*c) + 16*cos(6*d*x + 6*c)^2 + 12*(4*cos(2
*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + 36*cos(4*d*x + 4*c)^2 + 16*cos(2*d*x +
2*c)^2 + 4*(2*sin(6*d*x + 6*c) + 3*sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*s
in(8*d*x + 8*c) + sin(8*d*x + 8*c)^2 + 16*(3*sin(4*d*x + 4*c) + 2*sin(2*d*x
 + 2*c))*sin(6*d*x + 6*c) + 16*sin(6*d*x + 6*c)^2 + 36*sin(4*d*x + 4*c)^2 +
 48*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 16*sin(2*d*x + 2*c)^2 + 8*cos(2*d*x
 + 2*c) + 1)*d)
```

Fricas [A] time = 1.58469, size = 621, normalized size = 5.65

$$\left[\frac{3\,b^{\frac{3}{2}}\cos\left(dx+c\right)^{3}\log\left(-\frac{b\cos(dx+c)^{2}-2\sqrt{b}\sqrt{\frac{b}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)-2\,b}{\cos(dx+c)^{2}}\right)+\frac{2\left(3\,b\cos(dx+c)^{2}+2\,b\right)\sqrt{\frac{b}{\cos(dx+c)}}\sin(dx+c)}{\sqrt{\cos(dx+c)}}}{16\,d\cos\left(dx+c\right)^{3}},-\frac{3\sqrt{-b}b\,\mathrm{arc}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(7/2)*(b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [1/16*(3*b^(3/2)*cos(d*x + c)^3*log(-(b*cos(d*x + c)^2 - 2*sqrt(b)*sqrt(b/c
os(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) - 2*b)/cos(d*x + c)^2) + 2*(3*
```

```
b*cos(d*x + c)^2 + 2*b)*sqrt(b/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)
))/(d*cos(d*x + c)^3), -1/8*(3*sqrt(-b)*b*arctan(sqrt(-b)*sqrt(b/cos(d*x +
c))*sqrt(cos(d*x + c))*sin(d*x + c)/b)*cos(d*x + c)^3 - (3*b*cos(d*x + c)^2
 + 2*b)*sqrt(b/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x +
c)^3)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(7/2)*(b*sec(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b\sec(dx+c))^{\frac{3}{2}}\sec(dx+c)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(7/2)*(b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c))^(3/2)*sec(d*x + c)^(7/2), x)
```

3.143 $\int \sec^{\frac{5}{2}}(c+dx)(b\sec(c+dx))^{3/2}\,dx$

Optimal. Leaf size=72

$$\frac{b\sin^3(c+dx)\sec^{\frac{5}{2}}(c+dx)\sqrt{b\sec(c+dx)}}{3d}+\frac{b\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{b\sec(c+dx)}}{d}$$

```
[Out] (b*Sqrt[Sec[c + d*x]]*Sqrt[b*Sec[c + d*x]]*Sin[c + d*x])/d + (b*Sec[c + d*x
]^(5/2)*Sqrt[b*Sec[c + d*x]]*Sin[c + d*x]^3)/(3*d)
```

Rubi [A] time = 0.0169584, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {17, 3767}

$$\frac{b\sin^3(c+dx)\sec^{\frac{5}{2}}(c+dx)\sqrt{b\sec(c+dx)}}{3d}+\frac{b\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{b\sec(c+dx)}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(5/2)*(b*Sec[c + d*x])^(3/2),x]
```

```
[Out] (b*Sqrt[Sec[c + d*x]]*Sqrt[b*Sec[c + d*x]]*Sin[c + d*x])/d + (b*Sec[c + d*x
]^(5/2)*Sqrt[b*Sec[c + d*x]]*Sin[c + d*x]^3)/(3*d)
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(a^(m + 1/
2)*b^(n - 1/2)*Sqrt[b*v])/Sqrt[a*v], Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] && !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}\int \sec^{\frac{5}{2}}(c+dx)(b\sec(c+dx))^{3/2}\,dx &= \frac{\left(b\sqrt{b\sec(c+dx)}\right)\int \sec^4(c+dx)\,dx}{\sqrt{\sec(c+dx)}}\\ &= -\frac{\left(b\sqrt{b\sec(c+dx)}\right)\operatorname{Subst}\left(\int \left(1+x^2\right)\,dx,x,-\tan(c+dx)\right)}{d\sqrt{\sec(c+dx)}}\\ &= \frac{b\sqrt{\sec(c+dx)}\sqrt{b\sec(c+dx)}\sin(c+dx)}{d}+\frac{b\sec^{\frac{5}{2}}(c+dx)\sqrt{b\sec(c+dx)}\sin^3(c+}{3d}\end{aligned}$$

Mathematica [A] time = 0.088353, size = 45, normalized size = 0.62

$$\frac{\left(\frac{1}{3}\tan^3(c+dx)+\tan(c+dx)\right)(b\sec(c+dx))^{3/2}}{d\sec^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^(5/2)*(b*Sec[c + d*x])^(3/2),x]
```

```
[Out] ((b*Sec[c + d*x])^(3/2)*(Tan[c + d*x] + Tan[c + d*x]^3/3))/(d*Sec[c + d*x]^
(3/2))
```

Maple [A] time = 0.118, size = 52, normalized size = 0.7

$$\frac{\left(2\left(\cos\left(dx+c\right)\right)^{2}+1\right)\cos\left(dx+c\right)\sin\left(dx+c\right)}{3d}\left(\left(\cos\left(dx+c\right)\right)^{-1}\right)^{\frac{5}{2}}\left(\frac{b}{\cos\left(dx+c\right)}\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(5/2)*(b*sec(d*x+c))^(3/2),x)
```

```
[Out] 1/3/d*(2*cos(d*x+c)^2+1)*cos(d*x+c)*sin(d*x+c)*(1/cos(d*x+c))^(5/2)*(b/cos(
d*x+c))^(3/2)
```

Maxima [B] time = 2.10283, size = 404, normalized size = 5.61

$$-\frac{4\left(3\,b\cos\left(6\,dx+6\,c\right)\sin\left(\right.\right.}{3\left(2\left(3\,\cos\left(4\,dx+4\,c\right)+3\,\cos\left(2\,dx+2\,c\right)+1\right)\cos\left(6\,dx+6\,c\right)+\cos\left(6\,dx+6\,c\right)^{2}+6\left(3\,\cos\left(2\,dx+2\,c\right)+1\right)\cos\left(4\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)*(b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] -4/3*(3*b*cos(6*d*x + 6*c)*sin(2*d*x + 2*c) + 9*b*cos(4*d*x + 4*c)*sin(2*d*
x + 2*c) - (3*b*cos(2*d*x + 2*c) + b)*sin(6*d*x + 6*c) - 3*(3*b*cos(2*d*x +
 2*c) + b)*sin(4*d*x + 4*c))*sqrt(b)/((2*(3*cos(4*d*x + 4*c) + 3*cos(2*d*x
+ 2*c) + 1)*cos(6*d*x + 6*c) + cos(6*d*x + 6*c)^2 + 6*(3*cos(2*d*x + 2*c) +
 1)*cos(4*d*x + 4*c) + 9*cos(4*d*x + 4*c)^2 + 9*cos(2*d*x + 2*c)^2 + 6*(sin
(4*d*x + 4*c) + sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + sin(6*d*x + 6*c)^2 + 9
*sin(4*d*x + 4*c)^2 + 18*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 9*sin(2*d*x +
2*c)^2 + 6*cos(2*d*x + 2*c) + 1)*d)
```

Fricas [A] time = 1.35406, size = 117, normalized size = 1.62

$$\frac{\left(2\,b\cos\left(dx+c\right)^{2}+b\right)\sqrt{\frac{b}{\cos\left(dx+c\right)}}\sin\left(dx+c\right)}{3\,d\cos\left(dx+c\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)*(b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/3*(2*b*cos(d*x + c)^2 + b)*sqrt(b/cos(d*x + c))*sin(d*x + c)/(d*cos(d*x +
 c)^(5/2))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(5/2)*(b*sec(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b\sec(dx+c))^{\frac{3}{2}}\sec(dx+c)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)*(b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c))^(3/2)*sec(d*x + c)^(5/2), x)
```

3.144 $\int \sec^{\frac{3}{2}}(c+dx)(b\sec(c+dx))^{3/2}\,dx$

Optimal. Leaf size=74

$$\frac{b\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\sqrt{b\sec(c+dx)}}{2d}+\frac{b\sqrt{b\sec(c+dx)}\tanh^{-1}(\sin(c+dx))}{2d\sqrt{\sec(c+dx)}}$$

```
[Out] (b*ArcTanh[Sin[c + d*x]]*Sqrt[b*Sec[c + d*x]])/(2*d*Sqrt[Sec[c + d*x]]) + (
b*Sec[c + d*x]^(3/2)*Sqrt[b*Sec[c + d*x]]*Sin[c + d*x])/(2*d)
```

Rubi [A] time = 0.0207255, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {17, 3768, 3770}

$$\frac{b\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\sqrt{b\sec(c+dx)}}{2d}+\frac{b\sqrt{b\sec(c+dx)}\tanh^{-1}(\sin(c+dx))}{2d\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(3/2)*(b*Sec[c + d*x])^(3/2),x]
```

```
[Out] (b*ArcTanh[Sin[c + d*x]]*Sqrt[b*Sec[c + d*x]])/(2*d*Sqrt[Sec[c + d*x]]) + (
b*Sec[c + d*x]^(3/2)*Sqrt[b*Sec[c + d*x]]*Sin[c + d*x])/(2*d)
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(a^(m + 1/
2)*b^(n - 1/2)*Sqrt[b*v])/Sqrt[a*v], Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] && !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \sec^{\frac{3}{2}}(c+dx)(b\sec(c+dx))^{3/2}\,dx &= \frac{\left(b\sqrt{b\sec(c+dx)}\right)\int \sec^3(c+dx)\,dx}{\sqrt{\sec(c+dx)}}\\ &= \frac{b\sec^{\frac{3}{2}}(c+dx)\sqrt{b\sec(c+dx)}\sin(c+dx)}{2d}+\frac{\left(b\sqrt{b\sec(c+dx)}\right)\int \sec(c+dx)\,dx}{2\sqrt{\sec(c+dx)}}\\ &= \frac{b\tanh^{-1}(\sin(c+dx))\sqrt{b\sec(c+dx)}}{2d\sqrt{\sec(c+dx)}}+\frac{b\sec^{\frac{3}{2}}(c+dx)\sqrt{b\sec(c+dx)}\sin(c+dx)}{2d}\end{aligned}$$

Mathematica [A] time = 0.0584711, size = 50, normalized size = 0.68

$$\frac{(b\sec(c+dx))^{3/2}\left(\tanh^{-1}(\sin(c+dx))+\tan(c+dx)\sec(c+dx)\right)}{2d\sec^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^(3/2)*(b*Sec[c + d*x])^(3/2),x]
```

```
[Out] ((b*Sec[c + d*x])^(3/2)*(ArcTanh[Sin[c + d*x]] + Sec[c + d*x]*Tan[c + d*x])
)/(2*d*Sec[c + d*x]^(3/2))
```

Maple [A] time = 0.117, size = 112, normalized size = 1.5

$$\frac{\cos(dx+c)}{2d}\left(\ln\left(-\frac{-1+\cos(dx+c)-\sin(dx+c)}{\sin(dx+c)}\right)(\cos(dx+c))^2-\ln\left(-\frac{-1+\cos(dx+c)+\sin(dx+c)}{\sin(dx+c)}\right)(\cos(dx+c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(3/2)*(b*sec(d*x+c))^(3/2),x)
```

```
[Out] 1/2/d*(ln(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))*cos(d*x+c)^2-ln(-(-1+cos(
d*x+c)+sin(d*x+c))/sin(d*x+c))*cos(d*x+c)^2+sin(d*x+c))*cos(d*x+c)*(1/cos(d
*x+c))^(3/2)*(b/cos(d*x+c))^(3/2)
```

Maxima [B] time = 2.19128, size = 933, normalized size = 12.61

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] -1/4*(4*(b*sin(4*d*x + 4*c) + 2*b*sin(2*d*x + 2*c))*cos(3/2*arctan2(sin(2*d
*x + 2*c), cos(2*d*x + 2*c))) - 4*(b*sin(4*d*x + 4*c) + 2*b*sin(2*d*x + 2*c
))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - (b*cos(4*d*x + 4*
c)^2 + 4*b*cos(2*d*x + 2*c)^2 + b*sin(4*d*x + 4*c)^2 + 4*b*sin(4*d*x + 4*c)
*sin(2*d*x + 2*c) + 4*b*sin(2*d*x + 2*c)^2 + 2*(2*b*cos(2*d*x + 2*c) + b)*c
os(4*d*x + 4*c) + 4*b*cos(2*d*x + 2*c) + b)*log(cos(1/2*arctan2(sin(2*d*x +
 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
 2*c)))^2 + 2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) + (
b*cos(4*d*x + 4*c)^2 + 4*b*cos(2*d*x + 2*c)^2 + b*sin(4*d*x + 4*c)^2 + 4*b*
sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*b*sin(2*d*x + 2*c)^2 + 2*(2*b*cos(2*d
*x + 2*c) + b)*cos(4*d*x + 4*c) + 4*b*cos(2*d*x + 2*c) + b)*log(cos(1/2*arc
tan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2
*c), cos(2*d*x + 2*c)))^2 - 2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
 2*c))) + 1) - 4*(b*cos(4*d*x + 4*c) + 2*b*cos(2*d*x + 2*c) + b)*sin(3/2*ar
ctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*(b*cos(4*d*x + 4*c) + 2*b*co
s(2*d*x + 2*c) + b)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*s
qrt(b)/((2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 +
 4*cos(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x +
 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c) + 1)*d)
```

Fricas [A] time = 1.64844, size = 536, normalized size = 7.24

$$\left[\frac{b^{\frac{3}{2}}\cos\left(dx+c\right)\log\left(-\frac{b\cos(dx+c)^{2}-2\sqrt{b}\sqrt{\frac{b}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)-2\,b}{\cos(dx+c)^{2}}\right)+\frac{2\,b\sqrt{\frac{b}{\cos(dx+c)}}\sin(dx+c)}{\sqrt{\cos(dx+c)}}}{4\,d\cos\left(dx+c\right)}, -\frac{\sqrt{-b}b\arctan\left(\frac{\sqrt{-b}\sqrt{\frac{b}{\cos(dx}}}{}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*(b^(3/2)*cos(d*x + c)*log(-(b*cos(d*x + c)^2 - 2*sqrt(b)*sqrt(b/cos(d*
x + c))*sqrt(cos(d*x + c))*sin(d*x + c) - 2*b)/cos(d*x + c)^2) + 2*b*sqrt(b
/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x + c)), -1/2*(sqr
t(-b)*b*arctan(sqrt(-b)*sqrt(b/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c
)/b)*cos(d*x + c) - b*sqrt(b/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))
/(d*cos(d*x + c))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(3/2)*(b*sec(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sec\left(dx+c\right)\right)^{\frac{3}{2}}\sec\left(dx+c\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c))^(3/2)*sec(d*x + c)^(3/2), x)
```

3.145 $\int \sqrt{\sec(c+dx)}(b\sec(c+dx))^{3/2}\,dx$

Optimal. Leaf size=33

$$\frac{b\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{b\sec(c+dx)}}{d}$$

```
[Out] (b*Sqrt[Sec[c + d*x]]*Sqrt[b*Sec[c + d*x]]*Sin[c + d*x])/d
```

Rubi [A] time = 0.0113825, antiderivative size = 33, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {17, 3767, 8}

$$\frac{b\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{b\sec(c+dx)}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Sec[c + d*x]]*(b*Sec[c + d*x])^(3/2),x]
```

```
[Out] (b*Sqrt[Sec[c + d*x]]*Sqrt[b*Sec[c + d*x]]*Sin[c + d*x])/d
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(a^(m + 1/
2)*b^(n - 1/2)*Sqrt[b*v])/Sqrt[a*v], Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}
\int \sqrt{\sec(c+dx)}(b\sec(c+dx))^{3/2}\,dx &= \frac{\left(b\sqrt{b\sec(c+dx)}\right)\int \sec^2(c+dx)\,dx}{\sqrt{\sec(c+dx)}}\\
&= -\frac{\left(b\sqrt{b\sec(c+dx)}\right)\text{Subst}\left(\int 1\,dx, x, -\tan(c+dx)\right)}{d\sqrt{\sec(c+dx)}}\\
&= \frac{b\sqrt{\sec(c+dx)}\sqrt{b\sec(c+dx)}\sin(c+dx)}{d}
\end{aligned}$$

Mathematica [A] time = 0.0203905, size = 32, normalized size = 0.97

$$\frac{\sin(c+dx)(b\sec(c+dx))^{3/2}}{d\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Sec[c + d*x]]*(b*Sec[c + d*x])^(3/2),x]
```

```
[Out] ((b*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(d*Sqrt[Sec[c + d*x]])
```

Maple [A] time = 0.113, size = 39, normalized size = 1.2

$$\frac{\cos(dx+c)\sin(dx+c)}{d}\sqrt{(\cos(dx+c))^{-1}}\left(\frac{b}{\cos(dx+c)}\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(1/2)*(b*sec(d*x+c))^(3/2),x)
```

```
[Out] 1/d*(1/cos(d*x+c))^(1/2)*(b/cos(d*x+c))^(3/2)*cos(d*x+c)*sin(d*x+c)
```

Maxima [A] time = 2.03621, size = 73, normalized size = 2.21

$$\frac{2\,b^{\frac{3}{2}}\sin(2\,dx+2\,c)}{\left(\cos(2\,dx+2\,c)^{2}+\sin(2\,dx+2\,c)^{2}+2\,\cos(2\,dx+2\,c)+1\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)*(b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] 2*b^(3/2)*sin(2*d*x + 2*c)/((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*co
s(2*d*x + 2*c) + 1)*d)
```

Fricas [A] time = 1.41589, size = 81, normalized size = 2.45

$$\frac{b\sqrt{\frac{b}{\cos(dx+c)}}\sin(dx+c)}{d\sqrt{\cos(dx+c)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)*(b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] b*sqrt(b/cos(d*x + c))*sin(d*x + c)/(d*sqrt(cos(d*x + c)))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(1/2)*(b*sec(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b\sec(dx+c))^{\frac{3}{2}}\sqrt{\sec(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)*(b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c))^(3/2)*sqrt(sec(d*x + c)), x)
```

3.146 $\int \frac{(b\sec(c+dx))^{3/2}}{\sqrt{\sec(c+dx)}}\,dx$

Optimal. Leaf size=34

$$\frac{b\sqrt{b\sec(c+dx)}\tanh^{-1}(\sin(c+dx))}{d\sqrt{\sec(c+dx)}}$$

```
[Out] (b*ArcTanh[Sin[c + d*x]]*Sqrt[b*Sec[c + d*x]])/(d*Sqrt[Sec[c + d*x]])
```

Rubi [A] time = 0.0074038, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {17, 3770}

$$\frac{b\sqrt{b\sec(c+dx)}\tanh^{-1}(\sin(c+dx))}{d\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Sec[c + d*x])^(3/2)/Sqrt[Sec[c + d*x]],x]
```

```
[Out] (b*ArcTanh[Sin[c + d*x]]*Sqrt[b*Sec[c + d*x]])/(d*Sqrt[Sec[c + d*x]])
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(a^(m + 1/
2)*b^(n - 1/2)*Sqrt[b*v])/Sqrt[a*v], Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] && !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{(b\sec(c+dx))^{3/2}}{\sqrt{\sec(c+dx)}}\,dx &= \frac{\left(b\sqrt{b\sec(c+dx)}\right)\int \sec(c+dx)\,dx}{\sqrt{\sec(c+dx)}}\\ &= \frac{b\tanh^{-1}(\sin(c+dx))\sqrt{b\sec(c+dx)}}{d\sqrt{\sec(c+dx)}}\end{aligned}$$

Mathematica [A] time = 0.0210147, size = 33, normalized size = 0.97

$$\frac{(b\sec(c+dx))^{3/2}\tanh^{-1}(\sin(c+dx))}{d\sec^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Sec[c + d*x])^(3/2)/Sqrt[Sec[c + d*x]],x]
```

```
[Out] (ArcTanh[Sin[c + d*x]]*(b*Sec[c + d*x])^(3/2))/(d*Sec[c + d*x]^(3/2))
```

Maple [A] time = 0.117, size = 52, normalized size = 1.5

$$-2\,\frac{\cos\left(dx+c\right)}{d\sqrt{\left(\cos\left(dx+c\right)\right)^{-1}}}\mathrm{Artanh}\left(\frac{-1+\cos\left(dx+c\right)}{\sin\left(dx+c\right)}\right)\left(\frac{b}{\cos\left(dx+c\right)}\right)^{3/2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sec(d*x+c))^(3/2)/sec(d*x+c)^(1/2),x)
```

```
[Out] -2/d*arctanh((-1+cos(d*x+c))/sin(d*x+c))*cos(d*x+c)*(b/cos(d*x+c))^(3/2)/(1
/cos(d*x+c))^(1/2)
```

Maxima [B] time = 2.06124, size = 92, normalized size = 2.71

$$\frac{\left(b\log\left(\cos\left(dx+c\right)^{2}+\sin\left(dx+c\right)^{2}+2\,\sin\left(dx+c\right)+1\right)-b\log\left(\cos\left(dx+c\right)^{2}+\sin\left(dx+c\right)^{2}-2\,\sin\left(dx+c\right)+1\right)\right)\sqrt{b}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))^(3/2)/sec(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*(b*log(cos(d*x + c)^2 + sin(d*x + c)^2 + 2*sin(d*x + c) + 1) - b*log(co
s(d*x + c)^2 + sin(d*x + c)^2 - 2*sin(d*x + c) + 1))*sqrt(b)/d
```

Fricas [A] time = 1.78313, size = 293, normalized size = 8.62

$$\left[\frac{b^{\frac{3}{2}}\log\left(-\frac{b\cos(dx+c)^{2}-2\,\sqrt{b}\sqrt{\frac{b}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)-2\,b}{\cos(dx+c)^{2}}\right)}{2\,d},-\frac{\sqrt{-b}b\arctan\left(\frac{\sqrt{-b}\sqrt{\frac{b}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)}{b}\right)}{d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))^(3/2)/sec(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*b^(3/2)*log(-(b*cos(d*x + c)^2 - 2*sqrt(b)*sqrt(b/cos(d*x + c))*sqrt(c
os(d*x + c))*sin(d*x + c) - 2*b)/cos(d*x + c)^2)/d, -sqrt(-b)*b*arctan(sqrt
(-b)*sqrt(b/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/b)/d]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(b\sec\left(c+dx\right)\right)^{\frac{3}{2}}}{\sqrt{\sec\left(c+dx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))**(3/2)/sec(d*x+c)**(1/2),x)
```

```
[Out] Integral((b*sec(c + d*x))**(3/2)/sqrt(sec(c + d*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\sec(dx+c))^{\frac{3}{2}}}{\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))^(3/2)/sec(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c))^(3/2)/sqrt(sec(d*x + c)), x)
```

3.147 $\int \frac{(b\sec(c+dx))^{3/2}}{\sec^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=25

$$\frac{bx\sqrt{b\sec(c+dx)}}{\sqrt{\sec(c+dx)}}$$

```
[Out] (b*x*Sqrt[b*Sec[c + d*x]])/Sqrt[Sec[c + d*x]]
```

Rubi [A] time = 0.002594, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {17, 8}

$$\frac{bx\sqrt{b\sec(c+dx)}}{\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Sec[c + d*x])^(3/2)/Sec[c + d*x]^(3/2),x]
```

```
[Out] (b*x*Sqrt[b*Sec[c + d*x]])/Sqrt[Sec[c + d*x]]
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(a^(m + 1/
2)*b^(n - 1/2)*Sqrt[b*v])/Sqrt[a*v], Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] && !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \frac{(b\sec(c+dx))^{3/2}}{\sec^{\frac{3}{2}}(c+dx)}\,dx &= \frac{\left(b\sqrt{b\sec(c+dx)}\right)\int 1\,dx}{\sqrt{\sec(c+dx)}}\\ &= \frac{bx\sqrt{b\sec(c+dx)}}{\sqrt{\sec(c+dx)}}\end{aligned}$$

Mathematica [A] time = 0.0245737, size = 24, normalized size = 0.96

$$\frac{x(b\sec(c+dx))^{3/2}}{\sec^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Sec[c + d*x])^(3/2)/Sec[c + d*x]^(3/2),x]
```

```
[Out] (x*(b*Sec[c + d*x])^(3/2))/Sec[c + d*x]^(3/2)
```

Maple [A] time = 0.089, size = 32, normalized size = 1.3

$$\frac{dx+c}{d}\left(\frac{b}{\cos\left(dx+c\right)}\right)^{\frac{3}{2}}\left(\left(\cos\left(dx+c\right)\right)^{-1}\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sec(d*x+c))^(3/2)/sec(d*x+c)^(3/2),x)
```

```
[Out] 1/d*(d*x+c)*(b/cos(d*x+c))^(3/2)/(1/cos(d*x+c))^(3/2)
```

Maxima [A] time = 1.78698, size = 35, normalized size = 1.4

$$\frac{2\,b^{\frac{3}{2}}\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))^(3/2)/sec(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] 2*b^(3/2)*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/d
```

Fricas [A] time = 1.85659, size = 269, normalized size = 10.76

$$\left[\frac{\sqrt{-b}b\log\left(-2\sqrt{-b}\sqrt{\frac{b}{\cos(dx+c)}}\cos\left(dx+c\right)^{\frac{3}{2}}\sin\left(dx+c\right)+2\,b\cos\left(dx+c\right)^{2}-b\right)}{2\,d}, \frac{b^{\frac{3}{2}}\arctan\left(\frac{\sqrt{\frac{b}{\cos(dx+c)}}\sin(dx+c)}{\sqrt{b}\sqrt{\cos(dx+c)}}\right)}{d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))^(3/2)/sec(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*sqrt(-b)*b*log(-2*sqrt(-b)*sqrt(b/cos(d*x + c))*cos(d*x + c)^(3/2)*sin
(d*x + c) + 2*b*cos(d*x + c)^2 - b)/d, b^(3/2)*arctan(sqrt(b/cos(d*x + c))*
sin(d*x + c)/(sqrt(b)*sqrt(cos(d*x + c))))/d]
```

Sympy [A] time = 38.5588, size = 5, normalized size = 0.2

$$b^{\frac{3}{2}}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))**(3/2)/sec(d*x+c)**(3/2),x)
```

```
[Out] b**(3/2)*x
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\sec(dx+c))^{\frac{3}{2}}}{\sec(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))^(3/2)/sec(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c))^(3/2)/sec(d*x + c)^(3/2), x)
```

3.148 $\int \frac{(b\sec(c+dx))^{3/2}}{\sec^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=33

$$\frac{b\sin(c+dx)\sqrt{b\sec(c+dx)}}{d\sqrt{\sec(c+dx)}}$$

```
[Out] (b*Sqrt[b*Sec[c + d*x]]*Sin[c + d*x])/(d*Sqrt[Sec[c + d*x]])
```

Rubi [A] time = 0.0074591, antiderivative size = 33, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {17, 2637}

$$\frac{b\sin(c+dx)\sqrt{b\sec(c+dx)}}{d\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Sec[c + d*x])^(3/2)/Sec[c + d*x]^(5/2),x]
```

```
[Out] (b*Sqrt[b*Sec[c + d*x]]*Sin[c + d*x])/(d*Sqrt[Sec[c + d*x]])
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(a^(m + 1/
2)*b^(n - 1/2)*Sqrt[b*v])/Sqrt[a*v], Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{(b\sec(c+dx))^{3/2}}{\sec^{\frac{5}{2}}(c+dx)}\,dx &= \frac{\left(b\sqrt{b\sec(c+dx)}\right)\int \cos(c+dx)\,dx}{\sqrt{\sec(c+dx)}}\\ &= \frac{b\sqrt{b\sec(c+dx)}\sin(c+dx)}{d\sqrt{\sec(c+dx)}}\end{aligned}$$

Mathematica [A] time = 0.0440589, size = 32, normalized size = 0.97

$$\frac{\sin(c+dx)(b\sec(c+dx))^{3/2}}{d\sec^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Sec[c + d*x])^(3/2)/Sec[c + d*x]^(5/2),x]
```

```
[Out] ((b*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(d*Sec[c + d*x]^(3/2))
```

Maple [A] time = 0.114, size = 41, normalized size = 1.2

$$\frac{\sin\left(dx+c\right)}{d\cos\left(dx+c\right)}\left(\frac{b}{\cos\left(dx+c\right)}\right)^{\frac{3}{2}}\left(\left(\cos\left(dx+c\right)\right)^{-1}\right)^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sec(d*x+c))^(3/2)/sec(d*x+c)^(5/2),x)
```

```
[Out] 1/d*sin(d*x+c)*(b/cos(d*x+c))^(3/2)/(1/cos(d*x+c))^(5/2)/cos(d*x+c)
```

Maxima [A] time = 2.00161, size = 18, normalized size = 0.55

$$\frac{b^{\frac{3}{2}}\sin\left(dx+c\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))^(3/2)/sec(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] b^(3/2)*sin(d*x + c)/d
```

Fricas [A] time = 1.57804, size = 78, normalized size = 2.36

$$\frac{b\sqrt{\frac{b}{\cos(dx+c)}}\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))^(3/2)/sec(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] b*sqrt(b/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/d
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))**(3/2)/sec(d*x+c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(b\sec\left(dx+c\right)\right)^{\frac{3}{2}}}{\sec\left(dx+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))^(3/2)/sec(d*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c))^(3/2)/sec(d*x + c)^(5/2), x)
```

3.149 $\int \frac{(b\sec(c+dx))^{3/2}}{\sec^{\frac{7}{2}}(c+dx)}\,dx$

Optimal. Leaf size=65

$$\frac{bx\sqrt{b\sec(c+dx)}}{2\sqrt{\sec(c+dx)}}+\frac{b\sin(c+dx)\sqrt{b\sec(c+dx)}}{2d\sec^{\frac{3}{2}}(c+dx)}$$

```
[Out] (b*x*Sqrt[b*Sec[c + d*x]])/(2*Sqrt[Sec[c + d*x]]) + (b*Sqrt[b*Sec[c + d*x]]
*Sin[c + d*x])/(2*d*Sec[c + d*x]^(3/2))
```

Rubi [A] time = 0.0146445, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {17, 2635, 8}

$$\frac{bx\sqrt{b\sec(c+dx)}}{2\sqrt{\sec(c+dx)}}+\frac{b\sin(c+dx)\sqrt{b\sec(c+dx)}}{2d\sec^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Sec[c + d*x])^(3/2)/Sec[c + d*x]^(7/2),x]
```

```
[Out] (b*x*Sqrt[b*Sec[c + d*x]])/(2*Sqrt[Sec[c + d*x]]) + (b*Sqrt[b*Sec[c + d*x]]
*Sin[c + d*x])/(2*d*Sec[c + d*x]^(3/2))
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(a^(m + 1/
2)*b^(n - 1/2)*Sqrt[b*v])/Sqrt[a*v], Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{(b\sec(c+dx))^{3/2}}{\sec^{\frac{7}{2}}(c+dx)}\,dx &= \frac{\left(b\sqrt{b\sec(c+dx)}\right)\int \cos^2(c+dx)\,dx}{\sqrt{\sec(c+dx)}}\\
&= \frac{b\sqrt{b\sec(c+dx)}\sin(c+dx)}{2d\sec^{\frac{3}{2}}(c+dx)}+\frac{\left(b\sqrt{b\sec(c+dx)}\right)\int 1\,dx}{2\sqrt{\sec(c+dx)}}\\
&= \frac{bx\sqrt{b\sec(c+dx)}}{2\sqrt{\sec(c+dx)}}+\frac{b\sqrt{b\sec(c+dx)}\sin(c+dx)}{2d\sec^{\frac{3}{2}}(c+dx)}
\end{aligned}$$

Mathematica [A] time = 0.0792508, size = 45, normalized size = 0.69

$$\frac{(2(c+dx)+\sin(2(c+dx)))(b\sec(c+dx))^{3/2}}{4d\sec^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Sec[c + d*x])^(3/2)/Sec[c + d*x]^(7/2),x]
```

```
[Out] ((b*Sec[c + d*x])^(3/2)*(2*(c + d*x) + Sin[2*(c + d*x)]))/(4*d*Sec[c + d*x]
^(3/2))
```

Maple [A] time = 0.124, size = 54, normalized size = 0.8

$$\frac{\cos(dx+c)\sin(dx+c)+dx+c}{2\,d\,(\cos(dx+c))^{2}}\left(\frac{b}{\cos(dx+c)}\right)^{\frac{3}{2}}\left((\cos(dx+c))^{-1}\right)^{-\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sec(d*x+c))^(3/2)/sec(d*x+c)^(7/2),x)
```

```
[Out] 1/2/d*(cos(d*x+c)*sin(d*x+c)+d*x+c)*(b/cos(d*x+c))^(3/2)/cos(d*x+c)^2/(1/co
s(d*x+c))^(7/2)
```

Maxima [A] time = 2.03789, size = 38, normalized size = 0.58

$$\frac{(2\,(dx+c)b+b\sin(2\,dx+2\,c))\sqrt{b}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))^(3/2)/sec(d*x+c)^(7/2),x, algorithm="maxima")
```

```
[Out] 1/4*(2*(d*x + c)*b + b*sin(2*d*x + 2*c))*sqrt(b)/d
```

Fricas [A] time = 1.96696, size = 436, normalized size = 6.71

$$\left[\frac{2\,b\sqrt{\frac{b}{\cos(dx+c)}}\cos(dx+c)^{\frac{3}{2}}\sin(dx+c)+\sqrt{-b}b\log\left(-2\,\sqrt{-b}\sqrt{\frac{b}{\cos(dx+c)}}\cos(dx+c)^{\frac{3}{2}}\sin(dx+c)+2\,b\cos(dx+c)^{2}\right.}{4\,d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))^(3/2)/sec(d*x+c)^(7/2),x, algorithm="fricas")
```

```
[Out] [1/4*(2*b*sqrt(b/cos(d*x + c))*cos(d*x + c)^(3/2)*sin(d*x + c) + sqrt(-b)*b
*log(-2*sqrt(-b)*sqrt(b/cos(d*x + c))*cos(d*x + c)^(3/2)*sin(d*x + c) + 2*b
```

```
*cos(d*x + c)^2 - b))/d, 1/2*(b*sqrt(b/cos(d*x + c))*cos(d*x + c)^(3/2)*sin
(d*x + c) + b^(3/2)*arctan(sqrt(b/cos(d*x + c))*sin(d*x + c)/(sqrt(b)*sqrt(
cos(d*x + c)))))/d]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))**(3/2)/sec(d*x+c)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\sec(dx+c))^{\frac{3}{2}}}{\sec(dx+c)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))^(3/2)/sec(d*x+c)^(7/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c))^(3/2)/sec(d*x + c)^(7/2), x)
```

3.150 $\int \frac{(b\sec(c+dx))^{3/2}}{\sec^{\frac{9}{2}}(c+dx)}\,dx$

Optimal. Leaf size=72

$$\frac{b\sin(c+dx)\sqrt{b\sec(c+dx)}}{d\sqrt{\sec(c+dx)}}-\frac{b\sin^3(c+dx)\sqrt{b\sec(c+dx)}}{3d\sqrt{\sec(c+dx)}}$$

```
[Out] (b*Sqrt[b*Sec[c + d*x]]*Sin[c + d*x])/(d*Sqrt[Sec[c + d*x]]) - (b*Sqrt[b*Se
c[c + d*x]]*Sin[c + d*x]^3)/(3*d*Sqrt[Sec[c + d*x]])
```

Rubi [A] time = 0.0170607, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {17, 2633}

$$\frac{b\sin(c+dx)\sqrt{b\sec(c+dx)}}{d\sqrt{\sec(c+dx)}}-\frac{b\sin^3(c+dx)\sqrt{b\sec(c+dx)}}{3d\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Sec[c + d*x])^(3/2)/Sec[c + d*x]^(9/2),x]
```

```
[Out] (b*Sqrt[b*Sec[c + d*x]]*Sin[c + d*x])/(d*Sqrt[Sec[c + d*x]]) - (b*Sqrt[b*Se
c[c + d*x]]*Sin[c + d*x]^3)/(3*d*Sqrt[Sec[c + d*x]])
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(a^(m + 1/
2)*b^(n - 1/2)*Sqrt[b*v])/Sqrt[a*v], Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] && !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(b\sec(c+dx))^{3/2}}{\sec^{\frac{9}{2}}(c+dx)}\,dx &= \frac{\left(b\sqrt{b\sec(c+dx)}\right)\int \cos^3(c+dx)\,dx}{\sqrt{\sec(c+dx)}}\\ &= -\frac{\left(b\sqrt{b\sec(c+dx)}\right)\operatorname{Subst}\left(\int \left(1-x^2\right)\,dx,x,-\sin(c+dx)\right)}{d\sqrt{\sec(c+dx)}}\\ &= \frac{b\sqrt{b\sec(c+dx)}\sin(c+dx)}{d\sqrt{\sec(c+dx)}}-\frac{b\sqrt{b\sec(c+dx)}\sin^3(c+dx)}{3d\sqrt{\sec(c+dx)}}\end{aligned}$$

Mathematica [A] time = 0.138329, size = 45, normalized size = 0.62

$$\frac{\sin(c+dx)(\cos(2(c+dx))+5)(b\sec(c+dx))^{3/2}}{6d\sec^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Sec[c + d*x])^(3/2)/Sec[c + d*x]^(9/2),x]
```

```
[Out] ((5 + Cos[2*(c + d*x)])*(b*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(6*d*Sec[c + d
*x]^(3/2))
```

Maple [A] time = 0.109, size = 52, normalized size = 0.7

$$\frac{\left((\cos(dx+c))^2+2\right)\sin(dx+c)}{3\,d\,(\cos(dx+c))^3}\left(\frac{b}{\cos(dx+c)}\right)^{\frac{3}{2}}\left((\cos(dx+c))^{-1}\right)^{-\frac{9}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sec(d*x+c))^(3/2)/sec(d*x+c)^(9/2),x)
```

```
[Out] 1/3/d*(cos(d*x+c)^2+2)*sin(d*x+c)*(b/cos(d*x+c))^(3/2)/cos(d*x+c)^3/(1/cos(
d*x+c))^(9/2)
```

Maxima [A] time = 2.08281, size = 61, normalized size = 0.85

$$\frac{\left(b\sin(3\,dx+3\,c)+9\,b\sin\left(\frac{1}{3}\,\arctan(\sin(3\,dx+3\,c)\,,\cos(3\,dx+3\,c))\right)\right)\sqrt{b}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))^(3/2)/sec(d*x+c)^(9/2),x, algorithm="maxima")
```

```
[Out] 1/12*(b*sin(3*d*x + 3*c) + 9*b*sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x
+ 3*c))))*sqrt(b)/d
```

Fricas [A] time = 1.66819, size = 135, normalized size = 1.88

$$\frac{\left(b\cos(dx+c)^3+2\,b\cos(dx+c)\right)\sqrt{\frac{b}{\cos(dx+c)}}\sin(dx+c)}{3\,d\sqrt{\cos(dx+c)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))^(3/2)/sec(d*x+c)^(9/2),x, algorithm="fricas")
```

```
[Out] 1/3*(b*cos(d*x + c)^3 + 2*b*cos(d*x + c))*sqrt(b/cos(d*x + c))*sin(d*x + c)
/(d*sqrt(cos(d*x + c)))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))**(3/2)/sec(d*x+c)**(9/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\sec(dx+c))^{\frac{3}{2}}}{\sec(dx+c)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))^(3/2)/sec(d*x+c)^(9/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c))^(3/2)/sec(d*x + c)^(9/2), x)
```

3.151 $\int \frac{(b\sec(c+dx))^{3/2}}{\sec^{\frac{11}{2}}(c+dx)}\,dx$

Optimal. Leaf size=101

$$\frac{3bx\sqrt{b\sec(c+dx)}}{8\sqrt{\sec(c+dx)}}+\frac{3b\sin(c+dx)\sqrt{b\sec(c+dx)}}{8d\sec^{\frac{3}{2}}(c+dx)}+\frac{b\sin(c+dx)\sqrt{b\sec(c+dx)}}{4d\sec^{\frac{7}{2}}(c+dx)}$$

```
[Out] (3*b*x*Sqrt[b*Sec[c + d*x]])/(8*Sqrt[Sec[c + d*x]]) + (b*Sqrt[b*Sec[c + d*x
]]*Sin[c + d*x])/(4*d*Sec[c + d*x]^(7/2)) + (3*b*Sqrt[b*Sec[c + d*x]]*Sin[c
 + d*x])/(8*d*Sec[c + d*x]^(3/2))
```

Rubi [A] time = 0.0275267, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {17, 2635, 8}

$$\frac{3bx\sqrt{b\sec(c+dx)}}{8\sqrt{\sec(c+dx)}}+\frac{3b\sin(c+dx)\sqrt{b\sec(c+dx)}}{8d\sec^{\frac{3}{2}}(c+dx)}+\frac{b\sin(c+dx)\sqrt{b\sec(c+dx)}}{4d\sec^{\frac{7}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Sec[c + d*x])^(3/2)/Sec[c + d*x]^(11/2),x]
```

```
[Out] (3*b*x*Sqrt[b*Sec[c + d*x]])/(8*Sqrt[Sec[c + d*x]]) + (b*Sqrt[b*Sec[c + d*x
]]*Sin[c + d*x])/(4*d*Sec[c + d*x]^(7/2)) + (3*b*Sqrt[b*Sec[c + d*x]]*Sin[c
 + d*x])/(8*d*Sec[c + d*x]^(3/2))
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(a^(m + 1/
2)*b^(n - 1/2)*Sqrt[b*v])/Sqrt[a*v], Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(b\sec(c+dx))^{3/2}}{\sec^{\frac{11}{2}}(c+dx)}\,dx &= \frac{\left(b\sqrt{b\sec(c+dx)}\right)\int \cos^4(c+dx)\,dx}{\sqrt{\sec(c+dx)}} \\
&= \frac{b\sqrt{b\sec(c+dx)}\sin(c+dx)}{4d\sec^{\frac{7}{2}}(c+dx)} + \frac{\left(3b\sqrt{b\sec(c+dx)}\right)\int \cos^2(c+dx)\,dx}{4\sqrt{\sec(c+dx)}} \\
&= \frac{b\sqrt{b\sec(c+dx)}\sin(c+dx)}{4d\sec^{\frac{7}{2}}(c+dx)} + \frac{3b\sqrt{b\sec(c+dx)}\sin(c+dx)}{8d\sec^{\frac{3}{2}}(c+dx)} + \frac{\left(3b\sqrt{b\sec(c+dx)}\right)\int 1\,dx}{8\sqrt{\sec(c+dx)}} \\
&= \frac{3bx\sqrt{b\sec(c+dx)}}{8\sqrt{\sec(c+dx)}} + \frac{b\sqrt{b\sec(c+dx)}\sin(c+dx)}{4d\sec^{\frac{7}{2}}(c+dx)} + \frac{3b\sqrt{b\sec(c+dx)}\sin(c+dx)}{8d\sec^{\frac{3}{2}}(c+dx)}
\end{aligned}
$$

Mathematica [A] time = 0.154344, size = 55, normalized size = 0.54

$$
\frac{(12(c+dx)+8\sin(2(c+dx))+\sin(4(c+dx)))(b\sec(c+dx))^{3/2}}{32d\sec^{\frac{3}{2}}(c+dx)}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Sec[c + d*x])^(3/2)/Sec[c + d*x]^(11/2),x]
```

```
[Out] ((b*Sec[c + d*x])^(3/2)*(12*(c + d*x) + 8*Sin[2*(c + d*x)] + Sin[4*(c + d*x
)]))/(32*d*Sec[c + d*x]^(3/2))
```

Maple [A] time = 0.123, size = 74, normalized size = 0.7

$$
\frac{2\,(\cos(dx+c))^3\sin(dx+c)+3\,\cos(dx+c)\sin(dx+c)+3\,dx+3\,c}{8\,d\,(\cos(dx+c))^4}\left(\frac{b}{\cos(dx+c)}\right)^{\frac{3}{2}}\left((\cos(dx+c))^{-1}\right)^{-\frac{11}{2}}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sec(d*x+c))^(3/2)/sec(d*x+c)^(11/2),x)
```

```
[Out] 1/8/d*(2*cos(d*x+c)^3*sin(d*x+c)+3*cos(d*x+c)*sin(d*x+c)+3*d*x+3*c)*(b/cos(
d*x+c))^(3/2)/(1/cos(d*x+c))^(11/2)/cos(d*x+c)^4
```

Maxima [A] time = 2.153, size = 72, normalized size = 0.71

$$
\frac{\left(12\,(dx+c)b+b\sin(4\,dx+4\,c)+8\,b\sin\left(\frac{1}{2}\,\arctan(\sin(4\,dx+4\,c),\cos(4\,dx+4\,c))\right)\right)\sqrt{b}}{32\,d}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))^(3/2)/sec(d*x+c)^(11/2),x, algorithm="maxima")
```

```
[Out] 1/32*(12*(d*x + c)*b + b*sin(4*d*x + 4*c) + 8*b*sin(1/2*arctan2(sin(4*d*x +
 4*c), cos(4*d*x + 4*c))))*sqrt(b)/d
```

Fricas [A] time = 2.0165, size = 551, normalized size = 5.46

$$\left[\frac{3\sqrt{-b}b\log\left(-2\sqrt{-b}\sqrt{\frac{b}{\cos(dx+c)}}\cos(dx+c)^{\frac{3}{2}}\sin(dx+c)+2b\cos(dx+c)^2-b\right)+\frac{2\left(2b\cos(dx+c)^4+3b\cos(dx+c)^2\right)\sqrt{\frac{b}{\cos(dx+c)}}\sin}{\sqrt{\cos(dx+c)}}}{16d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))^(3/2)/sec(d*x+c)^(11/2),x, algorithm="fricas")
```

```
[Out] [1/16*(3*sqrt(-b)*b*log(-2*sqrt(-b)*sqrt(b/cos(d*x + c))*cos(d*x + c)^(3/2)
*sin(d*x + c) + 2*b*cos(d*x + c)^2 - b) + 2*(2*b*cos(d*x + c)^4 + 3*b*cos(d
*x + c)^2)*sqrt(b/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/d, 1/8*(3*
b^(3/2)*arctan(sqrt(b/cos(d*x + c))*sin(d*x + c)/(sqrt(b)*sqrt(cos(d*x + c)
))) + (2*b*cos(d*x + c)^4 + 3*b*cos(d*x + c)^2)*sqrt(b/cos(d*x + c))*sin(d*
x + c)/sqrt(cos(d*x + c)))/d]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))**(3/2)/sec(d*x+c)**(11/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\sec(dx+c))^{\frac{3}{2}}}{\sec(dx+c)^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))^(3/2)/sec(d*x+c)^(11/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c))^(3/2)/sec(d*x + c)^(11/2), x)
```

3.152 $\int \sec^{\frac{7}{2}}(c+dx)(b\sec(c+dx))^{5/2}\,dx$

Optimal. Leaf size=116

$$\frac{b^2\sin^5(c+dx)\sec^{\frac{9}{2}}(c+dx)\sqrt{b\sec(c+dx)}}{5d}+\frac{2b^2\sin^3(c+dx)\sec^{\frac{5}{2}}(c+dx)\sqrt{b\sec(c+dx)}}{3d}+\frac{b^2\sin(c+dx)\sqrt{\sec(c+dx)}\ldots}{d}$$

```
[Out] (b^2*Sqrt[Sec[c + d*x]]*Sqrt[b*Sec[c + d*x]]*Sin[c + d*x])/d + (2*b^2*Sec[c
 + d*x]^(5/2)*Sqrt[b*Sec[c + d*x]]*Sin[c + d*x]^3)/(3*d) + (b^2*Sec[c + d*x
]^(9/2)*Sqrt[b*Sec[c + d*x]]*Sin[c + d*x]^5)/(5*d)
```

Rubi [A] time = 0.0239371, antiderivative size = 116, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {17, 3767}

$$\frac{b^2\sin^5(c+dx)\sec^{\frac{9}{2}}(c+dx)\sqrt{b\sec(c+dx)}}{5d}+\frac{2b^2\sin^3(c+dx)\sec^{\frac{5}{2}}(c+dx)\sqrt{b\sec(c+dx)}}{3d}+\frac{b^2\sin(c+dx)\sqrt{\sec(c+dx)}\ldots}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(7/2)*(b*Sec[c + d*x])^(5/2),x]
```

```
[Out] (b^2*Sqrt[Sec[c + d*x]]*Sqrt[b*Sec[c + d*x]]*Sin[c + d*x])/d + (2*b^2*Sec[c
 + d*x]^(5/2)*Sqrt[b*Sec[c + d*x]]*Sin[c + d*x]^3)/(3*d) + (b^2*Sec[c + d*x
]^(9/2)*Sqrt[b*Sec[c + d*x]]*Sin[c + d*x]^5)/(5*d)
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(a^(m + 1/
2)*b^(n - 1/2)*Sqrt[b*v])/Sqrt[a*v], Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}\int \sec^{\frac{7}{2}}(c+dx)(b\sec(c+dx))^{5/2}\,dx &= \frac{\left(b^2\sqrt{b\sec(c+dx)}\right)\int \sec^6(c+dx)\,dx}{\sqrt{\sec(c+dx)}}\\ &= -\frac{\left(b^2\sqrt{b\sec(c+dx)}\right)\operatorname{Subst}\left(\int\left(1+2x^2+x^4\right)dx,x,-\tan(c+dx)\right)}{d\sqrt{\sec(c+dx)}}\\ &= \frac{b^2\sqrt{\sec(c+dx)}\sqrt{b\sec(c+dx)}\sin(c+dx)}{d}+\frac{2b^2\sec^{\frac{5}{2}}(c+dx)\sqrt{b\sec(c+dx)}\sin\ldots}{3d}\end{aligned}$$

Mathematica [A] time = 0.216542, size = 57, normalized size = 0.49

$$\frac{\left(\frac{1}{5}\tan^5(c+dx)+\frac{2}{3}\tan^3(c+dx)+\tan(c+dx)\right)(b\sec(c+dx))^{5/2}}{d\sec^{\frac{5}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^(7/2)*(b*Sec[c + d*x])^(5/2),x]
```

```
[Out] ((b*Sec[c + d*x])^(5/2)*(Tan[c + d*x] + (2*Tan[c + d*x]^3)/3 + Tan[c + d*x]
^5/5))/(d*Sec[c + d*x]^(5/2))
```

Maple [A] time = 0.119, size = 62, normalized size = 0.5

$$\frac{\left(8\left(\cos\left(dx+c\right)\right)^{4}+4\left(\cos\left(dx+c\right)\right)^{2}+3\right)\cos\left(dx+c\right)\sin\left(dx+c\right)}{15\,d}\left(\left(\cos\left(dx+c\right)\right)^{-1}\right)^{\frac{7}{2}}\left(\frac{b}{\cos\left(dx+c\right)}\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(7/2)*(b*sec(d*x+c))^(5/2),x)
```

```
[Out] 1/15/d*(8*cos(d*x+c)^4+4*cos(d*x+c)^2+3)*(1/cos(d*x+c))^(7/2)*(b/cos(d*x+c)
)^(5/2)*cos(d*x+c)*sin(d*x+c)
```

Maxima [B] time = 2.53733, size = 952, normalized size = 8.21

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(7/2)*(b*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] -16/15*(5*(2*b^2*sin(4*d*x + 4*c) + b^2*sin(2*d*x + 2*c))*cos(10*d*x + 10*c
) + 25*(2*b^2*sin(4*d*x + 4*c) + b^2*sin(2*d*x + 2*c))*cos(8*d*x + 8*c) + 5
0*(2*b^2*sin(4*d*x + 4*c) + b^2*sin(2*d*x + 2*c))*cos(6*d*x + 6*c) - (10*b^
2*cos(4*d*x + 4*c) + 5*b^2*cos(2*d*x + 2*c) + b^2)*sin(10*d*x + 10*c) - 5*(
10*b^2*cos(4*d*x + 4*c) + 5*b^2*cos(2*d*x + 2*c) + b^2)*sin(8*d*x + 8*c) -
10*(10*b^2*cos(4*d*x + 4*c) + 5*b^2*cos(2*d*x + 2*c) + b^2)*sin(6*d*x + 6*c
))*sqrt(b)/((2*(5*cos(8*d*x + 8*c) + 10*cos(6*d*x + 6*c) + 10*cos(4*d*x + 4
*c) + 5*cos(2*d*x + 2*c) + 1)*cos(10*d*x + 10*c) + cos(10*d*x + 10*c)^2 + 1
0*(10*cos(6*d*x + 6*c) + 10*cos(4*d*x + 4*c) + 5*cos(2*d*x + 2*c) + 1)*cos(
8*d*x + 8*c) + 25*cos(8*d*x + 8*c)^2 + 20*(10*cos(4*d*x + 4*c) + 5*cos(2*d*
x + 2*c) + 1)*cos(6*d*x + 6*c) + 100*cos(6*d*x + 6*c)^2 + 20*(5*cos(2*d*x +
 2*c) + 1)*cos(4*d*x + 4*c) + 100*cos(4*d*x + 4*c)^2 + 25*cos(2*d*x + 2*c)^
2 + 10*(sin(8*d*x + 8*c) + 2*sin(6*d*x + 6*c) + 2*sin(4*d*x + 4*c) + sin(2*
d*x + 2*c))*sin(10*d*x + 10*c) + sin(10*d*x + 10*c)^2 + 50*(2*sin(6*d*x + 6
*c) + 2*sin(4*d*x + 4*c) + sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + 25*sin(8*d*
x + 8*c)^2 + 100*(2*sin(4*d*x + 4*c) + sin(2*d*x + 2*c))*sin(6*d*x + 6*c) +
 100*sin(6*d*x + 6*c)^2 + 100*sin(4*d*x + 4*c)^2 + 100*sin(4*d*x + 4*c)*sin
(2*d*x + 2*c) + 25*sin(2*d*x + 2*c)^2 + 10*cos(2*d*x + 2*c) + 1)*d)
```

Fricas [A] time = 1.72835, size = 158, normalized size = 1.36

$$\frac{\left(8\,b^{2}\cos\left(dx+c\right)^{4}+4\,b^{2}\cos\left(dx+c\right)^{2}+3\,b^{2}\right)\sqrt{\frac{b}{\cos\left(dx+c\right)}}\sin\left(dx+c\right)}{15\,d\cos\left(dx+c\right)^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(7/2)*(b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/15*(8*b^2*cos(d*x + c)^4 + 4*b^2*cos(d*x + c)^2 + 3*b^2)*sqrt(b/cos(d*x +
 c))*sin(d*x + c)/(d*cos(d*x + c)^(9/2))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(7/2)*(b*sec(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b\sec(dx+c))^{\frac{5}{2}}\sec(dx+c)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(7/2)*(b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c))^(5/2)*sec(d*x + c)^(7/2), x)
```

3.153 $\int \sec^{\frac{3}{2}}(c+dx)(b\sec(c+dx))^{5/2}\,dx$

Optimal. Leaf size=76

$$\frac{b^2\sin^3(c+dx)\sec^{\frac{5}{2}}(c+dx)\sqrt{b\sec(c+dx)}}{3d}+\frac{b^2\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{b\sec(c+dx)}}{d}$$

```
[Out] (b^2*Sqrt[Sec[c + d*x]]*Sqrt[b*Sec[c + d*x]]*Sin[c + d*x])/d + (b^2*Sec[c +
 d*x]^(5/2)*Sqrt[b*Sec[c + d*x]]*Sin[c + d*x]^3)/(3*d)
```

Rubi [A] time = 0.0174445, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {17, 3767}

$$\frac{b^2\sin^3(c+dx)\sec^{\frac{5}{2}}(c+dx)\sqrt{b\sec(c+dx)}}{3d}+\frac{b^2\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{b\sec(c+dx)}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(3/2)*(b*Sec[c + d*x])^(5/2),x]
```

```
[Out] (b^2*Sqrt[Sec[c + d*x]]*Sqrt[b*Sec[c + d*x]]*Sin[c + d*x])/d + (b^2*Sec[c +
 d*x]^(5/2)*Sqrt[b*Sec[c + d*x]]*Sin[c + d*x]^3)/(3*d)
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(a^(m + 1/
2)*b^(n - 1/2)*Sqrt[b*v])/Sqrt[a*v], Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}\int \sec^{\frac{3}{2}}(c+dx)(b\sec(c+dx))^{5/2}\,dx&=\frac{\left(b^2\sqrt{b\sec(c+dx)}\right)\int \sec^4(c+dx)\,dx}{\sqrt{\sec(c+dx)}}\\&=-\frac{\left(b^2\sqrt{b\sec(c+dx)}\right)\mathrm{Subst}\left(\int\left(1+x^2\right)\,dx,x,-\tan(c+dx)\right)}{d\sqrt{\sec(c+dx)}}\\&=\frac{b^2\sqrt{\sec(c+dx)}\sqrt{b\sec(c+dx)}\sin(c+dx)}{d}+\frac{b^2\sec^{\frac{5}{2}}(c+dx)\sqrt{b\sec(c+dx)}\sin^3(c}{3d}\end{aligned}$$

Mathematica [A] time = 0.0789691, size = 45, normalized size = 0.59

$$\frac{\left(\frac{1}{3}\tan^3(c+dx)+\tan(c+dx)\right)(b\sec(c+dx))^{5/2}}{d\sec^{\frac{5}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^(3/2)*(b*Sec[c + d*x])^(5/2),x]
```

```
[Out] ((b*Sec[c + d*x])^(5/2)*(Tan[c + d*x] + Tan[c + d*x]^3/3))/(d*Sec[c + d*x]^
(5/2))
```

Maple [A] time = 0.106, size = 52, normalized size = 0.7

$$\frac{\left(2\left(\cos\left(dx+c\right)\right)^{2}+1\right)\cos\left(dx+c\right)\sin\left(dx+c\right)}{3d}\left(\left(\cos\left(dx+c\right)\right)^{-1}\right)^{\frac{3}{2}}\left(\frac{b}{\cos\left(dx+c\right)}\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(3/2)*(b*sec(d*x+c))^(5/2),x)
```

```
[Out] 1/3/d*(2*cos(d*x+c)^2+1)*(1/cos(d*x+c))^(3/2)*(b/cos(d*x+c))^(5/2)*cos(d*x+
c)*sin(d*x+c)
```

Maxima [B] time = 1.9785, size = 420, normalized size = 5.53

$$-\frac{4\left(3\,b^{2}\cos\left(6\,dx+6\,c\right)\sin\left(2\right.\right.}{3\left(2\left(3\,\cos\left(4\,dx+4\,c\right)+3\,\cos\left(2\,dx+2\,c\right)+1\right)\cos\left(6\,dx+6\,c\right)+\cos\left(6\,dx+6\,c\right)^{2}+6\left(3\,\cos\left(2\,dx+2\,c\right)+1\right)\cos\left(4\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(b*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] -4/3*(3*b^2*cos(6*d*x + 6*c)*sin(2*d*x + 2*c) + 9*b^2*cos(4*d*x + 4*c)*sin(
2*d*x + 2*c) - (3*b^2*cos(2*d*x + 2*c) + b^2)*sin(6*d*x + 6*c) - 3*(3*b^2*c
os(2*d*x + 2*c) + b^2)*sin(4*d*x + 4*c))*sqrt(b)/((2*(3*cos(4*d*x + 4*c) +
3*cos(2*d*x + 2*c) + 1)*cos(6*d*x + 6*c) + cos(6*d*x + 6*c)^2 + 6*(3*cos(2*
d*x + 2*c) + 1)*cos(4*d*x + 4*c) + 9*cos(4*d*x + 4*c)^2 + 9*cos(2*d*x + 2*c
)^2 + 6*(sin(4*d*x + 4*c) + sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + sin(6*d*x
+ 6*c)^2 + 9*sin(4*d*x + 4*c)^2 + 18*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 9*
sin(2*d*x + 2*c)^2 + 6*cos(2*d*x + 2*c) + 1)*d)
```

Fricas [A] time = 1.6932, size = 123, normalized size = 1.62

$$\frac{\left(2\,b^{2}\cos\left(dx+c\right)^{2}+b^{2}\right)\sqrt{\frac{b}{\cos\left(dx+c\right)}}\sin\left(dx+c\right)}{3\,d\cos\left(dx+c\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/3*(2*b^2*cos(d*x + c)^2 + b^2)*sqrt(b/cos(d*x + c))*sin(d*x + c)/(d*cos(d
*x + c)^(5/2))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(3/2)*(b*sec(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b\sec(dx+c))^{\frac{5}{2}}\sec(dx+c)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c))^(5/2)*sec(d*x + c)^(3/2), x)
```

3.154 $\int \sqrt{\sec(c+dx)}(b\sec(c+dx))^{5/2}\,dx$

Optimal. Leaf size=78

$$\frac{b^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\sqrt{b\sec(c+dx)}}{2d}+\frac{b^2\sqrt{b\sec(c+dx)}\tanh^{-1}(\sin(c+dx))}{2d\sqrt{\sec(c+dx)}}$$

```
[Out] (b^2*ArcTanh[Sin[c + d*x]]*Sqrt[b*Sec[c + d*x]])/(2*d*Sqrt[Sec[c + d*x]]) +
 (b^2*Sec[c + d*x]^(3/2)*Sqrt[b*Sec[c + d*x]]*Sin[c + d*x])/(2*d)
```

Rubi [A] time = 0.020254, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {17, 3768, 3770}

$$\frac{b^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\sqrt{b\sec(c+dx)}}{2d}+\frac{b^2\sqrt{b\sec(c+dx)}\tanh^{-1}(\sin(c+dx))}{2d\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Sec[c + d*x]]*(b*Sec[c + d*x])^(5/2),x]
```

```
[Out] (b^2*ArcTanh[Sin[c + d*x]]*Sqrt[b*Sec[c + d*x]])/(2*d*Sqrt[Sec[c + d*x]]) +
 (b^2*Sec[c + d*x]^(3/2)*Sqrt[b*Sec[c + d*x]]*Sin[c + d*x])/(2*d)
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(a^(m + 1/
2)*b^(n - 1/2)*Sqrt[b*v])/Sqrt[a*v], Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \sqrt{\sec(c+dx)}(b\sec(c+dx))^{5/2}\,dx &= \frac{\left(b^2\sqrt{b\sec(c+dx)}\right)\int \sec^3(c+dx)\,dx}{\sqrt{\sec(c+dx)}}\\
&= \frac{b^2\sec^{\frac{3}{2}}(c+dx)\sqrt{b\sec(c+dx)}\sin(c+dx)}{2d}+\frac{\left(b^2\sqrt{b\sec(c+dx)}\right)\int \sec(c+dx)\,}{2\sqrt{\sec(c+dx)}}\\
&= \frac{b^2\tanh^{-1}(\sin(c+dx))\sqrt{b\sec(c+dx)}}{2d\sqrt{\sec(c+dx)}}+\frac{b^2\sec^{\frac{3}{2}}(c+dx)\sqrt{b\sec(c+dx)}\sin(c+}{2d}
\end{aligned}$$

Mathematica [A] time = 0.0767913, size = 50, normalized size = 0.64

$$\frac{(b\sec(c+dx))^{5/2}\left(\tanh^{-1}(\sin(c+dx))+\tan(c+dx)\sec(c+dx)\right)}{2d\sec^{\frac{5}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Sec[c + d*x]]*(b*Sec[c + d*x])^(5/2),x]
```

```
[Out] ((b*Sec[c + d*x])^(5/2)*(ArcTanh[Sin[c + d*x]] + Sec[c + d*x]*Tan[c + d*x])
)/(2*d*Sec[c + d*x]^(5/2))
```

Maple [A] time = 0.132, size = 112, normalized size = 1.4

$$\frac{\cos(dx+c)}{2d}\left(\ln\left(-\frac{-1+\cos(dx+c)-\sin(dx+c)}{\sin(dx+c)}\right)(\cos(dx+c))^2-\ln\left(-\frac{-1+\cos(dx+c)+\sin(dx+c)}{\sin(dx+c)}\right)(\cos(dx+c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(1/2)*(b*sec(d*x+c))^(5/2),x)
```

```
[Out] 1/2/d*(ln(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))*cos(d*x+c)^2-ln(-(-1+cos(
d*x+c)+sin(d*x+c))/sin(d*x+c))*cos(d*x+c)^2+sin(d*x+c))*cos(d*x+c)*(1/cos(d
*x+c))^(1/2)*(b/cos(d*x+c))^(5/2)
```

Maxima [B] time = 2.19142, size = 1008, normalized size = 12.92

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)*(b*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] -1/4*(4*(b^2*sin(4*d*x + 4*c) + 2*b^2*sin(2*d*x + 2*c))*cos(3/2*arctan2(sin
(2*d*x + 2*c), cos(2*d*x + 2*c))) - 4*(b^2*sin(4*d*x + 4*c) + 2*b^2*sin(2*d
*x + 2*c))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - (b^2*cos(
4*d*x + 4*c)^2 + 4*b^2*cos(2*d*x + 2*c)^2 + b^2*sin(4*d*x + 4*c)^2 + 4*b^2*
sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*b^2*sin(2*d*x + 2*c)^2 + 4*b^2*cos(2*
d*x + 2*c) + b^2 + 2*(2*b^2*cos(2*d*x + 2*c) + b^2)*cos(4*d*x + 4*c))*log(c
os(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2(sin
(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/2*arctan2(sin(2*d*x + 2*c), c
os(2*d*x + 2*c))) + 1) + (b^2*cos(4*d*x + 4*c)^2 + 4*b^2*cos(2*d*x + 2*c)^2
 + b^2*sin(4*d*x + 4*c)^2 + 4*b^2*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*b^2
*sin(2*d*x + 2*c)^2 + 4*b^2*cos(2*d*x + 2*c) + b^2 + 2*(2*b^2*cos(2*d*x + 2
*c) + b^2)*cos(4*d*x + 4*c))*log(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*
x + 2*c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*s
in(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) - 4*(b^2*cos(4*d*x
 + 4*c) + 2*b^2*cos(2*d*x + 2*c) + b^2)*sin(3/2*arctan2(sin(2*d*x + 2*c), c
os(2*d*x + 2*c))) + 4*(b^2*cos(4*d*x + 4*c) + 2*b^2*cos(2*d*x + 2*c) + b^2)
*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*sqrt(b)/((2*(2*cos(2
*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)
^2 + sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x
```

```
+ 2*c)^2 + 4*cos(2*d*x + 2*c) + 1)*d)
```

Fricas [A] time = 1.97983, size = 544, normalized size = 6.97

$$\left[\frac{b^{\frac{5}{2}}\cos\left(dx+c\right)\log\left(-\frac{b\cos(dx+c)^2-2\sqrt{b}\sqrt{\frac{b}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)-2b}{\cos(dx+c)^2}\right)+\frac{2b^2\sqrt{\frac{b}{\cos(dx+c)}}\sin(dx+c)}{\sqrt{\cos(dx+c)}}}{4d\cos\left(dx+c\right)}, -\frac{\sqrt{-b}b^2\arctan\left(\frac{\sqrt{-b}\sqrt{\frac{b}{\cos(\ldots}}}{\ldots}\right.}{\ldots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)*(b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] [1/4*(b^(5/2)*cos(d*x + c)*log(-(b*cos(d*x + c)^2 - 2*sqrt(b)*sqrt(b/cos(d*
x + c))*sqrt(cos(d*x + c))*sin(d*x + c) - 2*b)/cos(d*x + c)^2) + 2*b^2*sqrt
(b/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x + c)), -1/2*(s
qrt(-b)*b^2*arctan(sqrt(-b)*sqrt(b/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x
 + c)/b)*cos(d*x + c) - b^2*sqrt(b/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x
+ c)))/(d*cos(d*x + c))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(1/2)*(b*sec(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sec\left(dx+c\right)\right)^{\frac{5}{2}}\sqrt{\sec\left(dx+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)*(b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c))^(5/2)*sqrt(sec(d*x + c)), x)
```

3.155 $\int \frac{(b\sec(c+dx))^{5/2}}{\sqrt{\sec(c+dx)}}\,dx$

Optimal. Leaf size=35

$$\frac{b^2\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{b\sec(c+dx)}}{d}$$

```
[Out] (b^2*Sqrt[Sec[c + d*x]]*Sqrt[b*Sec[c + d*x]]*Sin[c + d*x])/d
```

Rubi [A] time = 0.011894, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {17, 3767, 8}

$$\frac{b^2\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{b\sec(c+dx)}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Sec[c + d*x])^(5/2)/Sqrt[Sec[c + d*x]],x]
```

```
[Out] (b^2*Sqrt[Sec[c + d*x]]*Sqrt[b*Sec[c + d*x]]*Sin[c + d*x])/d
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(a^(m + 1/
2)*b^(n - 1/2)*Sqrt[b*v])/Sqrt[a*v], Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \frac{(b\sec(c+dx))^{5/2}}{\sqrt{\sec(c+dx)}}\,dx &= \frac{\left(b^2\sqrt{b\sec(c+dx)}\right)\int \sec^2(c+dx)\,dx}{\sqrt{\sec(c+dx)}}\\ &= -\frac{\left(b^2\sqrt{b\sec(c+dx)}\right)\operatorname{Subst}(\int 1\,dx, x, -\tan(c+dx))}{d\sqrt{\sec(c+dx)}}\\ &= \frac{b^2\sqrt{\sec(c+dx)}\sqrt{b\sec(c+dx)}\sin(c+dx)}{d}\end{aligned}$$

Mathematica [A] time = 0.0309176, size = 32, normalized size = 0.91

$$\frac{\sin(c+dx)(b\sec(c+dx))^{5/2}}{d\sec^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Sec[c + d*x])^(5/2)/Sqrt[Sec[c + d*x]],x]
```

```
[Out] ((b*Sec[c + d*x])^(5/2)*Sin[c + d*x])/(d*Sec[c + d*x]^(3/2))
```

Maple [A] time = 0.127, size = 39, normalized size = 1.1

$$\frac{\cos(dx+c)\sin(dx+c)}{d}\left(\frac{b}{\cos(dx+c)}\right)^{\frac{5}{2}}\frac{1}{\sqrt{(\cos(dx+c))^{-1}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sec(d*x+c))^(5/2)/sec(d*x+c)^(1/2),x)
```

```
[Out] 1/d*(b/cos(d*x+c))^(5/2)*cos(d*x+c)*sin(d*x+c)/(1/cos(d*x+c))^(1/2)
```

Maxima [A] time = 2.06034, size = 73, normalized size = 2.09

$$\frac{2\,b^{\frac{5}{2}}\sin(2\,dx+2\,c)}{\left(\cos(2\,dx+2\,c)^2+\sin(2\,dx+2\,c)^2+2\,\cos(2\,dx+2\,c)+1\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))^(5/2)/sec(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] 2*b^(5/2)*sin(2*d*x + 2*c)/((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*co
s(2*d*x + 2*c) + 1)*d)
```

Fricas [A] time = 1.63597, size = 84, normalized size = 2.4

$$\frac{b^2\sqrt{\frac{b}{\cos(dx+c)}}\sin(dx+c)}{d\sqrt{\cos(dx+c)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))^(5/2)/sec(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] b^2*sqrt(b/cos(d*x + c))*sin(d*x + c)/(d*sqrt(cos(d*x + c)))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))**(5/2)/sec(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\sec(dx+c))^{\frac{5}{2}}}{\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))^(5/2)/sec(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c))^(5/2)/sqrt(sec(d*x + c)), x)
```

3.156 $\int \frac{(b\sec(c+dx))^{5/2}}{\sec^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=36

$$\frac{b^2\sqrt{b\sec(c+dx)}\tanh^{-1}(\sin(c+dx))}{d\sqrt{\sec(c+dx)}}$$

```
[Out] (b^2*ArcTanh[Sin[c + d*x]]*Sqrt[b*Sec[c + d*x]])/(d*Sqrt[Sec[c + d*x]])
```

Rubi [A] time = 0.00772, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {17, 3770}

$$\frac{b^2\sqrt{b\sec(c+dx)}\tanh^{-1}(\sin(c+dx))}{d\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Sec[c + d*x])^(5/2)/Sec[c + d*x]^(3/2),x]
```

```
[Out] (b^2*ArcTanh[Sin[c + d*x]]*Sqrt[b*Sec[c + d*x]])/(d*Sqrt[Sec[c + d*x]])
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(a^(m + 1/
2)*b^(n - 1/2)*Sqrt[b*v])/Sqrt[a*v], Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{(b\sec(c+dx))^{5/2}}{\sec^{\frac{3}{2}}(c+dx)}\,dx &= \frac{\left(b^2\sqrt{b\sec(c+dx)}\right)\int \sec(c+dx)\,dx}{\sqrt{\sec(c+dx)}}\\
&= \frac{b^2\tanh^{-1}(\sin(c+dx))\sqrt{b\sec(c+dx)}}{d\sqrt{\sec(c+dx)}}
\end{aligned}$$

Mathematica [A] time = 0.0342288, size = 33, normalized size = 0.92

$$\frac{(b\sec(c+dx))^{5/2}\tanh^{-1}(\sin(c+dx))}{d\sec^{\frac{5}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Sec[c + d*x])^(5/2)/Sec[c + d*x]^(3/2),x]
```

```
[Out] (ArcTanh[Sin[c + d*x]]*(b*Sec[c + d*x])^(5/2))/(d*Sec[c + d*x]^(5/2))
```

Maple [A] time = 0.097, size = 52, normalized size = 1.4

$$-2\,{\frac {\cos \left(dx+c \right) }{d \left(\left(\cos \left(dx+c \right) \right) ^{-1} \right) ^{3/2}}{\rm Artanh} \left({\frac {-1+\cos \left(dx+c \right) }{\sin \left(dx+c \right) }} \right) \left({\frac {b}{\cos \left(dx+c \right) }} \right) ^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sec(d*x+c))^(5/2)/sec(d*x+c)^(3/2),x)
```

```
[Out] -2/d*arctanh((-1+cos(d*x+c))/sin(d*x+c))*cos(d*x+c)*(b/cos(d*x+c))^(5/2)/(1
/cos(d*x+c))^(3/2)
```

Maxima [B] time = 1.91586, size = 97, normalized size = 2.69

$$\frac{\left(b^2 \log\left(\cos\left(dx+c\right)^2+\sin\left(dx+c\right)^2+2\,\sin\left(dx+c\right)+1\right)-b^2 \log\left(\cos\left(dx+c\right)^2+\sin\left(dx+c\right)^2-2\,\sin\left(dx+c\right)+1\right)\right)\sqrt{b}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))^(5/2)/sec(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/2*(b^2*log(cos(d*x + c)^2 + sin(d*x + c)^2 + 2*sin(d*x + c) + 1) - b^2*lo
g(cos(d*x + c)^2 + sin(d*x + c)^2 - 2*sin(d*x + c) + 1))*sqrt(b)/d
```

Fricas [A] time = 1.84508, size = 296, normalized size = 8.22

$$\left[\frac{b^{\frac{5}{2}} \log\left(-\frac{b \cos(dx+c)^2-2\,\sqrt{b}\sqrt{\frac{b}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)-2\,b}{\cos(dx+c)^2}\right)}{2\,d}, -\frac{\sqrt{-b}b^2 \arctan\left(\frac{\sqrt{-b}\sqrt{\frac{b}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)}{b}\right)}{d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))^(5/2)/sec(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*b^(5/2)*log(-(b*cos(d*x + c)^2 - 2*sqrt(b)*sqrt(b/cos(d*x + c))*sqrt(c
os(d*x + c))*sin(d*x + c) - 2*b)/cos(d*x + c)^2)/d, -sqrt(-b)*b^2*arctan(sq
rt(-b)*sqrt(b/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/b)/d]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))**(5/2)/sec(d*x+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\sec{(dx+c)})^{\frac{5}{2}}}{\sec{(dx+c)}^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))^(5/2)/sec(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c))^(5/2)/sec(d*x + c)^(3/2), x)
```

3.157 $\int \frac{(b\sec(c+dx))^{5/2}}{\sec^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=27

$$\frac{b^2x\sqrt{b\sec(c+dx)}}{\sqrt{\sec(c+dx)}}$$

```
[Out] (b^2*x*Sqrt[b*Sec[c + d*x]])/Sqrt[Sec[c + d*x]]
```

Rubi [A] time = 0.0026389, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {17, 8}

$$\frac{b^2x\sqrt{b\sec(c+dx)}}{\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Sec[c + d*x])^(5/2)/Sec[c + d*x]^(5/2),x]
```

```
[Out] (b^2*x*Sqrt[b*Sec[c + d*x]])/Sqrt[Sec[c + d*x]]
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(a^(m + 1/
2)*b^(n - 1/2)*Sqrt[b*v])/Sqrt[a*v], Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \frac{(b\sec(c+dx))^{5/2}}{\sec^{\frac{5}{2}}(c+dx)}\,dx &= \frac{\left(b^2\sqrt{b\sec(c+dx)}\right)\int 1\,dx}{\sqrt{\sec(c+dx)}}\\ &= \frac{b^2x\sqrt{b\sec(c+dx)}}{\sqrt{\sec(c+dx)}}\end{aligned}$$

Mathematica [A] time = 0.0194576, size = 24, normalized size = 0.89

$$\frac{x(b\sec(c+dx))^{5/2}}{\sec^{\frac{5}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Sec[c + d*x])^(5/2)/Sec[c + d*x]^(5/2),x]
```

```
[Out] (x*(b*Sec[c + d*x])^(5/2))/Sec[c + d*x]^(5/2)
```

Maple [A] time = 0.091, size = 32, normalized size = 1.2

$$\frac{dx+c}{d}\left(\frac{b}{\cos\left(dx+c\right)}\right)^{\frac{5}{2}}\left(\left(\cos\left(dx+c\right)\right)^{-1}\right)^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sec(d*x+c))^(5/2)/sec(d*x+c)^(5/2),x)
```

```
[Out] 1/d*(d*x+c)*(b/cos(d*x+c))^(5/2)/(1/cos(d*x+c))^(5/2)
```

Maxima [A] time = 1.82461, size = 35, normalized size = 1.3

$$\frac{2\,b^{\frac{5}{2}}\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))^(5/2)/sec(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] 2*b^(5/2)*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/d
```

Fricas [A] time = 1.97332, size = 271, normalized size = 10.04

$$\left[\frac{\sqrt{-b}b^{2}\log\left(-2\,\sqrt{-b}\sqrt{\frac{b}{\cos(dx+c)}}\cos\left(dx+c\right)^{\frac{3}{2}}\sin\left(dx+c\right)+2\,b\cos\left(dx+c\right)^{2}-b\right)}{2\,d},\ \frac{b^{\frac{5}{2}}\arctan\left(\frac{\sqrt{\frac{b}{\cos(dx+c)}}\sin(dx+c)}{\sqrt{b}\sqrt{\cos(dx+c)}}\right)}{d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))^(5/2)/sec(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/2*sqrt(-b)*b^2*log(-2*sqrt(-b)*sqrt(b/cos(d*x + c))*cos(d*x + c)^(3/2)*s
in(d*x + c) + 2*b*cos(d*x + c)^2 - b)/d, b^(5/2)*arctan(sqrt(b/cos(d*x + c)
)*sin(d*x + c)/(sqrt(b)*sqrt(cos(d*x + c))))/d]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))**(5/2)/sec(d*x+c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b\sec\left(dx+c\right)\right)^{\frac{5}{2}}}{\sec\left(dx+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))^(5/2)/sec(d*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c))^(5/2)/sec(d*x + c)^(5/2), x)
```

3.158 $\int \frac{(b\sec(c+dx))^{5/2}}{\sec^{\frac{7}{2}}(c+dx)}\,dx$

Optimal. Leaf size=35

$$\frac{b^2\sin(c+dx)\sqrt{b\sec(c+dx)}}{d\sqrt{\sec(c+dx)}}$$

```
[Out] (b^2*Sqrt[b*Sec[c + d*x]]*Sin[c + d*x])/(d*Sqrt[Sec[c + d*x]])
```

Rubi [A] time = 0.0070795, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {17, 2637}

$$\frac{b^2\sin(c+dx)\sqrt{b\sec(c+dx)}}{d\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Sec[c + d*x])^(5/2)/Sec[c + d*x]^(7/2),x]
```

```
[Out] (b^2*Sqrt[b*Sec[c + d*x]]*Sin[c + d*x])/(d*Sqrt[Sec[c + d*x]])
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(a^(m + 1/
2)*b^(n - 1/2)*Sqrt[b*v])/Sqrt[a*v], Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{(b\sec(c+dx))^{5/2}}{\sec^{\frac{7}{2}}(c+dx)}\,dx &= \frac{\left(b^2\sqrt{b\sec(c+dx)}\right)\int \cos(c+dx)\,dx}{\sqrt{\sec(c+dx)}}\\ &= \frac{b^2\sqrt{b\sec(c+dx)}\sin(c+dx)}{d\sqrt{\sec(c+dx)}}\end{aligned}$$

Mathematica [A] time = 0.0652117, size = 32, normalized size = 0.91

$$\frac{\sin(c+dx)(b\sec(c+dx))^{5/2}}{d\sec^{\frac{5}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Sec[c + d*x])^(5/2)/Sec[c + d*x]^(7/2),x]
```

```
[Out] ((b*Sec[c + d*x])^(5/2)*Sin[c + d*x])/(d*Sec[c + d*x]^(5/2))
```

Maple [A] time = 0.105, size = 41, normalized size = 1.2

$$\frac{\sin (dx+c)}{d\cos (dx+c)}\left(\frac{b}{\cos (dx+c)}\right)^{\frac{5}{2}}\left((\cos (dx+c))^{-1}\right)^{-\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sec(d*x+c))^(5/2)/sec(d*x+c)^(7/2),x)
```

```
[Out] 1/d*sin(d*x+c)*(b/cos(d*x+c))^(5/2)/(1/cos(d*x+c))^(7/2)/cos(d*x+c)
```

Maxima [A] time = 1.87907, size = 18, normalized size = 0.51

$$\frac{b^{\frac{5}{2}}\sin (dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))^(5/2)/sec(d*x+c)^(7/2),x, algorithm="maxima")
```

```
[Out] b^(5/2)*sin(d*x + c)/d
```

Fricas [A] time = 1.66211, size = 81, normalized size = 2.31

$$\frac{b^2\sqrt{\frac{b}{\cos(dx+c)}}\sqrt{\cos (dx+c)}\sin (dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))^(5/2)/sec(d*x+c)^(7/2),x, algorithm="fricas")
```

```
[Out] b^2*sqrt(b/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/d
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))**(5/2)/sec(d*x+c)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\sec (dx+c))^{\frac{5}{2}}}{\sec (dx+c)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))^(5/2)/sec(d*x+c)^(7/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c))^(5/2)/sec(d*x + c)^(7/2), x)
```

3.159 $\int \frac{(b\sec(c+dx))^{5/2}}{\sec^{\frac{9}{2}}(c+dx)}\,dx$

Optimal. Leaf size=69

$$\frac{b^2x\sqrt{b\sec(c+dx)}}{2\sqrt{\sec(c+dx)}}+\frac{b^2\sin(c+dx)\sqrt{b\sec(c+dx)}}{2d\sec^{\frac{3}{2}}(c+dx)}$$

```
[Out] (b^2*x*Sqrt[b*Sec[c + d*x]])/(2*Sqrt[Sec[c + d*x]]) + (b^2*Sqrt[b*Sec[c + d
*x]]*Sin[c + d*x])/(2*d*Sec[c + d*x]^(3/2))
```

Rubi [A] time = 0.0154454, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {17, 2635, 8}

$$\frac{b^2x\sqrt{b\sec(c+dx)}}{2\sqrt{\sec(c+dx)}}+\frac{b^2\sin(c+dx)\sqrt{b\sec(c+dx)}}{2d\sec^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Sec[c + d*x])^(5/2)/Sec[c + d*x]^(9/2),x]
```

```
[Out] (b^2*x*Sqrt[b*Sec[c + d*x]])/(2*Sqrt[Sec[c + d*x]]) + (b^2*Sqrt[b*Sec[c + d
*x]]*Sin[c + d*x])/(2*d*Sec[c + d*x]^(3/2))
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(a^(m + 1/
2)*b^(n - 1/2)*Sqrt[b*v])/Sqrt[a*v], Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \frac{(b\sec(c+dx))^{5/2}}{\sec^{\frac{9}{2}}(c+dx)}\,dx &= \frac{\left(b^2\sqrt{b\sec(c+dx)}\right)\int \cos^2(c+dx)\,dx}{\sqrt{\sec(c+dx)}}\\ &= \frac{b^2\sqrt{b\sec(c+dx)}\sin(c+dx)}{2d\sec^{\frac{3}{2}}(c+dx)}+\frac{\left(b^2\sqrt{b\sec(c+dx)}\right)\int 1\,dx}{2\sqrt{\sec(c+dx)}}\\ &= \frac{b^2x\sqrt{b\sec(c+dx)}}{2\sqrt{\sec(c+dx)}}+\frac{b^2\sqrt{b\sec(c+dx)}\sin(c+dx)}{2d\sec^{\frac{3}{2}}(c+dx)}\end{aligned}$$

Mathematica [A] time = 0.1127, size = 45, normalized size = 0.65

$$\frac{(2(c+dx)+\sin(2(c+dx)))(b\sec(c+dx))^{5/2}}{4d\sec^{\frac{5}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Sec[c + d*x])^(5/2)/Sec[c + d*x]^(9/2),x]
```

```
[Out] ((b*Sec[c + d*x])^(5/2)*(2*(c + d*x) + Sin[2*(c + d*x)]))/(4*d*Sec[c + d*x]
^(5/2))
```

Maple [A] time = 0.105, size = 54, normalized size = 0.8

$$\frac{\cos(dx+c)\sin(dx+c)+dx+c}{2\,d\,(\cos(dx+c))^2}\left(\frac{b}{\cos(dx+c)}\right)^{\frac{5}{2}}\left((\cos(dx+c))^{-1}\right)^{-\frac{9}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sec(d*x+c))^(5/2)/sec(d*x+c)^(9/2),x)
```

```
[Out] 1/2/d*(cos(d*x+c)*sin(d*x+c)+d*x+c)*(b/cos(d*x+c))^(5/2)/cos(d*x+c)^2/(1/co
s(d*x+c))^(9/2)
```

Maxima [A] time = 2.12771, size = 43, normalized size = 0.62

$$\frac{\left(2\,(dx+c)b^2+b^2\sin(2\,dx+2\,c)\right)\sqrt{b}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))^(5/2)/sec(d*x+c)^(9/2),x, algorithm="maxima")
```

```
[Out] 1/4*(2*(d*x + c)*b^2 + b^2*sin(2*d*x + 2*c))*sqrt(b)/d
```

Fricas [A] time = 2.04022, size = 444, normalized size = 6.43

$$\left[\frac{2\,b^2\sqrt{\frac{b}{\cos(dx+c)}}\cos(dx+c)^{\frac{3}{2}}\sin(dx+c)+\sqrt{-b}b^2\log\left(-2\sqrt{-b}\sqrt{\frac{b}{\cos(dx+c)}}\cos(dx+c)^{\frac{3}{2}}\sin(dx+c)+2\,b\cos(dx+c\right.}{4\,d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))^(5/2)/sec(d*x+c)^(9/2),x, algorithm="fricas")
```

```
[Out] [1/4*(2*b^2*sqrt(b/cos(d*x + c))*cos(d*x + c)^(3/2)*sin(d*x + c) + sqrt(-b)
*b^2*log(-2*sqrt(-b)*sqrt(b/cos(d*x + c))*cos(d*x + c)^(3/2)*sin(d*x + c) +
```

```
 2*b*cos(d*x + c)^2 - b))/d, 1/2*(b^2*sqrt(b/cos(d*x + c))*cos(d*x + c)^(3/
2)*sin(d*x + c) + b^(5/2)*arctan(sqrt(b/cos(d*x + c))*sin(d*x + c)/(sqrt(b)
*sqrt(cos(d*x + c)))))/d]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))**(5/2)/sec(d*x+c)**(9/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\sec(dx+c))^{\frac{5}{2}}}{\sec(dx+c)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))^(5/2)/sec(d*x+c)^(9/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c))^(5/2)/sec(d*x + c)^(9/2), x)
```

3.160 $\int \frac{(b\sec(c+dx))^{5/2}}{\sec^{\frac{11}{2}}(c+dx)}\,dx$

Optimal. Leaf size=76

$$\frac{b^2\sin(c+dx)\sqrt{b\sec(c+dx)}}{d\sqrt{\sec(c+dx)}}-\frac{b^2\sin^3(c+dx)\sqrt{b\sec(c+dx)}}{3d\sqrt{\sec(c+dx)}}$$

```
[Out] (b^2*Sqrt[b*Sec[c + d*x]]*Sin[c + d*x])/(d*Sqrt[Sec[c + d*x]]) - (b^2*Sqrt[
b*Sec[c + d*x]]*Sin[c + d*x]^3)/(3*d*Sqrt[Sec[c + d*x]])
```

Rubi [A] time = 0.0168081, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {17, 2633}

$$\frac{b^2\sin(c+dx)\sqrt{b\sec(c+dx)}}{d\sqrt{\sec(c+dx)}}-\frac{b^2\sin^3(c+dx)\sqrt{b\sec(c+dx)}}{3d\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Sec[c + d*x])^(5/2)/Sec[c + d*x]^(11/2),x]
```

```
[Out] (b^2*Sqrt[b*Sec[c + d*x]]*Sin[c + d*x])/(d*Sqrt[Sec[c + d*x]]) - (b^2*Sqrt[
b*Sec[c + d*x]]*Sin[c + d*x]^3)/(3*d*Sqrt[Sec[c + d*x]])
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(a^(m + 1/
2)*b^(n - 1/2)*Sqrt[b*v])/Sqrt[a*v], Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(b\sec(c+dx))^{5/2}}{\sec^{\frac{11}{2}}(c+dx)}\,dx &= \frac{\left(b^2\sqrt{b\sec(c+dx)}\right)\int \cos^3(c+dx)\,dx}{\sqrt{\sec(c+dx)}}\\ &= -\frac{\left(b^2\sqrt{b\sec(c+dx)}\right)\operatorname{Subst}\left(\int \left(1-x^2\right)\,dx,x,-\sin(c+dx)\right)}{d\sqrt{\sec(c+dx)}}\\ &= \frac{b^2\sqrt{b\sec(c+dx)}\sin(c+dx)}{d\sqrt{\sec(c+dx)}}-\frac{b^2\sqrt{b\sec(c+dx)}\sin^3(c+dx)}{3d\sqrt{\sec(c+dx)}}\end{aligned}$$

Mathematica [A] time = 0.162517, size = 45, normalized size = 0.59

$$\frac{\sin(c+dx)(\cos(2(c+dx))+5)(b\sec(c+dx))^{5/2}}{6d\sec^{\frac{5}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Sec[c + d*x])^(5/2)/Sec[c + d*x]^(11/2),x]
```

```
[Out] ((5 + Cos[2*(c + d*x)])*(b*Sec[c + d*x])^(5/2)*Sin[c + d*x])/(6*d*Sec[c + d
*x]^(5/2))
```

Maple [A] time = 0.114, size = 52, normalized size = 0.7

$$\frac{\left(\left(\cos\left(dx+c\right)\right)^{2}+2\right)\sin\left(dx+c\right)}{3\,d\left(\cos\left(dx+c\right)\right)^{3}}\left(\frac{b}{\cos\left(dx+c\right)}\right)^{\frac{5}{2}}\left(\left(\cos\left(dx+c\right)\right)^{-1}\right)^{-\frac{11}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sec(d*x+c))^(5/2)/sec(d*x+c)^(11/2),x)
```

```
[Out] 1/3/d*(cos(d*x+c)^2+2)*sin(d*x+c)*(b/cos(d*x+c))^(5/2)/cos(d*x+c)^3/(1/cos(
d*x+c))^(11/2)
```

Maxima [A] time = 2., size = 66, normalized size = 0.87

$$\frac{\left(b^{2}\sin\left(3\,dx+3\,c\right)+9\,b^{2}\sin\left(\frac{1}{3}\,\arctan\left(\sin\left(3\,dx+3\,c\right),\cos\left(3\,dx+3\,c\right)\right)\right)\right)\sqrt{b}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))^(5/2)/sec(d*x+c)^(11/2),x, algorithm="maxima")
```

```
[Out] 1/12*(b^2*sin(3*d*x + 3*c) + 9*b^2*sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*
d*x + 3*c))))*sqrt(b)/d
```

Fricas [A] time = 1.64336, size = 140, normalized size = 1.84

$$\frac{\left(b^{2}\cos\left(dx+c\right)^{3}+2\,b^{2}\cos\left(dx+c\right)\right)\sqrt{\frac{b}{\cos(dx+c)}}\sin\left(dx+c\right)}{3\,d\sqrt{\cos\left(dx+c\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))^(5/2)/sec(d*x+c)^(11/2),x, algorithm="fricas")
```

```
[Out] 1/3*(b^2*cos(d*x + c)^3 + 2*b^2*cos(d*x + c))*sqrt(b/cos(d*x + c))*sin(d*x
+ c)/(d*sqrt(cos(d*x + c)))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))**(5/2)/sec(d*x+c)**(11/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\sec(dx+c))^{\frac{5}{2}}}{\sec(dx+c)^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))^(5/2)/sec(d*x+c)^(11/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c))^(5/2)/sec(d*x + c)^(11/2), x)
```

3.161 $\int \frac{\sec^{\frac{7}{2}}(c+dx)}{\sqrt{b\sec(c+dx)}}\,dx$

Optimal. Leaf size=72

$$\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{2d\sqrt{b\sec(c+dx)}}+\frac{\sqrt{\sec(c+dx)}\tanh^{-1}(\sin(c+dx))}{2d\sqrt{b\sec(c+dx)}}$$

```
[Out] (ArcTanh[Sin[c + d*x]]*Sqrt[Sec[c + d*x]])/(2*d*Sqrt[b*Sec[c + d*x]]) + (Se
c[c + d*x]^(5/2)*Sin[c + d*x])/(2*d*Sqrt[b*Sec[c + d*x]])
```

Rubi [A] time = 0.0192948, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {17, 3768, 3770}

$$\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{2d\sqrt{b\sec(c+dx)}}+\frac{\sqrt{\sec(c+dx)}\tanh^{-1}(\sin(c+dx))}{2d\sqrt{b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(7/2)/Sqrt[b*Sec[c + d*x]],x]
```

```
[Out] (ArcTanh[Sin[c + d*x]]*Sqrt[Sec[c + d*x]])/(2*d*Sqrt[b*Sec[c + d*x]]) + (Se
c[c + d*x]^(5/2)*Sin[c + d*x])/(2*d*Sqrt[b*Sec[c + d*x]])
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(a^(m + 1/
2)*b^(n - 1/2)*Sqrt[b*v])/Sqrt[a*v], Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sec^{\frac{7}{2}}(c+dx)}{\sqrt{b\sec(c+dx)}}\,dx &= \frac{\sqrt{\sec(c+dx)}\int \sec^3(c+dx)\,dx}{\sqrt{b\sec(c+dx)}}\\
&= \frac{\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{2d\sqrt{b\sec(c+dx)}}+\frac{\sqrt{\sec(c+dx)}\int \sec(c+dx)\,dx}{2\sqrt{b\sec(c+dx)}}\\
&= \frac{\tanh^{-1}(\sin(c+dx))\sqrt{\sec(c+dx)}}{2d\sqrt{b\sec(c+dx)}}+\frac{\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{2d\sqrt{b\sec(c+dx)}}
\end{aligned}$$

Mathematica [A] time = 0.0574469, size = 50, normalized size = 0.69

$$\frac{\sqrt{\sec(c+dx)}\left(\tanh^{-1}(\sin(c+dx))+\tan(c+dx)\sec(c+dx)\right)}{2d\sqrt{b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^(7/2)/Sqrt[b*Sec[c + d*x]],x]
```

```
[Out] (Sqrt[Sec[c + d*x]]*(ArcTanh[Sin[c + d*x]] + Sec[c + d*x]*Tan[c + d*x]))/(2
*d*Sqrt[b*Sec[c + d*x]])
```

Maple [A] time = 0.137, size = 112, normalized size = 1.6

$$\frac{\cos(dx+c)}{2d}\left(\ln\left(-\frac{-1+\cos(dx+c)-\sin(dx+c)}{\sin(dx+c)}\right)(\cos(dx+c))^2-\ln\left(-\frac{-1+\cos(dx+c)+\sin(dx+c)}{\sin(dx+c)}\right)(\cos(dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(7/2)/(b*sec(d*x+c))^(1/2),x)
```

```
[Out] 1/2/d*(ln(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))*cos(d*x+c)^2-ln(-(-1+cos(
d*x+c)+sin(d*x+c))/sin(d*x+c))*cos(d*x+c)^2+sin(d*x+c))*(1/cos(d*x+c))^(7/2
)*cos(d*x+c)/(b/cos(d*x+c))^(1/2)
```

Maxima [B] time = 2.02468, size = 892, normalized size = 12.39

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(7/2)/(b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] -1/4*(4*(sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*cos(3/2*arctan2(sin(2*d*x +
 2*c), cos(2*d*x + 2*c))) - 4*(sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*cos(1
/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - (2*(2*cos(2*d*x + 2*c) +
1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*x
 + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*
cos(2*d*x + 2*c) + 1)*log(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c
)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/2*
arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) + (2*(2*cos(2*d*x + 2*c)
+ 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d
*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 +
4*cos(2*d*x + 2*c) + 1)*log(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2
*c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sin(1/
2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) - 4*(cos(4*d*x + 4*c) +
 2*cos(2*d*x + 2*c) + 1)*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
)) + 4*(cos(4*d*x + 4*c) + 2*cos(2*d*x + 2*c) + 1)*sin(1/2*arctan2(sin(2*d*
x + 2*c), cos(2*d*x + 2*c))))/((2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c)
 + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*sin(4
*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c) +
1)*sqrt(b)*d)
```

Fricas [A] time = 1.96414, size = 533, normalized size = 7.4

$$\left[\frac{\sqrt{b}\cos\left(dx+c\right)\log\left(-\frac{b\cos(dx+c)^2-2\sqrt{b}\sqrt{\frac{b}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)-2b}{\cos(dx+c)^2}\right)+\frac{2\sqrt{\frac{b}{\cos(dx+c)}}\sin(dx+c)}{\sqrt{\cos(dx+c)}}}{4\,bd\cos\left(dx+c\right)}, -\frac{\sqrt{-b}\arctan\left(\frac{\sqrt{-b}\sqrt{\frac{b}{\cos(dx+c)}}\sqrt{\cos(dx+c)}}{b}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(7/2)/(b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(b)*cos(d*x + c)*log(-(b*cos(d*x + c)^2 - 2*sqrt(b)*sqrt(b/cos(d*
x + c))*sqrt(cos(d*x + c))*sin(d*x + c) - 2*b)/cos(d*x + c)^2) + 2*sqrt(b/c
os(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(b*d*cos(d*x + c)), -1/2*(sqr
t(-b)*arctan(sqrt(-b)*sqrt(b/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/
b)*cos(d*x + c) - sqrt(b/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(b*
d*cos(d*x + c))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(7/2)/(b*sec(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec\left(dx+c\right)^{\frac{7}{2}}}{\sqrt{b\sec\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(7/2)/(b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^(7/2)/sqrt(b*sec(d*x + c)), x)
```

3.162 $\int \frac{\sec^{\frac{5}{2}}(c+dx)}{\sqrt{b\sec(c+dx)}}\,dx$

Optimal. Leaf size=32

$$\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d\sqrt{b\sec(c+dx)}}$$

```
[Out] (Sec[c + d*x]^(3/2)*Sin[c + d*x])/(d*Sqrt[b*Sec[c + d*x]])
```

Rubi [A] time = 0.0114377, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {17, 3767, 8}

$$\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d\sqrt{b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(5/2)/Sqrt[b*Sec[c + d*x]],x]
```

```
[Out] (Sec[c + d*x]^(3/2)*Sin[c + d*x])/(d*Sqrt[b*Sec[c + d*x]])
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(a^(m + 1/
2)*b^(n - 1/2)*Sqrt[b*v])/Sqrt[a*v], Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec^{\frac{5}{2}}(c+dx)}{\sqrt{b\sec(c+dx)}}\,dx &= \frac{\sqrt{\sec(c+dx)}\int \sec^2(c+dx)\,dx}{\sqrt{b\sec(c+dx)}}\\ &= -\frac{\sqrt{\sec(c+dx)}\,\text{Subst}\left(\int 1\,dx, x, -\tan(c+dx)\right)}{d\sqrt{b\sec(c+dx)}}\\ &= \frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{d\sqrt{b\sec(c+dx)}}\end{aligned}$$

Mathematica [A] time = 0.0256689, size = 32, normalized size = 1.

$$\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d\sqrt{b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^(5/2)/Sqrt[b*Sec[c + d*x]],x]
```

```
[Out] (Sec[c + d*x]^(3/2)*Sin[c + d*x])/(d*Sqrt[b*Sec[c + d*x]])
```

Maple [A] time = 0.119, size = 39, normalized size = 1.2

$$\frac{\cos(dx+c)\sin(dx+c)}{d}\left((\cos(dx+c))^{-1}\right)^{\frac{5}{2}}\frac{1}{\sqrt{\frac{b}{\cos(dx+c)}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(5/2)/(b*sec(d*x+c))^(1/2),x)
```

```
[Out] 1/d*(1/cos(d*x+c))^(5/2)*cos(d*x+c)*sin(d*x+c)/(b/cos(d*x+c))^(1/2)
```

Maxima [B] time = 2.03861, size = 80, normalized size = 2.5

$$\frac{2\sqrt{b}\sin(2\,dx+2\,c)}{\left(b\cos(2\,dx+2\,c)^2+b\sin(2\,dx+2\,c)^2+2\,b\cos(2\,dx+2\,c)+b\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)/(b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] 2*sqrt(b)*sin(2*d*x + 2*c)/((b*cos(2*d*x + 2*c)^2 + b*sin(2*d*x + 2*c)^2 +
2*b*cos(2*d*x + 2*c) + b)*d)
```

Fricas [A] time = 1.64232, size = 81, normalized size = 2.53

$$\frac{\sqrt{\frac{b}{\cos(dx+c)}}\sin(dx+c)}{bd\sqrt{\cos(dx+c)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)/(b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] sqrt(b/cos(d*x + c))*sin(d*x + c)/(b*d*sqrt(cos(d*x + c)))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(5/2)/(b*sec(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)^{\frac{5}{2}}}{\sqrt{b\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)/(b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^(5/2)/sqrt(b*sec(d*x + c)), x)
```

3.163 $\int \frac{\sec^{\frac{3}{2}}(c+dx)}{\sqrt{b\sec(c+dx)}}\,dx$

Optimal. Leaf size=33

$$\frac{\sqrt{\sec(c+dx)}\tanh^{-1}(\sin(c+dx))}{d\sqrt{b\sec(c+dx)}}$$

```
[Out] (ArcTanh[Sin[c + d*x]]*Sqrt[Sec[c + d*x]])/(d*Sqrt[b*Sec[c + d*x]])
```

Rubi [A] time = 0.0074269, antiderivative size = 33, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {17, 3770}

$$\frac{\sqrt{\sec(c+dx)}\tanh^{-1}(\sin(c+dx))}{d\sqrt{b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(3/2)/Sqrt[b*Sec[c + d*x]],x]
```

```
[Out] (ArcTanh[Sin[c + d*x]]*Sqrt[Sec[c + d*x]])/(d*Sqrt[b*Sec[c + d*x]])
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(a^(m + 1/
2)*b^(n - 1/2)*Sqrt[b*v])/Sqrt[a*v], Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] && !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec^{\frac{3}{2}}(c+dx)}{\sqrt{b\sec(c+dx)}}\,dx &= \frac{\sqrt{\sec(c+dx)}\int \sec(c+dx)\,dx}{\sqrt{b\sec(c+dx)}}\\ &= \frac{\tanh^{-1}(\sin(c+dx))\sqrt{\sec(c+dx)}}{d\sqrt{b\sec(c+dx)}}\end{aligned}$$

Mathematica [A] time = 0.0164863, size = 33, normalized size = 1.

$$\frac{\sqrt{\sec(c+dx)}\tanh^{-1}(\sin(c+dx))}{d\sqrt{b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^(3/2)/Sqrt[b*Sec[c + d*x]],x]
```

```
[Out] (ArcTanh[Sin[c + d*x]]*Sqrt[Sec[c + d*x]])/(d*Sqrt[b*Sec[c + d*x]])
```

Maple [A] time = 0.12, size = 52, normalized size = 1.6

$$-2\frac{\left((\cos(dx+c))^{-1}\right)^{3/2}\cos(dx+c)}{d}\mathrm{Artanh}\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}\right)\frac{1}{\sqrt{\frac{b}{\cos(dx+c)}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(3/2)/(b*sec(d*x+c))^(1/2),x)
```

```
[Out] -2/d*arctanh((-1+cos(d*x+c))/sin(d*x+c))*(1/cos(d*x+c))^(3/2)*cos(d*x+c)/(b
/cos(d*x+c))^(1/2)
```

Maxima [B] time = 1.98168, size = 88, normalized size = 2.67

$$\frac{\log\left(\cos(dx+c)^2+\sin(dx+c)^2+2\sin(dx+c)+1\right)-\log\left(\cos(dx+c)^2+\sin(dx+c)^2-2\sin(dx+c)+1\right)}{2\sqrt{b}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)/(b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*(log(cos(d*x + c)^2 + sin(d*x + c)^2 + 2*sin(d*x + c) + 1) - log(cos(d*
x + c)^2 + sin(d*x + c)^2 - 2*sin(d*x + c) + 1))/(sqrt(b)*d)
```

Fricas [A] time = 1.90028, size = 298, normalized size = 9.03

$$\left[\frac{\log\left(-\frac{b\cos(dx+c)^2-2\sqrt{b}\sqrt{\frac{b}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)-2b}{\cos(dx+c)^2}\right)}{2\sqrt{b}d}, -\frac{\sqrt{-b}\arctan\left(\frac{\sqrt{-b}\sqrt{\frac{b}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)}{b}\right)}{bd}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)/(b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*log(-(b*cos(d*x + c)^2 - 2*sqrt(b)*sqrt(b/cos(d*x + c))*sqrt(cos(d*x +
 c))*sin(d*x + c) - 2*b)/cos(d*x + c)^2)/(sqrt(b)*d), -sqrt(-b)*arctan(sqrt
(-b)*sqrt(b/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/b)/(b*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sec^{\frac{3}{2}}(c+dx)}{\sqrt{b\sec(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(3/2)/(b*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral(sec(c + d*x)**(3/2)/sqrt(b*sec(c + d*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)^{\frac{3}{2}}}{\sqrt{b\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)/(b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^(3/2)/sqrt(b*sec(d*x + c)), x)
```

3.164 $\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{b\sec(c+dx)}}\,dx$

Optimal. Leaf size=24

$$\frac{x\sqrt{\sec(c+dx)}}{\sqrt{b\sec(c+dx)}}$$

```
[Out] (x*Sqrt[Sec[c + d*x]])/Sqrt[b*Sec[c + d*x]]
```

Rubi [A] time = 0.0022741, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {17, 8}

$$\frac{x\sqrt{\sec(c+dx)}}{\sqrt{b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Sec[c + d*x]]/Sqrt[b*Sec[c + d*x]],x]
```

```
[Out] (x*Sqrt[Sec[c + d*x]])/Sqrt[b*Sec[c + d*x]]
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(a^(m + 1/
2)*b^(n - 1/2)*Sqrt[b*v])/Sqrt[a*v], Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] && !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{b\sec(c+dx)}}\,dx &= \frac{\sqrt{\sec(c+dx)}\int 1\,dx}{\sqrt{b\sec(c+dx)}}\\ &= \frac{x\sqrt{\sec(c+dx)}}{\sqrt{b\sec(c+dx)}}\end{aligned}$$

Mathematica [A] time = 0.0147899, size = 24, normalized size = 1.

$$\frac{x\sqrt{\sec(c+dx)}}{\sqrt{b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Sec[c + d*x]]/Sqrt[b*Sec[c + d*x]],x]
```

```
[Out] (x*Sqrt[Sec[c + d*x]])/Sqrt[b*Sec[c + d*x]]
```

Maple [A] time = 0.092, size = 32, normalized size = 1.3

$$\frac{dx+c}{d}\sqrt{(\cos(dx+c))^{-1}}\frac{1}{\sqrt{\frac{b}{\cos(dx+c)}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(1/2)/(b*sec(d*x+c))^(1/2),x)
```

```
[Out] 1/d*(d*x+c)*(1/cos(d*x+c))^(1/2)/(b/cos(d*x+c))^(1/2)
```

Maxima [A] time = 1.57971, size = 35, normalized size = 1.46

$$\frac{2\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{\sqrt{b}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)/(b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] 2*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/(sqrt(b)*d)
```

Fricas [A] time = 1.97238, size = 274, normalized size = 11.42

$$\left[-\frac{\sqrt{-b}\log\left(2\sqrt{-b}\sqrt{\frac{b}{\cos(dx+c)}}\cos(dx+c)^{\frac{3}{2}}\sin(dx+c)+2b\cos(dx+c)^2-b\right)}{2bd},\ \frac{\arctan\left(\frac{\sqrt{\frac{b}{\cos(dx+c)}}\sin(dx+c)}{\sqrt{b}\sqrt{\cos(dx+c)}}\right)}{\sqrt{b}d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)/(b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/2*sqrt(-b)*log(2*sqrt(-b)*sqrt(b/cos(d*x + c))*cos(d*x + c)^(3/2)*sin(d
*x + c) + 2*b*cos(d*x + c)^2 - b)/(b*d), arctan(sqrt(b/cos(d*x + c))*sin(d*
x + c)/(sqrt(b)*sqrt(cos(d*x + c))))/(sqrt(b)*d)]
```

Sympy [A] time = 19.0768, size = 5, normalized size = 0.21

$$\frac{x}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(1/2)/(b*sec(d*x+c))**(1/2),x)
```

```
[Out] x/sqrt(b)
```

<u>Giac</u> [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\sec(dx+c)}}{\sqrt{b\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)/(b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(sec(d*x + c))/sqrt(b*sec(d*x + c)), x)
```

3.165 $\int \frac{1}{\sqrt{\sec(c+dx)}\sqrt{b\sec(c+dx)}}\,dx$

Optimal. Leaf size=32

$$\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{d\sqrt{b\sec(c+dx)}}$$

```
[Out] (Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(d*Sqrt[b*Sec[c + d*x]])
```

Rubi [A] time = 0.0066139, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {18, 2637}

$$\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{d\sqrt{b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[Sec[c + d*x]]*Sqrt[b*Sec[c + d*x]]),x]
```

```
[Out] (Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(d*Sqrt[b*Sec[c + d*x]])
```

Rule 18

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(a^(m - 1/
2)*b^(n + 1/2)*Sqrt[a*v])/Sqrt[b*v], Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] && !IntegerQ[m] && ILtQ[n - 1/2, 0] && IntegerQ[m + n]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{\sec(c+dx)}\sqrt{b\sec(c+dx)}}\,dx &= \frac{\sqrt{\sec(c+dx)}\int \cos(c+dx)\,dx}{\sqrt{b\sec(c+dx)}}\\ &= \frac{\sqrt{\sec(c+dx)}\sin(c+dx)}{d\sqrt{b\sec(c+dx)}}\end{aligned}$$

Mathematica [A] time = 0.0361462, size = 32, normalized size = 1.

$$\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{d\sqrt{b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[Sec[c + d*x]]*Sqrt[b*Sec[c + d*x]]),x]
```

```
[Out] (Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(d*Sqrt[b*Sec[c + d*x]])
```

Maple [A] time = 0.154, size = 41, normalized size = 1.3

$$\frac{\sin (dx + c)}{d \cos (dx + c)} \frac{1}{\sqrt{(\cos (dx + c))^{-1}}} \frac{1}{\sqrt{\frac{b}{\cos(dx+c)}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sec(d*x+c)^(1/2)/(b*sec(d*x+c))^(1/2),x)
```

```
[Out] 1/d*sin(d*x+c)/(1/cos(d*x+c))^(1/2)/(b/cos(d*x+c))^(1/2)/cos(d*x+c)
```

Maxima [A] time = 2.09274, size = 18, normalized size = 0.56

$$\frac{\sin (dx + c)}{\sqrt{b}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(1/2)/(b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] sin(d*x + c)/(sqrt(b)*d)
```

Fricas [A] time = 1.7484, size = 81, normalized size = 2.53

$$\frac{\sqrt{\frac{b}{\cos(dx+c)}} \sqrt{\cos (dx + c)} \sin (dx + c)}{bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(1/2)/(b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] sqrt(b/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(b*d)
```

Sympy [A] time = 25.9602, size = 36, normalized size = 1.12

$$\begin{cases} \frac{\tan (c+dx)}{\sqrt{b}d \sec (c+dx)} & \text{for}\ d \neq 0 \\ \frac{x}{\sqrt{b \sec (c)} \sqrt{\sec (c)}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)**(1/2)/(b*sec(d*x+c))**(1/2),x)
```

```
[Out] Piecewise((tan(c + d*x)/(sqrt(b)*d*sec(c + d*x)), Ne(d, 0)), (x/(sqrt(b*sec
(c))*sqrt(sec(c))), True))
```

<u>Giac</u> [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{b\sec(dx+c)}\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(1/2)/(b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(b*sec(d*x + c))*sqrt(sec(d*x + c))), x)
```

3.166 $\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)\sqrt{b\sec(c+dx)}}\,dx$

Optimal. Leaf size=63

$$\frac{x\sqrt{\sec(c+dx)}}{2\sqrt{b\sec(c+dx)}}+\frac{\sin(c+dx)}{2d\sqrt{\sec(c+dx)}\sqrt{b\sec(c+dx)}}$$

```
[Out] (x*Sqrt[Sec[c + d*x]])/(2*Sqrt[b*Sec[c + d*x]]) + Sin[c + d*x]/(2*d*Sqrt[Se
c[c + d*x]]*Sqrt[b*Sec[c + d*x]])
```

Rubi [A] time = 0.0135085, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {18, 2635, 8}

$$\frac{x\sqrt{\sec(c+dx)}}{2\sqrt{b\sec(c+dx)}}+\frac{\sin(c+dx)}{2d\sqrt{\sec(c+dx)}\sqrt{b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sec[c + d*x]^(3/2)*Sqrt[b*Sec[c + d*x]]),x]
```

```
[Out] (x*Sqrt[Sec[c + d*x]])/(2*Sqrt[b*Sec[c + d*x]]) + Sin[c + d*x]/(2*d*Sqrt[Se
c[c + d*x]]*Sqrt[b*Sec[c + d*x]])
```

Rule 18

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(a^(m - 1/
2)*b^(n + 1/2)*Sqrt[a*v])/Sqrt[b*v], Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && ILtQ[n - 1/2, 0] && IntegerQ[m + n]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)\sqrt{b\sec(c+dx)}}\,dx &= \frac{\sqrt{\sec(c+dx)}\int \cos^2(c+dx)\,dx}{\sqrt{b\sec(c+dx)}}\\ &= \frac{\sin(c+dx)}{2d\sqrt{\sec(c+dx)}\sqrt{b\sec(c+dx)}}+\frac{\sqrt{\sec(c+dx)}\int 1\,dx}{2\sqrt{b\sec(c+dx)}}\\ &= \frac{x\sqrt{\sec(c+dx)}}{2\sqrt{b\sec(c+dx)}}+\frac{\sin(c+dx)}{2d\sqrt{\sec(c+dx)}\sqrt{b\sec(c+dx)}}\end{aligned}$$

Mathematica [A] time = 0.0769065, size = 45, normalized size = 0.71

$$\frac{(2(c+dx)+\sin(2(c+dx)))\sqrt{\sec(c+dx)}}{4d\sqrt{b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sec[c + d*x]^(3/2)*Sqrt[b*Sec[c + d*x]]),x]
```

```
[Out] (Sqrt[Sec[c + d*x]]*(2*(c + d*x) + Sin[2*(c + d*x)]))/(4*d*Sqrt[b*Sec[c + d
*x]])
```

Maple [A] time = 0.135, size = 54, normalized size = 0.9

$$\frac{\cos(dx+c)\sin(dx+c)+dx+c}{2d(\cos(dx+c))^2}\left((\cos(dx+c))^{-1}\right)^{-\frac{3}{2}}\frac{1}{\sqrt{\frac{b}{\cos(dx+c)}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sec(d*x+c)^(3/2)/(b*sec(d*x+c))^(1/2),x)
```

```
[Out] 1/2/d*(cos(d*x+c)*sin(d*x+c)+d*x+c)/cos(d*x+c)^2/(1/cos(d*x+c))^(3/2)/(b/co
s(d*x+c))^(1/2)
```

Maxima [A] time = 2.05379, size = 34, normalized size = 0.54

$$\frac{2dx+2c+\sin(2dx+2c)}{4\sqrt{b}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(3/2)/(b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] 1/4*(2*d*x + 2*c + sin(2*d*x + 2*c))/(sqrt(b)*d)
```

Fricas [A] time = 2.00488, size = 437, normalized size = 6.94

$$\left[\frac{2\sqrt{\frac{b}{\cos(dx+c)}}\cos(dx+c)^{\frac{3}{2}}\sin(dx+c)-\sqrt{-b}\log\left(2\sqrt{-b}\sqrt{\frac{b}{\cos(dx+c)}}\cos(dx+c)^{\frac{3}{2}}\sin(dx+c)+2b\cos(dx+c)^2-b\right)\sqrt{}}{4bd},-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(3/2)/(b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(2*sqrt(b/cos(d*x + c))*cos(d*x + c)^(3/2)*sin(d*x + c) - sqrt(-b)*log
(2*sqrt(-b)*sqrt(b/cos(d*x + c))*cos(d*x + c)^(3/2)*sin(d*x + c) + 2*b*cos(
```

```
d*x + c)^2 - b))/(b*d), 1/2*(sqrt(b/cos(d*x + c))*cos(d*x + c)^(3/2)*sin(d*
x + c) + sqrt(b)*arctan(sqrt(b/cos(d*x + c))*sin(d*x + c)/(sqrt(b)*sqrt(cos
(d*x + c)))))/(b*d)]
```

Sympy [A] time = 44.2609, size = 82, normalized size = 1.3

$$\begin{cases} \frac{x\tan^2(c+dx)}{2\sqrt{b}\sec^2(c+dx)} + \frac{x}{2\sqrt{b}\sec^2(c+dx)} + \frac{\tan(c+dx)}{2\sqrt{b}d\sec^2(c+dx)} & \text{for } d \neq 0 \\ \frac{x}{\sqrt{b\sec(c)}\sec^{\frac{3}{2}}(c)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)**(3/2)/(b*sec(d*x+c))**(1/2),x)
```

```
[Out] Piecewise((x*tan(c + d*x)**2/(2*sqrt(b)*sec(c + d*x)**2) + x/(2*sqrt(b)*sec
(c + d*x)**2) + tan(c + d*x)/(2*sqrt(b)*d*sec(c + d*x)**2), Ne(d, 0)), (x/(
sqrt(b*sec(c))*sec(c)**(3/2)), True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{b\sec(dx+c)}\sec(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(3/2)/(b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(b*sec(d*x + c))*sec(d*x + c)^(3/2)), x)
```

3.167 $\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)\sqrt{b\sec(c+dx)}}\,dx$

Optimal. Leaf size=70

$$\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{d\sqrt{b\sec(c+dx)}}-\frac{\sin^3(c+dx)\sqrt{\sec(c+dx)}}{3d\sqrt{b\sec(c+dx)}}$$

```
[Out] (Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(d*Sqrt[b*Sec[c + d*x]]) - (Sqrt[Sec[c +
d*x]]*Sin[c + d*x]^3)/(3*d*Sqrt[b*Sec[c + d*x]])
```

Rubi [A] time = 0.0157183, antiderivative size = 70, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {18, 2633}

$$\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{d\sqrt{b\sec(c+dx)}}-\frac{\sin^3(c+dx)\sqrt{\sec(c+dx)}}{3d\sqrt{b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sec[c + d*x]^(5/2)*Sqrt[b*Sec[c + d*x]]),x]
```

```
[Out] (Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(d*Sqrt[b*Sec[c + d*x]]) - (Sqrt[Sec[c +
d*x]]*Sin[c + d*x]^3)/(3*d*Sqrt[b*Sec[c + d*x]])
```

Rule 18

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(a^(m - 1/
2)*b^(n + 1/2)*Sqrt[a*v])/Sqrt[b*v], Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && ILtQ[n - 1/2, 0] && IntegerQ[m + n]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)\sqrt{b\sec(c+dx)}}\,dx &= \frac{\sqrt{\sec(c+dx)}\int \cos^3(c+dx)\,dx}{\sqrt{b\sec(c+dx)}}\\ &= -\frac{\sqrt{\sec(c+dx)}\,\text{Subst}\left(\int \left(1-x^2\right)\,dx,x,-\sin(c+dx)\right)}{d\sqrt{b\sec(c+dx)}}\\ &= \frac{\sqrt{\sec(c+dx)}\sin(c+dx)}{d\sqrt{b\sec(c+dx)}}-\frac{\sqrt{\sec(c+dx)}\sin^3(c+dx)}{3d\sqrt{b\sec(c+dx)}}\end{aligned}$$

Mathematica [A] time = 0.0942708, size = 45, normalized size = 0.64

$$\frac{\sin(c+dx)(\cos(2(c+dx))+5)\sqrt{\sec(c+dx)}}{6d\sqrt{b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sec[c + d*x]^(5/2)*Sqrt[b*Sec[c + d*x]]),x]
```

```
[Out] ((5 + Cos[2*(c + d*x)])*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(6*d*Sqrt[b*Sec[c
+ d*x]])
```

Maple [A] time = 0.124, size = 52, normalized size = 0.7

$$\frac{\sin(dx+c)\left((\cos(dx+c))^2+2\right)}{3d(\cos(dx+c))^3}\left((\cos(dx+c))^{-1}\right)^{-\frac{5}{2}}\frac{1}{\sqrt{\frac{b}{\cos(dx+c)}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sec(d*x+c)^(5/2)/(b*sec(d*x+c))^(1/2),x)
```

```
[Out] 1/3/d*sin(d*x+c)*(cos(d*x+c)^2+2)/cos(d*x+c)^3/(1/cos(d*x+c))^(5/2)/(b/cos(
d*x+c))^(1/2)
```

Maxima [A] time = 2.12168, size = 57, normalized size = 0.81

$$\frac{\sin(3dx+3c)+9\sin\left(\frac{1}{3}\arctan(\sin(3dx+3c),\cos(3dx+3c))\right)}{12\sqrt{b}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(5/2)/(b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] 1/12*(sin(3*d*x + 3*c) + 9*sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*
c))))/(sqrt(b)*d)
```

Fricas [A] time = 1.6854, size = 132, normalized size = 1.89

$$\frac{\left(\cos(dx+c)^3+2\cos(dx+c)\right)\sqrt{\frac{b}{\cos(dx+c)}}\sin(dx+c)}{3bd\sqrt{\cos(dx+c)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(5/2)/(b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3*(cos(d*x + c)^3 + 2*cos(d*x + c))*sqrt(b/cos(d*x + c))*sin(d*x + c)/(b*
d*sqrt(cos(d*x + c)))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)**(5/2)/(b*sec(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{b\sec{(dx+c)}}\sec{(dx+c)}^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(5/2)/(b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(b*sec(d*x + c))*sec(d*x + c)^(5/2)), x)
```

3.168 $\int \frac{\sec^{\frac{9}{2}}(c+dx)}{(b\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=78

$$\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{2bd\sqrt{b\sec(c+dx)}}+\frac{\sqrt{\sec(c+dx)}\tanh^{-1}(\sin(c+dx))}{2bd\sqrt{b\sec(c+dx)}}$$

```
[Out] (ArcTanh[Sin[c + d*x]]*Sqrt[Sec[c + d*x]])/(2*b*d*Sqrt[b*Sec[c + d*x]]) + (
Sec[c + d*x]^(5/2)*Sin[c + d*x])/(2*b*d*Sqrt[b*Sec[c + d*x]])
```

Rubi [A] time = 0.0201799, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {17, 3768, 3770}

$$\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{2bd\sqrt{b\sec(c+dx)}}+\frac{\sqrt{\sec(c+dx)}\tanh^{-1}(\sin(c+dx))}{2bd\sqrt{b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(9/2)/(b*Sec[c + d*x])^(3/2),x]
```

```
[Out] (ArcTanh[Sin[c + d*x]]*Sqrt[Sec[c + d*x]])/(2*b*d*Sqrt[b*Sec[c + d*x]]) + (
Sec[c + d*x]^(5/2)*Sin[c + d*x])/(2*b*d*Sqrt[b*Sec[c + d*x]])
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(a^(m + 1/
2)*b^(n - 1/2)*Sqrt[b*v])/Sqrt[a*v], Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sec^{\frac{9}{2}}(c+dx)}{(b\sec(c+dx))^{3/2}}\,dx &= \frac{\sqrt{\sec(c+dx)}\int \sec^3(c+dx)\,dx}{b\sqrt{b\sec(c+dx)}}\\
&= \frac{\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{2bd\sqrt{b\sec(c+dx)}}+\frac{\sqrt{\sec(c+dx)}\int \sec(c+dx)\,dx}{2b\sqrt{b\sec(c+dx)}}\\
&= \frac{\tanh^{-1}(\sin(c+dx))\sqrt{\sec(c+dx)}}{2bd\sqrt{b\sec(c+dx)}}+\frac{\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{2bd\sqrt{b\sec(c+dx)}}
\end{aligned}$$

Mathematica [A] time = 0.0574739, size = 50, normalized size = 0.64

$$\frac{\sec^{\frac{3}{2}}(c+dx)\left(\tanh^{-1}(\sin(c+dx))+\tan(c+dx)\sec(c+dx)\right)}{2d(b\sec(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^(9/2)/(b*Sec[c + d*x])^(3/2),x]
```

```
[Out] (Sec[c + d*x]^(3/2)*(ArcTanh[Sin[c + d*x]] + Sec[c + d*x]*Tan[c + d*x]))/(2
*d*(b*Sec[c + d*x])^(3/2))
```

Maple [A] time = 0.117, size = 112, normalized size = 1.4

$$\frac{\cos(dx+c)}{2d}\left(\ln\left(-\frac{-1+\cos(dx+c)-\sin(dx+c)}{\sin(dx+c)}\right)(\cos(dx+c))^2-\ln\left(-\frac{-1+\cos(dx+c)+\sin(dx+c)}{\sin(dx+c)}\right)(\cos(dx+c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(9/2)/(b*sec(d*x+c))^(3/2),x)
```

```
[Out] 1/2/d*(ln(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))*cos(d*x+c)^2-ln(-(-1+cos(
d*x+c)+sin(d*x+c))/sin(d*x+c))*cos(d*x+c)^2+sin(d*x+c))*(1/cos(d*x+c))^(9/2
)*cos(d*x+c)/(b/cos(d*x+c))^(3/2)
```

Maxima [B] time = 2.14968, size = 905, normalized size = 11.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(9/2)/(b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] -1/4*(4*(sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*cos(3/2*arctan2(sin(2*d*x +
 2*c), cos(2*d*x + 2*c))) - 4*(sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*cos(1
/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - (2*(2*cos(2*d*x + 2*c) +
1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*x
 + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*
cos(2*d*x + 2*c) + 1)*log(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c
)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/2*
arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) + (2*(2*cos(2*d*x + 2*c)
+ 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d
*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 +
4*cos(2*d*x + 2*c) + 1)*log(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2
*c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sin(1/
2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) - 4*(cos(4*d*x + 4*c) +
 2*cos(2*d*x + 2*c) + 1)*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
)) + 4*(cos(4*d*x + 4*c) + 2*cos(2*d*x + 2*c) + 1)*sin(1/2*arctan2(sin(2*d*
x + 2*c), cos(2*d*x + 2*c))))/((b*cos(4*d*x + 4*c)^2 + 4*b*cos(2*d*x + 2*c)
^2 + b*sin(4*d*x + 4*c)^2 + 4*b*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*b*sin
(2*d*x + 2*c)^2 + 2*(2*b*cos(2*d*x + 2*c) + b)*cos(4*d*x + 4*c) + 4*b*cos(2
*d*x + 2*c) + b)*sqrt(b)*d)
```

Fricas [A] time = 1.95467, size = 539, normalized size = 6.91

$$\left[\frac{\sqrt{b}\cos\left(dx+c\right)\log\left(-\frac{b\cos(dx+c)^2-2\sqrt{b}\sqrt{\frac{b}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)-2b}{\cos(dx+c)^2}\right)+\frac{2\sqrt{\frac{b}{\cos(dx+c)}}\sin(dx+c)}{\sqrt{\cos(dx+c)}}}{4b^2d\cos\left(dx+c\right)}, -\frac{\sqrt{-b}\arctan\left(\frac{\sqrt{-b}\sqrt{\frac{b}{\cos(dx+c)}}\cdots}{\cdots}\right)\cdots}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(9/2)/(b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(b)*cos(d*x + c)*log(-(b*cos(d*x + c)^2 - 2*sqrt(b)*sqrt(b/cos(d*
x + c))*sqrt(cos(d*x + c))*sin(d*x + c) - 2*b)/cos(d*x + c)^2) + 2*sqrt(b/c
os(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(b^2*d*cos(d*x + c)), -1/2*(s
qrt(-b)*arctan(sqrt(-b)*sqrt(b/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c
)/b)*cos(d*x + c) - sqrt(b/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(
b^2*d*cos(d*x + c))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(9/2)/(b*sec(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec\left(dx+c\right)^{\frac{9}{2}}}{\left(b\sec\left(dx+c\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(9/2)/(b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^(9/2)/(b*sec(d*x + c))^(3/2), x)
```

3.169 $\int \frac{\sec^{\frac{7}{2}}(c+dx)}{(b\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=35

$$\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{bd\sqrt{b\sec(c+dx)}}$$

```
[Out] (Sec[c + d*x]^(3/2)*Sin[c + d*x])/(b*d*Sqrt[b*Sec[c + d*x]])
```

Rubi [A] time = 0.0118306, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {17, 3767, 8}

$$\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{bd\sqrt{b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(7/2)/(b*Sec[c + d*x])^(3/2),x]
```

```
[Out] (Sec[c + d*x]^(3/2)*Sin[c + d*x])/(b*d*Sqrt[b*Sec[c + d*x]])
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(a^(m + 1/
2)*b^(n - 1/2)*Sqrt[b*v])/Sqrt[a*v], Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sec^{\frac{7}{2}}(c+dx)}{(b\sec(c+dx))^{3/2}}\,dx &= \frac{\sqrt{\sec(c+dx)}\int \sec^2(c+dx)\,dx}{b\sqrt{b\sec(c+dx)}}\\
&= -\frac{\sqrt{\sec(c+dx)}\,\text{Subst}\left(\int 1\,dx, x, -\tan(c+dx)\right)}{bd\sqrt{b\sec(c+dx)}}\\
&= \frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{bd\sqrt{b\sec(c+dx)}}
\end{aligned}$$

Mathematica [A] time = 0.0353204, size = 32, normalized size = 0.91

$$\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{d(b\sec(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^(7/2)/(b*Sec[c + d*x])^(3/2),x]
```

```
[Out] (Sec[c + d*x]^(5/2)*Sin[c + d*x])/(d*(b*Sec[c + d*x])^(3/2))
```

Maple [A] time = 0.101, size = 39, normalized size = 1.1

$$\frac{\cos(dx+c)\sin(dx+c)}{d}\left((\cos(dx+c))^{-1}\right)^{\frac{7}{2}}\left(\frac{b}{\cos(dx+c)}\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(7/2)/(b*sec(d*x+c))^(3/2),x)
```

```
[Out] 1/d*(1/cos(d*x+c))^(7/2)*cos(d*x+c)*sin(d*x+c)/(b/cos(d*x+c))^(3/2)
```

Maxima [B] time = 2.1738, size = 90, normalized size = 2.57

$$\frac{2\sqrt{b}\sin(2dx+2c)}{\left(b^2\cos(2dx+2c)^2+b^2\sin(2dx+2c)^2+2b^2\cos(2dx+2c)+b^2\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(7/2)/(b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] 2*sqrt(b)*sin(2*d*x + 2*c)/((b^2*cos(2*d*x + 2*c)^2 + b^2*sin(2*d*x + 2*c)^
2 + 2*b^2*cos(2*d*x + 2*c) + b^2)*d)
```

Fricas [A] time = 1.67221, size = 84, normalized size = 2.4

$$\frac{\sqrt{\frac{b}{\cos(dx+c)}}\sin(dx+c)}{b^2d\sqrt{\cos(dx+c)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(7/2)/(b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] sqrt(b/cos(d*x + c))*sin(d*x + c)/(b^2*d*sqrt(cos(d*x + c)))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(7/2)/(b*sec(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)^{\frac{7}{2}}}{(b\sec(dx+c))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(7/2)/(b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^(7/2)/(b*sec(d*x + c))^(3/2), x)
```

3.170 $\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(b\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=36

$$\frac{\sqrt{\sec(c+dx)}\tanh^{-1}(\sin(c+dx))}{bd\sqrt{b\sec(c+dx)}}$$

```
[Out] (ArcTanh[Sin[c + d*x]]*Sqrt[Sec[c + d*x]])/(b*d*Sqrt[b*Sec[c + d*x]])
```

Rubi [A] time = 0.0074694, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {17, 3770}

$$\frac{\sqrt{\sec(c+dx)}\tanh^{-1}(\sin(c+dx))}{bd\sqrt{b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(5/2)/(b*Sec[c + d*x])^(3/2),x]
```

```
[Out] (ArcTanh[Sin[c + d*x]]*Sqrt[Sec[c + d*x]])/(b*d*Sqrt[b*Sec[c + d*x]])
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(a^(m + 1/
2)*b^(n - 1/2)*Sqrt[b*v])/Sqrt[a*v], Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(b\sec(c+dx))^{3/2}}\,dx &= \frac{\sqrt{\sec(c+dx)}\int \sec(c+dx)\,dx}{b\sqrt{b\sec(c+dx)}}\\ &= \frac{\tanh^{-1}(\sin(c+dx))\sqrt{\sec(c+dx)}}{bd\sqrt{b\sec(c+dx)}}\end{aligned}$$

Mathematica [A] time = 0.0262157, size = 33, normalized size = 0.92

$$\frac{\sec^{\frac{3}{2}}(c+dx)\tanh^{-1}(\sin(c+dx))}{d(b\sec(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^(5/2)/(b*Sec[c + d*x])^(3/2),x]
```

```
[Out] (ArcTanh[Sin[c + d*x]]*Sec[c + d*x]^(3/2))/(d*(b*Sec[c + d*x])^(3/2))
```

Maple [A] time = 0.092, size = 52, normalized size = 1.4

$$-2\frac{\cos\left(dx+c\right)\left(\left(\cos\left(dx+c\right)\right)^{-1}\right)^{5/2}}{d}\mathrm{Artanh}\left(\frac{-1+\cos\left(dx+c\right)}{\sin\left(dx+c\right)}\right)\left(\frac{b}{\cos\left(dx+c\right)}\right)^{-3/2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(5/2)/(b*sec(d*x+c))^(3/2),x)
```

```
[Out] -2/d*arctanh((-1+cos(d*x+c))/sin(d*x+c))*cos(d*x+c)*(1/cos(d*x+c))^(5/2)/(b
/cos(d*x+c))^(3/2)
```

Maxima [B] time = 2.05257, size = 88, normalized size = 2.44

$$\frac{\log\left(\cos\left(dx+c\right)^{2}+\sin\left(dx+c\right)^{2}+2\,\sin\left(dx+c\right)+1\right)-\log\left(\cos\left(dx+c\right)^{2}+\sin\left(dx+c\right)^{2}-2\,\sin\left(dx+c\right)+1\right)}{2\,b^{\frac{3}{2}}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)/(b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] 1/2*(log(cos(d*x + c)^2 + sin(d*x + c)^2 + 2*sin(d*x + c) + 1) - log(cos(d*
x + c)^2 + sin(d*x + c)^2 - 2*sin(d*x + c) + 1))/(b^(3/2)*d)
```

Fricas [A] time = 1.88568, size = 301, normalized size = 8.36

$$\left[\frac{\log\left(-\frac{b\cos\left(dx+c\right)^{2}-2\,\sqrt{b}\sqrt{\frac{b}{\cos\left(dx+c\right)}}\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)-2\,b}{\cos\left(dx+c\right)^{2}}\right)}{2\,b^{\frac{3}{2}}d},-\frac{\sqrt{-b}\arctan\left(\frac{\sqrt{-b}\sqrt{\frac{b}{\cos\left(dx+c\right)}}\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)}{b}\right)}{b^{2}d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)/(b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*log(-(b*cos(d*x + c)^2 - 2*sqrt(b)*sqrt(b/cos(d*x + c))*sqrt(cos(d*x +
 c))*sin(d*x + c) - 2*b)/cos(d*x + c)^2)/(b^(3/2)*d), -sqrt(-b)*arctan(sqrt
(-b)*sqrt(b/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/b)/(b^2*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(5/2)/(b*sec(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)^{\frac{5}{2}}}{(b\sec(dx+c))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)/(b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^(5/2)/(b*sec(d*x + c))^(3/2), x)
```

3.171 $\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(b\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=27

$$\frac{x\sqrt{\sec(c+dx)}}{b\sqrt{b\sec(c+dx)}}$$

```
[Out] (x*Sqrt[Sec[c + d*x]])/(b*Sqrt[b*Sec[c + d*x]])
```

Rubi [A] time = 0.0027357, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {17, 8}

$$\frac{x\sqrt{\sec(c+dx)}}{b\sqrt{b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(3/2)/(b*Sec[c + d*x])^(3/2),x]
```

```
[Out] (x*Sqrt[Sec[c + d*x]])/(b*Sqrt[b*Sec[c + d*x]])
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(a^(m + 1/
2)*b^(n - 1/2)*Sqrt[b*v])/Sqrt[a*v], Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(b\sec(c+dx))^{3/2}}\,dx &= \frac{\sqrt{\sec(c+dx)}\int 1\,dx}{b\sqrt{b\sec(c+dx)}}\\ &= \frac{x\sqrt{\sec(c+dx)}}{b\sqrt{b\sec(c+dx)}}\end{aligned}$$

Mathematica [A] time = 0.025473, size = 24, normalized size = 0.89

$$\frac{x\sec^{\frac{3}{2}}(c+dx)}{(b\sec(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^(3/2)/(b*Sec[c + d*x])^(3/2),x]
```

```
[Out] (x*Sec[c + d*x]^(3/2))/(b*Sec[c + d*x])^(3/2)
```

Maple [A] time = 0.083, size = 32, normalized size = 1.2

$$\frac{dx+c}{d}\left(\left(\cos\left(dx+c\right)\right)^{-1}\right)^{\frac{3}{2}}\left(\frac{b}{\cos\left(dx+c\right)}\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(3/2)/(b*sec(d*x+c))^(3/2),x)
```

```
[Out] 1/d*(d*x+c)*(1/cos(d*x+c))^(3/2)/(b/cos(d*x+c))^(3/2)
```

Maxima [A] time = 1.64326, size = 35, normalized size = 1.3

$$\frac{2\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{b^{\frac{3}{2}}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)/(b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] 2*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/(b^(3/2)*d)
```

Fricas [A] time = 1.93745, size = 277, normalized size = 10.26

$$\left[-\frac{\sqrt{-b}\log\left(2\sqrt{-b}\sqrt{\frac{b}{\cos(dx+c)}}\cos\left(dx+c\right)^{\frac{3}{2}}\sin\left(dx+c\right)+2\,b\cos\left(dx+c\right)^{2}-b\right)}{2\,b^{2}d},\ \frac{\arctan\left(\frac{\sqrt{\frac{b}{\cos(dx+c)}}\sin(dx+c)}{\sqrt{b}\sqrt{\cos(dx+c)}}\right)}{b^{\frac{3}{2}}d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)/(b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/2*sqrt(-b)*log(2*sqrt(-b)*sqrt(b/cos(d*x + c))*cos(d*x + c)^(3/2)*sin(d
*x + c) + 2*b*cos(d*x + c)^2 - b)/(b^2*d), arctan(sqrt(b/cos(d*x + c))*sin(
d*x + c)/(sqrt(b)*sqrt(cos(d*x + c))))/(b^(3/2)*d)]
```

Sympy [A] time = 133.827, size = 5, normalized size = 0.19

$$\frac{x}{b^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(3/2)/(b*sec(d*x+c))**(3/2),x)
```

```
[Out] x/b**(3/2)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)^{\frac{3}{2}}}{(b\sec(dx+c))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)/(b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^(3/2)/(b*sec(d*x + c))^(3/2), x)
```

3.172 $\int \frac{\sqrt{\sec(c+dx)}}{(b\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=35

$$\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{bd\sqrt{b\sec(c+dx)}}$$

```
[Out] (Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(b*d*Sqrt[b*Sec[c + d*x]])
```

Rubi [A] time = 0.007553, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {17, 2637}

$$\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{bd\sqrt{b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Sec[c + d*x]]/(b*Sec[c + d*x])^(3/2),x]
```

```
[Out] (Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(b*d*Sqrt[b*Sec[c + d*x]])
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(a^(m + 1/
2)*b^(n - 1/2)*Sqrt[b*v])/Sqrt[a*v], Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{\sec(c+dx)}}{(b\sec(c+dx))^{3/2}}\,dx &= \frac{\sqrt{\sec(c+dx)}\int \cos(c+dx)\,dx}{b\sqrt{b\sec(c+dx)}}\\ &= \frac{\sqrt{\sec(c+dx)}\sin(c+dx)}{bd\sqrt{b\sec(c+dx)}}\end{aligned}$$

Mathematica [A] time = 0.0410732, size = 32, normalized size = 0.91

$$\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d(b\sec(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Sec[c + d*x]]/(b*Sec[c + d*x])^(3/2),x]
```

```
[Out] (Sec[c + d*x]^(3/2)*Sin[c + d*x])/(d*(b*Sec[c + d*x])^(3/2))
```

Maple [A] time = 0.105, size = 41, normalized size = 1.2

$$\frac{\sin(dx+c)}{d\cos(dx+c)}\sqrt{(\cos(dx+c))^{-1}}\left(\frac{b}{\cos(dx+c)}\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(1/2)/(b*sec(d*x+c))^(3/2),x)
```

```
[Out] 1/d*sin(d*x+c)*(1/cos(d*x+c))^(1/2)/(b/cos(d*x+c))^(3/2)/cos(d*x+c)
```

Maxima [A] time = 1.93042, size = 18, normalized size = 0.51

$$\frac{\sin(dx+c)}{b^{\frac{3}{2}}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)/(b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] sin(d*x + c)/(b^(3/2)*d)
```

Fricas [A] time = 1.65839, size = 84, normalized size = 2.4

$$\frac{\sqrt{\frac{b}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)}{b^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)/(b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] sqrt(b/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(b^2*d)
```

Sympy [A] time = 19.5518, size = 36, normalized size = 1.03

$$\begin{cases}\frac{\tan(c+dx)}{b^{\frac{3}{2}}d\sec(c+dx)} & \text{for } d\neq 0\\ \frac{x\sqrt{\sec(c)}}{(b\sec(c))^{\frac{3}{2}}} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(1/2)/(b*sec(d*x+c))**(3/2),x)
```

```
[Out] Piecewise((tan(c + d*x)/(b**(3/2)*d*sec(c + d*x)), Ne(d, 0)), (x*sqrt(sec(c
))/(b*sec(c))**(3/2), True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\sec(dx+c)}}{(b\sec(dx+c))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)/(b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(sec(d*x + c))/(b*sec(d*x + c))^(3/2), x)
```

3.173 $\int \frac{1}{\sqrt{\sec(c+dx)}(b\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=69

$$\frac{x\sqrt{\sec(c+dx)}}{2b\sqrt{b\sec(c+dx)}}+\frac{\sin(c+dx)}{2bd\sqrt{\sec(c+dx)}\sqrt{b\sec(c+dx)}}$$

```
[Out] (x*Sqrt[Sec[c + d*x]])/(2*b*Sqrt[b*Sec[c + d*x]]) + Sin[c + d*x]/(2*b*d*Sqr
t[Sec[c + d*x]]*Sqrt[b*Sec[c + d*x]])
```

Rubi [A] time = 0.0141337, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {18, 2635, 8}

$$\frac{x\sqrt{\sec(c+dx)}}{2b\sqrt{b\sec(c+dx)}}+\frac{\sin(c+dx)}{2bd\sqrt{\sec(c+dx)}\sqrt{b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[Sec[c + d*x]]*(b*Sec[c + d*x])^(3/2)),x]
```

```
[Out] (x*Sqrt[Sec[c + d*x]])/(2*b*Sqrt[b*Sec[c + d*x]]) + Sin[c + d*x]/(2*b*d*Sqr
t[Sec[c + d*x]]*Sqrt[b*Sec[c + d*x]])
```

Rule 18

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(a^(m - 1/
2)*b^(n + 1/2)*Sqrt[a*v])/Sqrt[b*v], Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && ILtQ[n - 1/2, 0] && IntegerQ[m + n]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{\sec(c+dx)}(b\sec(c+dx))^{3/2}}\,dx &= \frac{\sqrt{\sec(c+dx)}\int \cos^2(c+dx)\,dx}{b\sqrt{b\sec(c+dx)}}\\ &= \frac{\sin(c+dx)}{2bd\sqrt{\sec(c+dx)}\sqrt{b\sec(c+dx)}}+\frac{\sqrt{\sec(c+dx)}\int 1\,dx}{2b\sqrt{b\sec(c+dx)}}\\ &= \frac{x\sqrt{\sec(c+dx)}}{2b\sqrt{b\sec(c+dx)}}+\frac{\sin(c+dx)}{2bd\sqrt{\sec(c+dx)}\sqrt{b\sec(c+dx)}}\end{aligned}$$

Mathematica [A] time = 0.0659757, size = 45, normalized size = 0.65

$$\frac{(2(c+dx)+\sin(2(c+dx)))\sec^{\frac{3}{2}}(c+dx)}{4d(b\sec(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[Sec[c + d*x]]*(b*Sec[c + d*x])^(3/2)),x]
```

```
[Out] (Sec[c + d*x]^(3/2)*(2*(c + d*x) + Sin[2*(c + d*x)]))/(4*d*(b*Sec[c + d*x])
^(3/2))
```

Maple [A] time = 0.123, size = 54, normalized size = 0.8

$$\frac{\cos(dx+c)\sin(dx+c)+dx+c}{2\,d\,(\cos(dx+c))^2}\frac{1}{\sqrt{(\cos(dx+c))^{-1}}}\left(\frac{b}{\cos(dx+c)}\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sec(d*x+c)^(1/2)/(b*sec(d*x+c))^(3/2),x)
```

```
[Out] 1/2/d*(cos(d*x+c)*sin(d*x+c)+d*x+c)/cos(d*x+c)^2/(1/cos(d*x+c))^(1/2)/(b/co
s(d*x+c))^(3/2)
```

Maxima [A] time = 2.04361, size = 34, normalized size = 0.49

$$\frac{2\,dx+2\,c+\sin(2\,dx+2\,c)}{4\,b^{\frac{3}{2}}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(1/2)/(b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] 1/4*(2*d*x + 2*c + sin(2*d*x + 2*c))/(b^(3/2)*d)
```

Fricas [A] time = 1.97708, size = 443, normalized size = 6.42

$$\left[\frac{2\sqrt{\frac{b}{\cos(dx+c)}}\cos(dx+c)^{\frac{3}{2}}\sin(dx+c)-\sqrt{-b}\log\left(2\sqrt{-b}\sqrt{\frac{b}{\cos(dx+c)}}\cos(dx+c)^{\frac{3}{2}}\sin(dx+c)+2\,b\cos(dx+c)^2-b\right)}{4\,b^2d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(1/2)/(b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*(2*sqrt(b/cos(d*x + c))*cos(d*x + c)^(3/2)*sin(d*x + c) - sqrt(-b)*log
(2*sqrt(-b)*sqrt(b/cos(d*x + c))*cos(d*x + c)^(3/2)*sin(d*x + c) + 2*b*cos(
d*x + c)^2 - b))/(b^2*d), 1/2*(sqrt(b/cos(d*x + c))*cos(d*x + c)^(3/2)*sin(
d*x + c) + sqrt(b)*arctan(sqrt(b/cos(d*x + c))*sin(d*x + c)/(sqrt(b)*sqrt(c
os(d*x + c)))))/(b^2*d)]
```

Sympy [A] time = 49.1731, size = 82, normalized size = 1.19

$$\begin{cases} \frac{x\tan^2(c+dx)}{2b^{\frac{3}{2}}\sec^2(c+dx)} + \frac{x}{2b^{\frac{3}{2}}\sec^2(c+dx)} + \frac{\tan(c+dx)}{2b^{\frac{3}{2}}d\sec^2(c+dx)} & \text{for } d \neq 0 \\ \frac{x}{(b\sec(c))^{\frac{3}{2}}\sqrt{\sec(c)}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)**(1/2)/(b*sec(d*x+c))**(3/2),x)
```

```
[Out] Piecewise((x*tan(c + d*x)**2/(2*b**(3/2)*sec(c + d*x)**2) + x/(2*b**(3/2)*s
ec(c + d*x)**2) + tan(c + d*x)/(2*b**(3/2)*d*sec(c + d*x)**2), Ne(d, 0)), (
x/((b*sec(c))**(3/2)*sqrt(sec(c))), True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b\sec(dx+c))^{\frac{3}{2}}\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(1/2)/(b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*sec(d*x + c))^(3/2)*sqrt(sec(d*x + c))), x)
```

3.174 $\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(b\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=76

$$\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{bd\sqrt{b\sec(c+dx)}}-\frac{\sin^3(c+dx)\sqrt{\sec(c+dx)}}{3bd\sqrt{b\sec(c+dx)}}$$

```
[Out] (Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(b*d*Sqrt[b*Sec[c + d*x]]) - (Sqrt[Sec[c
+ d*x]]*Sin[c + d*x]^3)/(3*b*d*Sqrt[b*Sec[c + d*x]])
```

Rubi [A] time = 0.0168956, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {18, 2633}

$$\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{bd\sqrt{b\sec(c+dx)}}-\frac{\sin^3(c+dx)\sqrt{\sec(c+dx)}}{3bd\sqrt{b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sec[c + d*x]^(3/2)*(b*Sec[c + d*x])^(3/2)),x]
```

```
[Out] (Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(b*d*Sqrt[b*Sec[c + d*x]]) - (Sqrt[Sec[c
+ d*x]]*Sin[c + d*x]^3)/(3*b*d*Sqrt[b*Sec[c + d*x]])
```

Rule 18

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(a^(m - 1/
2)*b^(n + 1/2)*Sqrt[a*v])/Sqrt[b*v], Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && ILtQ[n - 1/2, 0] && IntegerQ[m + n]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(b\sec(c+dx))^{3/2}}\,dx &= \frac{\sqrt{\sec(c+dx)}\int \cos^3(c+dx)\,dx}{b\sqrt{b\sec(c+dx)}}\\ &= -\frac{\sqrt{\sec(c+dx)}\,\text{Subst}\left(\int \left(1-x^2\right)\,dx, x, -\sin(c+dx)\right)}{bd\sqrt{b\sec(c+dx)}}\\ &= \frac{\sqrt{\sec(c+dx)}\sin(c+dx)}{bd\sqrt{b\sec(c+dx)}}-\frac{\sqrt{\sec(c+dx)}\sin^3(c+dx)}{3bd\sqrt{b\sec(c+dx)}}\end{aligned}$$

Mathematica [A] time = 0.0848518, size = 45, normalized size = 0.59

$$\frac{\sin(c+dx)(\cos(2(c+dx))+5)\sec^{\frac{3}{2}}(c+dx)}{6d(b\sec(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sec[c + d*x]^(3/2)*(b*Sec[c + d*x])^(3/2)),x]
```

```
[Out] ((5 + Cos[2*(c + d*x)])*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(6*d*(b*Sec[c + d*
x])^(3/2))
```

Maple [A] time = 0.107, size = 52, normalized size = 0.7

$$\frac{\sin(dx+c)\left((\cos(dx+c))^2+2\right)}{3\,d\,(\cos(dx+c))^3}\left((\cos(dx+c))^{-1}\right)^{-\frac{3}{2}}\left(\frac{b}{\cos(dx+c)}\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sec(d*x+c)^(3/2)/(b*sec(d*x+c))^(3/2),x)
```

```
[Out] 1/3/d*sin(d*x+c)*(cos(d*x+c)^2+2)/cos(d*x+c)^3/(1/cos(d*x+c))^(3/2)/(b/cos(
d*x+c))^(3/2)
```

Maxima [A] time = 2.04891, size = 57, normalized size = 0.75

$$\frac{\sin(3\,dx+3\,c)+9\,\sin\left(\frac{1}{3}\,\arctan(\sin(3\,dx+3\,c),\cos(3\,dx+3\,c))\right)}{12\,b^{\frac{3}{2}}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(3/2)/(b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] 1/12*(sin(3*d*x + 3*c) + 9*sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*
c))))/(b^(3/2)*d)
```

Fricas [A] time = 1.71806, size = 135, normalized size = 1.78

$$\frac{\left(\cos(dx+c)^3+2\,\cos(dx+c)\right)\sqrt{\frac{b}{\cos(dx+c)}}\,\sin(dx+c)}{3\,b^2d\sqrt{\cos(dx+c)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(3/2)/(b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/3*(cos(d*x + c)^3 + 2*cos(d*x + c))*sqrt(b/cos(d*x + c))*sin(d*x + c)/(b^
2*d*sqrt(cos(d*x + c)))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)**(3/2)/(b*sec(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b\sec{(dx+c)})^{\frac{3}{2}}\sec{(dx+c)}^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(3/2)/(b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*sec(d*x + c))^(3/2)*sec(d*x + c)^(3/2)), x)
```

3.175 $\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)(b\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=107

$$\frac{3x\sqrt{\sec(c+dx)}}{8b\sqrt{b\sec(c+dx)}}+\frac{\sin(c+dx)}{4bd\sec^{\frac{5}{2}}(c+dx)\sqrt{b\sec(c+dx)}}+\frac{3\sin(c+dx)}{8bd\sqrt{\sec(c+dx)}\sqrt{b\sec(c+dx)}}$$

```
[Out] (3*x*Sqrt[Sec[c + d*x]])/(8*b*Sqrt[b*Sec[c + d*x]]) + Sin[c + d*x]/(4*b*d*S
ec[c + d*x]^(5/2)*Sqrt[b*Sec[c + d*x]]) + (3*Sin[c + d*x])/(8*b*d*Sqrt[Sec[
c + d*x]]*Sqrt[b*Sec[c + d*x]])
```

Rubi [A] time = 0.027397, antiderivative size = 107, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {18, 2635, 8}

$$\frac{3x\sqrt{\sec(c+dx)}}{8b\sqrt{b\sec(c+dx)}}+\frac{\sin(c+dx)}{4bd\sec^{\frac{5}{2}}(c+dx)\sqrt{b\sec(c+dx)}}+\frac{3\sin(c+dx)}{8bd\sqrt{\sec(c+dx)}\sqrt{b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sec[c + d*x]^(5/2)*(b*Sec[c + d*x])^(3/2)),x]
```

```
[Out] (3*x*Sqrt[Sec[c + d*x]])/(8*b*Sqrt[b*Sec[c + d*x]]) + Sin[c + d*x]/(4*b*d*S
ec[c + d*x]^(5/2)*Sqrt[b*Sec[c + d*x]]) + (3*Sin[c + d*x])/(8*b*d*Sqrt[Sec[
c + d*x]]*Sqrt[b*Sec[c + d*x]])
```

Rule 18

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(a^(m - 1/
2)*b^(n + 1/2)*Sqrt[a*v])/Sqrt[b*v], Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && ILtQ[n - 1/2, 0] && IntegerQ[m + n]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)(b\sec(c+dx))^{3/2}}\,dx = \frac{\sqrt{\sec(c+dx)}\int \cos^4(c+dx)\,dx}{b\sqrt{b\sec(c+dx)}}$$
$$= \frac{\sin(c+dx)}{4bd\sec^{\frac{5}{2}}(c+dx)\sqrt{b\sec(c+dx)}} + \frac{\left(3\sqrt{\sec(c+dx)}\right)\int \cos^2(c+dx)\,dx}{4b\sqrt{b\sec(c+dx)}}$$
$$= \frac{\sin(c+dx)}{4bd\sec^{\frac{5}{2}}(c+dx)\sqrt{b\sec(c+dx)}} + \frac{3\sin(c+dx)}{8bd\sqrt{\sec(c+dx)}\sqrt{b\sec(c+dx)}} + \frac{\left(3\sqrt{\sec(\right.}{8b\sqrt{b}}$$
$$= \frac{3x\sqrt{\sec(c+dx)}}{8b\sqrt{b\sec(c+dx)}} + \frac{\sin(c+dx)}{4bd\sec^{\frac{5}{2}}(c+dx)\sqrt{b\sec(c+dx)}} + \frac{3\sin(c+dx)}{8bd\sqrt{\sec(c+dx)}\sqrt{b\sec}}$$

Mathematica [A] time = 0.121046, size = 55, normalized size = 0.51

$$\frac{(12(c+dx)+8\sin(2(c+dx))+\sin(4(c+dx)))\sec^{\frac{3}{2}}(c+dx)}{32d(b\sec(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sec[c + d*x]^(5/2)*(b*Sec[c + d*x])^(3/2)),x]
```

```
[Out] (Sec[c + d*x]^(3/2)*(12*(c + d*x) + 8*Sin[2*(c + d*x)] + Sin[4*(c + d*x)]))
/(32*d*(b*Sec[c + d*x])^(3/2))
```

Maple [A] time = 0.139, size = 74, normalized size = 0.7

$$\frac{2(\cos(dx+c))^3\sin(dx+c)+3\cos(dx+c)\sin(dx+c)+3dx+3c}{8d(\cos(dx+c))^4}\left((\cos(dx+c))^{-1}\right)^{-\frac{5}{2}}\left(\frac{b}{\cos(dx+c)}\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sec(d*x+c)^(5/2)/(b*sec(d*x+c))^(3/2),x)
```

```
[Out] 1/8/d*(2*cos(d*x+c)^3*sin(d*x+c)+3*cos(d*x+c)*sin(d*x+c)+3*d*x+3*c)/(1/cos(
d*x+c))^(5/2)/(b/cos(d*x+c))^(3/2)/cos(d*x+c)^4
```

Maxima [A] time = 1.87057, size = 66, normalized size = 0.62

$$\frac{12\,dx+12\,c+\sin(4\,dx+4\,c)+8\,\sin\left(\frac{1}{2}\arctan(\sin(4\,dx+4\,c),\cos(4\,dx+4\,c))\right)}{32\,b^{\frac{3}{2}}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(5/2)/(b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] 1/32*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(1/2*arctan2(sin(4*d*x + 4*c)
, cos(4*d*x + 4*c))))/(b^(3/2)*d)
```

Fricas [A] time = 2.15391, size = 552, normalized size = 5.16

$$\left[\frac{\frac{2\left(2\cos(dx+c)^4+3\cos(dx+c)^2\right)\sqrt{\frac{b}{\cos(dx+c)}}\sin(dx+c)}{\sqrt{\cos(dx+c)}}-3\sqrt{-b}\log\left(2\sqrt{-b}\sqrt{\frac{b}{\cos(dx+c)}}\cos(dx+c)^{\frac{3}{2}}\sin(dx+c)+2b\cos(dx+c)^2-b\right)}{16b^2d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(5/2)/(b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [1/16*(2*(2*cos(d*x + c)^4 + 3*cos(d*x + c)^2)*sqrt(b/cos(d*x + c))*sin(d*x
 + c)/sqrt(cos(d*x + c)) - 3*sqrt(-b)*log(2*sqrt(-b)*sqrt(b/cos(d*x + c))*c
os(d*x + c)^(3/2)*sin(d*x + c) + 2*b*cos(d*x + c)^2 - b))/(b^2*d), 1/8*((2*
cos(d*x + c)^4 + 3*cos(d*x + c)^2)*sqrt(b/cos(d*x + c))*sin(d*x + c)/sqrt(c
os(d*x + c)) + 3*sqrt(b)*arctan(sqrt(b/cos(d*x + c))*sin(d*x + c)/(sqrt(b)*
sqrt(cos(d*x + c)))))/(b^2*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)**(5/2)/(b*sec(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b\sec(dx+c))^{\frac{3}{2}}\sec(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(5/2)/(b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*sec(d*x + c))^(3/2)*sec(d*x + c)^(5/2)), x)
```

3.176 $\int \frac{\sec^{\frac{11}{2}}(c+dx)}{(b\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=78

$$\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{2b^2d\sqrt{b\sec(c+dx)}}+\frac{\sqrt{\sec(c+dx)}\tanh^{-1}(\sin(c+dx))}{2b^2d\sqrt{b\sec(c+dx)}}$$

```
[Out] (ArcTanh[Sin[c + d*x]]*Sqrt[Sec[c + d*x]])/(2*b^2*d*Sqrt[b*Sec[c + d*x]]) +
 (Sec[c + d*x]^(5/2)*Sin[c + d*x])/(2*b^2*d*Sqrt[b*Sec[c + d*x]])
```

Rubi [A] time = 0.0202177, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {17, 3768, 3770}

$$\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{2b^2d\sqrt{b\sec(c+dx)}}+\frac{\sqrt{\sec(c+dx)}\tanh^{-1}(\sin(c+dx))}{2b^2d\sqrt{b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(11/2)/(b*Sec[c + d*x])^(5/2),x]
```

```
[Out] (ArcTanh[Sin[c + d*x]]*Sqrt[Sec[c + d*x]])/(2*b^2*d*Sqrt[b*Sec[c + d*x]]) +
 (Sec[c + d*x]^(5/2)*Sin[c + d*x])/(2*b^2*d*Sqrt[b*Sec[c + d*x]])
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(a^(m + 1/
2)*b^(n - 1/2)*Sqrt[b*v])/Sqrt[a*v], Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec^{\frac{11}{2}}(c+dx)}{(b\sec(c+dx))^{5/2}}\,dx &= \frac{\sqrt{\sec(c+dx)}\int \sec^3(c+dx)\,dx}{b^2\sqrt{b\sec(c+dx)}}\\ &= \frac{\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{2b^2d\sqrt{b\sec(c+dx)}}+\frac{\sqrt{\sec(c+dx)}\int \sec(c+dx)\,dx}{2b^2\sqrt{b\sec(c+dx)}}\\ &= \frac{\tanh^{-1}(\sin(c+dx))\sqrt{\sec(c+dx)}}{2b^2d\sqrt{b\sec(c+dx)}}+\frac{\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{2b^2d\sqrt{b\sec(c+dx)}}\end{aligned}$$

Mathematica [A] time = 0.0506845, size = 53, normalized size = 0.68

$$\frac{\sqrt{\sec(c+dx)}\left(\tanh^{-1}(\sin(c+dx))+\tan(c+dx)\sec(c+dx)\right)}{2b^2d\sqrt{b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^(11/2)/(b*Sec[c + d*x])^(5/2),x]
```

```
[Out] (Sqrt[Sec[c + d*x]]*(ArcTanh[Sin[c + d*x]] + Sec[c + d*x]*Tan[c + d*x]))/(2
*b^2*d*Sqrt[b*Sec[c + d*x]])
```

Maple [A] time = 0.117, size = 112, normalized size = 1.4

$$\frac{\cos(dx+c)}{2d}\left(\ln\left(-\frac{-1+\cos(dx+c)-\sin(dx+c)}{\sin(dx+c)}\right)(\cos(dx+c))^2-\ln\left(-\frac{-1+\cos(dx+c)+\sin(dx+c)}{\sin(dx+c)}\right)(\cos(dx+c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(11/2)/(b*sec(d*x+c))^(5/2),x)
```

```
[Out] 1/2/d*(ln(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))*cos(d*x+c)^2-ln(-(-1+cos(
d*x+c)+sin(d*x+c))/sin(d*x+c))*cos(d*x+c)^2+sin(d*x+c))*(1/cos(d*x+c))^(11/
2)*cos(d*x+c)/(b/cos(d*x+c))^(5/2)
```

Maxima [B] time = 2.12037, size = 929, normalized size = 11.91

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(11/2)/(b*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] -1/4*(4*(sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*cos(3/2*arctan2(sin(2*d*x +
 2*c), cos(2*d*x + 2*c))) - 4*(sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*cos(1
/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - (2*(2*cos(2*d*x + 2*c) +
1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*x
 + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*
cos(2*d*x + 2*c) + 1)*log(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c
)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/2*
arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) + (2*(2*cos(2*d*x + 2*c)
+ 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d
*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 +
4*cos(2*d*x + 2*c) + 1)*log(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2
*c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sin(1/
2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) - 4*(cos(4*d*x + 4*c) +
 2*cos(2*d*x + 2*c) + 1)*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
)) + 4*(cos(4*d*x + 4*c) + 2*cos(2*d*x + 2*c) + 1)*sin(1/2*arctan2(sin(2*d*
x + 2*c), cos(2*d*x + 2*c))))/((b^2*cos(4*d*x + 4*c)^2 + 4*b^2*cos(2*d*x +
2*c)^2 + b^2*sin(4*d*x + 4*c)^2 + 4*b^2*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) +
 4*b^2*sin(2*d*x + 2*c)^2 + 4*b^2*cos(2*d*x + 2*c) + b^2 + 2*(2*b^2*cos(2*d
*x + 2*c) + b^2)*cos(4*d*x + 4*c))*sqrt(b)*d)
```

Fricas [A] time = 1.95703, size = 539, normalized size = 6.91

$$\left[\frac{\sqrt{b}\cos\left(dx+c\right)\log\left(-\frac{b\cos(dx+c)^{2}-2\sqrt{b}\sqrt{\frac{b}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)-2b}{\cos(dx+c)^{2}}\right)+\frac{2\sqrt{\frac{b}{\cos(dx+c)}}\sin(dx+c)}{\sqrt{\cos(dx+c)}}}{4b^{3}d\cos\left(dx+c\right)}, -\frac{\sqrt{-b}\arctan\left(\frac{\sqrt{-b}\sqrt{\frac{b}{\cos(dx+c}}}{}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(11/2)/(b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(b)*cos(d*x + c)*log(-(b*cos(d*x + c)^2 - 2*sqrt(b)*sqrt(b/cos(d*
x + c))*sqrt(cos(d*x + c))*sin(d*x + c) - 2*b)/cos(d*x + c)^2) + 2*sqrt(b/c
os(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(b^3*d*cos(d*x + c)), -1/2*(s
qrt(-b)*arctan(sqrt(-b)*sqrt(b/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c
)/b)*cos(d*x + c) - sqrt(b/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(
b^3*d*cos(d*x + c))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(11/2)/(b*sec(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec\left(dx+c\right)^{\frac{11}{2}}}{\left(b\sec\left(dx+c\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(11/2)/(b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^(11/2)/(b*sec(d*x + c))^(5/2), x)
```

3.177 $\int \frac{\sec^{\frac{9}{2}}(c+dx)}{(b\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=35

$$\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{b^2d\sqrt{b\sec(c+dx)}}$$

```
[Out] (Sec[c + d*x]^(3/2)*Sin[c + d*x])/(b^2*d*Sqrt[b*Sec[c + d*x]])
```

Rubi [A] time = 0.0115766, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {17, 3767, 8}

$$\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{b^2d\sqrt{b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(9/2)/(b*Sec[c + d*x])^(5/2),x]
```

```
[Out] (Sec[c + d*x]^(3/2)*Sin[c + d*x])/(b^2*d*Sqrt[b*Sec[c + d*x]])
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(a^(m + 1/
2)*b^(n - 1/2)*Sqrt[b*v])/Sqrt[a*v], Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec^{\frac{9}{2}}(c+dx)}{(b\sec(c+dx))^{5/2}}\,dx &= \frac{\sqrt{\sec(c+dx)}\int \sec^2(c+dx)\,dx}{b^2\sqrt{b\sec(c+dx)}}\\ &= -\frac{\sqrt{\sec(c+dx)}\,\text{Subst}\left(\int 1\,dx, x, -\tan(c+dx)\right)}{b^2d\sqrt{b\sec(c+dx)}}\\ &= \frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{b^2d\sqrt{b\sec(c+dx)}}\end{aligned}$$

Mathematica [A] time = 0.0327365, size = 32, normalized size = 0.91

$$\frac{\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{d(b\sec(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^(9/2)/(b*Sec[c + d*x])^(5/2),x]
```

```
[Out] (Sec[c + d*x]^(7/2)*Sin[c + d*x])/(d*(b*Sec[c + d*x])^(5/2))
```

Maple [A] time = 0.106, size = 39, normalized size = 1.1

$$\frac{\cos(dx+c)\sin(dx+c)}{d}\left((\cos(dx+c))^{-1}\right)^{\frac{9}{2}}\left(\frac{b}{\cos(dx+c)}\right)^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(9/2)/(b*sec(d*x+c))^(5/2),x)
```

```
[Out] 1/d*(1/cos(d*x+c))^(9/2)*cos(d*x+c)*sin(d*x+c)/(b/cos(d*x+c))^(5/2)
```

Maxima [B] time = 1.85632, size = 90, normalized size = 2.57

$$\frac{2\sqrt{b}\sin(2dx+2c)}{\left(b^3\cos(2dx+2c)^2+b^3\sin(2dx+2c)^2+2b^3\cos(2dx+2c)+b^3\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(9/2)/(b*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] 2*sqrt(b)*sin(2*d*x + 2*c)/((b^3*cos(2*d*x + 2*c)^2 + b^3*sin(2*d*x + 2*c)^
2 + 2*b^3*cos(2*d*x + 2*c) + b^3)*d)
```

Fricas [A] time = 1.63829, size = 84, normalized size = 2.4

$$\frac{\sqrt{\frac{b}{\cos(dx+c)}}\sin(dx+c)}{b^3d\sqrt{\cos(dx+c)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(9/2)/(b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] sqrt(b/cos(d*x + c))*sin(d*x + c)/(b^3*d*sqrt(cos(d*x + c)))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(9/2)/(b*sec(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

<u>Giac</u> [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)^{\frac{9}{2}}}{(b\sec(dx+c))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(9/2)/(b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^(9/2)/(b*sec(d*x + c))^(5/2), x)
```

3.178 $\int \frac{\sec^{\frac{7}{2}}(c+dx)}{(b\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=36

$$\frac{\sqrt{\sec(c+dx)}\tanh^{-1}(\sin(c+dx))}{b^2d\sqrt{b\sec(c+dx)}}$$

```
[Out] (ArcTanh[Sin[c + d*x]]*Sqrt[Sec[c + d*x]])/(b^2*d*Sqrt[b*Sec[c + d*x]])
```

Rubi [A] time = 0.0077274, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {17, 3770}

$$\frac{\sqrt{\sec(c+dx)}\tanh^{-1}(\sin(c+dx))}{b^2d\sqrt{b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(7/2)/(b*Sec[c + d*x])^(5/2),x]
```

```
[Out] (ArcTanh[Sin[c + d*x]]*Sqrt[Sec[c + d*x]])/(b^2*d*Sqrt[b*Sec[c + d*x]])
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(a^(m + 1/
2)*b^(n - 1/2)*Sqrt[b*v])/Sqrt[a*v], Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] && !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec^{\frac{7}{2}}(c+dx)}{(b\sec(c+dx))^{5/2}}\,dx &= \frac{\sqrt{\sec(c+dx)}\int \sec(c+dx)\,dx}{b^2\sqrt{b\sec(c+dx)}}\\ &= \frac{\tanh^{-1}(\sin(c+dx))\sqrt{\sec(c+dx)}}{b^2d\sqrt{b\sec(c+dx)}}\end{aligned}$$

Mathematica [A] time = 0.0219341, size = 33, normalized size = 0.92

$$\frac{\sec^{\frac{5}{2}}(c+dx)\tanh^{-1}(\sin(c+dx))}{d(b\sec(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^(7/2)/(b*Sec[c + d*x])^(5/2),x]
```

```
[Out] (ArcTanh[Sin[c + d*x]]*Sec[c + d*x]^(5/2))/(d*(b*Sec[c + d*x])^(5/2))
```

Maple [A] time = 0.088, size = 52, normalized size = 1.4

$$-2\frac{\left((\cos(dx+c))^{-1}\right)^{7/2}\cos(dx+c)}{d}\mathrm{Artanh}\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}\right)\left(\frac{b}{\cos(dx+c)}\right)^{-5/2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(7/2)/(b*sec(d*x+c))^(5/2),x)
```

```
[Out] -2/d*(1/cos(d*x+c))^(7/2)*arctanh((-1+cos(d*x+c))/sin(d*x+c))*cos(d*x+c)/(b
/cos(d*x+c))^(5/2)
```

Maxima [B] time = 1.9491, size = 88, normalized size = 2.44

$$\frac{\log\left(\cos(dx+c)^2+\sin(dx+c)^2+2\sin(dx+c)+1\right)-\log\left(\cos(dx+c)^2+\sin(dx+c)^2-2\sin(dx+c)+1\right)}{2b^{\frac{5}{2}}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(7/2)/(b*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] 1/2*(log(cos(d*x + c)^2 + sin(d*x + c)^2 + 2*sin(d*x + c) + 1) - log(cos(d*
x + c)^2 + sin(d*x + c)^2 - 2*sin(d*x + c) + 1))/(b^(5/2)*d)
```

Fricas [A] time = 1.90422, size = 301, normalized size = 8.36

$$\left[\frac{\log\left(-\frac{b\cos(dx+c)^2-2\sqrt{b}\sqrt{\frac{b}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)-2b}{\cos(dx+c)^2}\right)}{2b^{\frac{5}{2}}d},-\frac{\sqrt{-b}\arctan\left(\frac{\sqrt{-b}\sqrt{\frac{b}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)}{b}\right)}{b^3d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(7/2)/(b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] [1/2*log(-(b*cos(d*x + c)^2 - 2*sqrt(b)*sqrt(b/cos(d*x + c))*sqrt(cos(d*x +
 c))*sin(d*x + c) - 2*b)/cos(d*x + c)^2)/(b^(5/2)*d), -sqrt(-b)*arctan(sqrt
(-b)*sqrt(b/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/b)/(b^3*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(7/2)/(b*sec(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)^{\frac{7}{2}}}{(b\sec(dx+c))^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(7/2)/(b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^(7/2)/(b*sec(d*x + c))^(5/2), x)
```

3.179 $\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(b\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=27

$$\frac{x\sqrt{\sec(c+dx)}}{b^2\sqrt{b\sec(c+dx)}}$$

```
[Out] (x*Sqrt[Sec[c + d*x]])/(b^2*Sqrt[b*Sec[c + d*x]])
```

Rubi [A] time = 0.0026661, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {17, 8}

$$\frac{x\sqrt{\sec(c+dx)}}{b^2\sqrt{b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(5/2)/(b*Sec[c + d*x])^(5/2),x]
```

```
[Out] (x*Sqrt[Sec[c + d*x]])/(b^2*Sqrt[b*Sec[c + d*x]])
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(a^(m + 1/
2)*b^(n - 1/2)*Sqrt[b*v])/Sqrt[a*v], Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] && !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(b\sec(c+dx))^{5/2}}\,dx &= \frac{\sqrt{\sec(c+dx)}\int 1\,dx}{b^2\sqrt{b\sec(c+dx)}}\\ &= \frac{x\sqrt{\sec(c+dx)}}{b^2\sqrt{b\sec(c+dx)}}\end{aligned}$$

Mathematica [A] time = 0.0261257, size = 24, normalized size = 0.89

$$\frac{x\sec^{\frac{5}{2}}(c+dx)}{(b\sec(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^(5/2)/(b*Sec[c + d*x])^(5/2),x]
```

```
[Out] (x*Sec[c + d*x]^(5/2))/(b*Sec[c + d*x])^(5/2)
```

Maple [A] time = 0.077, size = 32, normalized size = 1.2

$$\frac{dx+c}{d}\left(\left(\cos\left(dx+c\right)\right)^{-1}\right)^{\frac{5}{2}}\left(\frac{b}{\cos\left(dx+c\right)}\right)^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(5/2)/(b*sec(d*x+c))^(5/2),x)
```

```
[Out] 1/d*(d*x+c)*(1/cos(d*x+c))^(5/2)/(b/cos(d*x+c))^(5/2)
```

Maxima [A] time = 1.77235, size = 35, normalized size = 1.3

$$\frac{2\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{b^{\frac{5}{2}}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)/(b*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] 2*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/(b^(5/2)*d)
```

Fricas [A] time = 1.95575, size = 277, normalized size = 10.26

$$\left[-\frac{\sqrt{-b}\log\left(2\sqrt{-b}\sqrt{\frac{b}{\cos(dx+c)}}\cos\left(dx+c\right)^{\frac{3}{2}}\sin\left(dx+c\right)+2\,b\cos\left(dx+c\right)^{2}-b\right)}{2\,b^{3}d}, \frac{\arctan\left(\frac{\sqrt{\frac{b}{\cos(dx+c)}}\sin(dx+c)}{\sqrt{b}\sqrt{\cos(dx+c)}}\right)}{b^{\frac{5}{2}}d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)/(b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/2*sqrt(-b)*log(2*sqrt(-b)*sqrt(b/cos(d*x + c))*cos(d*x + c)^(3/2)*sin(d
*x + c) + 2*b*cos(d*x + c)^2 - b)/(b^3*d), arctan(sqrt(b/cos(d*x + c))*sin(
d*x + c)/(sqrt(b)*sqrt(cos(d*x + c))))/(b^(5/2)*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(5/2)/(b*sec(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac **[F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)^{\frac{5}{2}}}{(b\sec(dx+c))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)/(b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^(5/2)/(b*sec(d*x + c))^(5/2), x)
```

3.180 $\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(b\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=35

$$\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{b^2 d\sqrt{b\sec(c+dx)}}$$

```
[Out] (Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(b^2*d*Sqrt[b*Sec[c + d*x]])
```

Rubi [A] time = 0.0069425, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {17, 2637}

$$\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{b^2 d\sqrt{b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(3/2)/(b*Sec[c + d*x])^(5/2),x]
```

```
[Out] (Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(b^2*d*Sqrt[b*Sec[c + d*x]])
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(a^(m + 1/
2)*b^(n - 1/2)*Sqrt[b*v])/Sqrt[a*v], Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] && !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(b\sec(c+dx))^{5/2}}\,dx &= \frac{\sqrt{\sec(c+dx)}\int \cos(c+dx)\,dx}{b^2\sqrt{b\sec(c+dx)}}\\ &= \frac{\sqrt{\sec(c+dx)}\sin(c+dx)}{b^2 d\sqrt{b\sec(c+dx)}}\end{aligned}$$

Mathematica [A] time = 0.0252233, size = 35, normalized size = 1.

$$\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{b^2 d\sqrt{b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^(3/2)/(b*Sec[c + d*x])^(5/2),x]
```

```
[Out] (Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(b^2*d*Sqrt[b*Sec[c + d*x]])
```

Maple [A] time = 0.094, size = 41, normalized size = 1.2

$$\frac{\sin\left(dx+c\right)}{d\cos\left(dx+c\right)}\left(\left(\cos\left(dx+c\right)\right)^{-1}\right)^{\frac{3}{2}}\left(\frac{b}{\cos\left(dx+c\right)}\right)^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(3/2)/(b*sec(d*x+c))^(5/2),x)
```

```
[Out] 1/d*sin(d*x+c)*(1/cos(d*x+c))^(3/2)/(b/cos(d*x+c))^(5/2)/cos(d*x+c)
```

Maxima [A] time = 1.94217, size = 18, normalized size = 0.51

$$\frac{\sin\left(dx+c\right)}{b^{\frac{5}{2}}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)/(b*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] sin(d*x + c)/(b^(5/2)*d)
```

Fricas [A] time = 1.64625, size = 84, normalized size = 2.4

$$\frac{\sqrt{\frac{b}{\cos(dx+c)}}\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)}{b^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)/(b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] sqrt(b/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(b^3*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(3/2)/(b*sec(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sec\left(dx+c\right)^{\frac{3}{2}}}{\left(b\sec\left(dx+c\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)/(b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^(3/2)/(b*sec(d*x + c))^(5/2), x)
```

3.181 $\int \frac{\sqrt{\sec(c+dx)}}{(b\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=69

$$\frac{x\sqrt{\sec(c+dx)}}{2b^2\sqrt{b\sec(c+dx)}}+\frac{\sin(c+dx)}{2b^2d\sqrt{\sec(c+dx)}\sqrt{b\sec(c+dx)}}$$

```
[Out] (x*Sqrt[Sec[c + d*x]])/(2*b^2*Sqrt[b*Sec[c + d*x]]) + Sin[c + d*x]/(2*b^2*d
*Sqrt[Sec[c + d*x]]*Sqrt[b*Sec[c + d*x]])
```

Rubi [A] time = 0.0146361, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {17, 2635, 8}

$$\frac{x\sqrt{\sec(c+dx)}}{2b^2\sqrt{b\sec(c+dx)}}+\frac{\sin(c+dx)}{2b^2d\sqrt{\sec(c+dx)}\sqrt{b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Sec[c + d*x]]/(b*Sec[c + d*x])^(5/2),x]
```

```
[Out] (x*Sqrt[Sec[c + d*x]])/(2*b^2*Sqrt[b*Sec[c + d*x]]) + Sin[c + d*x]/(2*b^2*d
*Sqrt[Sec[c + d*x]]*Sqrt[b*Sec[c + d*x]])
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(a^(m + 1/
2)*b^(n - 1/2)*Sqrt[b*v])/Sqrt[a*v], Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] && !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{\sec(c+dx)}}{(b\sec(c+dx))^{5/2}}\,dx &= \frac{\sqrt{\sec(c+dx)}\int \cos^2(c+dx)\,dx}{b^2\sqrt{b\sec(c+dx)}}\\ &= \frac{\sin(c+dx)}{2b^2d\sqrt{\sec(c+dx)}\sqrt{b\sec(c+dx)}}+\frac{\sqrt{\sec(c+dx)}\int 1\,dx}{2b^2\sqrt{b\sec(c+dx)}}\\ &= \frac{x\sqrt{\sec(c+dx)}}{2b^2\sqrt{b\sec(c+dx)}}+\frac{\sin(c+dx)}{2b^2d\sqrt{\sec(c+dx)}\sqrt{b\sec(c+dx)}}\end{aligned}$$

Mathematica [A] time = 0.0516438, size = 48, normalized size = 0.7

$$\frac{(2(c+dx)+\sin(2(c+dx)))\sqrt{\sec(c+dx)}}{4b^2d\sqrt{b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Sec[c + d*x]]/(b*Sec[c + d*x])^(5/2),x]
```

```
[Out] (Sqrt[Sec[c + d*x]]*(2*(c + d*x) + Sin[2*(c + d*x)]))/(4*b^2*d*Sqrt[b*Sec[c
 + d*x]])
```

Maple [A] time = 0.127, size = 54, normalized size = 0.8

$$\frac{\cos(dx+c)\sin(dx+c)+dx+c}{2d(\cos(dx+c))^2}\sqrt{(\cos(dx+c))^{-1}}\left(\frac{b}{\cos(dx+c)}\right)^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(1/2)/(b*sec(d*x+c))^(5/2),x)
```

```
[Out] 1/2/d*(cos(d*x+c)*sin(d*x+c)+d*x+c)*(1/cos(d*x+c))^(1/2)/cos(d*x+c)^2/(b/co
s(d*x+c))^(5/2)
```

Maxima [A] time = 2.1739, size = 34, normalized size = 0.49

$$\frac{2dx+2c+\sin(2dx+2c)}{4b^{\frac{5}{2}}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)/(b*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] 1/4*(2*d*x + 2*c + sin(2*d*x + 2*c))/(b^(5/2)*d)
```

Fricas [A] time = 1.98352, size = 443, normalized size = 6.42

$$\left[\frac{2\sqrt{\frac{b}{\cos(dx+c)}}\cos(dx+c)^{\frac{3}{2}}\sin(dx+c)-\sqrt{-b}\log\left(2\sqrt{-b}\sqrt{\frac{b}{\cos(dx+c)}}\cos(dx+c)^{\frac{3}{2}}\sin(dx+c)+2b\cos(dx+c)^2-b\right)}{4b^3d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)/(b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] [1/4*(2*sqrt(b/cos(d*x + c))*cos(d*x + c)^(3/2)*sin(d*x + c) - sqrt(-b)*log
(2*sqrt(-b)*sqrt(b/cos(d*x + c))*cos(d*x + c)^(3/2)*sin(d*x + c) + 2*b*cos(
```

```
d*x + c)^2 - b))/(b^3*d), 1/2*(sqrt(b/cos(d*x + c))*cos(d*x + c)^(3/2)*sin(
d*x + c) + sqrt(b)*arctan(sqrt(b/cos(d*x + c))*sin(d*x + c)/(sqrt(b)*sqrt(c
os(d*x + c)))))/(b^3*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(1/2)/(b*sec(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\sec(dx+c)}}{(b\sec(dx+c))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)/(b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(sec(d*x + c))/(b*sec(d*x + c))^(5/2), x)
```

3.182 $\int \frac{1}{\sqrt{\sec(c+dx)}(b\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=76

$$\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{b^2d\sqrt{b\sec(c+dx)}}-\frac{\sin^3(c+dx)\sqrt{\sec(c+dx)}}{3b^2d\sqrt{b\sec(c+dx)}}$$

```
[Out] (Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(b^2*d*Sqrt[b*Sec[c + d*x]]) - (Sqrt[Sec[
c + d*x]]*Sin[c + d*x]^3)/(3*b^2*d*Sqrt[b*Sec[c + d*x]])
```

Rubi [A] time = 0.0167266, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {18, 2633}

$$\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{b^2d\sqrt{b\sec(c+dx)}}-\frac{\sin^3(c+dx)\sqrt{\sec(c+dx)}}{3b^2d\sqrt{b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[Sec[c + d*x]]*(b*Sec[c + d*x])^(5/2)),x]
```

```
[Out] (Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(b^2*d*Sqrt[b*Sec[c + d*x]]) - (Sqrt[Sec[
c + d*x]]*Sin[c + d*x]^3)/(3*b^2*d*Sqrt[b*Sec[c + d*x]])
```

Rule 18

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(a^(m - 1/
2)*b^(n + 1/2)*Sqrt[a*v])/Sqrt[b*v], Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && ILtQ[n - 1/2, 0] && IntegerQ[m + n]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{\sec(c+dx)}(b\sec(c+dx))^{5/2}}\,dx &= \frac{\sqrt{\sec(c+dx)}\int \cos^3(c+dx)\,dx}{b^2\sqrt{b\sec(c+dx)}}\\ &= -\frac{\sqrt{\sec(c+dx)}\,\text{Subst}\left(\int \left(1-x^2\right)\,dx, x, -\sin(c+dx)\right)}{b^2d\sqrt{b\sec(c+dx)}}\\ &= \frac{\sqrt{\sec(c+dx)}\sin(c+dx)}{b^2d\sqrt{b\sec(c+dx)}}-\frac{\sqrt{\sec(c+dx)}\sin^3(c+dx)}{3b^2d\sqrt{b\sec(c+dx)}}\end{aligned}$$

Mathematica [A] time = 0.0724163, size = 48, normalized size = 0.63

$$\frac{\sin(c+dx)(\cos(2(c+dx))+5)\sqrt{\sec(c+dx)}}{6b^2d\sqrt{b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[Sec[c + d*x]]*(b*Sec[c + d*x])^(5/2)),x]
```

```
[Out] ((5 + Cos[2*(c + d*x)])*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(6*b^2*d*Sqrt[b*Se
c[c + d*x]])
```

Maple [A] time = 0.134, size = 52, normalized size = 0.7

$$\frac{\sin(dx+c)\left((\cos(dx+c))^2+2\right)}{3d(\cos(dx+c))^3}\frac{1}{\sqrt{(\cos(dx+c))^{-1}}}\left(\frac{b}{\cos(dx+c)}\right)^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sec(d*x+c)^(1/2)/(b*sec(d*x+c))^(5/2),x)
```

```
[Out] 1/3/d*sin(d*x+c)*(cos(d*x+c)^2+2)/cos(d*x+c)^3/(1/cos(d*x+c))^(1/2)/(b/cos(
d*x+c))^(5/2)
```

Maxima [A] time = 2.16079, size = 57, normalized size = 0.75

$$\frac{\sin(3\,dx+3\,c)+9\,\sin\left(\frac{1}{3}\,\arctan(\sin(3\,dx+3\,c),\cos(3\,dx+3\,c))\right)}{12\,b^{\frac{5}{2}}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(1/2)/(b*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] 1/12*(sin(3*d*x + 3*c) + 9*sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*
c))))/(b^(5/2)*d)
```

Fricas [A] time = 1.69798, size = 135, normalized size = 1.78

$$\frac{\left(\cos(dx+c)^3+2\,\cos(dx+c)\right)\sqrt{\frac{b}{\cos(dx+c)}}\sin(dx+c)}{3\,b^3d\sqrt{\cos(dx+c)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(1/2)/(b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/3*(cos(d*x + c)^3 + 2*cos(d*x + c))*sqrt(b/cos(d*x + c))*sin(d*x + c)/(b^
3*d*sqrt(cos(d*x + c)))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)**(1/2)/(b*sec(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b\sec(dx+c))^{\frac{5}{2}}\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(1/2)/(b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*sec(d*x + c))^(5/2)*sqrt(sec(d*x + c))), x)
```

3.183 $\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(b\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=107

$$\frac{3x\sqrt{\sec(c+dx)}}{8b^2\sqrt{b\sec(c+dx)}}+\frac{\sin(c+dx)}{4b^2d\sec^{\frac{5}{2}}(c+dx)\sqrt{b\sec(c+dx)}}+\frac{3\sin(c+dx)}{8b^2d\sqrt{\sec(c+dx)}\sqrt{b\sec(c+dx)}}$$

```
[Out] (3*x*Sqrt[Sec[c + d*x]])/(8*b^2*Sqrt[b*Sec[c + d*x]]) + Sin[c + d*x]/(4*b^2
*d*Sec[c + d*x]^(5/2)*Sqrt[b*Sec[c + d*x]]) + (3*Sin[c + d*x])/(8*b^2*d*Sqr
t[Sec[c + d*x]]*Sqrt[b*Sec[c + d*x]])
```

Rubi [A] time = 0.0284181, antiderivative size = 107, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {18, 2635, 8}

$$\frac{3x\sqrt{\sec(c+dx)}}{8b^2\sqrt{b\sec(c+dx)}}+\frac{\sin(c+dx)}{4b^2d\sec^{\frac{5}{2}}(c+dx)\sqrt{b\sec(c+dx)}}+\frac{3\sin(c+dx)}{8b^2d\sqrt{\sec(c+dx)}\sqrt{b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sec[c + d*x]^(3/2)*(b*Sec[c + d*x])^(5/2)),x]
```

```
[Out] (3*x*Sqrt[Sec[c + d*x]])/(8*b^2*Sqrt[b*Sec[c + d*x]]) + Sin[c + d*x]/(4*b^2
*d*Sec[c + d*x]^(5/2)*Sqrt[b*Sec[c + d*x]]) + (3*Sin[c + d*x])/(8*b^2*d*Sqr
t[Sec[c + d*x]]*Sqrt[b*Sec[c + d*x]])
```

Rule 18

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(a^(m - 1/
2)*b^(n + 1/2)*Sqrt[a*v])/Sqrt[b*v], Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] && !IntegerQ[m] && ILtQ[n - 1/2, 0] && IntegerQ[m + n]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(b\sec(c+dx))^{5/2}}\,dx = \frac{\sqrt{\sec(c+dx)}\int \cos^4(c+dx)\,dx}{b^2\sqrt{b\sec(c+dx)}}$$

$$= \frac{\sin(c+dx)}{4b^2d\sec^{\frac{5}{2}}(c+dx)\sqrt{b\sec(c+dx)}} + \frac{\left(3\sqrt{\sec(c+dx)}\right)\int \cos^2(c+dx)\,dx}{4b^2\sqrt{b\sec(c+dx)}}$$

$$= \frac{\sin(c+dx)}{4b^2d\sec^{\frac{5}{2}}(c+dx)\sqrt{b\sec(c+dx)}} + \frac{3\sin(c+dx)}{8b^2d\sqrt{\sec(c+dx)}\sqrt{b\sec(c+dx)}} + \frac{\left(3\sqrt{\mathrm{se}\right.}{8b^2}$$

$$= \frac{3x\sqrt{\sec(c+dx)}}{8b^2\sqrt{b\sec(c+dx)}} + \frac{\sin(c+dx)}{4b^2d\sec^{\frac{5}{2}}(c+dx)\sqrt{b\sec(c+dx)}} + \frac{3\sin(c+d}{8b^2d\sqrt{\sec(c+dx)}\sqrt{b}}$$

Mathematica [A] time = 0.0575848, size = 58, normalized size = 0.54

$$\frac{(12(c+dx)+8\sin(2(c+dx))+\sin(4(c+dx)))\sqrt{\sec(c+dx)}}{32b^2d\sqrt{b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sec[c + d*x]^(3/2)*(b*Sec[c + d*x])^(5/2)),x]
```

```
[Out] (Sqrt[Sec[c + d*x]]*(12*(c + d*x) + 8*Sin[2*(c + d*x)] + Sin[4*(c + d*x)]))
/(32*b^2*d*Sqrt[b*Sec[c + d*x]])
```

Maple [A] time = 0.127, size = 74, normalized size = 0.7

$$\frac{2(\cos(dx+c))^3\sin(dx+c)+3\cos(dx+c)\sin(dx+c)+3dx+3c}{8d(\cos(dx+c))^4}\left((\cos(dx+c))^{-1}\right)^{-\frac{3}{2}}\left(\frac{b}{\cos(dx+c)}\right)^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sec(d*x+c)^(3/2)/(b*sec(d*x+c))^(5/2),x)
```

```
[Out] 1/8/d*(2*cos(d*x+c)^3*sin(d*x+c)+3*cos(d*x+c)*sin(d*x+c)+3*d*x+3*c)/cos(d*x
+c)^4/(1/cos(d*x+c))^(3/2)/(b/cos(d*x+c))^(5/2)
```

Maxima [A] time = 1.97597, size = 66, normalized size = 0.62

$$\frac{12dx+12c+\sin(4dx+4c)+8\sin\left(\frac{1}{2}\arctan(\sin(4dx+4c),\cos(4dx+4c))\right)}{32b^{\frac{5}{2}}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(3/2)/(b*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] 1/32*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(1/2*arctan2(sin(4*d*x + 4*c)
, cos(4*d*x + 4*c))))/(b^(5/2)*d)
```

Fricas [A] time = 2.46266, size = 552, normalized size = 5.16

$$\left[\frac{\frac{2\left(2\cos(dx+c)^4+3\cos(dx+c)^2\right)\sqrt{\frac{b}{\cos(dx+c)}}\sin(dx+c)}{\sqrt{\cos(dx+c)}} - 3\sqrt{-b}\log\left(2\sqrt{-b}\sqrt{\frac{b}{\cos(dx+c)}}\cos\left(dx+c\right)^{\frac{3}{2}}\sin\left(dx+c\right)+2\,b\cos\left(dx+c\right)^2-b\right)}{16\,b^3d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(3/2)/(b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] [1/16*(2*(2*cos(d*x + c)^4 + 3*cos(d*x + c)^2)*sqrt(b/cos(d*x + c))*sin(d*x
 + c)/sqrt(cos(d*x + c)) - 3*sqrt(-b)*log(2*sqrt(-b)*sqrt(b/cos(d*x + c))*c
os(d*x + c)^(3/2)*sin(d*x + c) + 2*b*cos(d*x + c)^2 - b))/(b^3*d), 1/8*((2*
cos(d*x + c)^4 + 3*cos(d*x + c)^2)*sqrt(b/cos(d*x + c))*sin(d*x + c)/sqrt(c
os(d*x + c)) + 3*sqrt(b)*arctan(sqrt(b/cos(d*x + c))*sin(d*x + c)/(sqrt(b)*
sqrt(cos(d*x + c)))))/(b^3*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)**(3/2)/(b*sec(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b\sec\left(dx+c\right)\right)^{\frac{5}{2}}\sec\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(3/2)/(b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*sec(d*x + c))^(5/2)*sec(d*x + c)^(3/2)), x)
```

3.184 $\int \sec^2(c+dx)\sqrt[3]{b\sec(c+dx)}\,dx$

Optimal. Leaf size=58

$$\frac{3\sin(c+dx)(b\sec(c+dx))^{4/3}\text{Hypergeometric2F1}\left(-\frac{2}{3},\frac{1}{2},\frac{1}{3},\cos^2(c+dx)\right)}{4bd\sqrt{\sin^2(c+dx)}}$$

```
[Out] (3*Hypergeometric2F1[-2/3, 1/2, 1/3, Cos[c + d*x]^2]*(b*Sec[c + d*x])^(4/3)
*Sin[c + d*x])/(4*b*d*Sqrt[Sin[c + d*x]^2])
```

Rubi [A] time = 0.0325862, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {16, 3772, 2643}

$$\frac{3\sin(c+dx)(b\sec(c+dx))^{4/3}\,{}_2F_1\left(-\frac{2}{3},\frac{1}{2};\frac{1}{3};\cos^2(c+dx)\right)}{4bd\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2*(b*Sec[c + d*x])^(1/3),x]
```

```
[Out] (3*Hypergeometric2F1[-2/3, 1/2, 1/3, Cos[c + d*x]^2]*(b*Sec[c + d*x])^(4/3)
*Sin[c + d*x])/(4*b*d*Sqrt[Sin[c + d*x]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 3772

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}\int \sec^2(c+dx)\sqrt[3]{b\sec(c+dx)}\,dx &= \frac{\int (b\sec(c+dx))^{7/3}\,dx}{b^2}\\ &= \frac{\left(\sqrt[3]{\frac{\cos(c+dx)}{b}}\sqrt[3]{b\sec(c+dx)}\right)\int \frac{1}{\left(\frac{\cos(c+dx)}{b}\right)^{7/3}}\,dx}{b^2}\\ &= \frac{3\,{}_2F_1\left(-\frac{2}{3},\frac{1}{2};\frac{1}{3};\cos^2(c+dx)\right)(b\sec(c+dx))^{4/3}\sin(c+dx)}{4bd\sqrt{\sin^2(c+dx)}}\end{aligned}$$

Mathematica [A] time = 0.0749997, size = 60, normalized size = 1.03

$$\frac{3\sqrt{-\tan^2(c+dx)}\csc(c+dx)(b\sec(c+dx))^{4/3}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{7}{6},\frac{13}{6},\sec^2(c+dx)\right)}{7bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2*(b*Sec[c + d*x])^(1/3),x]
```

```
[Out] (3*Csc[c + d*x]*Hypergeometric2F1[1/2, 7/6, 13/6, Sec[c + d*x]^2]*(b*Sec[c
+ d*x])^(4/3)*Sqrt[-Tan[c + d*x]^2])/(7*b*d)
```

Maple [F] time = 0.067, size = 0, normalized size = 0.

$$\int (\sec(dx+c))^2 \sqrt[3]{b\sec(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(b*sec(d*x+c))^(1/3),x)
```

```
[Out] int(sec(d*x+c)^2*(b*sec(d*x+c))^(1/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b\sec(dx+c))^{\frac{1}{3}}\sec(dx+c)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(b*sec(d*x+c))^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c))^(1/3)*sec(d*x + c)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((b\sec(dx+c))^{\frac{1}{3}}\sec(dx+c)^2,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(b*sec(d*x+c))^(1/3),x, algorithm="fricas")
```

```
[Out] integral((b*sec(d*x + c))^(1/3)*sec(d*x + c)^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt[3]{b\sec(c+dx)}\sec^2(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(b*sec(d*x+c))**(1/3),x)
```

```
[Out] Integral((b*sec(c + d*x))**(1/3)*sec(c + d*x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b\sec(dx+c))^{\frac{1}{3}}\sec(dx+c)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(b*sec(d*x+c))^(1/3),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c))^(1/3)*sec(d*x + c)^2, x)
```

3.185 $\int \sec(c+dx)\sqrt[3]{b\sec(c+dx)}\,dx$

Optimal. Leaf size=53

$$\frac{3\sin(c+dx)\sqrt[3]{b\sec(c+dx)}\text{Hypergeometric2F1}\left(-\frac{1}{6},\frac{1}{2},\frac{5}{6},\cos^2(c+dx)\right)}{d\sqrt{\sin^2(c+dx)}}$$

```
[Out] (3*Hypergeometric2F1[-1/6, 1/2, 5/6, Cos[c + d*x]^2]*(b*Sec[c + d*x])^(1/3)
*Sin[c + d*x])/(d*Sqrt[Sin[c + d*x]^2])
```

Rubi [A] time = 0.0308292, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {16, 3772, 2643}

$$\frac{3\sin(c+dx)\sqrt[3]{b\sec(c+dx)}\,{}_2F_1\left(-\frac{1}{6},\frac{1}{2};\frac{5}{6};\cos^2(c+dx)\right)}{d\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*(b*Sec[c + d*x])^(1/3),x]
```

```
[Out] (3*Hypergeometric2F1[-1/6, 1/2, 5/6, Cos[c + d*x]^2]*(b*Sec[c + d*x])^(1/3)
*Sin[c + d*x])/(d*Sqrt[Sin[c + d*x]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 3772

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] && !IntegerQ[n]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}
\int \sec(c+dx)\sqrt[3]{b\sec(c+dx)}\,dx &= \frac{\int (b\sec(c+dx))^{4/3}\,dx}{b}\\
&= \frac{\left(\sqrt[3]{\frac{\cos(c+dx)}{b}}\sqrt[3]{b\sec(c+dx)}\right)\int \frac{1}{\left(\frac{\cos(c+dx)}{b}\right)^{4/3}}\,dx}{b}\\
&= \frac{3\,{}_2F_1\left(-\frac{1}{6},\frac{1}{2};\frac{5}{6};\cos^2(c+dx)\right)\sqrt[3]{b\sec(c+dx)}\sin(c+dx)}{d\sqrt{\sin^2(c+dx)}}
\end{aligned}$$

Mathematica [A] time = 0.0473121, size = 60, normalized size = 1.13

$$\frac{3\sqrt{-\tan^2(c+dx)}\cot(c+dx)(b\sec(c+dx))^{4/3}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{5}{3},\sec^2(c+dx)\right)}{4bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]*(b*Sec[c + d*x])^(1/3),x]
```

```
[Out] (3*Cot[c + d*x]*Hypergeometric2F1[1/2, 2/3, 5/3, Sec[c + d*x]^2]*(b*Sec[c +
 d*x])^(4/3)*Sqrt[-Tan[c + d*x]^2])/(4*b*d)
```

Maple [F] time = 0.063, size = 0, normalized size = 0.

$$\int \sec(dx+c)\sqrt[3]{b\sec(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(b*sec(d*x+c))^(1/3),x)
```

```
[Out] int(sec(d*x+c)*(b*sec(d*x+c))^(1/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b\sec(dx+c))^{\frac{1}{3}}\sec(dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(b*sec(d*x+c))^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c))^(1/3)*sec(d*x + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((b\sec(dx+c))^{\frac{1}{3}}\sec(dx+c), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(b*sec(d*x+c))^(1/3),x, algorithm="fricas")
```

```
[Out] integral((b*sec(d*x + c))^(1/3)*sec(d*x + c), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt[3]{b\sec(c+dx)}\sec(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(b*sec(d*x+c))**(1/3),x)
```

```
[Out] Integral((b*sec(c + d*x))**(1/3)*sec(c + d*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b\sec(dx+c))^{\frac{1}{3}}\sec(dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(b*sec(d*x+c))^(1/3),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c))^(1/3)*sec(d*x + c), x)
```

3.186 $\int \sqrt[3]{b\sec(c+dx)}\,dx$

Optimal. Leaf size=56

$$-\frac{3b\sin(c+dx)\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},\cos^2(c+dx)\right)}{2d\sqrt{\sin^2(c+dx)}(b\sec(c+dx))^{2/3}}$$

```
[Out] (-3*b*Hypergeometric2F1[1/3, 1/2, 4/3, Cos[c + d*x]^2]*Sin[c + d*x])/(2*d*(
b*Sec[c + d*x])^(2/3)*Sqrt[Sin[c + d*x]^2])
```

Rubi [A] time = 0.0249474, antiderivative size = 56, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3772, 2643}

$$-\frac{3b\sin(c+dx)\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};\cos^2(c+dx)\right)}{2d\sqrt{\sin^2(c+dx)}(b\sec(c+dx))^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Sec[c + d*x])^(1/3),x]
```

```
[Out] (-3*b*Hypergeometric2F1[1/3, 1/2, 4/3, Cos[c + d*x]^2]*Sin[c + d*x])/(2*d*(
b*Sec[c + d*x])^(2/3)*Sqrt[Sin[c + d*x]^2])
```

Rule 3772

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] && !IntegerQ[n]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}\int \sqrt[3]{b\sec(c+dx)}\,dx &= \sqrt[3]{\frac{\cos(c+dx)}{b}}\sqrt[3]{b\sec(c+dx)}\int \frac{1}{\sqrt[3]{\frac{\cos(c+dx)}{b}}}\,dx\\ &= -\frac{3\cos(c+dx)\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};\cos^2(c+dx)\right)\sqrt[3]{b\sec(c+dx)}\sin(c+dx)}{2d\sqrt{\sin^2(c+dx)}}\end{aligned}$$

Mathematica [A] time = 0.0415476, size = 55, normalized size = 0.98

$$\frac{3\sqrt{-\tan^2(c+dx)}\cot(c+dx)\sqrt[3]{b\sec(c+dx)}\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{1}{2},\frac{7}{6},\sec^2(c+dx)\right)}{d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(b*Sec[c + d*x])^(1/3),x]
```

```
[Out] (3*Cot[c + d*x]*Hypergeometric2F1[1/6, 1/2, 7/6, Sec[c + d*x]^2]*(b*Sec[c +
 d*x])^(1/3)*Sqrt[-Tan[c + d*x]^2])/d
```

Maple [F] time = 0.131, size = 0, normalized size = 0.

$$\int \sqrt[3]{b \sec(dx + c)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sec(d*x+c))^(1/3),x)
```

```
[Out] int((b*sec(d*x+c))^(1/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b \sec(dx + c))^{\frac{1}{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c))^(1/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left((b \sec(dx + c))^{\frac{1}{3}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))^(1/3),x, algorithm="fricas")
```

```
[Out] integral((b*sec(d*x + c))^(1/3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt[3]{b \sec(c + dx)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))**(1/3),x)
```

```
[Out] Integral((b*sec(c + d*x))**(1/3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b\sec(dx+c))^{\frac{1}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))^(1/3),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c))^(1/3), x)
```

3.187 $\int \cos(c+dx)\sqrt[3]{b\sec(c+dx)}\,dx$

Optimal. Leaf size=58

$$-\frac{3b^2\sin(c+dx)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{6},\frac{11}{6},\cos^2(c+dx)\right)}{5d\sqrt{\sin^2(c+dx)}(b\sec(c+dx))^{5/3}}$$

```
[Out] (-3*b^2*Hypergeometric2F1[1/2, 5/6, 11/6, Cos[c + d*x]^2]*Sin[c + d*x])/(5*
d*(b*Sec[c + d*x])^(5/3)*Sqrt[Sin[c + d*x]^2])
```

Rubi [A] time = 0.0358247, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {16, 3772, 2643}

$$-\frac{3b^2\sin(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{5}{6};\frac{11}{6};\cos^2(c+dx)\right)}{5d\sqrt{\sin^2(c+dx)}(b\sec(c+dx))^{5/3}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(b*Sec[c + d*x])^(1/3),x]
```

```
[Out] (-3*b^2*Hypergeometric2F1[1/2, 5/6, 11/6, Cos[c + d*x]^2]*Sin[c + d*x])/(5*
d*(b*Sec[c + d*x])^(5/3)*Sqrt[Sin[c + d*x]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 3772

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] && !IntegerQ[n]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}
\int \cos(c+dx)\sqrt[3]{b\sec(c+dx)}\,dx &= b\int \frac{1}{(b\sec(c+dx))^{2/3}}\,dx\\
&= \left(b\sqrt[3]{\frac{\cos(c+dx)}{b}}\sqrt[3]{b\sec(c+dx)}\right)\int \left(\frac{\cos(c+dx)}{b}\right)^{2/3}\,dx\\
&= -\frac{3\cos^2(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{5}{6};\frac{11}{6};\cos^2(c+dx)\right)\sqrt[3]{b\sec(c+dx)}\sin(c+dx)}{5d\sqrt{\sin^2(c+dx)}}
\end{aligned}$$

Mathematica [A] time = 0.0669936, size = 58, normalized size = 1.

$$-\frac{3b\sqrt{-\tan^2(c+dx)}\cot(c+dx)\text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{1}{2},\frac{2}{3},\sec^2(c+dx)\right)}{2d(b\sec(c+dx))^{2/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]*(b*Sec[c + d*x])^(1/3),x]
```

```
[Out] (-3*b*Cot[c + d*x]*Hypergeometric2F1[-1/3, 1/2, 2/3, Sec[c + d*x]^2]*Sqrt[-
Tan[c + d*x]^2])/(2*d*(b*Sec[c + d*x])^(2/3))
```

Maple [F] time = 0.11, size = 0, normalized size = 0.

$$\int \cos(dx+c)\sqrt[3]{b\sec(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(b*sec(d*x+c))^(1/3),x)
```

```
[Out] int(cos(d*x+c)*(b*sec(d*x+c))^(1/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b\sec(dx+c))^{\frac{1}{3}}\cos(dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(b*sec(d*x+c))^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c))^(1/3)*cos(d*x + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((b\sec(dx+c))^{\frac{1}{3}}\cos(dx+c),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(b*sec(d*x+c))^(1/3),x, algorithm="fricas")
```

```
[Out] integral((b*sec(d*x + c))^(1/3)*cos(d*x + c), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt[3]{b\sec(c+dx)}\cos(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(b*sec(d*x+c))**(1/3),x)
```

```
[Out] Integral((b*sec(c + d*x))**(1/3)*cos(c + d*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b\sec(dx+c))^{\frac{1}{3}}\cos(dx+c)\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(b*sec(d*x+c))^(1/3),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c))^(1/3)*cos(d*x + c), x)
```

3.188 $\int \cos^2(c+dx)\sqrt[3]{b\sec(c+dx)}\,dx$

Optimal. Leaf size=58

$$-\frac{3b^3\sin(c+dx)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{4}{3},\frac{7}{3},\cos^2(c+dx)\right)}{8d\sqrt{\sin^2(c+dx)}(b\sec(c+dx))^{8/3}}$$

```
[Out] (-3*b^3*Hypergeometric2F1[1/2, 4/3, 7/3, Cos[c + d*x]^2]*Sin[c + d*x])/(8*d
*(b*Sec[c + d*x])^(8/3)*Sqrt[Sin[c + d*x]^2])
```

Rubi [A] time = 0.0432759, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {16, 3772, 2643}

$$-\frac{3b^3\sin(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{4}{3};\frac{7}{3};\cos^2(c+dx)\right)}{8d\sqrt{\sin^2(c+dx)}(b\sec(c+dx))^{8/3}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(b*Sec[c + d*x])^(1/3),x]
```

```
[Out] (-3*b^3*Hypergeometric2F1[1/2, 4/3, 7/3, Cos[c + d*x]^2]*Sin[c + d*x])/(8*d
*(b*Sec[c + d*x])^(8/3)*Sqrt[Sin[c + d*x]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 3772

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] && !IntegerQ[n]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}\int \cos^2(c+dx)\sqrt[3]{b\sec(c+dx)}\,dx &= b^2\int \frac{1}{(b\sec(c+dx))^{5/3}}\,dx\\ &= \left(b^2\sqrt[3]{\frac{\cos(c+dx)}{b}}\sqrt[3]{b\sec(c+dx)}\right)\int \left(\frac{\cos(c+dx)}{b}\right)^{5/3}dx\\ &= -\frac{3\cos^3(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{4}{3};\frac{7}{3};\cos^2(c+dx)\right)\sqrt[3]{b\sec(c+dx)}\sin(c+dx)}{8d\sqrt{\sin^2(c+dx)}}\end{aligned}$$

Mathematica [A] time = 0.143094, size = 59, normalized size = 1.02

$$\frac{3\sin(2(c+dx))\sqrt[3]{b\sec(c+dx)}\text{Hypergeometric2F1}\left(-\frac{5}{6},\frac{1}{2},\frac{1}{6},\sec^2(c+dx)\right)}{10d\sqrt{-\tan^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*(b*Sec[c + d*x])^(1/3),x]
```

```
[Out] (3*Hypergeometric2F1[-5/6, 1/2, 1/6, Sec[c + d*x]^2]*(b*Sec[c + d*x])^(1/3)
*Sin[2*(c + d*x)])/(10*d*Sqrt[-Tan[c + d*x]^2])
```

Maple [F] time = 0.166, size = 0, normalized size = 0.

$$\int (\cos(dx+c))^2 \sqrt[3]{b\sec(dx+c)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(b*sec(d*x+c))^(1/3),x)
```

```
[Out] int(cos(d*x+c)^2*(b*sec(d*x+c))^(1/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b\sec(dx+c))^{\frac{1}{3}} \cos(dx+c)^2\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(b*sec(d*x+c))^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c))^(1/3)*cos(d*x + c)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((b\sec(dx+c))^{\frac{1}{3}} \cos(dx+c)^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(b*sec(d*x+c))^(1/3),x, algorithm="fricas")
```

```
[Out] integral((b*sec(d*x + c))^(1/3)*cos(d*x + c)^2, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(b*sec(d*x+c))**(1/3),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b\sec(dx+c))^{\frac{1}{3}}\cos(dx+c)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(b*sec(d*x+c))^(1/3),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c))^(1/3)*cos(d*x + c)^2, x)
```

3.189 $\int \sec^2(c+dx)(b\sec(c+dx))^{4/3}\,dx$

Optimal. Leaf size=58

$$\frac{3\sin(c+dx)(b\sec(c+dx))^{7/3}\text{Hypergeometric2F1}\left(-\frac{7}{6},\frac{1}{2},-\frac{1}{6},\cos^2(c+dx)\right)}{7bd\sqrt{\sin^2(c+dx)}}$$

```
[Out] (3*Hypergeometric2F1[-7/6, 1/2, -1/6, Cos[c + d*x]^2]*(b*Sec[c + d*x])^(7/3
)*Sin[c + d*x])/(7*b*d*Sqrt[Sin[c + d*x]^2])
```

Rubi [A] time = 0.0309513, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {16, 3772, 2643}

$$\frac{3\sin(c+dx)(b\sec(c+dx))^{7/3}\,{}_2F_1\left(-\frac{7}{6},\frac{1}{2};-\frac{1}{6};\cos^2(c+dx)\right)}{7bd\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2*(b*Sec[c + d*x])^(4/3),x]
```

```
[Out] (3*Hypergeometric2F1[-7/6, 1/2, -1/6, Cos[c + d*x]^2]*(b*Sec[c + d*x])^(7/3
)*Sin[c + d*x])/(7*b*d*Sqrt[Sin[c + d*x]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 3772

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}
\int \sec^2(c+dx)(b\sec(c+dx))^{4/3}\,dx &= \frac{\int (b\sec(c+dx))^{10/3}\,dx}{b^2}\\
&= \frac{\left(\sqrt[3]{\frac{\cos(c+dx)}{b}}\sqrt[3]{b\sec(c+dx)}\right)\int \frac{1}{\left(\frac{\cos(c+dx)}{b}\right)^{10/3}}\,dx}{b^2}\\
&= \frac{3b\,{}_2F_1\left(-\frac{7}{6},\frac{1}{2};-\frac{1}{6};\cos^2(c+dx)\right)\sec(c+dx)\sqrt[3]{b\sec(c+dx)}\tan(c+dx)}{7d\sqrt{\sin^2(c+dx)}}
\end{aligned}$$

Mathematica [A] time = 0.0796494, size = 60, normalized size = 1.03

$$\frac{3\sqrt{-\tan^2(c+dx)}\csc(c+dx)(b\sec(c+dx))^{7/3}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{3},\frac{8}{3},\sec^2(c+dx)\right)}{10bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2*(b*Sec[c + d*x])^(4/3),x]
```

```
[Out] (3*Csc[c + d*x]*Hypergeometric2F1[1/2, 5/3, 8/3, Sec[c + d*x]^2]*(b*Sec[c +
 d*x])^(7/3)*Sqrt[-Tan[c + d*x]^2])/(10*b*d)
```

Maple [F] time = 0.068, size = 0, normalized size = 0.

$$\int (\sec(dx+c))^2 (b\sec(dx+c))^{\frac{4}{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(b*sec(d*x+c))^(4/3),x)
```

```
[Out] int(sec(d*x+c)^2*(b*sec(d*x+c))^(4/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b\sec(dx+c))^{\frac{4}{3}} \sec(dx+c)^2\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(b*sec(d*x+c))^(4/3),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c))^(4/3)*sec(d*x + c)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((b\sec(dx+c))^{\frac{1}{3}}\, b\sec(dx+c)^3, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(b*sec(d*x+c))^(4/3),x, algorithm="fricas")
```

```
[Out] integral((b*sec(d*x + c))^(1/3)*b*sec(d*x + c)^3, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(b*sec(d*x+c))**(4/3),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b\sec(dx+c))^{\frac{4}{3}}\sec(dx+c)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(b*sec(d*x+c))^(4/3),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c))^(4/3)*sec(d*x + c)^2, x)
```

3.190 $\int \sec(c+dx)(b\sec(c+dx))^{4/3}\,dx$

Optimal. Leaf size=55

$$\frac{3\sin(c+dx)(b\sec(c+dx))^{4/3}\text{Hypergeometric2F1}\left(-\frac{2}{3},\frac{1}{2},\frac{1}{3},\cos^2(c+dx)\right)}{4d\sqrt{\sin^2(c+dx)}}$$

```
[Out] (3*Hypergeometric2F1[-2/3, 1/2, 1/3, Cos[c + d*x]^2]*(b*Sec[c + d*x])^(4/3)
*Sin[c + d*x])/(4*d*Sqrt[Sin[c + d*x]^2])
```

Rubi [A] time = 0.0302989, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {16, 3772, 2643}

$$\frac{3\sin(c+dx)(b\sec(c+dx))^{4/3}\,{}_2F_1\left(-\frac{2}{3},\frac{1}{2};\frac{1}{3};\cos^2(c+dx)\right)}{4d\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*(b*Sec[c + d*x])^(4/3),x]
```

```
[Out] (3*Hypergeometric2F1[-2/3, 1/2, 1/3, Cos[c + d*x]^2]*(b*Sec[c + d*x])^(4/3)
*Sin[c + d*x])/(4*d*Sqrt[Sin[c + d*x]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 3772

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}\int \sec(c+dx)(b\sec(c+dx))^{4/3}\,dx &= \frac{\int (b\sec(c+dx))^{7/3}\,dx}{b}\\ &= \frac{\left(\sqrt[3]{\frac{\cos(c+dx)}{b}}\sqrt[3]{b\sec(c+dx)}\right)\int \frac{1}{\left(\frac{\cos(c+dx)}{b}\right)^{7/3}}\,dx}{b}\\ &= \frac{3\,{}_2F_1\left(-\frac{2}{3},\frac{1}{2};\frac{1}{3};\cos^2(c+dx)\right)(b\sec(c+dx))^{4/3}\sin(c+dx)}{4d\sqrt{\sin^2(c+dx)}}\end{aligned}$$

Mathematica [A] time = 0.0490058, size = 60, normalized size = 1.09

$$\frac{3\sqrt{-\tan^2(c+dx)}\cot(c+dx)(b\sec(c+dx))^{7/3}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{7}{6},\frac{13}{6},\sec^2(c+dx)\right)}{7bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]*(b*Sec[c + d*x])^(4/3),x]
```

```
[Out] (3*Cot[c + d*x]*Hypergeometric2F1[1/2, 7/6, 13/6, Sec[c + d*x]^2]*(b*Sec[c
+ d*x])^(7/3)*Sqrt[-Tan[c + d*x]^2])/(7*b*d)
```

Maple [F] time = 0.064, size = 0, normalized size = 0.

$$\int \sec(dx+c)(b\sec(dx+c))^{\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(b*sec(d*x+c))^(4/3),x)
```

```
[Out] int(sec(d*x+c)*(b*sec(d*x+c))^(4/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b\sec(dx+c))^{\frac{4}{3}}\sec(dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(b*sec(d*x+c))^(4/3),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c))^(4/3)*sec(d*x + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((b\sec(dx+c))^{\frac{1}{3}}\,b\sec(dx+c)^2\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(b*sec(d*x+c))^(4/3),x, algorithm="fricas")
```

```
[Out] integral((b*sec(d*x + c))^(1/3)*b*sec(d*x + c)^2, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(b*sec(d*x+c))**(4/3),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b\sec(dx+c))^{\frac{4}{3}}\sec(dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(b*sec(d*x+c))^(4/3),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c))^(4/3)*sec(d*x + c), x)
```

3.191 $\int (b\sec(c+dx))^{4/3}\,dx$

Optimal. Leaf size=54

$$\frac{3b\sin(c+dx)\sqrt[3]{b\sec(c+dx)}\text{Hypergeometric2F1}\left(-\frac{1}{6},\frac{1}{2},\frac{5}{6},\cos^2(c+dx)\right)}{d\sqrt{\sin^2(c+dx)}}$$

```
[Out] (3*b*Hypergeometric2F1[-1/6, 1/2, 5/6, Cos[c + d*x]^2]*(b*Sec[c + d*x])^(1/
3)*Sin[c + d*x])/(d*Sqrt[Sin[c + d*x]^2])
```

Rubi [A] time = 0.026934, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3772, 2643}

$$\frac{3b\sin(c+dx)\sqrt[3]{b\sec(c+dx)}\,{}_2F_1\left(-\frac{1}{6},\frac{1}{2};\frac{5}{6};\cos^2(c+dx)\right)}{d\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Sec[c + d*x])^(4/3),x]
```

```
[Out] (3*b*Hypergeometric2F1[-1/6, 1/2, 5/6, Cos[c + d*x]^2]*(b*Sec[c + d*x])^(1/
3)*Sin[c + d*x])/(d*Sqrt[Sin[c + d*x]^2])
```

Rule 3772

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] && !IntegerQ[n]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}\int (b\sec(c+dx))^{4/3}\,dx &= \sqrt[3]{\frac{\cos(c+dx)}{b}}\sqrt[3]{b\sec(c+dx)}\int \frac{1}{\left(\frac{\cos(c+dx)}{b}\right)^{4/3}}\,dx\\ &= \frac{3b\,{}_2F_1\left(-\frac{1}{6},\frac{1}{2};\frac{5}{6};\cos^2(c+dx)\right)\sqrt[3]{b\sec(c+dx)}\sin(c+dx)}{d\sqrt{\sin^2(c+dx)}}\end{aligned}$$

Mathematica [A] time = 0.0082638, size = 57, normalized size = 1.06

$$\frac{3\sqrt{-\tan^2(c+dx)}\cot(c+dx)(b\sec(c+dx))^{4/3}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{5}{3},\sec^2(c+dx)\right)}{4d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(b*Sec[c + d*x])^(4/3),x]
```

```
[Out] (3*Cot[c + d*x]*Hypergeometric2F1[1/2, 2/3, 5/3, Sec[c + d*x]^2]*(b*Sec[c +
 d*x])^(4/3)*Sqrt[-Tan[c + d*x]^2])/(4*d)
```

Maple [F] time = 0.055, size = 0, normalized size = 0.

$$\int (b\sec(dx+c))^{\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sec(d*x+c))^(4/3),x)
```

```
[Out] int((b*sec(d*x+c))^(4/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b\sec(dx+c))^{\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))^(4/3),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c))^(4/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left((b\sec(dx+c))^{\frac{1}{3}}\,b\sec(dx+c),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))^(4/3),x, algorithm="fricas")
```

```
[Out] integral((b*sec(d*x + c))^(1/3)*b*sec(d*x + c), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))**(4/3),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b \sec (dx + c))^{\frac{4}{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))^(4/3),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c))^(4/3), x)
```

3.192 $\int \cos(c+dx)(b\sec(c+dx))^{4/3}\,dx$

Optimal. Leaf size=58

$$-\frac{3b^2\sin(c+dx)\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},\cos^2(c+dx)\right)}{2d\sqrt{\sin^2(c+dx)}(b\sec(c+dx))^{2/3}}$$

```
[Out] (-3*b^2*Hypergeometric2F1[1/3, 1/2, 4/3, Cos[c + d*x]^2]*Sin[c + d*x])/(2*d
*(b*Sec[c + d*x])^(2/3)*Sqrt[Sin[c + d*x]^2])
```

Rubi [A] time = 0.0381933, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {16, 3772, 2643}

$$-\frac{3b^2\sin(c+dx)\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};\cos^2(c+dx)\right)}{2d\sqrt{\sin^2(c+dx)}(b\sec(c+dx))^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(b*Sec[c + d*x])^(4/3),x]
```

```
[Out] (-3*b^2*Hypergeometric2F1[1/3, 1/2, 4/3, Cos[c + d*x]^2]*Sin[c + d*x])/(2*d
*(b*Sec[c + d*x])^(2/3)*Sqrt[Sin[c + d*x]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 3772

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] && !IntegerQ[n]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}
\int \cos(c+dx)(b\sec(c+dx))^{4/3}\,dx &= b\int \sqrt[3]{b\sec(c+dx)}\,dx\\
&= \left(b\sqrt[3]{\frac{\cos(c+dx)}{b}}\sqrt[3]{b\sec(c+dx)}\right)\int \frac{1}{\sqrt[3]{\frac{\cos(c+dx)}{b}}}\,dx\\
&= -\frac{3b\cos(c+dx)\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};\cos^2(c+dx)\right)\sqrt[3]{b\sec(c+dx)}\sin(c+dx)}{2d\sqrt{\sin^2(c+dx)}}
\end{aligned}$$

Mathematica [A] time = 0.009011, size = 56, normalized size = 0.97

$$\frac{3b\sqrt{-\tan^2(c+dx)}\cot(c+dx)\sqrt[3]{b\sec(c+dx)}\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{1}{2},\frac{7}{6},\sec^2(c+dx)\right)}{d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]*(b*Sec[c + d*x])^(4/3),x]
```

```
[Out] (3*b*Cot[c + d*x]*Hypergeometric2F1[1/6, 1/2, 7/6, Sec[c + d*x]^2]*(b*Sec[c
 + d*x])^(1/3)*Sqrt[-Tan[c + d*x]^2])/d
```

Maple [F] time = 0.106, size = 0, normalized size = 0.

$$\int \cos(dx+c)\,(b\sec(dx+c))^{\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(b*sec(d*x+c))^(4/3),x)
```

```
[Out] int(cos(d*x+c)*(b*sec(d*x+c))^(4/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b\sec(dx+c))^{\frac{4}{3}}\cos(dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(b*sec(d*x+c))^(4/3),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c))^(4/3)*cos(d*x + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((b\sec(dx+c))^{\frac{1}{3}}\,b\cos(dx+c)\sec(dx+c),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(b*sec(d*x+c))^(4/3),x, algorithm="fricas")
```

```
[Out] integral((b*sec(d*x + c))^(1/3)*b*cos(d*x + c)*sec(d*x + c), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(b*sec(d*x+c))**(4/3),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b\sec(dx+c))^{\frac{4}{3}}\cos(dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(b*sec(d*x+c))^(4/3),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c))^(4/3)*cos(d*x + c), x)
```

3.193 $\int \cos^2(c+dx)(b\sec(c+dx))^{4/3}\,dx$

Optimal. Leaf size=58

$$-\frac{3b^3\sin(c+dx)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{6},\frac{11}{6},\cos^2(c+dx)\right)}{5d\sqrt{\sin^2(c+dx)}(b\sec(c+dx))^{5/3}}$$

```
[Out] (-3*b^3*Hypergeometric2F1[1/2, 5/6, 11/6, Cos[c + d*x]^2]*Sin[c + d*x])/(5*
d*(b*Sec[c + d*x])^(5/3)*Sqrt[Sin[c + d*x]^2])
```

Rubi [A] time = 0.0460992, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {16, 3772, 2643}

$$-\frac{3b^3\sin(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{5}{6};\frac{11}{6};\cos^2(c+dx)\right)}{5d\sqrt{\sin^2(c+dx)}(b\sec(c+dx))^{5/3}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(b*Sec[c + d*x])^(4/3),x]
```

```
[Out] (-3*b^3*Hypergeometric2F1[1/2, 5/6, 11/6, Cos[c + d*x]^2]*Sin[c + d*x])/(5*
d*(b*Sec[c + d*x])^(5/3)*Sqrt[Sin[c + d*x]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 3772

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] && !IntegerQ[n]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}\int \cos^2(c+dx)(b\sec(c+dx))^{4/3}\,dx &= b^2\int \frac{1}{(b\sec(c+dx))^{2/3}}\,dx\\ &= \left(b^2\sqrt[3]{\frac{\cos(c+dx)}{b}}\sqrt[3]{b\sec(c+dx)}\right)\int \left(\frac{\cos(c+dx)}{b}\right)^{2/3}\,dx\\ &= -\frac{3b\cos^2(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{5}{6};\frac{11}{6};\cos^2(c+dx)\right)\sqrt[3]{b\sec(c+dx)}\sin(c+dx)}{5d\sqrt{\sin^2(c+dx)}}\end{aligned}$$

Mathematica [A] time = 0.054224, size = 60, normalized size = 1.03

$$-\frac{3b^2\sqrt{-\tan^2(c+dx)}\cot(c+dx)\text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{1}{2},\frac{2}{3},\sec^2(c+dx)\right)}{2d(b\sec(c+dx))^{2/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^2*(b*Sec[c + d*x])^(4/3),x]
```

```
[Out] (-3*b^2*Cot[c + d*x]*Hypergeometric2F1[-1/3, 1/2, 2/3, Sec[c + d*x]^2]*Sqrt
[-Tan[c + d*x]^2])/(2*d*(b*Sec[c + d*x])^(2/3))
```

Maple [F] time = 0.194, size = 0, normalized size = 0.

$$\int (\cos (dx+c))^2 (b\sec (dx+c))^{\frac{4}{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(b*sec(d*x+c))^(4/3),x)
```

```
[Out] int(cos(d*x+c)^2*(b*sec(d*x+c))^(4/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b\sec (dx+c))^{\frac{4}{3}} \cos (dx+c)^2\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(b*sec(d*x+c))^(4/3),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c))^(4/3)*cos(d*x + c)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((b\sec (dx+c))^{\frac{1}{3}}\, b\cos (dx+c)^2 \sec (dx+c)\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(b*sec(d*x+c))^(4/3),x, algorithm="fricas")
```

```
[Out] integral((b*sec(d*x + c))^(1/3)*b*cos(d*x + c)^2*sec(d*x + c), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(b*sec(d*x+c))**(4/3),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b\sec{(dx+c)})^{\frac{4}{3}}\cos{(dx+c)}^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(b*sec(d*x+c))^(4/3),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c))^(4/3)*cos(d*x + c)^2, x)
```

3.194 $\int \frac{\sec^2(c+dx)}{\sqrt[3]{b\sec(c+dx)}}\,dx$

Optimal. Leaf size=58

$$\frac{3\sin(c+dx)(b\sec(c+dx))^{2/3}\text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{1}{2},\frac{2}{3},\cos^2(c+dx)\right)}{2bd\sqrt{\sin^2(c+dx)}}$$

```
[Out] (3*Hypergeometric2F1[-1/3, 1/2, 2/3, Cos[c + d*x]^2]*(b*Sec[c + d*x])^(2/3)
*Sin[c + d*x])/(2*b*d*Sqrt[Sin[c + d*x]^2])
```

Rubi [A] time = 0.0311336, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {16, 3772, 2643}

$$\frac{3\sin(c+dx)(b\sec(c+dx))^{2/3}\,{}_2F_1\left(-\frac{1}{3},\frac{1}{2};\frac{2}{3};\cos^2(c+dx)\right)}{2bd\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2/(b*Sec[c + d*x])^(1/3),x]
```

```
[Out] (3*Hypergeometric2F1[-1/3, 1/2, 2/3, Cos[c + d*x]^2]*(b*Sec[c + d*x])^(2/3)
*Sin[c + d*x])/(2*b*d*Sqrt[Sin[c + d*x]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 3772

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] && !IntegerQ[n]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rubi steps

$$\int \frac{\sec^2(c+dx)}{\sqrt[3]{b\sec(c+dx)}}\,dx = \frac{\int (b\sec(c+dx))^{5/3}\,dx}{b^2}$$
$$= \frac{\left(\left(\frac{\cos(c+dx)}{b}\right)^{2/3}(b\sec(c+dx))^{2/3}\right)\int \frac{1}{\left(\frac{\cos(c+dx)}{b}\right)^{5/3}}\,dx}{b^2}$$
$$= \frac{3\,{}_2F_1\left(-\frac{1}{3},\frac{1}{2};\frac{2}{3};\cos^2(c+dx)\right)(b\sec(c+dx))^{2/3}\sin(c+dx)}{2bd\sqrt{\sin^2(c+dx)}}$$

Mathematica [A] time = 0.0595676, size = 60, normalized size = 1.03

$$\frac{3\sqrt{-\tan^2(c+dx)}\csc(c+dx)(b\sec(c+dx))^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{6},\frac{11}{6},\sec^2(c+dx)\right)}{5bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2/(b*Sec[c + d*x])^(1/3),x]
```

```
[Out] (3*Csc[c + d*x]*Hypergeometric2F1[1/2, 5/6, 11/6, Sec[c + d*x]^2]*(b*Sec[c
+ d*x])^(2/3)*Sqrt[-Tan[c + d*x]^2])/(5*b*d)
```

Maple [F] time = 0.065, size = 0, normalized size = 0.

$$\int (\sec(dx+c))^2 \frac{1}{\sqrt[3]{b\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2/(b*sec(d*x+c))^(1/3),x)
```

```
[Out] int(sec(d*x+c)^2/(b*sec(d*x+c))^(1/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)^2}{(b\sec(dx+c))^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(b*sec(d*x+c))^(1/3),x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)^2/(b*sec(d*x + c))^(1/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(b\sec(dx+c))^{\frac{2}{3}}\sec(dx+c)}{b}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(b*sec(d*x+c))^(1/3),x, algorithm="fricas")
```

```
[Out] integral((b*sec(d*x + c))^(2/3)*sec(d*x + c)/b, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec^2(c+dx)}{\sqrt[3]{b\sec(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2/(b*sec(d*x+c))**(1/3),x)
```

```
[Out] Integral(sec(c + d*x)**2/(b*sec(c + d*x))**(1/3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)^2}{(b\sec(dx+c))^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(b*sec(d*x+c))^(1/3),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^2/(b*sec(d*x + c))^(1/3), x)
```

3.195 $\int \frac{\sec(c+dx)}{\sqrt[3]{b\sec(c+dx)}}\,dx$

Optimal. Leaf size=53

$$-\frac{3\sin(c+dx)\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{1}{2},\frac{7}{6},\cos^2(c+dx)\right)}{d\sqrt{\sin^2(c+dx)}\sqrt[3]{b\sec(c+dx)}}$$

```
[Out] (-3*Hypergeometric2F1[1/6, 1/2, 7/6, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(b*Se
c[c + d*x])^(1/3)*Sqrt[Sin[c + d*x]^2])
```

Rubi [A] time = 0.0313008, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {16, 3772, 2643}

$$-\frac{3\sin(c+dx)\,{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{7}{6};\cos^2(c+dx)\right)}{d\sqrt{\sin^2(c+dx)}\sqrt[3]{b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]/(b*Sec[c + d*x])^(1/3),x]
```

```
[Out] (-3*Hypergeometric2F1[1/6, 1/2, 7/6, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(b*Se
c[c + d*x])^(1/3)*Sqrt[Sin[c + d*x]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 3772

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] && !IntegerQ[n]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rubi steps

$$\int \frac{\sec(c+dx)}{\sqrt[3]{b\sec(c+dx)}}\,dx = \frac{\int (b\sec(c+dx))^{2/3}\,dx}{b}$$
$$= \frac{\left(\left(\frac{\cos(c+dx)}{b}\right)^{2/3}(b\sec(c+dx))^{2/3}\right)\int \frac{1}{\left(\frac{\cos(c+dx)}{b}\right)^{2/3}}\,dx}{b}$$
$$= -\frac{3\cos(c+dx)\,{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{7}{6};\cos^2(c+dx)\right)(b\sec(c+dx))^{2/3}\sin(c+dx)}{bd\sqrt{\sin^2(c+dx)}}$$

Mathematica [A] time = 0.0474672, size = 60, normalized size = 1.13

$$\frac{3\sqrt{-\tan^2(c+dx)}\cot(c+dx)(b\sec(c+dx))^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},\sec^2(c+dx)\right)}{2bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]/(b*Sec[c + d*x])^(1/3),x]
```

```
[Out] (3*Cot[c + d*x]*Hypergeometric2F1[1/3, 1/2, 4/3, Sec[c + d*x]^2]*(b*Sec[c +
 d*x])^(2/3)*Sqrt[-Tan[c + d*x]^2])/(2*b*d)
```

Maple [F] time = 0.092, size = 0, normalized size = 0.

$$\int \sec(dx+c)\frac{1}{\sqrt[3]{b\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)/(b*sec(d*x+c))^(1/3),x)
```

```
[Out] int(sec(d*x+c)/(b*sec(d*x+c))^(1/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)}{(b\sec(dx+c))^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(b*sec(d*x+c))^(1/3),x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)/(b*sec(d*x + c))^(1/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(b\sec(dx+c))^{\frac{2}{3}}}{b},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(b*sec(d*x+c))^(1/3),x, algorithm="fricas")
```

```
[Out] integral((b*sec(d*x + c))^(2/3)/b, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(c+dx)}{\sqrt[3]{b\sec(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(b*sec(d*x+c))**(1/3),x)
```

```
[Out] Integral(sec(c + d*x)/(b*sec(c + d*x))**(1/3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)}{(b\sec(dx+c))^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(b*sec(d*x+c))^(1/3),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)/(b*sec(d*x + c))^(1/3), x)
```

3.196 $\int \frac{1}{\sqrt[3]{b\sec(c+dx)}}\,dx$

Optimal. Leaf size=56

$$-\frac{3b\sin(c+dx)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{5}{3},\cos^2(c+dx)\right)}{4d\sqrt{\sin^2(c+dx)}(b\sec(c+dx))^{4/3}}$$

```
[Out] (-3*b*Hypergeometric2F1[1/2, 2/3, 5/3, Cos[c + d*x]^2]*Sin[c + d*x])/(4*d*(
b*Sec[c + d*x])^(4/3)*Sqrt[Sin[c + d*x]^2])
```

Rubi [A] time = 0.025305, antiderivative size = 56, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3772, 2643}

$$-\frac{3b\sin(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};\cos^2(c+dx)\right)}{4d\sqrt{\sin^2(c+dx)}(b\sec(c+dx))^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Sec[c + d*x])^(-1/3),x]
```

```
[Out] (-3*b*Hypergeometric2F1[1/2, 2/3, 5/3, Cos[c + d*x]^2]*Sin[c + d*x])/(4*d*(
b*Sec[c + d*x])^(4/3)*Sqrt[Sin[c + d*x]^2])
```

Rule 3772

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] && !IntegerQ[n]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt[3]{b\sec(c+dx)}}\,dx &= \left(\frac{\cos(c+dx)}{b}\right)^{2/3}(b\sec(c+dx))^{2/3}\int \sqrt[3]{\frac{\cos(c+dx)}{b}}\,dx\\ &= -\frac{3\cos^2(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};\cos^2(c+dx)\right)(b\sec(c+dx))^{2/3}\sin(c+dx)}{4bd\sqrt{\sin^2(c+dx)}}\end{aligned}$$

Mathematica [A] time = 0.0542083, size = 55, normalized size = 0.98

$$-\frac{3\sqrt{-\tan^2(c+dx)}\cot(c+dx)\text{Hypergeometric2F1}\left(-\frac{1}{6},\frac{1}{2},\frac{5}{6},\sec^2(c+dx)\right)}{d\sqrt[3]{b\sec(c+dx)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(b*Sec[c + d*x])^(-1/3),x]
```

```
[Out] (-3*Cot[c + d*x]*Hypergeometric2F1[-1/6, 1/2, 5/6, Sec[c + d*x]^2]*Sqrt[-Ta
n[c + d*x]^2])/(d*(b*Sec[c + d*x])^(1/3))
```

Maple [F] time = 0.091, size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt[3]{b\sec\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*sec(d*x+c))^(1/3),x)
```

```
[Out] int(1/(b*sec(d*x+c))^(1/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b\sec\left(dx+c\right)\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*sec(d*x+c))^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c))^(-1/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(b\sec\left(dx+c\right)\right)^{\frac{2}{3}}}{b\sec\left(dx+c\right)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*sec(d*x+c))^(1/3),x, algorithm="fricas")
```

```
[Out] integral((b*sec(d*x + c))^(2/3)/(b*sec(d*x + c)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt[3]{b\sec\left(c+dx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*sec(d*x+c))**(1/3),x)
```

```
[Out] Integral((b*sec(c + d*x))**(-1/3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b\sec{(dx+c)})^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*sec(d*x+c))^(1/3),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c))^(-1/3), x)
```

3.197 $\int \frac{\cos(c+dx)}{\sqrt[3]{b\sec(c+dx)}}\,dx$

Optimal. Leaf size=58

$$-\frac{3b^2\sin(c+dx)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{7}{6},\frac{13}{6},\cos^2(c+dx)\right)}{7d\sqrt{\sin^2(c+dx)}(b\sec(c+dx))^{7/3}}$$

```
[Out] (-3*b^2*Hypergeometric2F1[1/2, 7/6, 13/6, Cos[c + d*x]^2]*Sin[c + d*x])/(7*
d*(b*Sec[c + d*x])^(7/3)*Sqrt[Sin[c + d*x]^2])
```

Rubi [A] time = 0.0374952, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {16, 3772, 2643}

$$-\frac{3b^2\sin(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{7}{6};\frac{13}{6};\cos^2(c+dx)\right)}{7d\sqrt{\sin^2(c+dx)}(b\sec(c+dx))^{7/3}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]/(b*Sec[c + d*x])^(1/3),x]
```

```
[Out] (-3*b^2*Hypergeometric2F1[1/2, 7/6, 13/6, Cos[c + d*x]^2]*Sin[c + d*x])/(7*
d*(b*Sec[c + d*x])^(7/3)*Sqrt[Sin[c + d*x]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 3772

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] && !IntegerQ[n]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos(c+dx)}{\sqrt[3]{b\sec(c+dx)}}\,dx &= b\int \frac{1}{(b\sec(c+dx))^{4/3}}\,dx\\
&= \left(b\left(\frac{\cos(c+dx)}{b}\right)^{2/3}(b\sec(c+dx))^{2/3}\right)\int\left(\frac{\cos(c+dx)}{b}\right)^{4/3}dx\\
&= -\frac{3\cos^3(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{7}{6};\frac{13}{6};\cos^2(c+dx)\right)(b\sec(c+dx))^{2/3}\sin(c+dx)}{7bd\sqrt{\sin^2(c+dx)}}
\end{aligned}$$

Mathematica [A] time = 0.114559, size = 58, normalized size = 1.

$$-\frac{3b\sqrt{-\tan^2(c+dx)}\cot(c+dx)\text{Hypergeometric2F1}\left(-\frac{2}{3},\frac{1}{2},\frac{1}{3},\sec^2(c+dx)\right)}{4d(b\sec(c+dx))^{4/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]/(b*Sec[c + d*x])^(1/3),x]
```

```
[Out] (-3*b*Cot[c + d*x]*Hypergeometric2F1[-2/3, 1/2, 1/3, Sec[c + d*x]^2]*Sqrt[-
Tan[c + d*x]^2])/(4*d*(b*Sec[c + d*x])^(4/3))
```

Maple [F] time = 0.068, size = 0, normalized size = 0.

$$\int \cos(dx+c)\frac{1}{\sqrt[3]{b\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)/(b*sec(d*x+c))^(1/3),x)
```

```
[Out] int(cos(d*x+c)/(b*sec(d*x+c))^(1/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)}{(b\sec(dx+c))^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(b*sec(d*x+c))^(1/3),x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)/(b*sec(d*x + c))^(1/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(b\sec(dx+c))^{\frac{2}{3}}\cos(dx+c)}{b\sec(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(b*sec(d*x+c))^(1/3),x, algorithm="fricas")
```

```
[Out] integral((b*sec(d*x + c))^(2/3)*cos(d*x + c)/(b*sec(d*x + c)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(c+dx)}{\sqrt[3]{b\sec(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(b*sec(d*x+c))**(1/3),x)
```

```
[Out] Integral(cos(c + d*x)/(b*sec(c + d*x))**(1/3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)}{(b\sec(dx+c))^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(b*sec(d*x+c))^(1/3),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)/(b*sec(d*x + c))^(1/3), x)
```

3.198 $\int \frac{\cos^2(c+dx)}{\sqrt[3]{b\sec(c+dx)}}\,dx$

Optimal. Leaf size=58

$$-\frac{3b^3\sin(c+dx)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{3},\frac{8}{3},\cos^2(c+dx)\right)}{10d\sqrt{\sin^2(c+dx)}(b\sec(c+dx))^{10/3}}$$

```
[Out] (-3*b^3*Hypergeometric2F1[1/2, 5/3, 8/3, Cos[c + d*x]^2]*Sin[c + d*x])/(10*
d*(b*Sec[c + d*x])^(10/3)*Sqrt[Sin[c + d*x]^2])
```

Rubi [A] time = 0.04462, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {16, 3772, 2643}

$$-\frac{3b^3\sin(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{5}{3};\frac{8}{3};\cos^2(c+dx)\right)}{10d\sqrt{\sin^2(c+dx)}(b\sec(c+dx))^{10/3}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2/(b*Sec[c + d*x])^(1/3),x]
```

```
[Out] (-3*b^3*Hypergeometric2F1[1/2, 5/3, 8/3, Cos[c + d*x]^2]*Sin[c + d*x])/(10*
d*(b*Sec[c + d*x])^(10/3)*Sqrt[Sin[c + d*x]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 3772

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos^2(c+dx)}{\sqrt[3]{b\sec(c+dx)}}\,dx &= b^2\int \frac{1}{(b\sec(c+dx))^{7/3}}\,dx\\
&= \left(b^2\left(\frac{\cos(c+dx)}{b}\right)^{2/3}(b\sec(c+dx))^{2/3}\right)\int \left(\frac{\cos(c+dx)}{b}\right)^{7/3}dx\\
&= -\frac{3\cos^4(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{5}{3};\frac{8}{3};\cos^2(c+dx)\right)(b\sec(c+dx))^{2/3}\sin(c+dx)}{10bd\sqrt{\sin^2(c+dx)}}
\end{aligned}$$

Mathematica [A] time = 0.0697015, size = 60, normalized size = 1.03

$$-\frac{3b^2\sqrt{-\tan^2(c+dx)}\cot(c+dx)\text{Hypergeometric2F1}\left(-\frac{7}{6},\frac{1}{2},-\frac{1}{6},\sec^2(c+dx)\right)}{7d(b\sec(c+dx))^{7/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^2/(b*Sec[c + d*x])^(1/3),x]
```

```
[Out] (-3*b^2*Cot[c + d*x]*Hypergeometric2F1[-7/6, 1/2, -1/6, Sec[c + d*x]^2]*Sqr
t[-Tan[c + d*x]^2])/(7*d*(b*Sec[c + d*x])^(7/3))
```

Maple [F] time = 0.142, size = 0, normalized size = 0.

$$\int (\cos(dx+c))^2 \frac{1}{\sqrt[3]{b\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2/(b*sec(d*x+c))^(1/3),x)
```

```
[Out] int(cos(d*x+c)^2/(b*sec(d*x+c))^(1/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)^2}{(b\sec(dx+c))^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(b*sec(d*x+c))^(1/3),x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)^2/(b*sec(d*x + c))^(1/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(b\sec(dx+c))^{\frac{2}{3}}\cos(dx+c)^2}{b\sec(dx+c)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(b*sec(d*x+c))^(1/3),x, algorithm="fricas")
```

```
[Out] integral((b*sec(d*x + c))^(2/3)*cos(d*x + c)^2/(b*sec(d*x + c)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos^2(c+dx)}{\sqrt[3]{b\sec(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2/(b*sec(d*x+c))**(1/3),x)
```

```
[Out] Integral(cos(c + d*x)**2/(b*sec(c + d*x))**(1/3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)^2}{(b\sec(dx+c))^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(b*sec(d*x+c))^(1/3),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^2/(b*sec(d*x + c))^(1/3), x)
```

3.199 $\int \frac{\sec^2(c+dx)}{(b\sec(c+dx))^{4/3}}\,dx$

Optimal. Leaf size=56

$$-\frac{3\sin(c+dx)\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{1}{2},\frac{7}{6},\cos^2(c+dx)\right)}{bd\sqrt{\sin^2(c+dx)}\sqrt[3]{b\sec(c+dx)}}$$

```
[Out] (-3*Hypergeometric2F1[1/6, 1/2, 7/6, Cos[c + d*x]^2]*Sin[c + d*x])/(b*d*(b*
Sec[c + d*x])^(1/3)*Sqrt[Sin[c + d*x]^2])
```

Rubi [A] time = 0.031286, antiderivative size = 56, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {16, 3772, 2643}

$$-\frac{3\sin(c+dx)\,{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{7}{6};\cos^2(c+dx)\right)}{bd\sqrt{\sin^2(c+dx)}\sqrt[3]{b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2/(b*Sec[c + d*x])^(4/3),x]
```

```
[Out] (-3*Hypergeometric2F1[1/6, 1/2, 7/6, Cos[c + d*x]^2]*Sin[c + d*x])/(b*d*(b*
Sec[c + d*x])^(1/3)*Sqrt[Sin[c + d*x]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 3772

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] && !IntegerQ[n]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rubi steps

$$\int \frac{\sec^2(c+dx)}{(b\sec(c+dx))^{4/3}}\,dx = \frac{\int (b\sec(c+dx))^{2/3}\,dx}{b^2}$$
$$= \frac{\left(\left(\frac{\cos(c+dx)}{b}\right)^{2/3}(b\sec(c+dx))^{2/3}\right)\int \frac{1}{\left(\frac{\cos(c+dx)}{b}\right)^{2/3}}\,dx}{b^2}$$
$$= -\frac{3\cos(c+dx)\,{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{7}{6};\cos^2(c+dx)\right)(b\sec(c+dx))^{2/3}\sin(c+dx)}{b^2d\sqrt{\sin^2(c+dx)}}$$

Mathematica [A] time = 0.0426375, size = 60, normalized size = 1.07

$$\frac{3\sqrt{-\tan^2(c+dx)}\cot(c+dx)(b\sec(c+dx))^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},\sec^2(c+dx)\right)}{2b^2d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]^2/(b*Sec[c + d*x])^(4/3),x]
```

```
[Out] (3*Cot[c + d*x]*Hypergeometric2F1[1/3, 1/2, 4/3, Sec[c + d*x]^2]*(b*Sec[c +
 d*x])^(2/3)*Sqrt[-Tan[c + d*x]^2])/(2*b^2*d)
```

Maple [F] time = 0.087, size = 0, normalized size = 0.

$$\int (\sec(dx+c))^2\,(b\sec(dx+c))^{-\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2/(b*sec(d*x+c))^(4/3),x)
```

```
[Out] int(sec(d*x+c)^2/(b*sec(d*x+c))^(4/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)^2}{(b\sec(dx+c))^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(b*sec(d*x+c))^(4/3),x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)^2/(b*sec(d*x + c))^(4/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(b\sec(dx+c))^{\frac{2}{3}}}{b^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(b*sec(d*x+c))^(4/3),x, algorithm="fricas")
```

```
[Out] integral((b*sec(d*x + c))^(2/3)/b^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec^2(c+dx)}{(b\sec(c+dx))^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2/(b*sec(d*x+c))**(4/3),x)
```

```
[Out] Integral(sec(c + d*x)**2/(b*sec(c + d*x))**(4/3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)^2}{(b\sec(dx+c))^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(b*sec(d*x+c))^(4/3),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^2/(b*sec(d*x + c))^(4/3), x)
```

3.200 $\int \frac{\sec(c+dx)}{(b\sec(c+dx))^{4/3}}\,dx$

Optimal. Leaf size=55

$$-\frac{3\sin(c+dx)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{5}{3},\cos^2(c+dx)\right)}{4d\sqrt{\sin^2(c+dx)}(b\sec(c+dx))^{4/3}}$$

```
[Out] (-3*Hypergeometric2F1[1/2, 2/3, 5/3, Cos[c + d*x]^2]*Sin[c + d*x])/(4*d*(b*
Sec[c + d*x])^(4/3)*Sqrt[Sin[c + d*x]^2])
```

Rubi [A] time = 0.02957, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {16, 3772, 2643}

$$-\frac{3\sin(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};\cos^2(c+dx)\right)}{4d\sqrt{\sin^2(c+dx)}(b\sec(c+dx))^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]/(b*Sec[c + d*x])^(4/3),x]
```

```
[Out] (-3*Hypergeometric2F1[1/2, 2/3, 5/3, Cos[c + d*x]^2]*Sin[c + d*x])/(4*d*(b*
Sec[c + d*x])^(4/3)*Sqrt[Sin[c + d*x]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 3772

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rubi steps

$$\int \frac{\sec(c+dx)}{(b\sec(c+dx))^{4/3}}\,dx = \frac{\int \frac{1}{\sqrt[3]{b\sec(c+dx)}}\,dx}{b}$$
$$= \frac{\left(\left(\frac{\cos(c+dx)}{b}\right)^{2/3}(b\sec(c+dx))^{2/3}\right)\int \sqrt[3]{\frac{\cos(c+dx)}{b}}\,dx}{b}$$
$$= -\frac{3\cos^2(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};\cos^2(c+dx)\right)(b\sec(c+dx))^{2/3}\sin(c+dx)}{4b^2d\sqrt{\sin^2(c+dx)}}$$

Mathematica [A] time = 0.0151685, size = 58, normalized size = 1.05

$$-\frac{3\sqrt{-\tan^2(c+dx)}\cot(c+dx)\text{Hypergeometric2F1}\left(-\frac{1}{6},\frac{1}{2},\frac{5}{6},\sec^2(c+dx)\right)}{bd\sqrt[3]{b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]/(b*Sec[c + d*x])^(4/3),x]
```

```
[Out] (-3*Cot[c + d*x]*Hypergeometric2F1[-1/6, 1/2, 5/6, Sec[c + d*x]^2]*Sqrt[-Ta
n[c + d*x]^2])/(b*d*(b*Sec[c + d*x])^(1/3))
```

Maple [F] time = 0.085, size = 0, normalized size = 0.

$$\int \sec(dx+c)\,(b\sec(dx+c))^{-\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)/(b*sec(d*x+c))^(4/3),x)
```

```
[Out] int(sec(d*x+c)/(b*sec(d*x+c))^(4/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)}{(b\sec(dx+c))^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(b*sec(d*x+c))^(4/3),x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)/(b*sec(d*x + c))^(4/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(b\sec(dx+c))^{\frac{2}{3}}}{b^2\sec(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(b*sec(d*x+c))^(4/3),x, algorithm="fricas")
```

```
[Out] integral((b*sec(d*x + c))^(2/3)/(b^2*sec(d*x + c)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(c+dx)}{(b\sec(c+dx))^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(b*sec(d*x+c))**(4/3),x)
```

```
[Out] Integral(sec(c + d*x)/(b*sec(c + d*x))**(4/3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)}{(b\sec(dx+c))^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(b*sec(d*x+c))^(4/3),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)/(b*sec(d*x + c))^(4/3), x)
```

3.201 $\int \frac{1}{(b\sec(c+dx))^{4/3}}\,dx$

Optimal. Leaf size=56

$$-\frac{3b\sin(c+dx)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{7}{6},\frac{13}{6},\cos^2(c+dx)\right)}{7d\sqrt{\sin^2(c+dx)}(b\sec(c+dx))^{7/3}}$$

```
[Out] (-3*b*Hypergeometric2F1[1/2, 7/6, 13/6, Cos[c + d*x]^2]*Sin[c + d*x])/(7*d*
(b*Sec[c + d*x])^(7/3)*Sqrt[Sin[c + d*x]^2])
```

Rubi [A] time = 0.0259363, antiderivative size = 56, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3772, 2643}

$$-\frac{3b\sin(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{7}{6};\frac{13}{6};\cos^2(c+dx)\right)}{7d\sqrt{\sin^2(c+dx)}(b\sec(c+dx))^{7/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Sec[c + d*x])^(-4/3),x]
```

```
[Out] (-3*b*Hypergeometric2F1[1/2, 7/6, 13/6, Cos[c + d*x]^2]*Sin[c + d*x])/(7*d*
(b*Sec[c + d*x])^(7/3)*Sqrt[Sin[c + d*x]^2])
```

Rule 3772

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] && !IntegerQ[n]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(b\sec(c+dx))^{4/3}}\,dx &= \left(\frac{\cos(c+dx)}{b}\right)^{2/3}(b\sec(c+dx))^{2/3}\int\left(\frac{\cos(c+dx)}{b}\right)^{4/3}dx\\ &= -\frac{3\cos^3(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{7}{6};\frac{13}{6};\cos^2(c+dx)\right)(b\sec(c+dx))^{2/3}\sin(c+dx)}{7b^2d\sqrt{\sin^2(c+dx)}}\end{aligned}$$

Mathematica [A] time = 0.0035631, size = 57, normalized size = 1.02

$$-\frac{3\sqrt{-\tan^2(c+dx)}\cot(c+dx)\text{Hypergeometric2F1}\left(-\frac{2}{3},\frac{1}{2},\frac{1}{3},\sec^2(c+dx)\right)}{4d(b\sec(c+dx))^{4/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(b*Sec[c + d*x])^(-4/3),x]
```

```
[Out] (-3*Cot[c + d*x]*Hypergeometric2F1[-2/3, 1/2, 1/3, Sec[c + d*x]^2]*Sqrt[-Ta
n[c + d*x]^2])/(4*d*(b*Sec[c + d*x])^(4/3))
```

Maple [F] time = 0.059, size = 0, normalized size = 0.

$$\int (b\sec(dx+c))^{-\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*sec(d*x+c))^(4/3),x)
```

```
[Out] int(1/(b*sec(d*x+c))^(4/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b\sec(dx+c))^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*sec(d*x+c))^(4/3),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c))^(-4/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{(b\sec(dx+c))^{\frac{2}{3}}}{b^2\sec(dx+c)^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*sec(d*x+c))^(4/3),x, algorithm="fricas")
```

```
[Out] integral((b*sec(d*x + c))^(2/3)/(b^2*sec(d*x + c)^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b\sec(c+dx))^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*sec(d*x+c))**(4/3),x)
```

```
[Out] Integral((b*sec(c + d*x))**(-4/3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b\sec(dx+c))^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*sec(d*x+c))^(4/3),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c))^(-4/3), x)
```

3.202 $\int \frac{\cos(c+dx)}{(b\sec(c+dx))^{4/3}}\,dx$

Optimal. Leaf size=58

$$-\frac{3b^2\sin(c+dx)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{3},\frac{8}{3},\cos^2(c+dx)\right)}{10d\sqrt{\sin^2(c+dx)}(b\sec(c+dx))^{10/3}}$$

```
[Out] (-3*b^2*Hypergeometric2F1[1/2, 5/3, 8/3, Cos[c + d*x]^2]*Sin[c + d*x])/(10*
d*(b*Sec[c + d*x])^(10/3)*Sqrt[Sin[c + d*x]^2])
```

Rubi [A] time = 0.0380579, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {16, 3772, 2643}

$$-\frac{3b^2\sin(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{5}{3};\frac{8}{3};\cos^2(c+dx)\right)}{10d\sqrt{\sin^2(c+dx)}(b\sec(c+dx))^{10/3}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]/(b*Sec[c + d*x])^(4/3),x]
```

```
[Out] (-3*b^2*Hypergeometric2F1[1/2, 5/3, 8/3, Cos[c + d*x]^2]*Sin[c + d*x])/(10*
d*(b*Sec[c + d*x])^(10/3)*Sqrt[Sin[c + d*x]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 3772

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] && !IntegerQ[n]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos(c+dx)}{(b\sec(c+dx))^{4/3}}\,dx &= b\int \frac{1}{(b\sec(c+dx))^{7/3}}\,dx\\
&= \left(b\left(\frac{\cos(c+dx)}{b}\right)^{2/3}(b\sec(c+dx))^{2/3}\right)\int \left(\frac{\cos(c+dx)}{b}\right)^{7/3}dx\\
&= -\frac{3\cos^4(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{5}{3};\frac{8}{3};\cos^2(c+dx)\right)(b\sec(c+dx))^{2/3}\sin(c+dx)}{10b^2d\sqrt{\sin^2(c+dx)}}
\end{aligned}$$

Mathematica [A] time = 0.0170913, size = 58, normalized size = 1.

$$-\frac{3b\sqrt{-\tan^2(c+dx)}\cot(c+dx)\text{Hypergeometric2F1}\left(-\frac{7}{6},\frac{1}{2},-\frac{1}{6},\sec^2(c+dx)\right)}{7d(b\sec(c+dx))^{7/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]/(b*Sec[c + d*x])^(4/3),x]
```

```
[Out] (-3*b*Cot[c + d*x]*Hypergeometric2F1[-7/6, 1/2, -1/6, Sec[c + d*x]^2]*Sqrt[
-Tan[c + d*x]^2])/(7*d*(b*Sec[c + d*x])^(7/3))
```

Maple [F] time = 0.069, size = 0, normalized size = 0.

$$\int \cos(dx+c)\,(b\sec(dx+c))^{-\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)/(b*sec(d*x+c))^(4/3),x)
```

```
[Out] int(cos(d*x+c)/(b*sec(d*x+c))^(4/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)}{(b\sec(dx+c))^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(b*sec(d*x+c))^(4/3),x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)/(b*sec(d*x + c))^(4/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(b\sec(dx+c))^{\frac{2}{3}}\cos(dx+c)}{b^2\sec(dx+c)^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(b*sec(d*x+c))^(4/3),x, algorithm="fricas")
```

```
[Out] integral((b*sec(d*x + c))^(2/3)*cos(d*x + c)/(b^2*sec(d*x + c)^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(b*sec(d*x+c))**(4/3),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)}{(b\sec(dx+c))^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(b*sec(d*x+c))^(4/3),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)/(b*sec(d*x + c))^(4/3), x)
```

3.203 $\int \frac{\cos^2(c+dx)}{(b\sec(c+dx))^{4/3}}\,dx$

Optimal. Leaf size=58

$$-\frac{3b^3\sin(c+dx)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{13}{6},\frac{19}{6},\cos^2(c+dx)\right)}{13d\sqrt{\sin^2(c+dx)}(b\sec(c+dx))^{13/3}}$$

```
[Out] (-3*b^3*Hypergeometric2F1[1/2, 13/6, 19/6, Cos[c + d*x]^2]*Sin[c + d*x])/(1
3*d*(b*Sec[c + d*x])^(13/3)*Sqrt[Sin[c + d*x]^2])
```

Rubi [A] time = 0.0472666, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {16, 3772, 2643}

$$-\frac{3b^3\sin(c+dx)\,_2F_1\left(\frac{1}{2},\frac{13}{6};\frac{19}{6};\cos^2(c+dx)\right)}{13d\sqrt{\sin^2(c+dx)}(b\sec(c+dx))^{13/3}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2/(b*Sec[c + d*x])^(4/3),x]
```

```
[Out] (-3*b^3*Hypergeometric2F1[1/2, 13/6, 19/6, Cos[c + d*x]^2]*Sin[c + d*x])/(1
3*d*(b*Sec[c + d*x])^(13/3)*Sqrt[Sin[c + d*x]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 3772

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos^2(c+dx)}{(b\sec(c+dx))^{4/3}}\,dx &= b^2\int \frac{1}{(b\sec(c+dx))^{10/3}}\,dx\\
&= \left(b^2\left(\frac{\cos(c+dx)}{b}\right)^{2/3}(b\sec(c+dx))^{2/3}\right)\int \left(\frac{\cos(c+dx)}{b}\right)^{10/3}\,dx\\
&= -\frac{3\cos^5(c+dx)\,_2F_1\left(\frac{1}{2},\frac{13}{6};\frac{19}{6};\cos^2(c+dx)\right)(b\sec(c+dx))^{2/3}\sin(c+dx)}{13b^2d\sqrt{\sin^2(c+dx)}}
\end{aligned}$$

Mathematica [A] time = 0.0993077, size = 60, normalized size = 1.03

$$-\frac{3b^2\sqrt{-\tan^2(c+dx)}\cot(c+dx)\text{Hypergeometric2F1}\left(-\frac{5}{3},\frac{1}{2},-\frac{2}{3},\sec^2(c+dx)\right)}{10d(b\sec(c+dx))^{10/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^2/(b*Sec[c + d*x])^(4/3),x]
```

```
[Out] (-3*b^2*Cot[c + d*x]*Hypergeometric2F1[-5/3, 1/2, -2/3, Sec[c + d*x]^2]*Sqr
t[-Tan[c + d*x]^2])/(10*d*(b*Sec[c + d*x])^(10/3))
```

Maple [F] time = 0.138, size = 0, normalized size = 0.

$$\int (\cos(dx+c))^2 (b\sec(dx+c))^{-\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2/(b*sec(d*x+c))^(4/3),x)
```

```
[Out] int(cos(d*x+c)^2/(b*sec(d*x+c))^(4/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)^2}{(b\sec(dx+c))^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(b*sec(d*x+c))^(4/3),x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)^2/(b*sec(d*x + c))^(4/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(b\sec(dx+c))^{\frac{2}{3}}\cos(dx+c)^2}{b^2\sec(dx+c)^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(b*sec(d*x+c))^(4/3),x, algorithm="fricas")
```

```
[Out] integral((b*sec(d*x + c))^(2/3)*cos(d*x + c)^2/(b^2*sec(d*x + c)^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2/(b*sec(d*x+c))**(4/3),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos (dx+c)^2}{(b\sec (dx+c))^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(b*sec(d*x+c))^(4/3),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^2/(b*sec(d*x + c))^(4/3), x)
```

3.204 $\int \sec^m(c+dx)(b\sec(c+dx))^{4/3}\,dx$

Optimal. Leaf size=81

$$\frac{3b\sin(c+dx)\sqrt[3]{b\sec(c+dx)}\sec^m(c+dx)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{6}(-3m-1),\frac{1}{6}(5-3m),\cos^2(c+dx)\right)}{d(3m+1)\sqrt{\sin^2(c+dx)}}$$

```
[Out] (3*b*Hypergeometric2F1[1/2, (-1 - 3*m)/6, (5 - 3*m)/6, Cos[c + d*x]^2]*Sec[
c + d*x]^m*(b*Sec[c + d*x])^(1/3)*Sin[c + d*x])/(d*(1 + 3*m)*Sqrt[Sin[c + d
*x]^2])
```

Rubi [A] time = 0.0402926, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {20, 3772, 2643}

$$\frac{3b\sin(c+dx)\sqrt[3]{b\sec(c+dx)}\sec^m(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{6}(-3m-1);\frac{1}{6}(5-3m);\cos^2(c+dx)\right)}{d(3m+1)\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^m*(b*Sec[c + d*x])^(4/3),x]
```

```
[Out] (3*b*Hypergeometric2F1[1/2, (-1 - 3*m)/6, (5 - 3*m)/6, Cos[c + d*x]^2]*Sec[
c + d*x]^m*(b*Sec[c + d*x])^(1/3)*Sin[c + d*x])/(d*(1 + 3*m)*Sqrt[Sin[c + d
*x]^2])
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(b^IntPart
[n]*(b*v)^FracPart[n])/(a^IntPart[n]*(a*v)^FracPart[n]), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !
IntegerQ[m + n]
```

Rule 3772

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] && !IntegerQ[n]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rubi steps

$$\int \sec^m(c+dx)(b\sec(c+dx))^{4/3}\,dx = \frac{\left(b\sqrt[3]{b\sec(c+dx)}\right)\int \sec^{\frac{4}{3}+m}(c+dx)\,dx}{\sqrt[3]{\sec(c+dx)}}$$
$$= \left(b\cos^{\frac{1}{3}+m}(c+dx)\sec^m(c+dx)\sqrt[3]{b\sec(c+dx)}\right)\int \cos^{-\frac{4}{3}-m}(c+dx)\,dx$$
$$= \frac{3b\,{}_2F_1\left(\frac{1}{2},\frac{1}{6}(-1-3m);\frac{1}{6}(5-3m);\cos^2(c+dx)\right)\sec^m(c+dx)\sqrt[3]{b\sec(c+dx)}\sin(c\cdot}{d(1+3m)\sqrt{\sin^2(c+dx)}}$$

Mathematica [A] time = 0.092348, size = 83, normalized size = 1.02

$$\frac{\sqrt{-\tan^2(c+dx)}\csc(c+dx)(b\sec(c+dx))^{4/3}\sec^{m-1}(c+dx)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}\left(m+\frac{4}{3}\right),\frac{1}{2}\left(m+\frac{10}{3}\right),\sec^2(c+\cdot\right.}{d\left(m+\frac{4}{3}\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^m*(b*Sec[c + d*x])^(4/3),x]
```

```
[Out] (Csc[c + d*x]*Hypergeometric2F1[1/2, (4/3 + m)/2, (10/3 + m)/2, Sec[c + d*x
]^2]*Sec[c + d*x]^(-1 + m)*(b*Sec[c + d*x])^(4/3)*Sqrt[-Tan[c + d*x]^2])/(d
*(4/3 + m))
```

Maple [F] time = 0.102, size = 0, normalized size = 0.

$$\int (\sec(dx+c))^m (b\sec(dx+c))^{\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^m*(b*sec(d*x+c))^(4/3),x)
```

```
[Out] int(sec(d*x+c)^m*(b*sec(d*x+c))^(4/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b\sec(dx+c))^{\frac{4}{3}}\sec(dx+c)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^m*(b*sec(d*x+c))^(4/3),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c))^(4/3)*sec(d*x + c)^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((b\sec(dx+c))^{\frac{1}{3}}\,b\sec(dx+c)^m\sec(dx+c),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^m*(b*sec(d*x+c))^(4/3),x, algorithm="fricas")
```

```
[Out] integral((b*sec(d*x + c))^(1/3)*b*sec(d*x + c)^m*sec(d*x + c), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**m*(b*sec(d*x+c))**(4/3),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b\sec(dx+c))^{\frac{4}{3}}\sec(dx+c)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^m*(b*sec(d*x+c))^(4/3),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c))^(4/3)*sec(d*x + c)^m, x)
```

3.205 $\int \sec^m(c+dx)(b\sec(c+dx))^{2/3}\,dx$

Optimal. Leaf size=82

$$-\frac{3\sin(c+dx)(b\sec(c+dx))^{2/3}\sec^{m-1}(c+dx)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{6}(1-3m),\frac{1}{6}(7-3m),\cos^2(c+dx)\right)}{d(1-3m)\sqrt{\sin^2(c+dx)}}$$

```
[Out] (-3*Hypergeometric2F1[1/2, (1 - 3*m)/6, (7 - 3*m)/6, Cos[c + d*x]^2]*Sec[c
+ d*x]^(-1 + m)*(b*Sec[c + d*x])^(2/3)*Sin[c + d*x])/(d*(1 - 3*m)*Sqrt[Sin[
c + d*x]^2])
```

Rubi [A] time = 0.0386954, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {20, 3772, 2643}

$$-\frac{3\sin(c+dx)(b\sec(c+dx))^{2/3}\sec^{m-1}(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{6}(1-3m);\frac{1}{6}(7-3m);\cos^2(c+dx)\right)}{d(1-3m)\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^m*(b*Sec[c + d*x])^(2/3),x]
```

```
[Out] (-3*Hypergeometric2F1[1/2, (1 - 3*m)/6, (7 - 3*m)/6, Cos[c + d*x]^2]*Sec[c
+ d*x]^(-1 + m)*(b*Sec[c + d*x])^(2/3)*Sin[c + d*x])/(d*(1 - 3*m)*Sqrt[Sin[
c + d*x]^2])
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(b^IntPart
[n]*(b*v)^FracPart[n])/(a^IntPart[n]*(a*v)^FracPart[n]), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !
IntegerQ[m + n]
```

Rule 3772

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] && !IntegerQ[n]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rubi steps

$$\int \sec^m(c+dx)(b\sec(c+dx))^{2/3}\,dx = \frac{(b\sec(c+dx))^{2/3}\int \sec^{\frac{2}{3}+m}(c+dx)\,dx}{\sec^{\frac{2}{3}}(c+dx)}$$
$$= \left(\cos^{\frac{2}{3}+m}(c+dx)\sec^m(c+dx)(b\sec(c+dx))^{2/3}\right)\int \cos^{-\frac{2}{3}-m}(c+dx)\,dx$$
$$= -\frac{3\,{}_2F_1\left(\frac{1}{2},\frac{1}{6}(1-3m);\frac{1}{6}(7-3m);\cos^2(c+dx)\right)\sec^{-1+m}(c+dx)(b\sec(c+dx))^{2/}}{d(1-3m)\sqrt{\sin^2(c+dx)}}$$

Mathematica [A] time = 0.138423, size = 83, normalized size = 1.01

$$\frac{\sqrt{-\tan^2(c+dx)}\csc(c+dx)(b\sec(c+dx))^{2/3}\sec^{m-1}(c+dx)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}\left(m+\frac{2}{3}\right),\frac{1}{2}\left(m+\frac{8}{3}\right),\sec^2(c\right.}{d\left(m+\frac{2}{3}\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^m*(b*Sec[c + d*x])^(2/3),x]
```

```
[Out] (Csc[c + d*x]*Hypergeometric2F1[1/2, (2/3 + m)/2, (8/3 + m)/2, Sec[c + d*x]
^2]*Sec[c + d*x]^(-1 + m)*(b*Sec[c + d*x])^(2/3)*Sqrt[-Tan[c + d*x]^2])/(d*
(2/3 + m))
```

Maple [F] time = 0.094, size = 0, normalized size = 0.

$$\int (\sec(dx+c))^m\,(b\sec(dx+c))^{\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^m*(b*sec(d*x+c))^(2/3),x)
```

```
[Out] int(sec(d*x+c)^m*(b*sec(d*x+c))^(2/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b\sec(dx+c))^{\frac{2}{3}}\sec(dx+c)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^m*(b*sec(d*x+c))^(2/3),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c))^(2/3)*sec(d*x + c)^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((b\sec(dx+c))^{\frac{2}{3}}\sec(dx+c)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^m*(b*sec(d*x+c))^(2/3),x, algorithm="fricas")
```

```
[Out] integral((b*sec(d*x + c))^(2/3)*sec(d*x + c)^m, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (b\sec (c+dx))^{\frac{2}{3}} \sec^m (c+dx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**m*(b*sec(d*x+c))**(2/3),x)
```

```
[Out] Integral((b*sec(c + d*x))**(2/3)*sec(c + d*x)**m, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b\sec (dx+c))^{\frac{2}{3}} \sec (dx+c)^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^m*(b*sec(d*x+c))^(2/3),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c))^(2/3)*sec(d*x + c)^m, x)
```

3.206 $\int \sec^m(c+dx)\sqrt[3]{b\sec(c+dx)}\,dx$

Optimal. Leaf size=82

$$-\frac{3\sin(c+dx)\sqrt[3]{b\sec(c+dx)}\sec^{m-1}(c+dx)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{6}(2-3m),\frac{1}{6}(8-3m),\cos^2(c+dx)\right)}{d(2-3m)\sqrt{\sin^2(c+dx)}}$$

```
[Out] (-3*Hypergeometric2F1[1/2, (2 - 3*m)/6, (8 - 3*m)/6, Cos[c + d*x]^2]*Sec[c
+ d*x]^(-1 + m)*(b*Sec[c + d*x])^(1/3)*Sin[c + d*x])/(d*(2 - 3*m)*Sqrt[Sin[
c + d*x]^2])
```

Rubi [A] time = 0.0406077, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {20, 3772, 2643}

$$-\frac{3\sin(c+dx)\sqrt[3]{b\sec(c+dx)}\sec^{m-1}(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{6}(2-3m);\frac{1}{6}(8-3m);\cos^2(c+dx)\right)}{d(2-3m)\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^m*(b*Sec[c + d*x])^(1/3),x]
```

```
[Out] (-3*Hypergeometric2F1[1/2, (2 - 3*m)/6, (8 - 3*m)/6, Cos[c + d*x]^2]*Sec[c
+ d*x]^(-1 + m)*(b*Sec[c + d*x])^(1/3)*Sin[c + d*x])/(d*(2 - 3*m)*Sqrt[Sin[
c + d*x]^2])
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(b^IntPart
[n]*(b*v)^FracPart[n])/(a^IntPart[n]*(a*v)^FracPart[n]), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !
IntegerQ[m + n]
```

Rule 3772

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] && !IntegerQ[n]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rubi steps

$$\int \sec^m(c+dx)\sqrt[3]{b\sec(c+dx)}\,dx = \frac{\sqrt[3]{b\sec(c+dx)}\int \sec^{\frac{1}{3}+m}(c+dx)\,dx}{\sqrt[3]{\sec(c+dx)}}$$
$$= \left(\cos^{\frac{1}{3}+m}(c+dx)\sec^m(c+dx)\sqrt[3]{b\sec(c+dx)}\right)\int \cos^{-\frac{1}{3}-m}(c+dx)\,dx$$
$$= -\frac{3\,{}_2F_1\left(\frac{1}{2},\frac{1}{6}(2-3m);\frac{1}{6}(8-3m);\cos^2(c+dx)\right)\sec^{-1+m}(c+dx)\sqrt[3]{b\sec(c+dx)}\sin(c\cdot}{d(2-3m)\sqrt{\sin^2(c+dx)}}$$

Mathematica [A] time = 0.140574, size = 83, normalized size = 1.01

$$\frac{\sqrt{-\tan^2(c+dx)}\csc(c+dx)\sqrt[3]{b\sec(c+dx)}\sec^{m-1}(c+dx)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}\left(m+\frac{1}{3}\right),\frac{1}{2}\left(m+\frac{7}{3}\right),\sec^2(c+dx)\right)}{d\left(m+\frac{1}{3}\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^m*(b*Sec[c + d*x])^(1/3),x]
```

```
[Out] (Csc[c + d*x]*Hypergeometric2F1[1/2, (1/3 + m)/2, (7/3 + m)/2, Sec[c + d*x]
^2]*Sec[c + d*x]^(-1 + m)*(b*Sec[c + d*x])^(1/3)*Sqrt[-Tan[c + d*x]^2])/(d*
(1/3 + m))
```

Maple [F] time = 0.086, size = 0, normalized size = 0.

$$\int (\sec(dx+c))^m \sqrt[3]{b\sec(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^m*(b*sec(d*x+c))^(1/3),x)
```

```
[Out] int(sec(d*x+c)^m*(b*sec(d*x+c))^(1/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b\sec(dx+c))^{\frac{1}{3}}\sec(dx+c)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^m*(b*sec(d*x+c))^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c))^(1/3)*sec(d*x + c)^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((b\sec(dx+c))^{\frac{1}{3}}\sec(dx+c)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^m*(b*sec(d*x+c))^(1/3),x, algorithm="fricas")
```

```
[Out] integral((b*sec(d*x + c))^(1/3)*sec(d*x + c)^m, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt[3]{b\sec(c+dx)}\sec^m(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**m*(b*sec(d*x+c))**(1/3),x)
```

```
[Out] Integral((b*sec(c + d*x))**(1/3)*sec(c + d*x)**m, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b\sec(dx+c))^{\frac{1}{3}}\sec(dx+c)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^m*(b*sec(d*x+c))^(1/3),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c))^(1/3)*sec(d*x + c)^m, x)
```

3.207 $\int \frac{\sec^m(c+dx)}{\sqrt[3]{b\sec(c+dx)}}\,dx$

Optimal. Leaf size=82

$$-\frac{3\sin(c+dx)\sec^{m-1}(c+dx)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{6}(4-3m),\frac{1}{6}(10-3m),\cos^2(c+dx)\right)}{d(4-3m)\sqrt{\sin^2(c+dx)}\sqrt[3]{b\sec(c+dx)}}$$

```
[Out] (-3*Hypergeometric2F1[1/2, (4 - 3*m)/6, (10 - 3*m)/6, Cos[c + d*x]^2]*Sec[c
 + d*x]^(-1 + m)*Sin[c + d*x])/(d*(4 - 3*m)*(b*Sec[c + d*x])^(1/3)*Sqrt[Sin
[c + d*x]^2])
```

Rubi [A] time = 0.0390033, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {20, 3772, 2643}

$$-\frac{3\sin(c+dx)\sec^{m-1}(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{6}(4-3m);\frac{1}{6}(10-3m);\cos^2(c+dx)\right)}{d(4-3m)\sqrt{\sin^2(c+dx)}\sqrt[3]{b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^m/(b*Sec[c + d*x])^(1/3),x]
```

```
[Out] (-3*Hypergeometric2F1[1/2, (4 - 3*m)/6, (10 - 3*m)/6, Cos[c + d*x]^2]*Sec[c
 + d*x]^(-1 + m)*Sin[c + d*x])/(d*(4 - 3*m)*(b*Sec[c + d*x])^(1/3)*Sqrt[Sin
[c + d*x]^2])
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(b^IntPart
[n]*(b*v)^FracPart[n])/(a^IntPart[n]*(a*v)^FracPart[n]), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !
IntegerQ[m + n]
```

Rule 3772

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] && !IntegerQ[n]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rubi steps

$$\int \frac{\sec^m(c+dx)}{\sqrt[3]{b\sec(c+dx)}}\,dx = \frac{\sqrt[3]{\sec(c+dx)}\int \sec^{-\frac{1}{3}+m}(c+dx)\,dx}{\sqrt[3]{b\sec(c+dx)}}$$
$$= \frac{\left(\cos^{\frac{2}{3}+m}(c+dx)\sec^{1+m}(c+dx)\right)\int \cos^{\frac{1}{3}-m}(c+dx)\,dx}{\sqrt[3]{b\sec(c+dx)}}$$
$$= -\frac{3\,{}_2F_1\left(\frac{1}{2},\frac{1}{6}(4-3m);\frac{1}{6}(10-3m);\cos^2(c+dx)\right)\sec^{-1+m}(c+dx)\sin(c+dx)}{d(4-3m)\sqrt[3]{b\sec(c+dx)}\sqrt{\sin^2(c+dx)}}$$

Mathematica [A] time = 0.140716, size = 83, normalized size = 1.01

$$\frac{\sqrt{-\tan^2(c+dx)}\csc(c+dx)\sec^{m-1}(c+dx)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}\left(m-\frac{1}{3}\right),\frac{1}{2}\left(m+\frac{5}{3}\right),\sec^2(c+dx)\right)}{d\left(m-\frac{1}{3}\right)\sqrt[3]{b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^m/(b*Sec[c + d*x])^(1/3),x]
```

```
[Out] (Csc[c + d*x]*Hypergeometric2F1[1/2, (-1/3 + m)/2, (5/3 + m)/2, Sec[c + d*x
]^2]*Sec[c + d*x]^(-1 + m)*Sqrt[-Tan[c + d*x]^2])/(d*(-1/3 + m)*(b*Sec[c +
d*x])^(1/3))
```

Maple [F] time = 0.082, size = 0, normalized size = 0.

$$\int (\sec(dx+c))^m \frac{1}{\sqrt[3]{b\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^m/(b*sec(d*x+c))^(1/3),x)
```

```
[Out] int(sec(d*x+c)^m/(b*sec(d*x+c))^(1/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)^m}{(b\sec(dx+c))^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^m/(b*sec(d*x+c))^(1/3),x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)^m/(b*sec(d*x + c))^(1/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(b\sec(dx+c))^{\frac{2}{3}}\sec(dx+c)^m}{b\sec(dx+c)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^m/(b*sec(d*x+c))^(1/3),x, algorithm="fricas")
```

```
[Out] integral((b*sec(d*x + c))^(2/3)*sec(d*x + c)^m/(b*sec(d*x + c)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec^m(c+dx)}{\sqrt[3]{b\sec(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**m/(b*sec(d*x+c))**(1/3),x)
```

```
[Out] Integral(sec(c + d*x)**m/(b*sec(c + d*x))**(1/3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)^m}{(b\sec(dx+c))^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^m/(b*sec(d*x+c))^(1/3),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^m/(b*sec(d*x + c))^(1/3), x)
```

3.208 $\int \frac{\sec^m(c+dx)}{(b\sec(c+dx))^{2/3}}\,dx$

Optimal. Leaf size=82

$$-\frac{3\sin(c+dx)\sec^{m-1}(c+dx)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{6}(5-3m),\frac{1}{6}(11-3m),\cos^2(c+dx)\right)}{d(5-3m)\sqrt{\sin^2(c+dx)}(b\sec(c+dx))^{2/3}}$$

```
[Out] (-3*Hypergeometric2F1[1/2, (5 - 3*m)/6, (11 - 3*m)/6, Cos[c + d*x]^2]*Sec[c
 + d*x]^(-1 + m)*Sin[c + d*x])/(d*(5 - 3*m)*(b*Sec[c + d*x])^(2/3)*Sqrt[Sin
[c + d*x]^2])
```

Rubi [A] time = 0.0406255, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {20, 3772, 2643}

$$-\frac{3\sin(c+dx)\sec^{m-1}(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{6}(5-3m);\frac{1}{6}(11-3m);\cos^2(c+dx)\right)}{d(5-3m)\sqrt{\sin^2(c+dx)}(b\sec(c+dx))^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^m/(b*Sec[c + d*x])^(2/3),x]
```

```
[Out] (-3*Hypergeometric2F1[1/2, (5 - 3*m)/6, (11 - 3*m)/6, Cos[c + d*x]^2]*Sec[c
 + d*x]^(-1 + m)*Sin[c + d*x])/(d*(5 - 3*m)*(b*Sec[c + d*x])^(2/3)*Sqrt[Sin
[c + d*x]^2])
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(b^IntPart
[n]*(b*v)^FracPart[n])/(a^IntPart[n]*(a*v)^FracPart[n]), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !
IntegerQ[m + n]
```

Rule 3772

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] && !IntegerQ[n]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rubi steps

$$\int \frac{\sec^m(c+dx)}{(b\sec(c+dx))^{2/3}}\,dx = \frac{\sec^{\frac{2}{3}}(c+dx)\int \sec^{-\frac{2}{3}+m}(c+dx)\,dx}{(b\sec(c+dx))^{2/3}}$$
$$= \frac{\left(\cos^{\frac{1}{3}+m}(c+dx)\sec^{1+m}(c+dx)\right)\int \cos^{\frac{2}{3}-m}(c+dx)\,dx}{(b\sec(c+dx))^{2/3}}$$
$$= -\frac{3\,{}_2F_1\left(\frac{1}{2},\frac{1}{6}(5-3m);\frac{1}{6}(11-3m);\cos^2(c+dx)\right)\sec^{-1+m}(c+dx)\sin(c+dx)}{d(5-3m)(b\sec(c+dx))^{2/3}\sqrt{\sin^2(c+dx)}}$$

Mathematica [A] time = 0.130089, size = 83, normalized size = 1.01

$$\frac{\sqrt{-\tan^2(c+dx)}\csc(c+dx)\sec^{m-1}(c+dx)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}\left(m-\frac{2}{3}\right),\frac{1}{2}\left(m+\frac{4}{3}\right),\sec^2(c+dx)\right)}{d\left(m-\frac{2}{3}\right)(b\sec(c+dx))^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^m/(b*Sec[c + d*x])^(2/3),x]
```

```
[Out] (Csc[c + d*x]*Hypergeometric2F1[1/2, (-2/3 + m)/2, (4/3 + m)/2, Sec[c + d*x
]^2]*Sec[c + d*x]^(-1 + m)*Sqrt[-Tan[c + d*x]^2])/(d*(-2/3 + m)*(b*Sec[c +
d*x])^(2/3))
```

Maple [F] time = 0.091, size = 0, normalized size = 0.

$$\int (\sec(dx+c))^m (b\sec(dx+c))^{-\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^m/(b*sec(d*x+c))^(2/3),x)
```

```
[Out] int(sec(d*x+c)^m/(b*sec(d*x+c))^(2/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)^m}{(b\sec(dx+c))^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^m/(b*sec(d*x+c))^(2/3),x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)^m/(b*sec(d*x + c))^(2/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(b\sec(dx+c))^{\frac{1}{3}}\sec(dx+c)^m}{b\sec(dx+c)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^m/(b*sec(d*x+c))^(2/3),x, algorithm="fricas")
```

```
[Out] integral((b*sec(d*x + c))^(1/3)*sec(d*x + c)^m/(b*sec(d*x + c)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec^{m}(c+dx)}{(b\sec(c+dx))^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**m/(b*sec(d*x+c))**(2/3),x)
```

```
[Out] Integral(sec(c + d*x)**m/(b*sec(c + d*x))**(2/3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)^{m}}{(b\sec(dx+c))^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^m/(b*sec(d*x+c))^(2/3),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^m/(b*sec(d*x + c))^(2/3), x)
```

3.209 $\int \frac{\sec^m(c+dx)}{(b\sec(c+dx))^{4/3}}\,dx$

Optimal. Leaf size=85

$$-\frac{3\sin(c+dx)\sec^{m-2}(c+dx)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{6}(7-3m),\frac{1}{6}(13-3m),\cos^2(c+dx)\right)}{bd(7-3m)\sqrt{\sin^2(c+dx)}\sqrt[3]{b\sec(c+dx)}}$$

```
[Out] (-3*Hypergeometric2F1[1/2, (7 - 3*m)/6, (13 - 3*m)/6, Cos[c + d*x]^2]*Sec[c
 + d*x]^(-2 + m)*Sin[c + d*x])/(b*d*(7 - 3*m)*(b*Sec[c + d*x])^(1/3)*Sqrt[S
in[c + d*x]^2])
```

Rubi [A] time = 0.0429287, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {20, 3772, 2643}

$$-\frac{3\sin(c+dx)\sec^{m-2}(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{6}(7-3m);\frac{1}{6}(13-3m);\cos^2(c+dx)\right)}{bd(7-3m)\sqrt{\sin^2(c+dx)}\sqrt[3]{b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^m/(b*Sec[c + d*x])^(4/3),x]
```

```
[Out] (-3*Hypergeometric2F1[1/2, (7 - 3*m)/6, (13 - 3*m)/6, Cos[c + d*x]^2]*Sec[c
 + d*x]^(-2 + m)*Sin[c + d*x])/(b*d*(7 - 3*m)*(b*Sec[c + d*x])^(1/3)*Sqrt[S
in[c + d*x]^2])
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(b^IntPart
[n]*(b*v)^FracPart[n])/(a^IntPart[n]*(a*v)^FracPart[n]), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !
IntegerQ[m + n]
```

Rule 3772

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] && !IntegerQ[n]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rubi steps

$$\int \frac{\sec^m(c+dx)}{(b\sec(c+dx))^{4/3}}\,dx = \frac{\sqrt[3]{\sec(c+dx)}\int \sec^{-\frac{4}{3}+m}(c+dx)\,dx}{b\sqrt[3]{b\sec(c+dx)}}$$
$$= \frac{\left(\cos^{\frac{2}{3}+m}(c+dx)\sec^{1+m}(c+dx)\right)\int \cos^{\frac{4}{3}-m}(c+dx)\,dx}{b\sqrt[3]{b\sec(c+dx)}}$$
$$= -\frac{3\,{}_2F_1\left(\frac{1}{2},\frac{1}{6}(7-3m);\frac{1}{6}(13-3m);\cos^2(c+dx)\right)\sec^{-2+m}(c+dx)\sin(c+dx)}{bd(7-3m)\sqrt[3]{b\sec(c+dx)}\sqrt{\sin^2(c+dx)}}$$

Mathematica [A] time = 0.183404, size = 83, normalized size = 0.98

$$\frac{\sqrt{-\tan^2(c+dx)}\csc(c+dx)\sec^{m-1}(c+dx)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}\left(m-\frac{4}{3}\right),\frac{1}{2}\left(m+\frac{2}{3}\right),\sec^2(c+dx)\right)}{d\left(m-\frac{4}{3}\right)(b\sec(c+dx))^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^m/(b*Sec[c + d*x])^(4/3),x]
```

```
[Out] (Csc[c + d*x]*Hypergeometric2F1[1/2, (-4/3 + m)/2, (2/3 + m)/2, Sec[c + d*x
]^2]*Sec[c + d*x]^(-1 + m)*Sqrt[-Tan[c + d*x]^2])/(d*(-4/3 + m)*(b*Sec[c +
d*x])^(4/3))
```

Maple [F] time = 0.082, size = 0, normalized size = 0.

$$\int (\sec(dx+c))^m (b\sec(dx+c))^{-\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^m/(b*sec(d*x+c))^(4/3),x)
```

```
[Out] int(sec(d*x+c)^m/(b*sec(d*x+c))^(4/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)^m}{(b\sec(dx+c))^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^m/(b*sec(d*x+c))^(4/3),x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)^m/(b*sec(d*x + c))^(4/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(b\sec(dx+c))^{\frac{2}{3}}\sec(dx+c)^m}{b^2\sec(dx+c)^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^m/(b*sec(d*x+c))^(4/3),x, algorithm="fricas")
```

```
[Out] integral((b*sec(d*x + c))^(2/3)*sec(d*x + c)^m/(b^2*sec(d*x + c)^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec^m(c+dx)}{(b\sec(c+dx))^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**m/(b*sec(d*x+c))**(4/3),x)
```

```
[Out] Integral(sec(c + d*x)**m/(b*sec(c + d*x))**(4/3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)^m}{(b\sec(dx+c))^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^m/(b*sec(d*x+c))^(4/3),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^m/(b*sec(d*x + c))^(4/3), x)
```

3.210 $\int \sec^m(c+dx)(b\sec(c+dx))^n\,dx$

Optimal. Leaf size=89

$$-\frac{\sin(c+dx)\sec^{m-1}(c+dx)(b\sec(c+dx))^n\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}(-m-n+1),\frac{1}{2}(-m-n+3),\cos^2(c+dx)\right)}{d(-m-n+1)\sqrt{\sin^2(c+dx)}}$$

```
[Out] -((Hypergeometric2F1[1/2, (1 - m - n)/2, (3 - m - n)/2, Cos[c + d*x]^2]*Sec
[c + d*x]^(-1 + m)*(b*Sec[c + d*x])^n*Sin[c + d*x])/(d*(1 - m - n)*Sqrt[Sin
[c + d*x]^2]))
```

Rubi [A] time = 0.0385914, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {20, 3772, 2643}

$$-\frac{\sin(c+dx)\sec^{m-1}(c+dx)(b\sec(c+dx))^n\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(-m-n+1);\frac{1}{2}(-m-n+3);\cos^2(c+dx)\right)}{d(-m-n+1)\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^m*(b*Sec[c + d*x])^n,x]
```

```
[Out] -((Hypergeometric2F1[1/2, (1 - m - n)/2, (3 - m - n)/2, Cos[c + d*x]^2]*Sec
[c + d*x]^(-1 + m)*(b*Sec[c + d*x])^n*Sin[c + d*x])/(d*(1 - m - n)*Sqrt[Sin
[c + d*x]^2]))
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(b^IntPart
[n]*(b*v)^FracPart[n])/(a^IntPart[n]*(a*v)^FracPart[n]), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !
IntegerQ[m + n]
```

Rule 3772

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] && !IntegerQ[n]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rubi steps

$$\int \sec^m(c+dx)(b\sec(c+dx))^n\,dx = (\sec^{-n}(c+dx)(b\sec(c+dx))^n)\int \sec^{m+n}(c+dx)\,dx$$
$$= (\cos^{m+n}(c+dx)\sec^m(c+dx)(b\sec(c+dx))^n)\int \cos^{-m-n}(c+dx)\,dx$$
$$= -\frac{{}_2F_1\left(\frac{1}{2},\frac{1}{2}(1-m-n);\frac{1}{2}(3-m-n);\cos^2(c+dx)\right)\sec^{-1+m}(c+dx)(b\sec(c+dx))^n \cdots}{d(1-m-n)\sqrt{\sin^2(c+dx)}}$$

Mathematica [A] time = 0.0650672, size = 76, normalized size = 0.85

$$\frac{\sqrt{-\tan^2(c+dx)}\csc(c+dx)\sec^{m-1}(c+dx)(b\sec(c+dx))^n\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+n}{2},\frac{1}{2}(m+n+2),\sec^2(c+dx)\right)}{d(m+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^m*(b*Sec[c + d*x])^n,x]
```

```
[Out] (Csc[c + d*x]*Hypergeometric2F1[1/2, (m + n)/2, (2 + m + n)/2, Sec[c + d*x]
^2]*Sec[c + d*x]^(-1 + m)*(b*Sec[c + d*x])^n*Sqrt[-Tan[c + d*x]^2])/(d*(m +
 n))
```

Maple [F] time = 0.594, size = 0, normalized size = 0.

$$\int (\sec(dx+c))^m (b\sec(dx+c))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^m*(b*sec(d*x+c))^n,x)
```

```
[Out] int(sec(d*x+c)^m*(b*sec(d*x+c))^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b\sec(dx+c))^n \sec(dx+c)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^m*(b*sec(d*x+c))^n,x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c))^n*sec(d*x + c)^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((b\sec(dx+c))^n \sec(dx+c)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^m*(b*sec(d*x+c))^n,x, algorithm="fricas")
```

```
[Out] integral((b*sec(d*x + c))^n*sec(d*x + c)^m, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (b\sec(c+dx))^n \sec^m(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**m*(b*sec(d*x+c))**n,x)
```

```
[Out] Integral((b*sec(c + d*x))**n*sec(c + d*x)**m, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b\sec(dx+c))^n \sec(dx+c)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^m*(b*sec(d*x+c))^n,x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c))^n*sec(d*x + c)^m, x)
```

3.211 $\int \sec^2(c+dx)(b\sec(c+dx))^n\,dx$

Optimal. Leaf size=72

$$\frac{\sin(c+dx)(b\sec(c+dx))^{n+1}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}(-n-1),\frac{1-n}{2},\cos^2(c+dx)\right)}{bd(n+1)\sqrt{\sin^2(c+dx)}}$$

```
[Out] (Hypergeometric2F1[1/2, (-1 - n)/2, (1 - n)/2, Cos[c + d*x]^2]*(b*Sec[c + d
*x])^(1 + n)*Sin[c + d*x])/(b*d*(1 + n)*Sqrt[Sin[c + d*x]^2])
```

Rubi [A] time = 0.0442525, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {16, 3772, 2643}

$$\frac{\sin(c+dx)(b\sec(c+dx))^{n+1}\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(-n-1);\frac{1-n}{2};\cos^2(c+dx)\right)}{bd(n+1)\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2*(b*Sec[c + d*x])^n,x]
```

```
[Out] (Hypergeometric2F1[1/2, (-1 - n)/2, (1 - n)/2, Cos[c + d*x]^2]*(b*Sec[c + d
*x])^(1 + n)*Sin[c + d*x])/(b*d*(1 + n)*Sqrt[Sin[c + d*x]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 3772

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] && !IntegerQ[n]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}\int \sec^2(c+dx)(b\sec(c+dx))^n\,dx &= \frac{\int (b\sec(c+dx))^{2+n}\,dx}{b^2}\\ &= \frac{\left(\left(\frac{\cos(c+dx)}{b}\right)^n(b\sec(c+dx))^n\right)\int \left(\frac{\cos(c+dx)}{b}\right)^{-2-n}dx}{b^2}\\ &= \frac{{}_2F_1\left(\frac{1}{2},\frac{1}{2}(-1-n);\frac{1-n}{2};\cos^2(c+dx)\right)(b\sec(c+dx))^{1+n}\sin(c+dx)}{bd(1+n)\sqrt{\sin^2(c+dx)}}\end{aligned}$$

Mathematica [A] time = 0.047058, size = 71, normalized size = 0.99

$$\frac{\sqrt{-\tan^2(c+dx)}\csc(c+dx)\sec(c+dx)(b\sec(c+dx))^n\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{n+2}{2},\frac{n+4}{2},\sec^2(c+dx)\right)}{d(n+2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2*(b*Sec[c + d*x])^n,x]
```

```
[Out] (Csc[c + d*x]*Hypergeometric2F1[1/2, (2 + n)/2, (4 + n)/2, Sec[c + d*x]^2]*
Sec[c + d*x]*(b*Sec[c + d*x])^n*Sqrt[-Tan[c + d*x]^2])/(d*(2 + n))
```

Maple [F] time = 0.328, size = 0, normalized size = 0.

$$\int (\sec(dx+c))^2 (b\sec(dx+c))^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(b*sec(d*x+c))^n,x)
```

```
[Out] int(sec(d*x+c)^2*(b*sec(d*x+c))^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b\sec(dx+c))^n \sec(dx+c)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(b*sec(d*x+c))^n,x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c))^n*sec(d*x + c)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((b\sec(dx+c))^n \sec(dx+c)^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(b*sec(d*x+c))^n,x, algorithm="fricas")
```

```
[Out] integral((b*sec(d*x + c))^n*sec(d*x + c)^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (b\sec(c+dx))^n \sec^2(c+dx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(b*sec(d*x+c))**n,x)
```

```
[Out] Integral((b*sec(c + d*x))**n*sec(c + d*x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b\sec(dx+c))^n \sec(dx+c)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(b*sec(d*x+c))^n,x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c))^n*sec(d*x + c)^2, x)
```

3.212 $\int \sec(c+dx)(b\sec(c+dx))^n\,dx$

Optimal. Leaf size=61

$$\frac{\sin(c+dx)(b\sec(c+dx))^n \text{Hypergeometric2F1}\left(\frac{1}{2},-\frac{n}{2},\frac{2-n}{2},\cos^2(c+dx)\right)}{dn\sqrt{\sin^2(c+dx)}}$$

```
[Out] (Hypergeometric2F1[1/2, -n/2, (2 - n)/2, Cos[c + d*x]^2]*(b*Sec[c + d*x])^n
*Sin[c + d*x])/(d*n*Sqrt[Sin[c + d*x]^2])
```

Rubi [A] time = 0.0366607, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {16, 3772, 2643}

$$\frac{\sin(c+dx)(b\sec(c+dx))^n\,{}_2F_1\left(\frac{1}{2},-\frac{n}{2};\frac{2-n}{2};\cos^2(c+dx)\right)}{dn\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*(b*Sec[c + d*x])^n,x]
```

```
[Out] (Hypergeometric2F1[1/2, -n/2, (2 - n)/2, Cos[c + d*x]^2]*(b*Sec[c + d*x])^n
*Sin[c + d*x])/(d*n*Sqrt[Sin[c + d*x]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 3772

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}
\int \sec(c+dx)(b\sec(c+dx))^n\,dx &= \frac{\int (b\sec(c+dx))^{1+n}\,dx}{b}\\
&= \frac{\left(\left(\frac{\cos(c+dx)}{b}\right)^n (b\sec(c+dx))^n\right)\int \left(\frac{\cos(c+dx)}{b}\right)^{-1-n}\,dx}{b}\\
&= \frac{{}_2F_1\left(\frac{1}{2},-\frac{n}{2};\frac{2-n}{2};\cos^2(c+dx)\right)(b\sec(c+dx))^n\sin(c+dx)}{dn\sqrt{\sin^2(c+dx)}}
\end{aligned}$$

Mathematica [A] time = 0.0401088, size = 65, normalized size = 1.07

$$\frac{\sqrt{-\tan^2(c+dx)}\csc(c+dx)(b\sec(c+dx))^n\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{n+1}{2},\frac{n+3}{2},\sec^2(c+dx)\right)}{d(n+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]*(b*Sec[c + d*x])^n,x]
```

```
[Out] (Csc[c + d*x]*Hypergeometric2F1[1/2, (1 + n)/2, (3 + n)/2, Sec[c + d*x]^2]*
(b*Sec[c + d*x])^n*Sqrt[-Tan[c + d*x]^2])/(d*(1 + n))
```

Maple [F] time = 0.396, size = 0, normalized size = 0.

$$\int \sec(dx+c)\,(b\sec(dx+c))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(b*sec(d*x+c))^n,x)
```

```
[Out] int(sec(d*x+c)*(b*sec(d*x+c))^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b\sec(dx+c))^n \sec(dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(b*sec(d*x+c))^n,x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c))^n*sec(d*x + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((b\sec(dx+c))^n \sec(dx+c), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(b*sec(d*x+c))^n,x, algorithm="fricas")
```

```
[Out] integral((b*sec(d*x + c))^n*sec(d*x + c), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (b\sec(c+dx))^n \sec(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(b*sec(d*x+c))**n,x)
```

```
[Out] Integral((b*sec(c + d*x))**n*sec(c + d*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b\sec(dx+c))^n \sec(dx+c)\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(b*sec(d*x+c))^n,x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c))^n*sec(d*x + c), x)
```

3.213 $\int (b\sec(c+dx))^n\,dx$

Optimal. Leaf size=73

$$-\frac{b\sin(c+dx)(b\sec(c+dx))^{n-1}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1-n}{2},\frac{3-n}{2},\cos^2(c+dx)\right)}{d(1-n)\sqrt{\sin^2(c+dx)}}$$

```
[Out] -((b*Hypergeometric2F1[1/2, (1 - n)/2, (3 - n)/2, Cos[c + d*x]^2]*(b*Sec[c
+ d*x])^(-1 + n)*Sin[c + d*x])/(d*(1 - n)*Sqrt[Sin[c + d*x]^2]))
```

Rubi [A] time = 0.0336746, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {3772, 2643}

$$-\frac{b\sin(c+dx)(b\sec(c+dx))^{n-1}\,{}_2F_1\left(\frac{1}{2},\frac{1-n}{2};\frac{3-n}{2};\cos^2(c+dx)\right)}{d(1-n)\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Sec[c + d*x])^n,x]
```

```
[Out] -((b*Hypergeometric2F1[1/2, (1 - n)/2, (3 - n)/2, Cos[c + d*x]^2]*(b*Sec[c
+ d*x])^(-1 + n)*Sin[c + d*x])/(d*(1 - n)*Sqrt[Sin[c + d*x]^2]))
```

Rule 3772

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] && !IntegerQ[n]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}\int (b\sec(c+dx))^n\,dx &= \left(\frac{\cos(c+dx)}{b}\right)^n (b\sec(c+dx))^n \int \left(\frac{\cos(c+dx)}{b}\right)^{-n} dx\\ &= -\frac{\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1-n}{2};\frac{3-n}{2};\cos^2(c+dx)\right)(b\sec(c+dx))^n\sin(c+dx)}{d(1-n)\sqrt{\sin^2(c+dx)}}\end{aligned}$$

Mathematica [A] time = 0.0421139, size = 61, normalized size = 0.84

$$\frac{\sqrt{-\tan^2(c+dx)}\cot(c+dx)(b\sec(c+dx))^n\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{n}{2},\frac{n+2}{2},\sec^2(c+dx)\right)}{dn}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(b*Sec[c + d*x])^n,x]
```

```
[Out] (Cot[c + d*x]*Hypergeometric2F1[1/2, n/2, (2 + n)/2, Sec[c + d*x]^2]*(b*Sec
[c + d*x])^n*Sqrt[-Tan[c + d*x]^2])/(d*n)
```

Maple [F] time = 0.243, size = 0, normalized size = 0.

$$\int (b\sec(dx+c))^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sec(d*x+c))^n,x)
```

```
[Out] int((b*sec(d*x+c))^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b\sec(dx+c))^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))^n,x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c))^n, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left((b\sec(dx+c))^n, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))^n,x, algorithm="fricas")
```

```
[Out] integral((b*sec(d*x + c))^n, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (b\sec(c+dx))^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))**n,x)
```

```
[Out] Integral((b*sec(c + d*x))**n, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b\sec(dx+c))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))^n,x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c))^n, x)
```

3.214 $\int \cos(c+dx)(b\sec(c+dx))^n\,dx$

Optimal. Leaf size=75

$$-\frac{b^2\sin(c+dx)(b\sec(c+dx))^{n-2}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2-n}{2},\frac{4-n}{2},\cos^2(c+dx)\right)}{d(2-n)\sqrt{\sin^2(c+dx)}}$$

```
[Out] -((b^2*Hypergeometric2F1[1/2, (2 - n)/2, (4 - n)/2, Cos[c + d*x]^2]*(b*Sec[
c + d*x])^(-2 + n)*Sin[c + d*x])/(d*(2 - n)*Sqrt[Sin[c + d*x]^2]))
```

Rubi [A] time = 0.0492554, antiderivative size = 75, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {16, 3772, 2643}

$$-\frac{b^2\sin(c+dx)(b\sec(c+dx))^{n-2}\,{}_2F_1\left(\frac{1}{2},\frac{2-n}{2};\frac{4-n}{2};\cos^2(c+dx)\right)}{d(2-n)\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(b*Sec[c + d*x])^n,x]
```

```
[Out] -((b^2*Hypergeometric2F1[1/2, (2 - n)/2, (4 - n)/2, Cos[c + d*x]^2]*(b*Sec[
c + d*x])^(-2 + n)*Sin[c + d*x])/(d*(2 - n)*Sqrt[Sin[c + d*x]^2]))
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 3772

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}\int \cos(c+dx)(b\sec(c+dx))^n\,dx &= b\int (b\sec(c+dx))^{-1+n}\,dx\\ &= \left(b\left(\frac{\cos(c+dx)}{b}\right)^n(b\sec(c+dx))^n\right)\int\left(\frac{\cos(c+dx)}{b}\right)^{1-n}dx\\ &= -\frac{\cos^2(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{2-n}{2};\frac{4-n}{2};\cos^2(c+dx)\right)(b\sec(c+dx))^n\sin(c+dx)}{d(2-n)\sqrt{\sin^2(c+dx)}}\end{aligned}$$

Mathematica [A] time = 0.0938096, size = 68, normalized size = 0.91

$$\frac{b\sqrt{-\tan^2(c+dx)}\cot(c+dx)(b\sec(c+dx))^{n-1}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{n-1}{2},\frac{n+1}{2},\sec^2(c+dx)\right)}{d(n-1)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]*(b*Sec[c + d*x])^n,x]
```

```
[Out] (b*Cot[c + d*x]*Hypergeometric2F1[1/2, (-1 + n)/2, (1 + n)/2, Sec[c + d*x]^
2]*(b*Sec[c + d*x])^(-1 + n)*Sqrt[-Tan[c + d*x]^2])/(d*(-1 + n))
```

Maple [F] time = 0.859, size = 0, normalized size = 0.

$$\int \cos(dx+c)\,(b\sec(dx+c))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(b*sec(d*x+c))^n,x)
```

```
[Out] int(cos(d*x+c)*(b*sec(d*x+c))^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b\sec(dx+c))^n\cos(dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(b*sec(d*x+c))^n,x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c))^n*cos(d*x + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((b\sec(dx+c))^n\cos(dx+c),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(b*sec(d*x+c))^n,x, algorithm="fricas")
```

```
[Out] integral((b*sec(d*x + c))^n*cos(d*x + c), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (b\sec(c+dx))^n\cos(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(b*sec(d*x+c))**n,x)
```

```
[Out] Integral((b*sec(c + d*x))**n*cos(c + d*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b\sec(dx+c))^n \cos(dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(b*sec(d*x+c))^n,x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c))^n*cos(d*x + c), x)
```

3.215 $\int \cos^2(c+dx)(b\sec(c+dx))^n\,dx$

Optimal. Leaf size=75

$$-\frac{b^3\sin(c+dx)(b\sec(c+dx))^{n-3}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3-n}{2},\frac{5-n}{2},\cos^2(c+dx)\right)}{d(3-n)\sqrt{\sin^2(c+dx)}}$$

```
[Out] -((b^3*Hypergeometric2F1[1/2, (3 - n)/2, (5 - n)/2, Cos[c + d*x]^2]*(b*Sec[
c + d*x])^(-3 + n)*Sin[c + d*x])/(d*(3 - n)*Sqrt[Sin[c + d*x]^2]))
```

Rubi [A] time = 0.0553529, antiderivative size = 75, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {16, 3772, 2643}

$$-\frac{b^3\sin(c+dx)(b\sec(c+dx))^{n-3}\,{}_2F_1\left(\frac{1}{2},\frac{3-n}{2};\frac{5-n}{2};\cos^2(c+dx)\right)}{d(3-n)\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(b*Sec[c + d*x])^n,x]
```

```
[Out] -((b^3*Hypergeometric2F1[1/2, (3 - n)/2, (5 - n)/2, Cos[c + d*x]^2]*(b*Sec[
c + d*x])^(-3 + n)*Sin[c + d*x])/(d*(3 - n)*Sqrt[Sin[c + d*x]^2]))
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 3772

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}
\int \cos^2(c+dx)(b\sec(c+dx))^n\,dx &= b^2\int (b\sec(c+dx))^{-2+n}\,dx\\
&= \left(b^2\left(\frac{\cos(c+dx)}{b}\right)^n(b\sec(c+dx))^n\right)\int \left(\frac{\cos(c+dx)}{b}\right)^{2-n}\,dx\\
&= -\frac{\cos^3(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{3-n}{2};\frac{5-n}{2};\cos^2(c+dx)\right)(b\sec(c+dx))^n\sin(c+dx)}{d(3-n)\sqrt{\sin^2(c+dx)}}
\end{aligned}$$

Mathematica [A] time = 0.0951624, size = 71, normalized size = 0.95

$$\frac{\cos^2(c+dx)\sqrt{-\tan^2(c+dx)}\cot(c+dx)(b\sec(c+dx))^n\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{n-2}{2},\frac{n}{2},\sec^2(c+dx)\right)}{d(n-2)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^2*(b*Sec[c + d*x])^n,x]
```

```
[Out] (Cos[c + d*x]^2*Cot[c + d*x]*Hypergeometric2F1[1/2, (-2 + n)/2, n/2, Sec[c
+ d*x]^2]*(b*Sec[c + d*x])^n*Sqrt[-Tan[c + d*x]^2])/(d*(-2 + n))
```

Maple [F] time = 0.968, size = 0, normalized size = 0.

$$\int (\cos(dx+c))^2 (b\sec(dx+c))^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(b*sec(d*x+c))^n,x)
```

```
[Out] int(cos(d*x+c)^2*(b*sec(d*x+c))^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b\sec(dx+c))^n \cos(dx+c)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(b*sec(d*x+c))^n,x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c))^n*cos(d*x + c)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((b\sec(dx+c))^n \cos(dx+c)^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(b*sec(d*x+c))^n,x, algorithm="fricas")
```

```
[Out] integral((b*sec(d*x + c))^n*cos(d*x + c)^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (b\sec(c+dx))^n \cos^2(c+dx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(b*sec(d*x+c))**n,x)
```

```
[Out] Integral((b*sec(c + d*x))**n*cos(c + d*x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b\sec(dx+c))^n \cos(dx+c)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(b*sec(d*x+c))^n,x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c))^n*cos(d*x + c)^2, x)
```

3.216 $\int \cos^3(c+dx)(b\sec(c+dx))^n\,dx$

Optimal. Leaf size=75

$$-\frac{b^4\sin(c+dx)(b\sec(c+dx))^{n-4}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{4-n}{2},\frac{6-n}{2},\cos^2(c+dx)\right)}{d(4-n)\sqrt{\sin^2(c+dx)}}$$

```
[Out] -((b^4*Hypergeometric2F1[1/2, (4 - n)/2, (6 - n)/2, Cos[c + d*x]^2]*(b*Sec[
c + d*x])^(-4 + n)*Sin[c + d*x])/(d*(4 - n)*Sqrt[Sin[c + d*x]^2]))
```

Rubi [A] time = 0.0558594, antiderivative size = 75, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {16, 3772, 2643}

$$-\frac{b^4\sin(c+dx)(b\sec(c+dx))^{n-4}\,{}_2F_1\left(\frac{1}{2},\frac{4-n}{2};\frac{6-n}{2};\cos^2(c+dx)\right)}{d(4-n)\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*(b*Sec[c + d*x])^n,x]
```

```
[Out] -((b^4*Hypergeometric2F1[1/2, (4 - n)/2, (6 - n)/2, Cos[c + d*x]^2]*(b*Sec[
c + d*x])^(-4 + n)*Sin[c + d*x])/(d*(4 - n)*Sqrt[Sin[c + d*x]^2]))
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 3772

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] && !IntegerQ[n]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}
\int \cos^3(c+dx)(b\sec(c+dx))^n\,dx &= b^3\int (b\sec(c+dx))^{-3+n}\,dx\\
&= \left(b^3\left(\frac{\cos(c+dx)}{b}\right)^n(b\sec(c+dx))^n\right)\int \left(\frac{\cos(c+dx)}{b}\right)^{3-n}\,dx\\
&= -\frac{\cos^4(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{4-n}{2};\frac{6-n}{2};\cos^2(c+dx)\right)(b\sec(c+dx))^n\sin(c+dx)}{d(4-n)\sqrt{\sin^2(c+dx)}}
\end{aligned}$$

Mathematica [A] time = 0.107992, size = 73, normalized size = 0.97

$$\frac{\cos^3(c+dx)\sqrt{-\tan^2(c+dx)}\cot(c+dx)(b\sec(c+dx))^n\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{n-3}{2},\frac{n-1}{2},\sec^2(c+dx)\right)}{d(n-3)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^3*(b*Sec[c + d*x])^n,x]
```

```
[Out] (Cos[c + d*x]^3*Cot[c + d*x]*Hypergeometric2F1[1/2, (-3 + n)/2, (-1 + n)/2,
 Sec[c + d*x]^2]*(b*Sec[c + d*x])^n*Sqrt[-Tan[c + d*x]^2])/(d*(-3 + n))
```

Maple [F] time = 1.35, size = 0, normalized size = 0.

$$\int (\cos(dx+c))^3 (b\sec(dx+c))^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(b*sec(d*x+c))^n,x)
```

```
[Out] int(cos(d*x+c)^3*(b*sec(d*x+c))^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b\sec(dx+c))^n \cos(dx+c)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(b*sec(d*x+c))^n,x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c))^n*cos(d*x + c)^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((b\sec(dx+c))^n \cos(dx+c)^3, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(b*sec(d*x+c))^n,x, algorithm="fricas")
```

```
[Out] integral((b*sec(d*x + c))^n*cos(d*x + c)^3, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(b*sec(d*x+c))**n,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b\sec(dx+c))^n \cos(dx+c)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(b*sec(d*x+c))^n,x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c))^n*cos(d*x + c)^3, x)
```

3.217 $\int \sec^{\frac{5}{2}}(c+dx)(b\sec(c+dx))^n\,dx$

Optimal. Leaf size=80

$$\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)(b\sec(c+dx))^n\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(-2n-3),\frac{1}{4}(1-2n),\cos^2(c+dx)\right)}{d(2n+3)\sqrt{\sin^2(c+dx)}}$$

```
[Out] (2*Hypergeometric2F1[1/2, (-3 - 2*n)/4, (1 - 2*n)/4, Cos[c + d*x]^2]*Sec[c
+ d*x]^(3/2)*(b*Sec[c + d*x])^n*Sin[c + d*x])/(d*(3 + 2*n)*Sqrt[Sin[c + d*x
]^2])
```

Rubi [A] time = 0.0400818, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {20, 3772, 2643}

$$\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)(b\sec(c+dx))^n\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(-2n-3);\frac{1}{4}(1-2n);\cos^2(c+dx)\right)}{d(2n+3)\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(5/2)*(b*Sec[c + d*x])^n,x]
```

```
[Out] (2*Hypergeometric2F1[1/2, (-3 - 2*n)/4, (1 - 2*n)/4, Cos[c + d*x]^2]*Sec[c
+ d*x]^(3/2)*(b*Sec[c + d*x])^n*Sin[c + d*x])/(d*(3 + 2*n)*Sqrt[Sin[c + d*x
]^2])
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(b^IntPart
[n]*(b*v)^FracPart[n])/(a^IntPart[n]*(a*v)^FracPart[n]), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !
IntegerQ[m + n]
```

Rule 3772

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] && !IntegerQ[n]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rubi steps

$$\int \sec^{\frac{5}{2}}(c+dx)(b\sec(c+dx))^n\,dx = (\sec^{-n}(c+dx)(b\sec(c+dx))^n)\int \sec^{\frac{5}{2}+n}(c+dx)\,dx$$
$$= \left(\cos^{\frac{1}{2}+n}(c+dx)\sqrt{\sec(c+dx)}(b\sec(c+dx))^n\right)\int \cos^{-\frac{5}{2}-n}(c+dx)\,dx$$
$$= \frac{2\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(-3-2n);\frac{1}{4}(1-2n);\cos^2(c+dx)\right)\sec^{\frac{3}{2}}(c+dx)(b\sec(c+dx))^n\sin(c}{d(3+2n)\sqrt{\sin^2(c+dx)}}$$

Mathematica [A] time = 0.103719, size = 81, normalized size = 1.01

$$\frac{\sqrt{-\tan^2(c+dx)}\csc(c+dx)\sec^{\frac{3}{2}}(c+dx)(b\sec(c+dx))^n\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}\left(n+\frac{5}{2}\right),\frac{1}{2}\left(n+\frac{9}{2}\right),\sec^2(c+dx\right.}{d\left(n+\frac{5}{2}\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^(5/2)*(b*Sec[c + d*x])^n,x]
```

```
[Out] (Csc[c + d*x]*Hypergeometric2F1[1/2, (5/2 + n)/2, (9/2 + n)/2, Sec[c + d*x]
^2]*Sec[c + d*x]^(3/2)*(b*Sec[c + d*x])^n*Sqrt[-Tan[c + d*x]^2])/(d*(5/2 +
n))
```

Maple [F] time = 0.12, size = 0, normalized size = 0.

$$\int (\sec(dx+c))^{\frac{5}{2}}(b\sec(dx+c))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(5/2)*(b*sec(d*x+c))^n,x)
```

```
[Out] int(sec(d*x+c)^(5/2)*(b*sec(d*x+c))^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b\sec(dx+c))^n\sec(dx+c)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)*(b*sec(d*x+c))^n,x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c))^n*sec(d*x + c)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((b\sec(dx+c))^n\sec(dx+c)^{\frac{5}{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)*(b*sec(d*x+c))^n,x, algorithm="fricas")
```

```
[Out] integral((b*sec(d*x + c))^n*sec(d*x + c)^(5/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(5/2)*(b*sec(d*x+c))**n,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b \sec (dx + c))^n \sec (dx + c)^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)*(b*sec(d*x+c))^n,x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c))^n*sec(d*x + c)^(5/2), x)
```

3.218 $\int \sec^{\frac{3}{2}}(c+dx)(b\sec(c+dx))^n\,dx$

Optimal. Leaf size=80

$$\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}(b\sec(c+dx))^n\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(-2n-1),\frac{1}{4}(3-2n),\cos^2(c+dx)\right)}{d(2n+1)\sqrt{\sin^2(c+dx)}}$$

```
[Out] (2*Hypergeometric2F1[1/2, (-1 - 2*n)/4, (3 - 2*n)/4, Cos[c + d*x]^2]*Sqrt[S
ec[c + d*x]]*(b*Sec[c + d*x])^n*Sin[c + d*x])/(d*(1 + 2*n)*Sqrt[Sin[c + d*x
]^2])
```

Rubi [A] time = 0.0382578, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {20, 3772, 2643}

$$\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}(b\sec(c+dx))^n\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(-2n-1);\frac{1}{4}(3-2n);\cos^2(c+dx)\right)}{d(2n+1)\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(3/2)*(b*Sec[c + d*x])^n,x]
```

```
[Out] (2*Hypergeometric2F1[1/2, (-1 - 2*n)/4, (3 - 2*n)/4, Cos[c + d*x]^2]*Sqrt[S
ec[c + d*x]]*(b*Sec[c + d*x])^n*Sin[c + d*x])/(d*(1 + 2*n)*Sqrt[Sin[c + d*x
]^2])
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(b^IntPart
[n]*(b*v)^FracPart[n])/(a^IntPart[n]*(a*v)^FracPart[n]), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !
IntegerQ[m + n]
```

Rule 3772

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] && !IntegerQ[n]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rubi steps

$$\int \sec^{\frac{3}{2}}(c+dx)(b\sec(c+dx))^n\,dx = (\sec^{-n}(c+dx)(b\sec(c+dx))^n)\int \sec^{\frac{3}{2}+n}(c+dx)\,dx$$
$$= \left(\cos^{\frac{1}{2}+n}(c+dx)\sqrt{\sec(c+dx)}(b\sec(c+dx))^n\right)\int \cos^{-\frac{3}{2}-n}(c+dx)\,dx$$
$$= \frac{2\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(-1-2n);\frac{1}{4}(3-2n);\cos^2(c+dx)\right)\sqrt{\sec(c+dx)}(b\sec(c+dx))^n\sin(c+d}{d(1+2n)\sqrt{\sin^2(c+dx)}}$$

Mathematica [A] time = 0.0922359, size = 81, normalized size = 1.01

$$\frac{\sqrt{-\tan^2(c+dx)}\csc(c+dx)\sqrt{\sec(c+dx)}(b\sec(c+dx))^n\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}\left(n+\frac{3}{2}\right),\frac{1}{2}\left(n+\frac{7}{2}\right),\sec^2(c+dx)\right)}{d\left(n+\frac{3}{2}\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^(3/2)*(b*Sec[c + d*x])^n,x]
```

```
[Out] (Csc[c + d*x]*Hypergeometric2F1[1/2, (3/2 + n)/2, (7/2 + n)/2, Sec[c + d*x]
^2]*Sqrt[Sec[c + d*x]]*(b*Sec[c + d*x])^n*Sqrt[-Tan[c + d*x]^2])/(d*(3/2 +
n))
```

Maple [F] time = 0.117, size = 0, normalized size = 0.

$$\int (\sec(dx+c))^{\frac{3}{2}}(b\sec(dx+c))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(3/2)*(b*sec(d*x+c))^n,x)
```

```
[Out] int(sec(d*x+c)^(3/2)*(b*sec(d*x+c))^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b\sec(dx+c))^n\sec(dx+c)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(b*sec(d*x+c))^n,x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c))^n*sec(d*x + c)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((b\sec(dx+c))^n\sec(dx+c)^{\frac{3}{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(b*sec(d*x+c))^n,x, algorithm="fricas")
```

```
[Out] integral((b*sec(d*x + c))^n*sec(d*x + c)^(3/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(3/2)*(b*sec(d*x+c))**n,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b \sec (dx+c))^n \sec (dx+c)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(b*sec(d*x+c))^n,x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c))^n*sec(d*x + c)^(3/2), x)
```

3.219 $\int \sqrt{\sec(c+dx)}(b\sec(c+dx))^n\,dx$

Optimal. Leaf size=80

$$-\frac{2\sin(c+dx)(b\sec(c+dx))^n\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(1-2n),\frac{1}{4}(5-2n),\cos^2(c+dx)\right)}{d(1-2n)\sqrt{\sin^2(c+dx)}\sqrt{\sec(c+dx)}}$$

```
[Out] (-2*Hypergeometric2F1[1/2, (1 - 2*n)/4, (5 - 2*n)/4, Cos[c + d*x]^2]*(b*Sec
[c + d*x])^n*Sin[c + d*x])/(d*(1 - 2*n)*Sqrt[Sec[c + d*x]]*Sqrt[Sin[c + d*x
]^2])
```

Rubi [A] time = 0.0381516, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {20, 3772, 2643}

$$-\frac{2\sin(c+dx)(b\sec(c+dx))^n\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(1-2n);\frac{1}{4}(5-2n);\cos^2(c+dx)\right)}{d(1-2n)\sqrt{\sin^2(c+dx)}\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Sec[c + d*x]]*(b*Sec[c + d*x])^n,x]
```

```
[Out] (-2*Hypergeometric2F1[1/2, (1 - 2*n)/4, (5 - 2*n)/4, Cos[c + d*x]^2]*(b*Sec
[c + d*x])^n*Sin[c + d*x])/(d*(1 - 2*n)*Sqrt[Sec[c + d*x]]*Sqrt[Sin[c + d*x
]^2])
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(b^IntPart
[n]*(b*v)^FracPart[n])/(a^IntPart[n]*(a*v)^FracPart[n]), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !
IntegerQ[m + n]
```

Rule 3772

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] && !IntegerQ[n]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rubi steps

$$\int \sqrt{\sec(c+dx)}(b\sec(c+dx))^n\,dx = \left(\sec^{-n}(c+dx)(b\sec(c+dx))^n\right)\int \sec^{\frac{1}{2}+n}(c+dx)\,dx$$
$$= \left(\cos^{\frac{1}{2}+n}(c+dx)\sqrt{\sec(c+dx)}(b\sec(c+dx))^n\right)\int \cos^{-\frac{1}{2}-n}(c+dx)\,dx$$
$$= -\frac{2\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(1-2n);\frac{1}{4}(5-2n);\cos^2(c+dx)\right)(b\sec(c+dx))^n\sin(c+dx)}{d(1-2n)\sqrt{\sec(c+dx)}\sqrt{\sin^2(c+dx)}}$$

Mathematica [A] time = 0.101078, size = 81, normalized size = 1.01

$$\frac{\sqrt{-\tan^2(c+dx)}\csc(c+dx)(b\sec(c+dx))^n\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}\left(n+\frac{1}{2}\right),\frac{1}{2}\left(n+\frac{5}{2}\right),\sec^2(c+dx)\right)}{d\left(n+\frac{1}{2}\right)\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Sec[c + d*x]]*(b*Sec[c + d*x])^n,x]
```

```
[Out] (Csc[c + d*x]*Hypergeometric2F1[1/2, (1/2 + n)/2, (5/2 + n)/2, Sec[c + d*x]
^2]*(b*Sec[c + d*x])^n*Sqrt[-Tan[c + d*x]^2])/(d*(1/2 + n)*Sqrt[Sec[c + d*x
]])
```

Maple [F] time = 0.128, size = 0, normalized size = 0.

$$\int (b\sec(dx+c))^n\sqrt{\sec(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sec(d*x+c))^n*sec(d*x+c)^(1/2),x)
```

```
[Out] int((b*sec(d*x+c))^n*sec(d*x+c)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b\sec(dx+c))^n\sqrt{\sec(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))^n*sec(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c))^n*sqrt(sec(d*x + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((b\sec(dx+c))^n\sqrt{\sec(dx+c)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))^n*sec(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((b*sec(d*x + c))^n*sqrt(sec(d*x + c)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (b \sec (c + dx))^n \sqrt{\sec (c + dx)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))**n*sec(d*x+c)**(1/2),x)
```

```
[Out] Integral((b*sec(c + d*x))**n*sqrt(sec(c + d*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b \sec (dx + c))^n \sqrt{\sec (dx + c)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))^n*sec(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c))^n*sqrt(sec(d*x + c)), x)
```

3.220 $\int \frac{(b\sec(c+dx))^n}{\sqrt{\sec(c+dx)}}\,dx$

Optimal. Leaf size=80

$$-\frac{2\sin(c+dx)(b\sec(c+dx))^n\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(3-2n),\frac{1}{4}(7-2n),\cos^2(c+dx)\right)}{d(3-2n)\sqrt{\sin^2(c+dx)}\sec^{\frac{3}{2}}(c+dx)}$$

```
[Out] (-2*Hypergeometric2F1[1/2, (3 - 2*n)/4, (7 - 2*n)/4, Cos[c + d*x]^2]*(b*Sec
[c + d*x])^n*Sin[c + d*x])/(d*(3 - 2*n)*Sec[c + d*x]^(3/2)*Sqrt[Sin[c + d*x
]^2])
```

Rubi [A] time = 0.0389261, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {20, 3772, 2643}

$$-\frac{2\sin(c+dx)(b\sec(c+dx))^n\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(3-2n);\frac{1}{4}(7-2n);\cos^2(c+dx)\right)}{d(3-2n)\sqrt{\sin^2(c+dx)}\sec^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Sec[c + d*x])^n/Sqrt[Sec[c + d*x]],x]
```

```
[Out] (-2*Hypergeometric2F1[1/2, (3 - 2*n)/4, (7 - 2*n)/4, Cos[c + d*x]^2]*(b*Sec
[c + d*x])^n*Sin[c + d*x])/(d*(3 - 2*n)*Sec[c + d*x]^(3/2)*Sqrt[Sin[c + d*x
]^2])
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(b^IntPart
[n]*(b*v)^FracPart[n])/(a^IntPart[n]*(a*v)^FracPart[n]), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !
IntegerQ[m + n]
```

Rule 3772

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] && !IntegerQ[n]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rubi steps

$$\int \frac{(b\sec(c+dx))^n}{\sqrt{\sec(c+dx)}}\,dx = \left(\sec^{-n}(c+dx)(b\sec(c+dx))^n\right)\int \sec^{-\frac{1}{2}+n}(c+dx)\,dx$$
$$= \left(\cos^{\frac{1}{2}+n}(c+dx)\sqrt{\sec(c+dx)}(b\sec(c+dx))^n\right)\int \cos^{\frac{1}{2}-n}(c+dx)\,dx$$
$$= -\frac{2\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(3-2n);\frac{1}{4}(7-2n);\cos^2(c+dx)\right)(b\sec(c+dx))^n\sin(c+dx)}{d(3-2n)\sec^{\frac{3}{2}}(c+dx)\sqrt{\sin^2(c+dx)}}$$

Mathematica [A] time = 0.117239, size = 81, normalized size = 1.01

$$\frac{\sqrt{-\tan^2(c+dx)}\csc(c+dx)(b\sec(c+dx))^n\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}\left(n-\frac{1}{2}\right),\frac{1}{2}\left(n+\frac{3}{2}\right),\sec^2(c+dx)\right)}{d\left(n-\frac{1}{2}\right)\sec^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Sec[c + d*x])^n/Sqrt[Sec[c + d*x]],x]
```

```
[Out] (Csc[c + d*x]*Hypergeometric2F1[1/2, (-1/2 + n)/2, (3/2 + n)/2, Sec[c + d*x
]^2]*(b*Sec[c + d*x])^n*Sqrt[-Tan[c + d*x]^2])/(d*(-1/2 + n)*Sec[c + d*x]^(
3/2))
```

Maple [F] time = 0.128, size = 0, normalized size = 0.

$$\int (b\sec(dx+c))^n \frac{1}{\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sec(d*x+c))^n/sec(d*x+c)^(1/2),x)
```

```
[Out] int((b*sec(d*x+c))^n/sec(d*x+c)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\sec(dx+c))^n}{\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))^n/sec(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c))^n/sqrt(sec(d*x + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(b\sec(dx+c))^n}{\sqrt{\sec(dx+c)}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))^n/sec(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((b*sec(d*x + c))^n/sqrt(sec(d*x + c)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\sec(c+dx))^n}{\sqrt{\sec(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))**n/sec(d*x+c)**(1/2),x)
```

```
[Out] Integral((b*sec(c + d*x))**n/sqrt(sec(c + d*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\sec(dx+c))^n}{\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))^n/sec(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c))^n/sqrt(sec(d*x + c)), x)
```

3.221 $\int \frac{(b\sec(c+dx))^n}{\sec^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=80

$$-\frac{2\sin(c+dx)(b\sec(c+dx))^n\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(5-2n),\frac{1}{4}(9-2n),\cos^2(c+dx)\right)}{d(5-2n)\sqrt{\sin^2(c+dx)}\sec^{\frac{5}{2}}(c+dx)}$$

```
[Out] (-2*Hypergeometric2F1[1/2, (5 - 2*n)/4, (9 - 2*n)/4, Cos[c + d*x]^2]*(b*Sec
[c + d*x])^n*Sin[c + d*x])/(d*(5 - 2*n)*Sec[c + d*x]^(5/2)*Sqrt[Sin[c + d*x
]^2])
```

Rubi [A] time = 0.0398434, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {20, 3772, 2643}

$$-\frac{2\sin(c+dx)(b\sec(c+dx))^n\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(5-2n);\frac{1}{4}(9-2n);\cos^2(c+dx)\right)}{d(5-2n)\sqrt{\sin^2(c+dx)}\sec^{\frac{5}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Sec[c + d*x])^n/Sec[c + d*x]^(3/2),x]
```

```
[Out] (-2*Hypergeometric2F1[1/2, (5 - 2*n)/4, (9 - 2*n)/4, Cos[c + d*x]^2]*(b*Sec
[c + d*x])^n*Sin[c + d*x])/(d*(5 - 2*n)*Sec[c + d*x]^(5/2)*Sqrt[Sin[c + d*x
]^2])
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(b^IntPart
[n]*(b*v)^FracPart[n])/(a^IntPart[n]*(a*v)^FracPart[n]), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !
IntegerQ[m + n]
```

Rule 3772

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] && !IntegerQ[n]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rubi steps

$$\int \frac{(b\sec(c+dx))^n}{\sec^{\frac{3}{2}}(c+dx)}\,dx = (\sec^{-n}(c+dx)(b\sec(c+dx))^n)\int \sec^{-\frac{3}{2}+n}(c+dx)\,dx$$
$$= \left(\cos^{\frac{1}{2}+n}(c+dx)\sqrt{\sec(c+dx)}(b\sec(c+dx))^n\right)\int \cos^{\frac{3}{2}-n}(c+dx)\,dx$$
$$= -\frac{2\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(5-2n);\frac{1}{4}(9-2n);\cos^2(c+dx)\right)(b\sec(c+dx))^n\sin(c+dx)}{d(5-2n)\sec^{\frac{5}{2}}(c+dx)\sqrt{\sin^2(c+dx)}}$$

Mathematica [A] time = 0.139254, size = 81, normalized size = 1.01

$$\frac{\sqrt{-\tan^2(c+dx)}\csc(c+dx)(b\sec(c+dx))^n\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}\left(n-\frac{3}{2}\right),\frac{1}{2}\left(n+\frac{1}{2}\right),\sec^2(c+dx)\right)}{d\left(n-\frac{3}{2}\right)\sec^{\frac{5}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Sec[c + d*x])^n/Sec[c + d*x]^(3/2),x]
```

```
[Out] (Csc[c + d*x]*Hypergeometric2F1[1/2, (-3/2 + n)/2, (1/2 + n)/2, Sec[c + d*x
]^2]*(b*Sec[c + d*x])^n*Sqrt[-Tan[c + d*x]^2])/(d*(-3/2 + n)*Sec[c + d*x]^(
5/2))
```

Maple [F] time = 0.12, size = 0, normalized size = 0.

$$\int (b\sec(dx+c))^n (\sec(dx+c))^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sec(d*x+c))^n/sec(d*x+c)^(3/2),x)
```

```
[Out] int((b*sec(d*x+c))^n/sec(d*x+c)^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\sec(dx+c))^n}{\sec(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))^n/sec(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c))^n/sec(d*x + c)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(b\sec(dx+c))^n}{\sec(dx+c)^{\frac{3}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))^n/sec(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b*sec(d*x + c))^n/sec(d*x + c)^(3/2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\sec{(c+dx)})^n}{\sec^{\frac{3}{2}}{(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))**n/sec(d*x+c)**(3/2),x)
```

```
[Out] Integral((b*sec(c + d*x))**n/sec(c + d*x)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\sec{(dx+c)})^n}{\sec{(dx+c)}^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))^n/sec(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c))^n/sec(d*x + c)^(3/2), x)
```

3.222 $\int \frac{(b\sec(c+dx))^n}{\sec^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=80

$$-\frac{2\sin(c+dx)(b\sec(c+dx))^n\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(7-2n),\frac{1}{4}(11-2n),\cos^2(c+dx)\right)}{d(7-2n)\sqrt{\sin^2(c+dx)}\sec^{\frac{7}{2}}(c+dx)}$$

```
[Out] (-2*Hypergeometric2F1[1/2, (7 - 2*n)/4, (11 - 2*n)/4, Cos[c + d*x]^2]*(b*Se
c[c + d*x])^n*Sin[c + d*x])/(d*(7 - 2*n)*Sec[c + d*x]^(7/2)*Sqrt[Sin[c + d*
x]^2])
```

Rubi [A] time = 0.038628, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {20, 3772, 2643}

$$-\frac{2\sin(c+dx)(b\sec(c+dx))^n\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(7-2n);\frac{1}{4}(11-2n);\cos^2(c+dx)\right)}{d(7-2n)\sqrt{\sin^2(c+dx)}\sec^{\frac{7}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Sec[c + d*x])^n/Sec[c + d*x]^(5/2),x]
```

```
[Out] (-2*Hypergeometric2F1[1/2, (7 - 2*n)/4, (11 - 2*n)/4, Cos[c + d*x]^2]*(b*Se
c[c + d*x])^n*Sin[c + d*x])/(d*(7 - 2*n)*Sec[c + d*x]^(7/2)*Sqrt[Sin[c + d*
x]^2])
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(b^IntPart
[n]*(b*v)^FracPart[n])/(a^IntPart[n]*(a*v)^FracPart[n]), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !
IntegerQ[m + n]
```

Rule 3772

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] && !IntegerQ[n]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rubi steps

$$\int \frac{(b\sec(c+dx))^n}{\sec^{\frac{5}{2}}(c+dx)}\,dx = (\sec^{-n}(c+dx)(b\sec(c+dx))^n)\int \sec^{-\frac{5}{2}+n}(c+dx)\,dx$$
$$= \left(\cos^{\frac{1}{2}+n}(c+dx)\sqrt{\sec(c+dx)}(b\sec(c+dx))^n\right)\int \cos^{\frac{5}{2}-n}(c+dx)\,dx$$
$$= -\frac{2\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(7-2n);\frac{1}{4}(11-2n);\cos^2(c+dx)\right)(b\sec(c+dx))^n\sin(c+dx)}{d(7-2n)\sec^{\frac{7}{2}}(c+dx)\sqrt{\sin^2(c+dx)}}$$

Mathematica [A] time = 0.180298, size = 81, normalized size = 1.01

$$\frac{\sqrt{-\tan^2(c+dx)}\csc(c+dx)(b\sec(c+dx))^n\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}\left(n-\frac{5}{2}\right),\frac{1}{2}\left(n-\frac{1}{2}\right),\sec^2(c+dx)\right)}{d\left(n-\frac{5}{2}\right)\sec^{\frac{7}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Sec[c + d*x])^n/Sec[c + d*x]^(5/2),x]
```

```
[Out] (Csc[c + d*x]*Hypergeometric2F1[1/2, (-5/2 + n)/2, (-1/2 + n)/2, Sec[c + d*
x]^2]*(b*Sec[c + d*x])^n*Sqrt[-Tan[c + d*x]^2])/(d*(-5/2 + n)*Sec[c + d*x]^
(7/2))
```

Maple [F] time = 0.123, size = 0, normalized size = 0.

$$\int (b\sec(dx+c))^n(\sec(dx+c))^{-\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sec(d*x+c))^n/sec(d*x+c)^(5/2),x)
```

```
[Out] int((b*sec(d*x+c))^n/sec(d*x+c)^(5/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\sec(dx+c))^n}{\sec(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))^n/sec(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c))^n/sec(d*x + c)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(b\sec(dx+c))^n}{\sec(dx+c)^{\frac{5}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))^n/sec(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] integral((b*sec(d*x + c))^n/sec(d*x + c)^(5/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))**n/sec(d*x+c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \sec (dx + c))^n}{\sec (dx + c)^{\frac{5}{2}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))^n/sec(d*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c))^n/sec(d*x + c)^(5/2), x)
```

3.223 $\int (d\sec(a+bx))^{7/2}\sin(a+bx)\,dx$

Optimal. Leaf size=20

$$\frac{2d(d\sec(a+bx))^{5/2}}{5b}$$

[Out] (2*d*(d*Sec[a + b*x])^(5/2))/(5*b)

Rubi [A] time = 0.0351707, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2622, 30}

$$\frac{2d(d\sec(a+bx))^{5/2}}{5b}$$

Antiderivative was successfully verified.

[In] Int[(d*Sec[a + b*x])^(7/2)*Sin[a + b*x],x]

[Out] (2*d*(d*Sec[a + b*x])^(5/2))/(5*b)

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] && !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int (d\sec(a+bx))^{7/2}\sin(a+bx)\,dx &= \frac{d\,\text{Subst}\left(\int x^{3/2}\,dx, x, d\sec(a+bx)\right)}{b}\\ &= \frac{2d(d\sec(a+bx))^{5/2}}{5b}\end{aligned}$$

Mathematica [A] time = 0.0545522, size = 20, normalized size = 1.

$$\frac{2d(d\sec(a+bx))^{5/2}}{5b}$$

Antiderivative was successfully verified.

[In] Integrate[(d*Sec[a + b*x])^(7/2)*Sin[a + b*x],x]

[Out] (2*d*(d*Sec[a + b*x])^(5/2))/(5*b)

Maple [A] time = 0.021, size = 17, normalized size = 0.9

$$\frac{2d}{5b}(d\sec(bx+a))^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sec(b*x+a))^(7/2)*sin(b*x+a),x)
```

```
[Out] 2/5*d*(d*sec(b*x+a))^(5/2)/b
```

Maxima [A] time = 1.1273, size = 31, normalized size = 1.55

$$\frac{2\left(\frac{d}{\cos(bx+a)}\right)^{\frac{7}{2}}\cos(bx+a)}{5b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(b*x+a))^(7/2)*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] 2/5*(d/cos(b*x + a))^(7/2)*cos(b*x + a)/b
```

Fricas [A] time = 1.63252, size = 66, normalized size = 3.3

$$\frac{2d^3\sqrt{\frac{d}{\cos(bx+a)}}}{5b\cos(bx+a)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(b*x+a))^(7/2)*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] 2/5*d^3*sqrt(d/cos(b*x + a))/(b*cos(b*x + a)^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(b*x+a))**(7/2)*sin(b*x+a),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.27447, size = 45, normalized size = 2.25

$$\frac{2d^4\mathrm{sgn}(\cos(bx+a))}{5\sqrt{d\cos(bx+a)}b\cos(bx+a)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(b*x+a))^(7/2)*sin(b*x+a),x, algorithm="giac")
```

```
[Out] 2/5*d^4*sgn(cos(b*x + a))/(sqrt(d*cos(b*x + a))*b*cos(b*x + a)^2)
```

3.224 $\int (d\sec(a+bx))^{5/2}\sin(a+bx)\,dx$

Optimal. Leaf size=20

$$\frac{2d(d\sec(a+bx))^{3/2}}{3b}$$

```
[Out] (2*d*(d*Sec[a + b*x])^(3/2))/(3*b)
```

Rubi [A] time = 0.0359804, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2622, 30}

$$\frac{2d(d\sec(a+bx))^{3/2}}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sec[a + b*x])^(5/2)*Sin[a + b*x],x]
```

```
[Out] (2*d*(d*Sec[a + b*x])^(3/2))/(3*b)
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] && !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int (d\sec(a+bx))^{5/2}\sin(a+bx)\,dx &= \frac{d\,\text{Subst}\left(\int \sqrt{x}\,dx, x, d\sec(a+bx)\right)}{b}\\ &= \frac{2d(d\sec(a+bx))^{3/2}}{3b}\end{aligned}$$

Mathematica [A] time = 0.0412481, size = 20, normalized size = 1.

$$\frac{2d(d\sec(a+bx))^{3/2}}{3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Sec[a + b*x])^(5/2)*Sin[a + b*x],x]
```

```
[Out] (2*d*(d*Sec[a + b*x])^(3/2))/(3*b)
```

Maple [A] time = 0.017, size = 17, normalized size = 0.9

$$\frac{2d}{3b}(d\sec(bx+a))^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sec(b*x+a))^(5/2)*sin(b*x+a),x)
```

```
[Out] 2/3*d*(d*sec(b*x+a))^(3/2)/b
```

Maxima [A] time = 1.14477, size = 31, normalized size = 1.55

$$\frac{2\left(\frac{d}{\cos(bx+a)}\right)^{\frac{5}{2}}\cos(bx+a)}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(b*x+a))^(5/2)*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] 2/3*(d/cos(b*x + a))^(5/2)*cos(b*x + a)/b
```

Fricas [A] time = 1.62174, size = 63, normalized size = 3.15

$$\frac{2d^2\sqrt{\frac{d}{\cos(bx+a)}}}{3b\cos(bx+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(b*x+a))^(5/2)*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] 2/3*d^2*sqrt(d/cos(b*x + a))/(b*cos(b*x + a))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(b*x+a))**(5/2)*sin(b*x+a),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.2794, size = 45, normalized size = 2.25

$$\frac{2d^3\mathrm{sgn}(\cos(bx+a))}{3\sqrt{d\cos(bx+a)}b\cos(bx+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(b*x+a))^(5/2)*sin(b*x+a),x, algorithm="giac")
```

```
[Out] 2/3*d^3*sgn(cos(b*x + a))/(sqrt(d*cos(b*x + a))*b*cos(b*x + a))
```

3.225 $\int (d\sec(a+bx))^{3/2}\sin(a+bx)\,dx$

Optimal. Leaf size=18

$$\frac{2d\sqrt{d\sec(a+bx)}}{b}$$

```
[Out] (2*d*Sqrt[d*Sec[a + b*x]])/b
```

Rubi [A] time = 0.0362843, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2622, 30}

$$\frac{2d\sqrt{d\sec(a+bx)}}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sec[a + b*x])^(3/2)*Sin[a + b*x],x]

[Out] (2*d*Sqrt[d*Sec[a + b*x]])/b
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int (d\sec(a+bx))^{3/2}\sin(a+bx)\,dx &= \frac{d\,\text{Subst}\left(\int \frac{1}{\sqrt{x}}\,dx, x, d\sec(a+bx)\right)}{b}\\ &= \frac{2d\sqrt{d\sec(a+bx)}}{b}\end{aligned}$$

Mathematica [A] time = 0.032755, size = 18, normalized size = 1.

$$\frac{2d\sqrt{d\sec(a+bx)}}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Sec[a + b*x])^(3/2)*Sin[a + b*x],x]

[Out] (2*d*Sqrt[d*Sec[a + b*x]])/b
```

Maple [A] time = 0.018, size = 17, normalized size = 0.9

$$2\,\frac{d\sqrt{d\sec\left(bx+a\right)}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sec(b*x+a))^(3/2)*sin(b*x+a),x)
```

```
[Out] 2*d*(d*sec(b*x+a))^(1/2)/b
```

Maxima [A] time = 1.1578, size = 31, normalized size = 1.72

$$\frac{2\left(\frac{d}{\cos(bx+a)}\right)^{\frac{3}{2}}\cos\left(bx+a\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(b*x+a))^(3/2)*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] 2*(d/cos(b*x + a))^(3/2)*cos(b*x + a)/b
```

Fricas [A] time = 1.66254, size = 38, normalized size = 2.11

$$\frac{2\,d\sqrt{\frac{d}{\cos(bx+a)}}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(b*x+a))^(3/2)*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] 2*d*sqrt(d/cos(b*x + a))/b
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(b*x+a))**(3/2)*sin(b*x+a),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.33896, size = 34, normalized size = 1.89

$$\frac{2\,d^{2}\mathrm{sgn}\left(\cos\left(bx+a\right)\right)}{\sqrt{d\cos\left(bx+a\right)}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(b*x+a))^(3/2)*sin(b*x+a),x, algorithm="giac")
```

```
[Out] 2*d^2*sgn(cos(b*x + a))/(sqrt(d*cos(b*x + a))*b)
```

3.226 $\int \sqrt{d \sec(a+bx)} \sin(a+bx)\, dx$

Optimal. Leaf size=18

$$-\frac{2d}{b\sqrt{d\sec(a+bx)}}$$

```
[Out] (-2*d)/(b*Sqrt[d*Sec[a + b*x]])
```

Rubi [A] time = 0.032692, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2622, 30}

$$-\frac{2d}{b\sqrt{d\sec(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d*Sec[a + b*x]]*Sin[a + b*x],x]
```

```
[Out] (-2*d)/(b*Sqrt[d*Sec[a + b*x]])
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] && !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \sqrt{d\sec(a+bx)}\sin(a+bx)\,dx &= \frac{d\,\text{Subst}\left(\int \frac{1}{x^{3/2}}\,dx, x, d\sec(a+bx)\right)}{b}\\ &= -\frac{2d}{b\sqrt{d\sec(a+bx)}}\end{aligned}$$

Mathematica [A] time = 0.0376551, size = 18, normalized size = 1.

$$-\frac{2d}{b\sqrt{d\sec(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d*Sec[a + b*x]]*Sin[a + b*x],x]
```

```
[Out] (-2*d)/(b*Sqrt[d*Sec[a + b*x]])
```

Maple [A] time = 0.025, size = 17, normalized size = 0.9

$$-2\frac{d}{b\sqrt{d\sec(bx+a)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sec(b*x+a))^(1/2)*sin(b*x+a),x)
```

```
[Out] -2*d/b/(d*sec(b*x+a))^(1/2)
```

Maxima [A] time = 1.05814, size = 31, normalized size = 1.72

$$-\frac{2\sqrt{\frac{d}{\cos(bx+a)}}\cos(bx+a)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(b*x+a))^(1/2)*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] -2*sqrt(d/cos(b*x + a))*cos(b*x + a)/b
```

Fricas [A] time = 1.6582, size = 54, normalized size = 3.

$$-\frac{2\sqrt{\frac{d}{\cos(bx+a)}}\cos(bx+a)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(b*x+a))^(1/2)*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] -2*sqrt(d/cos(b*x + a))*cos(b*x + a)/b
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{d\sec(a+bx)}\sin(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(b*x+a))**(1/2)*sin(b*x+a),x)
```

```
[Out] Integral(sqrt(d*sec(a + b*x))*sin(a + b*x), x)
```

Giac [A] time = 1.32468, size = 30, normalized size = 1.67

$$-\frac{2\sqrt{d\cos(bx+a)}\mathrm{sgn}(\cos(bx+a))}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(b*x+a))^(1/2)*sin(b*x+a),x, algorithm="giac")
```

```
[Out] -2*sqrt(d*cos(b*x + a))*sgn(cos(b*x + a))/b
```

3.227 $\int \frac{\sin(a+bx)}{\sqrt{d\sec(a+bx)}}\,dx$

Optimal. Leaf size=20

$$-\frac{2d}{3b(d\sec(a+bx))^{3/2}}$$

```
[Out] (-2*d)/(3*b*(d*Sec[a + b*x])^(3/2))
```

Rubi [A] time = 0.0326505, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2622, 30}

$$-\frac{2d}{3b(d\sec(a+bx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]/Sqrt[d*Sec[a + b*x]],x]
```

```
[Out] (-2*d)/(3*b*(d*Sec[a + b*x])^(3/2))
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] && !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\sin(a+bx)}{\sqrt{d\sec(a+bx)}}\,dx &= \frac{d\,\text{Subst}\left(\int \frac{1}{x^{5/2}}\,dx, x, d\sec(a+bx)\right)}{b}\\ &= -\frac{2d}{3b(d\sec(a+bx))^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.0529927, size = 20, normalized size = 1.

$$-\frac{2d}{3b(d\sec(a+bx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]/Sqrt[d*Sec[a + b*x]],x]
```

```
[Out] (-2*d)/(3*b*(d*Sec[a + b*x])^(3/2))
```

Maple [A] time = 0.025, size = 17, normalized size = 0.9

$$-\frac{2d}{3b}(d\sec(bx+a))^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)/(d*sec(b*x+a))^(1/2),x)
```

```
[Out] -2/3*d/b/(d*sec(b*x+a))^(3/2)
```

Maxima [A] time = 1.18467, size = 31, normalized size = 1.55

$$-\frac{2\cos(bx+a)}{3b\sqrt{\frac{d}{\cos(bx+a)}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)/(d*sec(b*x+a))^(1/2),x, algorithm="maxima")
```

```
[Out] -2/3*cos(b*x + a)/(b*sqrt(d/cos(b*x + a)))
```

Fricas [A] time = 1.70856, size = 65, normalized size = 3.25

$$-\frac{2\sqrt{\frac{d}{\cos(bx+a)}}\cos(bx+a)^2}{3bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)/(d*sec(b*x+a))^(1/2),x, algorithm="fricas")
```

```
[Out] -2/3*sqrt(d/cos(b*x + a))*cos(b*x + a)^2/(b*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sin(a+bx)}{\sqrt{d\sec(a+bx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)/(d*sec(b*x+a))**(1/2),x)
```

```
[Out] Integral(sin(a + b*x)/sqrt(d*sec(a + b*x)), x)
```

Giac [B] time = 3.62161, size = 47, normalized size = 2.35

$$-\frac{2\sqrt{d\cos(bx+a)}|b|\cos(bx+a)\,\mathrm{sgn}(b)\,\mathrm{sgn}(\cos(bx+a))}{3b^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)/(d*sec(b*x+a))^(1/2),x, algorithm="giac")
```

```
[Out] -2/3*sqrt(d*cos(b*x + a))*abs(b)*cos(b*x + a)*sgn(b)*sgn(cos(b*x + a))/(b^2
*d)
```

3.228 $\int (d\sec(a+bx))^{5/2}\sin^3(a+bx)\,dx$

Optimal. Leaf size=41

$$\frac{2d^3}{b\sqrt{d\sec(a+bx)}}+\frac{2d(d\sec(a+bx))^{3/2}}{3b}$$

```
[Out] (2*d^3)/(b*Sqrt[d*Sec[a + b*x]]) + (2*d*(d*Sec[a + b*x])^(3/2))/(3*b)
```

Rubi [A] time = 0.048539, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2622, 14}

$$\frac{2d^3}{b\sqrt{d\sec(a+bx)}}+\frac{2d(d\sec(a+bx))^{3/2}}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sec[a + b*x])^(5/2)*Sin[a + b*x]^3,x]
```

```
[Out] (2*d^3)/(b*Sqrt[d*Sec[a + b*x]]) + (2*d*(d*Sec[a + b*x])^(3/2))/(3*b)
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x], x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_) + (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int (d\sec(a+bx))^{5/2}\sin^3(a+bx)\,dx &= \frac{d^3\,\text{Subst}\left(\int \frac{-1+\frac{x^2}{d^2}}{x^{3/2}}\,dx,x,d\sec(a+bx)\right)}{b}\\ &= \frac{d^3\,\text{Subst}\left(\int \left(-\frac{1}{x^{3/2}}+\frac{\sqrt{x}}{d^2}\right)dx,x,d\sec(a+bx)\right)}{b}\\ &= \frac{2d^3}{b\sqrt{d\sec(a+bx)}}+\frac{2d(d\sec(a+bx))^{3/2}}{3b}\end{aligned}$$

Mathematica [A] time = 0.219513, size = 32, normalized size = 0.78

$$\frac{d(3\cos(2(a+bx))+5)(d\sec(a+bx))^{3/2}}{3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Sec[a + b*x])^(5/2)*Sin[a + b*x]^3,x]
```

```
[Out] (d*(5 + 3*Cos[2*(a + b*x)])*(d*Sec[a + b*x])^(3/2))/(3*b)
```

Maple [B] time = 0.199, size = 357, normalized size = 8.7

$$-\frac{(-1+\cos(bx+a))\cos(bx+a)}{6b(\sin(bx+a))^2}\left(12\sqrt{-\frac{\cos(bx+a)}{(\cos(bx+a)+1)^2}}(\cos(bx+a))^3+12\sqrt{-\frac{\cos(bx+a)}{(\cos(bx+a)+1)^2}}(\cos(bx+a))^2-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sec(b*x+a))^(5/2)*sin(b*x+a)^3,x)
```

```
[Out] -1/6/b*(-1+cos(b*x+a))*(12*(-cos(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)*cos(b*x+a)^
3+12*(-cos(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)*cos(b*x+a)^2-3*cos(b*x+a)^2*ln(-(
2*(-cos(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)*cos(b*x+a)^2-cos(b*x+a)^2-2*(-cos(b*
x+a)/(cos(b*x+a)+1)^2)^(1/2)+2*cos(b*x+a)-1)/sin(b*x+a)^2)+3*cos(b*x+a)^2*l
n(-2*(2*(-cos(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)*cos(b*x+a)^2-cos(b*x+a)^2-2*(-
cos(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)+2*cos(b*x+a)-1)/sin(b*x+a)^2)+4*cos(b*x+
a)*(-cos(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)+4*(-cos(b*x+a)/(cos(b*x+a)+1)^2)^(1
/2))*cos(b*x+a)*(d/cos(b*x+a))^(5/2)/(-cos(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)/s
in(b*x+a)^2
```

Maxima [A] time = 1.09725, size = 49, normalized size = 1.2

$$\frac{2\left(\frac{3d^2}{\sqrt{\frac{d}{\cos(bx+a)}}}+\left(\frac{d}{\cos(bx+a)}\right)^{\frac{3}{2}}\right)d}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(b*x+a))^(5/2)*sin(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 2/3*(3*d^2/sqrt(d/cos(b*x + a)) + (d/cos(b*x + a))^(3/2))*d/b
```

Fricas [A] time = 1.65811, size = 97, normalized size = 2.37

$$\frac{2\left(3d^2\cos(bx+a)^2+d^2\right)\sqrt{\frac{d}{\cos(bx+a)}}}{3b\cos(bx+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(b*x+a))^(5/2)*sin(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 2/3*(3*d^2*cos(b*x + a)^2 + d^2)*sqrt(d/cos(b*x + a))/(b*cos(b*x + a))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(b*x+a))**(5/2)*sin(b*x+a)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.18904, size = 66, normalized size = 1.61

$$\frac{2\left(3\sqrt{d\cos\left(bx+a\right)}d+\frac{d^2}{\sqrt{d\cos(bx+a)}\cos(bx+a)}\right)d\mathrm{sgn}\left(\cos\left(bx+a\right)\right)}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(b*x+a))^(5/2)*sin(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 2/3*(3*sqrt(d*cos(b*x + a))*d + d^2/(sqrt(d*cos(b*x + a))*cos(b*x + a)))*d*
sgn(cos(b*x + a))/b
```

3.229 $\int (d\sec(a+bx))^{9/2}\sin^3(a+bx)\,dx$

Optimal. Leaf size=43

$$\frac{2d(d\sec(a+bx))^{7/2}}{7b}-\frac{2d^3(d\sec(a+bx))^{3/2}}{3b}$$

```
[Out] (-2*d^3*(d*Sec[a + b*x])^(3/2))/(3*b) + (2*d*(d*Sec[a + b*x])^(7/2))/(7*b)
```

Rubi [A] time = 0.0485292, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2622, 14}

$$\frac{2d(d\sec(a+bx))^{7/2}}{7b}-\frac{2d^3(d\sec(a+bx))^{3/2}}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sec[a + b*x])^(9/2)*Sin[a + b*x]^3,x]
```

```
[Out] (-2*d^3*(d*Sec[a + b*x])^(3/2))/(3*b) + (2*d*(d*Sec[a + b*x])^(7/2))/(7*b)
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] && !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int (d\sec(a+bx))^{9/2}\sin^3(a+bx)\,dx &= \frac{d^3\,\text{Subst}\left(\int \sqrt{x}\left(-1+\frac{x^2}{d^2}\right)\,dx,x,d\sec(a+bx)\right)}{b}\\ &= \frac{d^3\,\text{Subst}\left(\int \left(-\sqrt{x}+\frac{x^{5/2}}{d^2}\right)\,dx,x,d\sec(a+bx)\right)}{b}\\ &= -\frac{2d^3(d\sec(a+bx))^{3/2}}{3b}+\frac{2d(d\sec(a+bx))^{7/2}}{7b}\end{aligned}$$

Mathematica [A] time = 0.102751, size = 42, normalized size = 0.98

$$-\frac{d^4(7\cos(2(a+bx))+1)\sec^3(a+bx)\sqrt{d\sec(a+bx)}}{21b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Sec[a + b*x])^(9/2)*Sin[a + b*x]^3,x]
```

```
[Out] -(d^4*(1 + 7*Cos[2*(a + b*x)])*Sec[a + b*x]^3*Sqrt[d*Sec[a + b*x]])/(21*b)
```

Maple [A] time = 0.149, size = 36, normalized size = 0.8

$$-\frac{\left(14\left(\cos\left(bx+a\right)\right)^{2}-6\right)\cos\left(bx+a\right)}{21\,b}\left(\frac{d}{\cos\left(bx+a\right)}\right)^{\frac{9}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sec(b*x+a))^(9/2)*sin(b*x+a)^3,x)
```

```
[Out] -2/21/b*(7*cos(b*x+a)^2-3)*(d/cos(b*x+a))^(9/2)*cos(b*x+a)
```

Maxima [A] time = 1.2052, size = 51, normalized size = 1.19

$$-\frac{2\left(7\,d^{2}\left(\frac{d}{\cos(bx+a)}\right)^{\frac{3}{2}}-3\left(\frac{d}{\cos(bx+a)}\right)^{\frac{7}{2}}\right)d}{21\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(b*x+a))^(9/2)*sin(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -2/21*(7*d^2*(d/cos(b*x + a))^(3/2) - 3*(d/cos(b*x + a))^(7/2))*d/b
```

Fricas [A] time = 1.69515, size = 105, normalized size = 2.44

$$-\frac{2\left(7\,d^{4}\cos\left(bx+a\right)^{2}-3\,d^{4}\right)\sqrt{\frac{d}{\cos(bx+a)}}}{21\,b\cos\left(bx+a\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(b*x+a))^(9/2)*sin(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -2/21*(7*d^4*cos(b*x + a)^2 - 3*d^4)*sqrt(d/cos(b*x + a))/(b*cos(b*x + a)^3
)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(b*x+a))**(9/2)*sin(b*x+a)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.3429, size = 66, normalized size = 1.53

$$-\frac{2\left(7\,d^5\cos\left(bx+a\right)^2-3\,d^5\right)\mathrm{sgn}\left(\cos\left(bx+a\right)\right)}{21\,\sqrt{d\cos\left(bx+a\right)}b\cos\left(bx+a\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(b*x+a))^(9/2)*sin(b*x+a)^3,x, algorithm="giac")
```

```
[Out] -2/21*(7*d^5*cos(b*x + a)^2 - 3*d^5)*sgn(cos(b*x + a))/(sqrt(d*cos(b*x + a)
)*b*cos(b*x + a)^3)
```

3.230 $\int (d\csc(a+bx))^{9/2}\sqrt{c\sec(a+bx)}\,dx$

Optimal. Leaf size=128

$$\frac{4d^4\sqrt{\sin(2a+2bx)}\text{EllipticF}\left(a+bx-\frac{\pi}{4},2\right)\sqrt{c\sec(a+bx)}\sqrt{d\csc(a+bx)}}{7b}-\frac{4cd^3(d\csc(a+bx))^{3/2}}{7b\sqrt{c\sec(a+bx)}}-\frac{2cd(d\csc(a+b}{7b\sqrt{c\sec(a+}}$$

```
[Out] (-4*c*d^3*(d*Csc[a + b*x])^(3/2))/(7*b*Sqrt[c*Sec[a + b*x]]) - (2*c*d*(d*Cs
c[a + b*x])^(7/2))/(7*b*Sqrt[c*Sec[a + b*x]]) + (4*d^4*Sqrt[d*Csc[a + b*x]]
*EllipticF[a - Pi/4 + b*x, 2]*Sqrt[c*Sec[a + b*x]]*Sqrt[Sin[2*a + 2*b*x]])/
(7*b)
```

Rubi [A] time = 0.203931, antiderivative size = 128, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {2625, 2630, 2573, 2641}

$$-\frac{4cd^3(d\csc(a+bx))^{3/2}}{7b\sqrt{c\sec(a+bx)}}+\frac{4d^4\sqrt{\sin(2a+2bx)}F\left(a+bx-\frac{\pi}{4}\middle|2\right)\sqrt{c\sec(a+bx)}\sqrt{d\csc(a+bx)}}{7b}-\frac{2cd(d\csc(a+bx))^{7/2}}{7b\sqrt{c\sec(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Csc[a + b*x])^(9/2)*Sqrt[c*Sec[a + b*x]],x]
```


```
[Out] (-4*c*d^3*(d*Csc[a + b*x])^(3/2))/(7*b*Sqrt[c*Sec[a + b*x]]) - (2*c*d*(d*Cs
c[a + b*x])^(7/2))/(7*b*Sqrt[c*Sec[a + b*x]]) + (4*d^4*Sqrt[d*Csc[a + b*x]]
*EllipticF[a - Pi/4 + b*x, 2]*Sqrt[c*Sec[a + b*x]]*Sqrt[Sin[2*a + 2*b*x]])/
(7*b)
```


Rule 2625

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> -Simp[(a*b*(a*Csc[e + f*x])^(m - 1)*(b*Sec[e + f*x])^(n -
 1))/(f*(m - 1)), x] + Dist[(a^2*(m + n - 2))/(m - 1), Int[(a*Csc[e + f*x])
^(m - 2)*(b*Sec[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m,
1] && IntegersQ[2*m, 2*n] &&  !GtQ[n, m]
```

Rule 2630

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Dist[(a*Csc[e + f*x])^m*(b*Sec[e + f*x])^n*(a*Sin[e + f*x]
)^m*(b*Cos[e + f*x])^n, Int[1/((a*Sin[e + f*x])^m*(b*Cos[e + f*x])^n), x],
x] /; FreeQ[{a, b, e, f, m, n}, x] && IntegerQ[m - 1/2] && IntegerQ[n - 1/2
]
```

Rule 2573

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (d\csc(a+bx))^{9/2}\sqrt{c\sec(a+bx)}\,dx = -\frac{2cd(d\csc(a+bx))^{7/2}}{7b\sqrt{c\sec(a+bx)}}+\frac{1}{7}\left(6d^2\right)\int (d\csc(a+bx))^{5/2}\sqrt{c\sec(a+bx)}\,dx$$
$$= -\frac{4cd^3(d\csc(a+bx))^{3/2}}{7b\sqrt{c\sec(a+bx)}}-\frac{2cd(d\csc(a+bx))^{7/2}}{7b\sqrt{c\sec(a+bx)}}+\frac{1}{7}\left(4d^4\right)\int \sqrt{d\csc(a+bx)}\sqrt{c\,s}$$
$$= -\frac{4cd^3(d\csc(a+bx))^{3/2}}{7b\sqrt{c\sec(a+bx)}}-\frac{2cd(d\csc(a+bx))^{7/2}}{7b\sqrt{c\sec(a+bx)}}+\frac{1}{7}\left(4d^4\sqrt{c\cos(a+bx)}\sqrt{d\csc(a}\right.$$
$$= -\frac{4cd^3(d\csc(a+bx))^{3/2}}{7b\sqrt{c\sec(a+bx)}}-\frac{2cd(d\csc(a+bx))^{7/2}}{7b\sqrt{c\sec(a+bx)}}+\frac{1}{7}\left(4d^4\sqrt{d\csc(a+bx)}\sqrt{c\sec(a}\right.$$
$$= -\frac{4cd^3(d\csc(a+bx))^{3/2}}{7b\sqrt{c\sec(a+bx)}}-\frac{2cd(d\csc(a+bx))^{7/2}}{7b\sqrt{c\sec(a+bx)}}+\frac{4d^4\sqrt{d\csc(a+bx)}F\left(a-\frac{\pi}{4}+\right.}{}$$

Mathematica [C] time = 1.48662, size = 122, normalized size = 0.95

$$\frac{2d^4\cos(2(a+bx))\cot(a+bx)\sqrt{c\sec(a+bx)}\sqrt{d\csc(a+bx)}\left((\cos(2(a+bx))-2)\csc^4(a+bx)-2\left(-\cot^2(a+bx)\right)^{3/4}\sec\right.}{7b\left(\csc^2(a+bx)-2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Csc[a + b*x])^(9/2)*Sqrt[c*Sec[a + b*x]],x]
```

```
[Out] (2*d^4*Cos[2*(a + b*x)]*Cot[a + b*x]*Sqrt[d*Csc[a + b*x]]*Sqrt[c*Sec[a + b*
x]]*((-2 + Cos[2*(a + b*x)])*Csc[a + b*x]^4 - 2*(-Cot[a + b*x]^2)^(3/4)*Hyp
ergeometric2F1[1/2, 3/4, 3/2, Csc[a + b*x]^2]*Sec[a + b*x]^2))/(7*b*(-2 + C
sc[a + b*x]^2))
```

Maple [B] time = 0.253, size = 542, normalized size = 4.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*csc(b*x+a))^(9/2)*(c*sec(b*x+a))^(1/2),x)
```

```
[Out] -1/7/b*2^(1/2)*(4*cos(b*x+a)^3*sin(b*x+a)*(-(-1+cos(b*x+a)-sin(b*x+a))/sin(
b*x+a))^(1/2)*((-1+cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a)
)/sin(b*x+a))^(1/2)*EllipticF((-(-1+cos(b*x+a)-sin(b*x+a))/sin(b*x+a))^(1/2
),1/2*2^(1/2))+4*cos(b*x+a)^2*sin(b*x+a)*(-(-1+cos(b*x+a)-sin(b*x+a))/sin(b
*x+a))^(1/2)*((-1+cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))
/sin(b*x+a))^(1/2)*EllipticF((-(-1+cos(b*x+a)-sin(b*x+a))/sin(b*x+a))^(1/2)
,1/2*2^(1/2))-4*cos(b*x+a)*sin(b*x+a)*(-(-1+cos(b*x+a)-sin(b*x+a))/sin(b*x+
a))^(1/2)*((-1+cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/si
n(b*x+a))^(1/2)*EllipticF((-(-1+cos(b*x+a)-sin(b*x+a))/sin(b*x+a))^(1/2),1/
2*2^(1/2))-4*sin(b*x+a)*(-(-1+cos(b*x+a)-sin(b*x+a))/sin(b*x+a))^(1/2)*((-1
+cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2
)*EllipticF((-(-1+cos(b*x+a)-sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))-2*c
os(b*x+a)^3*2^(1/2)+3*cos(b*x+a)*2^(1/2))*(d/sin(b*x+a))^(9/2)*(c/cos(b*x+a
))^(1/2)*sin(b*x+a)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (d\csc(bx+a))^{\frac{9}{2}}\sqrt{c\sec(bx+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(b*x+a))^(9/2)*(c*sec(b*x+a))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((d*csc(b*x + a))^(9/2)*sqrt(c*sec(b*x + a)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{d\csc(bx+a)}\sqrt{c\sec(bx+a)}d^4\csc(bx+a)^4, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(b*x+a))^(9/2)*(c*sec(b*x+a))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*csc(b*x + a))*sqrt(c*sec(b*x + a))*d^4*csc(b*x + a)^4, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(b*x+a))**(9/2)*(c*sec(b*x+a))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (d\csc(bx+a))^{\frac{9}{2}}\sqrt{c\sec(bx+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(b*x+a))^(9/2)*(c*sec(b*x+a))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((d*csc(b*x + a))^(9/2)*sqrt(c*sec(b*x + a)), x)
```

3.231 $\int (d\csc(a+bx))^{7/2}\sqrt{c\sec(a+bx)}\,dx$

Optimal. Leaf size=69

$$-\frac{8cd^3\sqrt{d\csc(a+bx)}}{5b\sqrt{c\sec(a+bx)}}-\frac{2cd(d\csc(a+bx))^{5/2}}{5b\sqrt{c\sec(a+bx)}}$$

```
[Out] (-8*c*d^3*Sqrt[d*Csc[a + b*x]])/(5*b*Sqrt[c*Sec[a + b*x]]) - (2*c*d*(d*Csc[
a + b*x])^(5/2))/(5*b*Sqrt[c*Sec[a + b*x]])
```

Rubi [A] time = 0.0970203, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.08, Rules used = {2625, 2619}

$$-\frac{8cd^3\sqrt{d\csc(a+bx)}}{5b\sqrt{c\sec(a+bx)}}-\frac{2cd(d\csc(a+bx))^{5/2}}{5b\sqrt{c\sec(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Csc[a + b*x])^(7/2)*Sqrt[c*Sec[a + b*x]],x]
```


```
[Out] (-8*c*d^3*Sqrt[d*Csc[a + b*x]])/(5*b*Sqrt[c*Sec[a + b*x]]) - (2*c*d*(d*Csc[
a + b*x])^(5/2))/(5*b*Sqrt[c*Sec[a + b*x]])
```


Rule 2625

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> -Simp[(a*b*(a*Csc[e + f*x])^(m - 1)*(b*Sec[e + f*x])^(n -
 1))/(f*(m - 1)), x] + Dist[(a^2*(m + n - 2))/(m - 1), Int[(a*Csc[e + f*x])
^(m - 2)*(b*Sec[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m,
1] && IntegersQ[2*m, 2*n] &&  !GtQ[n, m]
```

Rule 2619

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(a*b*(a*Csc[e + f*x])^(m - 1)*(b*Sec[e + f*x])^(n - 1
))/(f*(n - 1)), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n - 2, 0] &&
NeQ[n, 1]
```

Rubi steps

$$\begin{aligned}\int (d\csc(a+bx))^{7/2}\sqrt{c\sec(a+bx)}\,dx &= -\frac{2cd(d\csc(a+bx))^{5/2}}{5b\sqrt{c\sec(a+bx)}}+\frac{1}{5}\left(4d^2\right)\int (d\csc(a+bx))^{3/2}\sqrt{c\sec(a+bx)}\,dx\\ &= -\frac{8cd^3\sqrt{d\csc(a+bx)}}{5b\sqrt{c\sec(a+bx)}}-\frac{2cd(d\csc(a+bx))^{5/2}}{5b\sqrt{c\sec(a+bx)}}\end{aligned}$$

Mathematica [A] time = 0.116126, size = 56, normalized size = 0.81

$$-\frac{2d^3\sqrt{c\sec(a+bx)}\sqrt{d\csc(a+bx)}(4\cos(a+bx)+\cot(a+bx)\csc(a+bx))}{5b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Csc[a + b*x])^(7/2)*Sqrt[c*Sec[a + b*x]],x]
```

```
[Out] (-2*d^3*Sqrt[d*Csc[a + b*x]]*(4*Cos[a + b*x] + Cot[a + b*x]*Csc[a + b*x])*S
qrt[c*Sec[a + b*x]])/(5*b)
```

Maple [A] time = 0.191, size = 54, normalized size = 0.8

$$\frac{\left(8\,(\cos(bx+a))^2-10\right)\cos(bx+a)\sin(bx+a)}{5b}\left(\frac{d}{\sin(bx+a)}\right)^{\frac{7}{2}}\sqrt{\frac{c}{\cos(bx+a)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*csc(b*x+a))^(7/2)*(c*sec(b*x+a))^(1/2),x)
```

```
[Out] 2/5/b*(4*cos(b*x+a)^2-5)*cos(b*x+a)*(d/sin(b*x+a))^(7/2)*(c/cos(b*x+a))^(1/
2)*sin(b*x+a)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (d\csc(bx+a))^{\frac{7}{2}}\sqrt{c\sec(bx+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(b*x+a))^(7/2)*(c*sec(b*x+a))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((d*csc(b*x + a))^(7/2)*sqrt(c*sec(b*x + a)), x)
```

Fricas [A] time = 1.82622, size = 155, normalized size = 2.25

$$-\frac{2\left(4\,d^3\cos(bx+a)^3-5\,d^3\cos(bx+a)\right)\sqrt{\frac{c}{\cos(bx+a)}}\sqrt{\frac{d}{\sin(bx+a)}}}{5\left(b\cos(bx+a)^2-b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(b*x+a))^(7/2)*(c*sec(b*x+a))^(1/2),x, algorithm="fricas")
```

```
[Out] -2/5*(4*d^3*cos(b*x + a)^3 - 5*d^3*cos(b*x + a))*sqrt(c/cos(b*x + a))*sqrt(
d/sin(b*x + a))/(b*cos(b*x + a)^2 - b)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(b*x+a))**(7/2)*(c*sec(b*x+a))**(1/2),x)
```

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (d\csc(bx+a))^{\frac{7}{2}}\sqrt{c\sec(bx+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(b*x+a))^(7/2)*(c*sec(b*x+a))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((d*csc(b*x + a))^(7/2)*sqrt(c*sec(b*x + a)), x)
```

3.232 $\int (d\csc(a+bx))^{5/2}\sqrt{c\sec(a+bx)}\,dx$

Optimal. Leaf size=93

$$\frac{2d^2\sqrt{\sin(2a+2bx)}\text{EllipticF}\left(a+bx-\frac{\pi}{4},2\right)\sqrt{c\sec(a+bx)}\sqrt{d\csc(a+bx)}}{3b}-\frac{2cd(d\csc(a+bx))^{3/2}}{3b\sqrt{c\sec(a+bx)}}$$

```
[Out] (-2*c*d*(d*Csc[a + b*x])^(3/2))/(3*b*Sqrt[c*Sec[a + b*x]]) + (2*d^2*Sqrt[d*
Csc[a + b*x]]*EllipticF[a - Pi/4 + b*x, 2]*Sqrt[c*Sec[a + b*x]]*Sqrt[Sin[2*
a + 2*b*x]])/(3*b)
```

Rubi [A] time = 0.143491, antiderivative size = 93, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {2625, 2630, 2573, 2641}

$$\frac{2d^2\sqrt{\sin(2a+2bx)}F\left(a+bx-\frac{\pi}{4}\middle|2\right)\sqrt{c\sec(a+bx)}\sqrt{d\csc(a+bx)}}{3b}-\frac{2cd(d\csc(a+bx))^{3/2}}{3b\sqrt{c\sec(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Csc[a + b*x])^(5/2)*Sqrt[c*Sec[a + b*x]],x]
```

```
[Out] (-2*c*d*(d*Csc[a + b*x])^(3/2))/(3*b*Sqrt[c*Sec[a + b*x]]) + (2*d^2*Sqrt[d*
Csc[a + b*x]]*EllipticF[a - Pi/4 + b*x, 2]*Sqrt[c*Sec[a + b*x]]*Sqrt[Sin[2*
a + 2*b*x]])/(3*b)
```

Rule 2625

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> -Simp[(a*b*(a*Csc[e + f*x])^(m - 1)*(b*Sec[e + f*x])^(n -
 1))/(f*(m - 1)), x] + Dist[(a^2*(m + n - 2))/(m - 1), Int[(a*Csc[e + f*x])
^(m - 2)*(b*Sec[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m,
1] && IntegersQ[2*m, 2*n] &&  !GtQ[n, m]
```

Rule 2630

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Dist[(a*Csc[e + f*x])^m*(b*Sec[e + f*x])^n*(a*Sin[e + f*x]
)^m*(b*Cos[e + f*x])^n, Int[1/((a*Sin[e + f*x])^m*(b*Cos[e + f*x])^n), x],
x] /; FreeQ[{a, b, e, f, m, n}, x] && IntegerQ[m - 1/2] && IntegerQ[n - 1/2
]
```

Rule 2573

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (d\csc(a+bx))^{5/2}\sqrt{c\sec(a+bx)}\,dx = -\frac{2cd(d\csc(a+bx))^{3/2}}{3b\sqrt{c\sec(a+bx)}} + \frac{1}{3}\left(2d^2\right)\int \sqrt{d\csc(a+bx)}\sqrt{c\sec(a+bx)}\,dx$$
$$= -\frac{2cd(d\csc(a+bx))^{3/2}}{3b\sqrt{c\sec(a+bx)}} + \frac{1}{3}\left(2d^2\sqrt{c\cos(a+bx)}\sqrt{d\csc(a+bx)}\sqrt{c\sec(a+bx)}\sqrt{d\ldots}\right.$$
$$= -\frac{2cd(d\csc(a+bx))^{3/2}}{3b\sqrt{c\sec(a+bx)}} + \frac{1}{3}\left(2d^2\sqrt{d\csc(a+bx)}\sqrt{c\sec(a+bx)}\sqrt{\sin(2a+2bx)}\right)\int \ldots$$
$$= -\frac{2cd(d\csc(a+bx))^{3/2}}{3b\sqrt{c\sec(a+bx)}} + \frac{2d^2\sqrt{d\csc(a+bx)}F\left(a-\frac{\pi}{4}+bx\middle|2\right)\sqrt{c\sec(a+bx)}\sqrt{\sin\ldots}}{3b}$$

Mathematica [C] time = 0.85784, size = 109, normalized size = 1.17

$$-\frac{d(\cos(a+bx)+\cos(3(a+bx)))\sec^2(a+bx)\sqrt{c\sec(a+bx)}(d\csc(a+bx))^{3/2}\left(\left(-\cot^2(a+bx)\right)^{3/4}\text{Hypergeometric2F1}\ldots\right.}{3b\left(\csc^2(a+bx)-2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Csc[a + b*x])^(5/2)*Sqrt[c*Sec[a + b*x]],x]
```

```
[Out] -(d*(Cos[a + b*x] + Cos[3*(a + b*x)])*(d*Csc[a + b*x])^(3/2)*(Cot[a + b*x]^
2 + (-Cot[a + b*x]^2)^(3/4)*Hypergeometric2F1[1/2, 3/4, 3/2, Csc[a + b*x]^2
])*Sec[a + b*x]^2*Sqrt[c*Sec[a + b*x]])/(3*b*(-2 + Csc[a + b*x]^2))
```

Maple [B] time = 0.2, size = 285, normalized size = 3.1

$$\frac{\sqrt{2}\sin(bx+a)}{3b}\left(2\cos(bx+a)\sin(bx+a)\sqrt{-\frac{-1+\cos(bx+a)-\sin(bx+a)}{\sin(bx+a)}}\sqrt{\frac{-1+\cos(bx+a)+\sin(bx+a)}{\sin(bx+a)}}\sqrt{\frac{-1\ldots}{\mathrm{s}\ldots}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*csc(b*x+a))^(5/2)*(c*sec(b*x+a))^(1/2),x)
```

```
[Out] 1/3/b*2^(1/2)*(2*cos(b*x+a)*sin(b*x+a)*(-(-1+cos(b*x+a)-sin(b*x+a))/sin(b*x
+a))^(1/2)*((-1+cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/s
in(b*x+a))^(1/2)*EllipticF((-(-1+cos(b*x+a)-sin(b*x+a))/sin(b*x+a))^(1/2),1
/2*2^(1/2))+2*sin(b*x+a)*(-(-1+cos(b*x+a)-sin(b*x+a))/sin(b*x+a))^(1/2)*((-
1+cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/
2)*EllipticF((-(-1+cos(b*x+a)-sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))-co
s(b*x+a)*2^(1/2))*(d/sin(b*x+a))^(5/2)*(c/cos(b*x+a))^(1/2)*sin(b*x+a)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (d\csc(bx+a))^{\frac{5}{2}}\sqrt{c\sec(bx+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(b*x+a))^(5/2)*(c*sec(b*x+a))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((d*csc(b*x + a))^(5/2)*sqrt(c*sec(b*x + a)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{d\csc\left(bx+a\right)}\sqrt{c\sec\left(bx+a\right)}d^{2}\csc\left(bx+a\right)^{2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(b*x+a))^(5/2)*(c*sec(b*x+a))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*csc(b*x + a))*sqrt(c*sec(b*x + a))*d^2*csc(b*x + a)^2, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(b*x+a))**(5/2)*(c*sec(b*x+a))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(d\csc\left(bx+a\right)\right)^{\frac{5}{2}}\sqrt{c\sec\left(bx+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(b*x+a))^(5/2)*(c*sec(b*x+a))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((d*csc(b*x + a))^(5/2)*sqrt(c*sec(b*x + a)), x)
```

3.233 $\int (d\csc(a+bx))^{3/2}\sqrt{c\sec(a+bx)}\,dx$

Optimal. Leaf size=31

$$-\frac{2cd\sqrt{d\csc(a+bx)}}{b\sqrt{c\sec(a+bx)}}$$

```
[Out] (-2*c*d*Sqrt[d*Csc[a + b*x]])/(b*Sqrt[c*Sec[a + b*x]])
```

Rubi [A] time = 0.0486029, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.04, Rules used = {2619}

$$-\frac{2cd\sqrt{d\csc(a+bx)}}{b\sqrt{c\sec(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Csc[a + b*x])^(3/2)*Sqrt[c*Sec[a + b*x]],x]
```

```
[Out] (-2*c*d*Sqrt[d*Csc[a + b*x]])/(b*Sqrt[c*Sec[a + b*x]])
```

Rule 2619

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(a*b*(a*Csc[e + f*x])^(m - 1)*(b*Sec[e + f*x])^(n - 1
))/(f*(n - 1)), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n - 2, 0] &&
NeQ[n, 1]
```

Rubi steps

$$\int (d\csc(a+bx))^{3/2}\sqrt{c\sec(a+bx)}\,dx=-\frac{2cd\sqrt{d\csc(a+bx)}}{b\sqrt{c\sec(a+bx)}}$$

Mathematica [A] time = 0.0633391, size = 31, normalized size = 1.

$$-\frac{2cd\sqrt{d\csc(a+bx)}}{b\sqrt{c\sec(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Csc[a + b*x])^(3/2)*Sqrt[c*Sec[a + b*x]],x]
```

```
[Out] (-2*c*d*Sqrt[d*Csc[a + b*x]])/(b*Sqrt[c*Sec[a + b*x]])
```

Maple [A] time = 0.164, size = 42, normalized size = 1.4

$$-2\,\frac{\cos(bx+a)\sin(bx+a)}{b}\left(\frac{d}{\sin(bx+a)}\right)^{3/2}\sqrt{\frac{c}{\cos(bx+a)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*csc(b*x+a))^(3/2)*(c*sec(b*x+a))^(1/2),x)
```

```
[Out] -2/b*cos(b*x+a)*sin(b*x+a)*(d/sin(b*x+a))^(3/2)*(c/cos(b*x+a))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (d\csc(bx+a))^{\frac{3}{2}}\sqrt{c\sec(bx+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(b*x+a))^(3/2)*(c*sec(b*x+a))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((d*csc(b*x + a))^(3/2)*sqrt(c*sec(b*x + a)), x)
```

Fricas [A] time = 1.70862, size = 85, normalized size = 2.74

$$-\frac{2\,d\sqrt{\frac{c}{\cos(bx+a)}}\sqrt{\frac{d}{\sin(bx+a)}}\cos(bx+a)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(b*x+a))^(3/2)*(c*sec(b*x+a))^(1/2),x, algorithm="fricas")
```

```
[Out] -2*d*sqrt(c/cos(b*x + a))*sqrt(d/sin(b*x + a))*cos(b*x + a)/b
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(b*x+a))**(3/2)*(c*sec(b*x+a))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (d\csc(bx+a))^{\frac{3}{2}}\sqrt{c\sec(bx+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(b*x+a))^(3/2)*(c*sec(b*x+a))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((d*csc(b*x + a))^(3/2)*sqrt(c*sec(b*x + a)), x)
```

3.234 $\int \sqrt{d\csc(a+bx)}\sqrt{c\sec(a+bx)}\,dx$

Optimal. Leaf size=53

$$\frac{\sqrt{\sin(2a+2bx)}\text{EllipticF}\left(a+bx-\frac{\pi}{4},2\right)\sqrt{c\sec(a+bx)}\sqrt{d\csc(a+bx)}}{b}$$

```
[Out] (Sqrt[d*Csc[a + b*x]]*EllipticF[a - Pi/4 + b*x, 2]*Sqrt[c*Sec[a + b*x]]*Sqr
t[Sin[2*a + 2*b*x]])/b
```

Rubi [A] time = 0.0927454, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {2630, 2573, 2641}

$$\frac{\sqrt{\sin(2a+2bx)}F\left(a+bx-\frac{\pi}{4}\middle|2\right)\sqrt{c\sec(a+bx)}\sqrt{d\csc(a+bx)}}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d*Csc[a + b*x]]*Sqrt[c*Sec[a + b*x]],x]
```

```
[Out] (Sqrt[d*Csc[a + b*x]]*EllipticF[a - Pi/4 + b*x, 2]*Sqrt[c*Sec[a + b*x]]*Sqr
t[Sin[2*a + 2*b*x]])/b
```

Rule 2630

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Dist[(a*Csc[e + f*x])^m*(b*Sec[e + f*x])^n*(a*Sin[e + f*x]
)^m*(b*Cos[e + f*x])^n, Int[1/((a*Sin[e + f*x])^m*(b*Cos[e + f*x])^n), x],
x] /; FreeQ[{a, b, e, f, m, n}, x] && IntegerQ[m - 1/2] && IntegerQ[n - 1/2
]
```

Rule 2573

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \sqrt{d\csc(a+bx)}\sqrt{c\sec(a+bx)}\,dx &= \left(\sqrt{c\cos(a+bx)}\sqrt{d\csc(a+bx)}\sqrt{c\sec(a+bx)}\sqrt{d\sin(a+bx)}\right)\int \frac{1}{\sqrt{c\cos(a+bx)}\sqrt{d\sin(a+bx)}}\,dx\\ &= \left(\sqrt{d\csc(a+bx)}\sqrt{c\sec(a+bx)}\sqrt{\sin(2a+2bx)}\right)\int \frac{1}{\sqrt{\sin(2a+2bx)}}\,dx\\ &= \frac{\sqrt{d\csc(a+bx)}F\left(a-\frac{\pi}{4}+bx\middle|2\right)\sqrt{c\sec(a+bx)}\sqrt{\sin(2a+2bx)}}{b}\end{aligned}$$

Mathematica [C] time = 0.670832, size = 68, normalized size = 1.28

$$\frac{\tan^3(a+bx)\left(-\cot^2(a+bx)\right)^{7/4}\sqrt{c\sec(a+bx)}\sqrt{d\csc(a+bx)}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{3}{2},\csc^2(a+bx)\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d*Csc[a + b*x]]*Sqrt[c*Sec[a + b*x]],x]
```

```
[Out] ((-Cot[a + b*x]^2)^(7/4)*Sqrt[d*Csc[a + b*x]]*Hypergeometric2F1[1/2, 3/4, 3
/2, Csc[a + b*x]^2]*Sqrt[c*Sec[a + b*x]]*Tan[a + b*x]^3)/b
```

Maple [B] time = 0.196, size = 157, normalized size = 3.

$$-\frac{\sqrt{2}\left(\sin(bx+a)\right)^2}{b\left(-1+\cos(bx+a)\right)}\sqrt{\frac{c}{\cos(bx+a)}}\sqrt{\frac{d}{\sin(bx+a)}}\sqrt{-\frac{-1+\cos(bx+a)-\sin(bx+a)}{\sin(bx+a)}}\sqrt{\frac{-1+\cos(bx+a)+\sin(bx}{\sin(bx+a)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*csc(b*x+a))^(1/2)*(c*sec(b*x+a))^(1/2),x)
```

```
[Out] -1/b*2^(1/2)*(c/cos(b*x+a))^(1/2)*(d/sin(b*x+a))^(1/2)*sin(b*x+a)^2*(-(-1+c
os(b*x+a)-sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a)+sin(b*x+a))/sin(b*x
+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*EllipticF((-(-1+cos(b*x+a)-si
n(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))/(-1+cos(b*x+a))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{d\csc(bx+a)}\sqrt{c\sec(bx+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(b*x+a))^(1/2)*(c*sec(b*x+a))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*csc(b*x + a))*sqrt(c*sec(b*x + a)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{d\csc(bx+a)}\sqrt{c\sec(bx+a)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(b*x+a))^(1/2)*(c*sec(b*x+a))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*csc(b*x + a))*sqrt(c*sec(b*x + a)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{c \sec (a + bx)} \sqrt{d \csc (a + bx)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(b*x+a))**(1/2)*(c*sec(b*x+a))**(1/2),x)
```

```
[Out] Integral(sqrt(c*sec(a + b*x))*sqrt(d*csc(a + b*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{d \csc (bx + a)} \sqrt{c \sec (bx + a)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(b*x+a))^(1/2)*(c*sec(b*x+a))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*csc(b*x + a))*sqrt(c*sec(b*x + a)), x)
```

3.235 $\int \frac{\sqrt{c\sec(a+bx)}}{\sqrt{d\csc(a+bx)}}\,dx$

Optimal. Leaf size=270

$$-\frac{\tan^{-1}\left(1-\sqrt{2}\sqrt{\tan(a+bx)}\right)\sqrt{c\sec(a+bx)}}{\sqrt{2}b\sqrt{\tan(a+bx)}\sqrt{d\csc(a+bx)}}+\frac{\tan^{-1}\left(\sqrt{2}\sqrt{\tan(a+bx)}+1\right)\sqrt{c\sec(a+bx)}}{\sqrt{2}b\sqrt{\tan(a+bx)}\sqrt{d\csc(a+bx)}}+\frac{\sqrt{c\sec(a+bx)}\log\left(\mathrm{t}\right.}{2\sqrt{2}b\sqrt{\mathrm{ta}}}$$

```
[Out] -((ArcTan[1 - Sqrt[2]*Sqrt[Tan[a + b*x]]]*Sqrt[c*Sec[a + b*x]])/(Sqrt[2]*b*
Sqrt[d*Csc[a + b*x]]*Sqrt[Tan[a + b*x]])) + (ArcTan[1 + Sqrt[2]*Sqrt[Tan[a
+ b*x]]]*Sqrt[c*Sec[a + b*x]])/(Sqrt[2]*b*Sqrt[d*Csc[a + b*x]]*Sqrt[Tan[a +
 b*x]]) + (Log[1 - Sqrt[2]*Sqrt[Tan[a + b*x]] + Tan[a + b*x]]*Sqrt[c*Sec[a
+ b*x]])/(2*Sqrt[2]*b*Sqrt[d*Csc[a + b*x]]*Sqrt[Tan[a + b*x]]) - (Log[1 + S
qrt[2]*Sqrt[Tan[a + b*x]] + Tan[a + b*x]]*Sqrt[c*Sec[a + b*x]])/(2*Sqrt[2]*
b*Sqrt[d*Csc[a + b*x]]*Sqrt[Tan[a + b*x]])
```

Rubi [A] time = 0.140481, antiderivative size = 270, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.36, Rules used = {2629, 3476, 329, 297, 1162, 617, 204, 1165, 628}

$$-\frac{\tan^{-1}\left(1-\sqrt{2}\sqrt{\tan(a+bx)}\right)\sqrt{c\sec(a+bx)}}{\sqrt{2}b\sqrt{\tan(a+bx)}\sqrt{d\csc(a+bx)}}+\frac{\tan^{-1}\left(\sqrt{2}\sqrt{\tan(a+bx)}+1\right)\sqrt{c\sec(a+bx)}}{\sqrt{2}b\sqrt{\tan(a+bx)}\sqrt{d\csc(a+bx)}}+\frac{\sqrt{c\sec(a+bx)}\log\left(\mathrm{t}\right.}{2\sqrt{2}b\sqrt{\mathrm{ta}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c*Sec[a + b*x]]/Sqrt[d*Csc[a + b*x]],x]
```

```
[Out] -((ArcTan[1 - Sqrt[2]*Sqrt[Tan[a + b*x]]]*Sqrt[c*Sec[a + b*x]])/(Sqrt[2]*b*
Sqrt[d*Csc[a + b*x]]*Sqrt[Tan[a + b*x]])) + (ArcTan[1 + Sqrt[2]*Sqrt[Tan[a
+ b*x]]]*Sqrt[c*Sec[a + b*x]])/(Sqrt[2]*b*Sqrt[d*Csc[a + b*x]]*Sqrt[Tan[a +
 b*x]]) + (Log[1 - Sqrt[2]*Sqrt[Tan[a + b*x]] + Tan[a + b*x]]*Sqrt[c*Sec[a
+ b*x]])/(2*Sqrt[2]*b*Sqrt[d*Csc[a + b*x]]*Sqrt[Tan[a + b*x]]) - (Log[1 + S
qrt[2]*Sqrt[Tan[a + b*x]] + Tan[a + b*x]]*Sqrt[c*Sec[a + b*x]])/(2*Sqrt[2]*
b*Sqrt[d*Csc[a + b*x]]*Sqrt[Tan[a + b*x]])
```

Rule 2629

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Dist[((a*Csc[e + f*x])^m*(b*Sec[e + f*x])^n)/Tan[e + f*x]^
n, Int[Tan[e + f*x]^n, x], x] /; FreeQ[{a, b, e, f, m, n}, x] &&  !IntegerQ
[n] && EqQ[m + n, 0]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{\sqrt{c\sec(a+bx)}}{\sqrt{d\csc(a+bx)}}\,dx = \frac{\sqrt{c\sec(a+bx)}\int\sqrt{\tan(a+bx)}\,dx}{\sqrt{d\csc(a+bx)}\sqrt{\tan(a+bx)}}$$
$$= \frac{\sqrt{c\sec(a+bx)}\,\mathrm{Subst}\left(\int\frac{\sqrt{x}}{1+x^2}\,dx,x,\tan(a+bx)\right)}{b\sqrt{d\csc(a+bx)}\sqrt{\tan(a+bx)}}$$
$$= \frac{\left(2\sqrt{c\sec(a+bx)}\right)\mathrm{Subst}\left(\int\frac{x^2}{1+x^4}\,dx,x,\sqrt{\tan(a+bx)}\right)}{b\sqrt{d\csc(a+bx)}\sqrt{\tan(a+bx)}}$$
$$= -\frac{\sqrt{c\sec(a+bx)}\,\mathrm{Subst}\left(\int\frac{1-x^2}{1+x^4}\,dx,x,\sqrt{\tan(a+bx)}\right)}{b\sqrt{d\csc(a+bx)}\sqrt{\tan(a+bx)}} + \frac{\sqrt{c\sec(a+bx)}\,\mathrm{Subst}\left(\int\frac{1+x^2}{1+x^4}\,dx,x,\sqrt{\right.}{b\sqrt{d\csc(a+bx)}\sqrt{\tan(a+b}$$
$$= \frac{\sqrt{c\sec(a+bx)}\,\mathrm{Subst}\left(\int\frac{1}{1-\sqrt{2}x+x^2}\,dx,x,\sqrt{\tan(a+bx)}\right)}{2b\sqrt{d\csc(a+bx)}\sqrt{\tan(a+bx)}} + \frac{\sqrt{c\sec(a+bx)}\,\mathrm{Subst}\left(\int\frac{1}{1+\sqrt{2}x+x^2}\right.}{2b\sqrt{d\csc(a+bx)}\sqrt{\tan}}$$
$$= \frac{\log\left(1-\sqrt{2}\sqrt{\tan(a+bx)}+\tan(a+bx)\right)\sqrt{c\sec(a+bx)}}{2\sqrt{2}b\sqrt{d\csc(a+bx)}\sqrt{\tan(a+bx)}} - \frac{\log\left(1+\sqrt{2}\sqrt{\tan(a+bx)}+\tan(a\right.}{2\sqrt{2}b\sqrt{d\csc(a+bx)}\sqrt{}}$$
$$= -\frac{\tan^{-1}\left(1-\sqrt{2}\sqrt{\tan(a+bx)}\right)\sqrt{c\sec(a+bx)}}{\sqrt{2}b\sqrt{d\csc(a+bx)}\sqrt{\tan(a+bx)}} + \frac{\tan^{-1}\left(1+\sqrt{2}\sqrt{\tan(a+bx)}\right)\sqrt{c\sec(a+bx)}}{\sqrt{2}b\sqrt{d\csc(a+bx)}\sqrt{\tan(a+bx)}}$$

Mathematica [A] time = 1.32641, size = 171, normalized size = 0.63

$$\frac{\cot(a+bx)\sqrt{c\sec(a+bx)}\left(\log\left(\sqrt{\cot^2(a+bx)}-\sqrt{2}\sqrt[4]{\cot^2(a+bx)}+1\right)-\log\left(\sqrt{\cot^2(a+bx)}+\sqrt{2}\sqrt[4]{\cot^2(a+bx)}+\right.\right.}{2\sqrt{2}b\sqrt[4]{\cot^2(a+bx)}\sqrt{d\csc(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c*Sec[a + b*x]]/Sqrt[d*Csc[a + b*x]],x]
```

```
[Out] (Cot[a + b*x]*(2*ArcTan[1 - Sqrt[2]*(Cot[a + b*x]^2)^(1/4)] - 2*ArcTan[1 +
Sqrt[2]*(Cot[a + b*x]^2)^(1/4)] + Log[1 - Sqrt[2]*(Cot[a + b*x]^2)^(1/4) +
Sqrt[Cot[a + b*x]^2]] - Log[1 + Sqrt[2]*(Cot[a + b*x]^2)^(1/4) + Sqrt[Cot[a
 + b*x]^2]])*Sqrt[c*Sec[a + b*x]])/(2*Sqrt[2]*b*(Cot[a + b*x]^2)^(1/4)*Sqrt
[d*Csc[a + b*x]])
```

Maple [C] time = 0.148, size = 276, normalized size = 1.

$$\frac{\sqrt{2}\sin(bx+a)}{2b(-1+\cos(bx+a))}\sqrt{\frac{c}{\cos(bx+a)}}\sqrt{-\frac{-1+\cos(bx+a)-\sin(bx+a)}{\sin(bx+a)}}\sqrt{\frac{-1+\cos(bx+a)+\sin(bx+a)}{\sin(bx+a)}}\sqrt{\frac{-1+c}{\sin}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*sec(b*x+a))^(1/2)/(d*csc(b*x+a))^(1/2),x)
```

```
[Out] 1/2/b*2^(1/2)*(c/cos(b*x+a))^(1/2)*(-(-1+cos(b*x+a)-sin(b*x+a))/sin(b*x+a))
^(1/2)*((-1+cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b
*x+a))^(1/2)*(I*EllipticPi((-(-1+cos(b*x+a)-sin(b*x+a))/sin(b*x+a))^(1/2),1
/2+1/2*I,1/2*2^(1/2))-I*EllipticPi((-(-1+cos(b*x+a)-sin(b*x+a))/sin(b*x+a))
^(1/2),1/2-1/2*I,1/2*2^(1/2))+EllipticPi((-(-1+cos(b*x+a)-sin(b*x+a))/sin(b
```

```
*x+a))^(1/2),1/2+1/2*I,1/2*2^(1/2))+EllipticPi((-(-1+cos(b*x+a)-sin(b*x+a))
/sin(b*x+a))^(1/2),1/2-1/2*I,1/2*2^(1/2)))*sin(b*x+a)/(d/sin(b*x+a))^(1/2)/
(-1+cos(b*x+a))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c \sec (bx + a)}}{\sqrt{d \csc (bx + a)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sec(b*x+a))^(1/2)/(d*csc(b*x+a))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*sec(b*x + a))/sqrt(d*csc(b*x + a)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sec(b*x+a))^(1/2)/(d*csc(b*x+a))^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c \sec (a + bx)}}{\sqrt{d \csc (a + bx)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sec(b*x+a))**(1/2)/(d*csc(b*x+a))**(1/2),x)
```

```
[Out] Integral(sqrt(c*sec(a + b*x))/sqrt(d*csc(a + b*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c \sec (bx + a)}}{\sqrt{d \csc (bx + a)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sec(b*x+a))^(1/2)/(d*csc(b*x+a))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*sec(b*x + a))/sqrt(d*csc(b*x + a)), x)
```

3.236 $\int \frac{\sqrt{c\sec(a+bx)}}{(d\csc(a+bx))^{3/2}}\,dx$

Optimal. Leaf size=93

$$\frac{\sqrt{\sin(2a+2bx)}\text{EllipticF}\left(a+bx-\frac{\pi}{4},2\right)\sqrt{c\sec(a+bx)}\sqrt{d\csc(a+bx)}}{2bd^2}-\frac{c}{bd\sqrt{c\sec(a+bx)}\sqrt{d\csc(a+bx)}}$$

```
[Out] -(c/(b*d*Sqrt[d*Csc[a + b*x]]*Sqrt[c*Sec[a + b*x]])) + (Sqrt[d*Csc[a + b*x]
]*EllipticF[a - Pi/4 + b*x, 2]*Sqrt[c*Sec[a + b*x]]*Sqrt[Sin[2*a + 2*b*x]])
/(2*b*d^2)
```

Rubi [A] time = 0.147486, antiderivative size = 93, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {2627, 2630, 2573, 2641}

$$\frac{\sqrt{\sin(2a+2bx)}F\left(a+bx-\frac{\pi}{4}\middle|2\right)\sqrt{c\sec(a+bx)}\sqrt{d\csc(a+bx)}}{2bd^2}-\frac{c}{bd\sqrt{c\sec(a+bx)}\sqrt{d\csc(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c*Sec[a + b*x]]/(d*Csc[a + b*x])^(3/2),x]
```

```
[Out] -(c/(b*d*Sqrt[d*Csc[a + b*x]]*Sqrt[c*Sec[a + b*x]])) + (Sqrt[d*Csc[a + b*x]
]*EllipticF[a - Pi/4 + b*x, 2]*Sqrt[c*Sec[a + b*x]]*Sqrt[Sin[2*a + 2*b*x]])
/(2*b*d^2)
```

Rule 2627

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Simp[(b*(a*Csc[e + f*x])^(m + 1)*(b*Sec[e + f*x])^(n - 1)
)/(a*f*(m + n)), x] + Dist[(m + 1)/(a^2*(m + n)), Int[(a*Csc[e + f*x])^(m +
 2)*(b*Sec[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -1] &
& NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2630

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Dist[(a*Csc[e + f*x])^m*(b*Sec[e + f*x])^n*(a*Sin[e + f*x]
)^m*(b*Cos[e + f*x])^n, Int[1/((a*Sin[e + f*x])^m*(b*Cos[e + f*x])^n), x],
x] /; FreeQ[{a, b, e, f, m, n}, x] && IntegerQ[m - 1/2] && IntegerQ[n - 1/2
]
```

Rule 2573

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sqrt{c\sec(a+bx)}}{(d\csc(a+bx))^{3/2}}\,dx = -\frac{c}{bd\sqrt{d\csc(a+bx)}\sqrt{c\sec(a+bx)}} + \frac{\int \sqrt{d\csc(a+bx)}\sqrt{c\sec(a+bx)}\,dx}{2d^2}$$

$$= -\frac{c}{bd\sqrt{d\csc(a+bx)}\sqrt{c\sec(a+bx)}} + \frac{\left(\sqrt{c\cos(a+bx)}\sqrt{d\csc(a+bx)}\sqrt{c\sec(a+bx)}\sqrt{d\sin(a+\right.}{2d^2}$$

$$= -\frac{c}{bd\sqrt{d\csc(a+bx)}\sqrt{c\sec(a+bx)}} + \frac{\left(\sqrt{d\csc(a+bx)}\sqrt{c\sec(a+bx)}\sqrt{\sin(2a+2bx)}\right)\int \frac{1}{\sqrt{\sin(2a+}}}{2d^2}$$

$$= -\frac{c}{bd\sqrt{d\csc(a+bx)}\sqrt{c\sec(a+bx)}} + \frac{\sqrt{d\csc(a+bx)}F\left(a-\frac{\pi}{4}+bx\middle|2\right)\sqrt{c\sec(a+bx)}\sqrt{\sin(2a+}}{2bd^2}$$

Mathematica [C] time = 0.672873, size = 80, normalized size = 0.86

$$-\frac{(c\sec(a+bx))^{3/2}\left(\left(-\cot^2(a+bx)\right)^{3/4}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{3}{2},\csc^2(a+bx)\right)+\cos(2(a+bx))+1\right)}{2bcd\sqrt{d\csc(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c*Sec[a + b*x]]/(d*Csc[a + b*x])^(3/2),x]
```

```
[Out] -((1 + Cos[2*(a + b*x)] + (-Cot[a + b*x]^2)^(3/4)*Hypergeometric2F1[1/2, 3/
4, 3/2, Csc[a + b*x]^2])*(c*Sec[a + b*x])^(3/2))/(2*b*c*d*Sqrt[d*Csc[a + b*
x]])
```

Maple [A] time = 0.186, size = 188, normalized size = 2.

$$-\frac{\sqrt{2}}{2\,b\,(-1+\cos(bx+a))\sin(bx+a)}\left(\sin(bx+a)\sqrt{-\frac{-1+\cos(bx+a)-\sin(bx+a)}{\sin(bx+a)}}\sqrt{\frac{-1+\cos(bx+a)+\sin(bx+a)}{\sin(bx+a)}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*sec(b*x+a))^(1/2)/(d*csc(b*x+a))^(3/2),x)
```

```
[Out] -1/2/b*2^(1/2)*(sin(b*x+a)*(-(-1+cos(b*x+a)-sin(b*x+a))/sin(b*x+a))^(1/2)*(
(-1+cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(
1/2)*EllipticF((-(-1+cos(b*x+a)-sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))+
cos(b*x+a)^2*2^(1/2)-cos(b*x+a)*2^(1/2))*(c/cos(b*x+a))^(1/2)/(-1+cos(b*x+a
))/(d/sin(b*x+a))^(3/2)/sin(b*x+a)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c\sec(bx+a)}}{(d\csc(bx+a))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sec(b*x+a))^(1/2)/(d*csc(b*x+a))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*sec(b*x + a))/(d*csc(b*x + a))^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{d\csc\left(bx+a\right)}\sqrt{c\sec\left(bx+a\right)}}{d^{2}\csc\left(bx+a\right)^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sec(b*x+a))^(1/2)/(d*csc(b*x+a))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*csc(b*x + a))*sqrt(c*sec(b*x + a))/(d^2*csc(b*x + a)^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{c\sec\left(a+bx\right)}}{\left(d\csc\left(a+bx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sec(b*x+a))**(1/2)/(d*csc(b*x+a))**(3/2),x)
```

```
[Out] Integral(sqrt(c*sec(a + b*x))/(d*csc(a + b*x))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{c\sec\left(bx+a\right)}}{\left(d\csc\left(bx+a\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sec(b*x+a))^(1/2)/(d*csc(b*x+a))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*sec(b*x + a))/(d*csc(b*x + a))^(3/2), x)
```

3.237 $\int \frac{\sqrt{c\sec(a+bx)}}{(d\csc(a+bx))^{5/2}}\,dx$

Optimal. Leaf size=322

$$-\frac{3\tan^{-1}\left(1-\sqrt{2}\sqrt{\tan(a+bx)}\right)\sqrt{c\sec(a+bx)}}{4\sqrt{2}bd^2\sqrt{\tan(a+bx)}\sqrt{d\csc(a+bx)}}+\frac{3\tan^{-1}\left(\sqrt{2}\sqrt{\tan(a+bx)}+1\right)\sqrt{c\sec(a+bx)}}{4\sqrt{2}bd^2\sqrt{\tan(a+bx)}\sqrt{d\csc(a+bx)}}+\frac{3\sqrt{c\sec(a+bx)}\log\left(\right.}{8\sqrt{2}bd^2\sqrt{t}}$$

```
[Out] -c/(2*b*d*(d*Csc[a + b*x])^(3/2)*Sqrt[c*Sec[a + b*x]]) - (3*ArcTan[1 - Sqrt
[2]*Sqrt[Tan[a + b*x]]]*Sqrt[c*Sec[a + b*x]])/(4*Sqrt[2]*b*d^2*Sqrt[d*Csc[a
 + b*x]]*Sqrt[Tan[a + b*x]]) + (3*ArcTan[1 + Sqrt[2]*Sqrt[Tan[a + b*x]]]*Sq
rt[c*Sec[a + b*x]])/(4*Sqrt[2]*b*d^2*Sqrt[d*Csc[a + b*x]]*Sqrt[Tan[a + b*x]
]) + (3*Log[1 - Sqrt[2]*Sqrt[Tan[a + b*x]] + Tan[a + b*x]]*Sqrt[c*Sec[a + b
*x]])/(8*Sqrt[2]*b*d^2*Sqrt[d*Csc[a + b*x]]*Sqrt[Tan[a + b*x]]) - (3*Log[1
+ Sqrt[2]*Sqrt[Tan[a + b*x]] + Tan[a + b*x]]*Sqrt[c*Sec[a + b*x]])/(8*Sqrt[
2]*b*d^2*Sqrt[d*Csc[a + b*x]]*Sqrt[Tan[a + b*x]])
```

Rubi [A] time = 0.211808, antiderivative size = 322, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 10, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {2627, 2629, 3476, 329, 297, 1162, 617, 204, 1165, 628}

$$-\frac{3\tan^{-1}\left(1-\sqrt{2}\sqrt{\tan(a+bx)}\right)\sqrt{c\sec(a+bx)}}{4\sqrt{2}bd^2\sqrt{\tan(a+bx)}\sqrt{d\csc(a+bx)}}+\frac{3\tan^{-1}\left(\sqrt{2}\sqrt{\tan(a+bx)}+1\right)\sqrt{c\sec(a+bx)}}{4\sqrt{2}bd^2\sqrt{\tan(a+bx)}\sqrt{d\csc(a+bx)}}+\frac{3\sqrt{c\sec(a+bx)}\log\left(\right.}{8\sqrt{2}bd^2\sqrt{t}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c*Sec[a + b*x]]/(d*Csc[a + b*x])^(5/2),x]
```


```
[Out] -c/(2*b*d*(d*Csc[a + b*x])^(3/2)*Sqrt[c*Sec[a + b*x]]) - (3*ArcTan[1 - Sqrt
[2]*Sqrt[Tan[a + b*x]]]*Sqrt[c*Sec[a + b*x]])/(4*Sqrt[2]*b*d^2*Sqrt[d*Csc[a
 + b*x]]*Sqrt[Tan[a + b*x]]) + (3*ArcTan[1 + Sqrt[2]*Sqrt[Tan[a + b*x]]]*Sq
rt[c*Sec[a + b*x]])/(4*Sqrt[2]*b*d^2*Sqrt[d*Csc[a + b*x]]*Sqrt[Tan[a + b*x]
]) + (3*Log[1 - Sqrt[2]*Sqrt[Tan[a + b*x]] + Tan[a + b*x]]*Sqrt[c*Sec[a + b
*x]])/(8*Sqrt[2]*b*d^2*Sqrt[d*Csc[a + b*x]]*Sqrt[Tan[a + b*x]]) - (3*Log[1
+ Sqrt[2]*Sqrt[Tan[a + b*x]] + Tan[a + b*x]]*Sqrt[c*Sec[a + b*x]])/(8*Sqrt[
2]*b*d^2*Sqrt[d*Csc[a + b*x]]*Sqrt[Tan[a + b*x]])
```


Rule 2627

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Simp[(b*(a*Csc[e + f*x])^(m + 1)*(b*Sec[e + f*x])^(n - 1)
)/(a*f*(m + n)), x] + Dist[(m + 1)/(a^2*(m + n)), Int[(a*Csc[e + f*x])^(m +
 2)*(b*Sec[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -1] &
& NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2629

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Dist[((a*Csc[e + f*x])^m*(b*Sec[e + f*x])^n)/Tan[e + f*x]^
n, Int[Tan[e + f*x]^n, x], x] /; FreeQ[{a, b, e, f, m, n}, x] &&  !IntegerQ
[n] && EqQ[m + n, 0]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
```

```
IntegerQ[n]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{\sqrt{c\sec(a+bx)}}{(d\csc(a+bx))^{5/2}}\,dx = -\frac{c}{2bd(d\csc(a+bx))^{3/2}\sqrt{c\sec(a+bx)}} + \frac{3\int \frac{\sqrt{c\sec(a+bx)}}{\sqrt{d\csc(a+bx)}}\,dx}{4d^2}$$
$$= -\frac{c}{2bd(d\csc(a+bx))^{3/2}\sqrt{c\sec(a+bx)}} + \frac{\left(3\sqrt{c\sec(a+bx)}\right)\int \sqrt{\tan(a+bx)}\,dx}{4d^2\sqrt{d\csc(a+bx)}\sqrt{\tan(a+bx)}}$$
$$= -\frac{c}{2bd(d\csc(a+bx))^{3/2}\sqrt{c\sec(a+bx)}} + \frac{\left(3\sqrt{c\sec(a+bx)}\right)\text{Subst}\left(\int \frac{\sqrt{x}}{1+x^2}\,dx, x, \tan(a+bx)\right)}{4bd^2\sqrt{d\csc(a+bx)}\sqrt{\tan(a+bx)}}$$
$$= -\frac{c}{2bd(d\csc(a+bx))^{3/2}\sqrt{c\sec(a+bx)}} + \frac{\left(3\sqrt{c\sec(a+bx)}\right)\text{Subst}\left(\int \frac{x^2}{1+x^4}\,dx, x, \sqrt{\tan(a+bx)}\right)}{2bd^2\sqrt{d\csc(a+bx)}\sqrt{\tan(a+bx)}}$$
$$= -\frac{c}{2bd(d\csc(a+bx))^{3/2}\sqrt{c\sec(a+bx)}} - \frac{\left(3\sqrt{c\sec(a+bx)}\right)\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \sqrt{\tan(a+bx)}\right)}{4bd^2\sqrt{d\csc(a+bx)}\sqrt{\tan(a+bx)}}$$
$$= -\frac{c}{2bd(d\csc(a+bx))^{3/2}\sqrt{c\sec(a+bx)}} + \frac{\left(3\sqrt{c\sec(a+bx)}\right)\text{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}\,dx, x, \sqrt{\tan(a+bx)}\right)}{8bd^2\sqrt{d\csc(a+bx)}\sqrt{\tan(a+bx)}}$$
$$= -\frac{c}{2bd(d\csc(a+bx))^{3/2}\sqrt{c\sec(a+bx)}} + \frac{3\log\left(1-\sqrt{2}\sqrt{\tan(a+bx)}+\tan(a+bx)\right)\sqrt{c\sec(a+bx)}}{8\sqrt{2}bd^2\sqrt{d\csc(a+bx)}\sqrt{\tan(a+bx)}}$$
$$= -\frac{c}{2bd(d\csc(a+bx))^{3/2}\sqrt{c\sec(a+bx)}} - \frac{3\tan^{-1}\left(1-\sqrt{2}\sqrt{\tan(a+bx)}\right)\sqrt{c\sec(a+bx)}}{4\sqrt{2}bd^2\sqrt{d\csc(a+bx)}\sqrt{\tan(a+bx)}} + \frac{3\tan}{}$$

Mathematica [A] time = 1.78913, size = 222, normalized size = 0.69

$$\frac{c\sqrt{d\csc(a+bx)}\left(-4\sqrt[4]{\cot^2(a+bx)}+4\cos(2(a+bx))\sqrt[4]{\cot^2(a+bx)}+3\sqrt{2}\log\left(\sqrt{\cot^2(a+bx)}-\sqrt{2}\sqrt[4]{\cot^2(a+bx)}+1\right.\right.}{16bd^3\sqrt[4]{\cot^2(}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c*Sec[a + b*x]]/(d*Csc[a + b*x])^(5/2),x]
```

```
[Out] (c*Sqrt[d*Csc[a + b*x]]*(6*Sqrt[2]*ArcTan[1 - Sqrt[2]*(Cot[a + b*x]^2)^(1/4
)] - 6*Sqrt[2]*ArcTan[1 + Sqrt[2]*(Cot[a + b*x]^2)^(1/4)] - 4*(Cot[a + b*x]
^2)^(1/4) + 4*Cos[2*(a + b*x)]*(Cot[a + b*x]^2)^(1/4) + 3*Sqrt[2]*Log[1 - S
qrt[2]*(Cot[a + b*x]^2)^(1/4) + Sqrt[Cot[a + b*x]^2]] - 3*Sqrt[2]*Log[1 + S
qrt[2]*(Cot[a + b*x]^2)^(1/4) + Sqrt[Cot[a + b*x]^2]]))/(16*b*d^3*(Cot[a +
b*x]^2)^(1/4)*Sqrt[c*Sec[a + b*x]])
```

Maple [C] time = 0.225, size = 522, normalized size = 1.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*sec(b*x+a))^(1/2)/(d*csc(b*x+a))^(5/2),x)
```

```
[Out] -1/8/b*2^(1/2)*(3*I*(-(-1+cos(b*x+a)-sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos
(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*El
lipticPi((-(-1+cos(b*x+a)-sin(b*x+a))/sin(b*x+a))^(1/2),1/2-1/2*I,1/2*2^(1/
```

```
2))-3*I*(-(-1+cos(b*x+a)-sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a)+sin(
b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*EllipticPi((-(
-1+cos(b*x+a)-sin(b*x+a))/sin(b*x+a))^(1/2),1/2+1/2*I,1/2*2^(1/2))-3*(-(-1+
cos(b*x+a)-sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a)+sin(b*x+a))/sin(b*
x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*EllipticPi((-(-1+cos(b*x+a)-
sin(b*x+a))/sin(b*x+a))^(1/2),1/2-1/2*I,1/2*2^(1/2))-3*(-(-1+cos(b*x+a)-sin
(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((
-1+cos(b*x+a))/sin(b*x+a))^(1/2)*EllipticPi((-(-1+cos(b*x+a)-sin(b*x+a))/si
n(b*x+a))^(1/2),1/2+1/2*I,1/2*2^(1/2))+2*cos(b*x+a)^2*2^(1/2)-2*cos(b*x+a)*
2^(1/2))*(c/cos(b*x+a))^(1/2)/(-1+cos(b*x+a))/(d/sin(b*x+a))^(5/2)/sin(b*x+
a)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c\sec(bx+a)}}{(d\csc(bx+a))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sec(b*x+a))^(1/2)/(d*csc(b*x+a))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*sec(b*x + a))/(d*csc(b*x + a))^(5/2), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sec(b*x+a))^(1/2)/(d*csc(b*x+a))^(5/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sec(b*x+a))**(1/2)/(d*csc(b*x+a))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c\sec(bx+a)}}{(d\csc(bx+a))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sec(b*x+a))^(1/2)/(d*csc(b*x+a))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*sec(b*x + a))/(d*csc(b*x + a))^(5/2), x)
```

3.238 $\int (d\csc(a+bx))^{9/2}(c\sec(a+bx))^{3/2}\,dx$

Optimal. Leaf size=104

$$\frac{64cd^5\sqrt{c\sec(a+bx)}}{21b\sqrt{d\csc(a+bx)}}-\frac{16cd^3\sqrt{c\sec(a+bx)}(d\csc(a+bx))^{3/2}}{21b}-\frac{2cd\sqrt{c\sec(a+bx)}(d\csc(a+bx))^{7/2}}{7b}$$

```
[Out] (64*c*d^5*Sqrt[c*Sec[a + b*x]])/(21*b*Sqrt[d*Csc[a + b*x]]) - (16*c*d^3*(d*
Csc[a + b*x])^(3/2)*Sqrt[c*Sec[a + b*x]])/(21*b) - (2*c*d*(d*Csc[a + b*x])^
(7/2)*Sqrt[c*Sec[a + b*x]])/(7*b)
```

Rubi [A] time = 0.162464, antiderivative size = 104, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.08, Rules used = {2625, 2619}

$$\frac{64cd^5\sqrt{c\sec(a+bx)}}{21b\sqrt{d\csc(a+bx)}}-\frac{16cd^3\sqrt{c\sec(a+bx)}(d\csc(a+bx))^{3/2}}{21b}-\frac{2cd\sqrt{c\sec(a+bx)}(d\csc(a+bx))^{7/2}}{7b}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Csc[a + b*x])^(9/2)*(c*Sec[a + b*x])^(3/2),x]
```

```
[Out] (64*c*d^5*Sqrt[c*Sec[a + b*x]])/(21*b*Sqrt[d*Csc[a + b*x]]) - (16*c*d^3*(d*
Csc[a + b*x])^(3/2)*Sqrt[c*Sec[a + b*x]])/(21*b) - (2*c*d*(d*Csc[a + b*x])^
(7/2)*Sqrt[c*Sec[a + b*x]])/(7*b)
```

Rule 2625

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> -Simp[(a*b*(a*Csc[e + f*x])^(m - 1)*(b*Sec[e + f*x])^(n -
 1))/(f*(m - 1)), x] + Dist[(a^2*(m + n - 2))/(m - 1), Int[(a*Csc[e + f*x])
^(m - 2)*(b*Sec[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m,
1] && IntegersQ[2*m, 2*n] &&  !GtQ[n, m]
```

Rule 2619

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(a*b*(a*Csc[e + f*x])^(m - 1)*(b*Sec[e + f*x])^(n - 1
))/(f*(n - 1)), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n - 2, 0] &&
NeQ[n, 1]
```

Rubi steps

$$\begin{aligned}\int (d\csc(a+bx))^{9/2}(c\sec(a+bx))^{3/2}\,dx &= -\frac{2cd(d\csc(a+bx))^{7/2}\sqrt{c\sec(a+bx)}}{7b}+\frac{1}{7}\left(8d^2\right)\int (d\csc(a+bx))^{5/2}(c\sec(a\\ &= -\frac{16cd^3(d\csc(a+bx))^{3/2}\sqrt{c\sec(a+bx)}}{21b}-\frac{2cd(d\csc(a+bx))^{7/2}\sqrt{c\sec(a+b}}{7b}\\ &= \frac{64cd^5\sqrt{c\sec(a+bx)}}{21b\sqrt{d\csc(a+bx)}}-\frac{16cd^3(d\csc(a+bx))^{3/2}\sqrt{c\sec(a+bx)}}{21b}-\frac{2cd(d\csc(}{}\end{aligned}$$

Mathematica [A] time = 0.298261, size = 57, normalized size = 0.55

$$-\frac{2cd^5\left(3\csc^4(a+bx)+8\csc^2(a+bx)-32\right)\sqrt{c\sec(a+bx)}}{21b\sqrt{d\csc(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Csc[a + b*x])^(9/2)*(c*Sec[a + b*x])^(3/2),x]
```

```
[Out] (-2*c*d^5*(-32 + 8*Csc[a + b*x]^2 + 3*Csc[a + b*x]^4)*Sqrt[c*Sec[a + b*x]])
/(21*b*Sqrt[d*Csc[a + b*x]])
```

Maple [A] time = 0.196, size = 64, normalized size = 0.6

$$\frac{\left(64\left(\cos\left(bx+a\right)\right)^{4}-112\left(\cos\left(bx+a\right)\right)^{2}+42\right)\cos\left(bx+a\right)\sin\left(bx+a\right)}{21\,b}\left(\frac{d}{\sin\left(bx+a\right)}\right)^{\frac{9}{2}}\left(\frac{c}{\cos\left(bx+a\right)}\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*csc(b*x+a))^(9/2)*(c*sec(b*x+a))^(3/2),x)
```

```
[Out] 2/21/b*(32*cos(b*x+a)^4-56*cos(b*x+a)^2+21)*cos(b*x+a)*(d/sin(b*x+a))^(9/2)
*(c/cos(b*x+a))^(3/2)*sin(b*x+a)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\left(d\csc\left(bx+a\right)\right)^{\frac{9}{2}}\left(c\sec\left(bx+a\right)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(b*x+a))^(9/2)*(c*sec(b*x+a))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((d*csc(b*x + a))^(9/2)*(c*sec(b*x + a))^(3/2), x)
```

Fricas [A] time = 2.44703, size = 203, normalized size = 1.95

$$-\frac{2\left(32\,cd^{4}\cos\left(bx+a\right)^{4}-56\,cd^{4}\cos\left(bx+a\right)^{2}+21\,cd^{4}\right)\sqrt{\frac{c}{\cos(bx+a)}}\sqrt{\frac{d}{\sin(bx+a)}}}{21\left(b\cos\left(bx+a\right)^{2}-b\right)\sin\left(bx+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(b*x+a))^(9/2)*(c*sec(b*x+a))^(3/2),x, algorithm="fricas")
```

```
[Out] -2/21*(32*c*d^4*cos(b*x + a)^4 - 56*c*d^4*cos(b*x + a)^2 + 21*c*d^4)*sqrt(c
/cos(b*x + a))*sqrt(d/sin(b*x + a))/((b*cos(b*x + a)^2 - b)*sin(b*x + a))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(b*x+a))**(9/2)*(c*sec(b*x+a))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (d\csc(bx+a))^{\frac{9}{2}}(c\sec(bx+a))^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(b*x+a))^(9/2)*(c*sec(b*x+a))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((d*csc(b*x + a))^(9/2)*(c*sec(b*x + a))^(3/2), x)
```

3.239 $\int (d\csc(a+bx))^{7/2}(c\sec(a+bx))^{3/2}\,dx$

Optimal. Leaf size=166

$$-\frac{24c^2d^4E\left(a+bx-\frac{\pi}{4}\middle|2\right)}{5b\sqrt{\sin(2a+2bx)}\sqrt{c\sec(a+bx)}\sqrt{d\csc(a+bx)}}+\frac{24cd^5\sqrt{c\sec(a+bx)}}{5b(d\csc(a+bx))^{3/2}}-\frac{12cd^3\sqrt{c\sec(a+bx)}\sqrt{d\csc(a+bx)}}{5b}-\frac{2cd\sqrt{c}}{}$$

```
[Out] (24*c*d^5*Sqrt[c*Sec[a + b*x]])/(5*b*(d*Csc[a + b*x])^(3/2)) - (12*c*d^3*Sq
rt[d*Csc[a + b*x]]*Sqrt[c*Sec[a + b*x]])/(5*b) - (2*c*d*(d*Csc[a + b*x])^(5
/2)*Sqrt[c*Sec[a + b*x]])/(5*b) - (24*c^2*d^4*EllipticE[a - Pi/4 + b*x, 2])
/(5*b*Sqrt[d*Csc[a + b*x]]*Sqrt[c*Sec[a + b*x]]*Sqrt[Sin[2*a + 2*b*x]])
```

Rubi [A] time = 0.267906, antiderivative size = 166, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2625, 2626, 2630, 2572, 2639}

$$-\frac{24c^2d^4E\left(a+bx-\frac{\pi}{4}\middle|2\right)}{5b\sqrt{\sin(2a+2bx)}\sqrt{c\sec(a+bx)}\sqrt{d\csc(a+bx)}}+\frac{24cd^5\sqrt{c\sec(a+bx)}}{5b(d\csc(a+bx))^{3/2}}-\frac{12cd^3\sqrt{c\sec(a+bx)}\sqrt{d\csc(a+bx)}}{5b}-\frac{2cd\sqrt{c}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Csc[a + b*x])^(7/2)*(c*Sec[a + b*x])^(3/2),x]
```


```
[Out] (24*c*d^5*Sqrt[c*Sec[a + b*x]])/(5*b*(d*Csc[a + b*x])^(3/2)) - (12*c*d^3*Sq
rt[d*Csc[a + b*x]]*Sqrt[c*Sec[a + b*x]])/(5*b) - (2*c*d*(d*Csc[a + b*x])^(5
/2)*Sqrt[c*Sec[a + b*x]])/(5*b) - (24*c^2*d^4*EllipticE[a - Pi/4 + b*x, 2])
/(5*b*Sqrt[d*Csc[a + b*x]]*Sqrt[c*Sec[a + b*x]]*Sqrt[Sin[2*a + 2*b*x]])
```


Rule 2625

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> -Simp[(a*b*(a*Csc[e + f*x])^(m - 1)*(b*Sec[e + f*x])^(n -
 1))/(f*(m - 1)), x] + Dist[(a^2*(m + n - 2))/(m - 1), Int[(a*Csc[e + f*x])
^(m - 2)*(b*Sec[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m,
1] && IntegersQ[2*m, 2*n] &&  !GtQ[n, m]
```

Rule 2626

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*b*(a*Csc[e + f*x])^(m - 1)*(b*Sec[e + f*x])^(n -
1))/(f*(n - 1)), x] + Dist[(b^2*(m + n - 2))/(n - 1), Int[(a*Csc[e + f*x])^
m*(b*Sec[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1
] && IntegersQ[2*m, 2*n]
```

Rule 2630

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Dist[(a*Csc[e + f*x])^m*(b*Sec[e + f*x])^n*(a*Sin[e + f*x]
)^m*(b*Cos[e + f*x])^n, Int[1/((a*Sin[e + f*x])^m*(b*Cos[e + f*x])^n), x],
x] /; FreeQ[{a, b, e, f, m, n}, x] && IntegerQ[m - 1/2] && IntegerQ[n - 1/2
]
```

Rule 2572

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[(Sqrt[a*Sin[e + f*x]]*Sqrt[b*Cos[e + f*x]])/Sqrt[Sin[2*
```

```
e + 2*f*x]], Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (d\csc(a+bx))^{7/2}(c\sec(a+bx))^{3/2}\,dx = -\frac{2cd(d\csc(a+bx))^{5/2}\sqrt{c\sec(a+bx)}}{5b}+\frac{1}{5}\left(6d^2\right)\int (d\csc(a+bx))^{3/2}(c\sec(a$$
$$= -\frac{12cd^3\sqrt{d\csc(a+bx)}\sqrt{c\sec(a+bx)}}{5b}-\frac{2cd(d\csc(a+bx))^{5/2}\sqrt{c\sec(a+bx)}}{5b}$$
$$= \frac{24cd^5\sqrt{c\sec(a+bx)}}{5b(d\csc(a+bx))^{3/2}}-\frac{12cd^3\sqrt{d\csc(a+bx)}\sqrt{c\sec(a+bx)}}{5b}-\frac{2cd(d\csc(a+$$
$$= \frac{24cd^5\sqrt{c\sec(a+bx)}}{5b(d\csc(a+bx))^{3/2}}-\frac{12cd^3\sqrt{d\csc(a+bx)}\sqrt{c\sec(a+bx)}}{5b}-\frac{2cd(d\csc(a+$$
$$= \frac{24cd^5\sqrt{c\sec(a+bx)}}{5b(d\csc(a+bx))^{3/2}}-\frac{12cd^3\sqrt{d\csc(a+bx)}\sqrt{c\sec(a+bx)}}{5b}-\frac{2cd(d\csc(a+$$
$$= \frac{24cd^5\sqrt{c\sec(a+bx)}}{5b(d\csc(a+bx))^{3/2}}-\frac{12cd^3\sqrt{d\csc(a+bx)}\sqrt{c\sec(a+bx)}}{5b}-\frac{2cd(d\csc(a+$$

Mathematica [C] time = 1.11952, size = 114, normalized size = 0.69

$$-\frac{2cd^3\tan^2(a+bx)\sqrt{c\sec(a+bx)}\sqrt{d\csc(a+bx)}\left(12\cos^2(a+bx)\sqrt[4]{-\cot^2(a+bx)}\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{4},\frac{1}{2},\text{cs}\right.\right.}{5b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Csc[a + b*x])^(7/2)*(c*Sec[a + b*x])^(3/2),x]
```

```
[Out] (-2*c*d^3*Sqrt[d*Csc[a + b*x]]*(Cot[a + b*x]^2*(6*Cos[2*(a + b*x)] + Csc[a
+ b*x]^2) + 12*Cos[a + b*x]^2*(-Cot[a + b*x]^2)^(1/4)*Hypergeometric2F1[-1/
2, 1/4, 1/2, Csc[a + b*x]^2])*Sqrt[c*Sec[a + b*x]]*Tan[a + b*x]^2)/(5*b)
```

Maple [B] time = 0.207, size = 996, normalized size = 6.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*csc(b*x+a))^(7/2)*(c*sec(b*x+a))^(3/2),x)
```

```
[Out] -1/5/b*2^(1/2)*(24*cos(b*x+a)^3*(-(-1+cos(b*x+a)-sin(b*x+a))/sin(b*x+a))^(1
/2)*((-1+cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+
a))^(1/2)*EllipticE((-(-1+cos(b*x+a)-sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1
/2))-12*cos(b*x+a)^3*(-(-1+cos(b*x+a)-sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+co
s(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*E
llipticF((-(-1+cos(b*x+a)-sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))+24*cos
(b*x+a)^2*(-(-1+cos(b*x+a)-sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a)+si
```

```
n(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*EllipticE((-
(-1+cos(b*x+a)-sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))-12*cos(b*x+a)^2*(
-(-1+cos(b*x+a)-sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a)+sin(b*x+a))/s
in(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*EllipticF((-(-1+cos(b*x
+a)-sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))-24*cos(b*x+a)*(-(-1+cos(b*x+
a)-sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1
/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*EllipticE((-(-1+cos(b*x+a)-sin(b*x+a
))/sin(b*x+a))^(1/2),1/2*2^(1/2))+12*cos(b*x+a)*(-(-1+cos(b*x+a)-sin(b*x+a)
)/sin(b*x+a))^(1/2)*((-1+cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(
b*x+a))/sin(b*x+a))^(1/2)*EllipticF((-(-1+cos(b*x+a)-sin(b*x+a))/sin(b*x+a)
)^(1/2),1/2*2^(1/2))-12*cos(b*x+a)^3*2^(1/2)-24*(-(-1+cos(b*x+a)-sin(b*x+a)
)/sin(b*x+a))^(1/2)*((-1+cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(
b*x+a))/sin(b*x+a))^(1/2)*EllipticE((-(-1+cos(b*x+a)-sin(b*x+a))/sin(b*x+a)
)^(1/2),1/2*2^(1/2))+12*(-(-1+cos(b*x+a)-sin(b*x+a))/sin(b*x+a))^(1/2)*((-1
+cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2
)*EllipticF((-(-1+cos(b*x+a)-sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))+6*c
os(b*x+a)^2*2^(1/2)+12*cos(b*x+a)*2^(1/2)-5*2^(1/2))*cos(b*x+a)*(d/sin(b*x+
a))^(7/2)*(c/cos(b*x+a))^(3/2)*sin(b*x+a)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (d \csc (bx+a))^{\frac{7}{2}} (c \sec (bx+a))^{\frac{3}{2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(b*x+a))^(7/2)*(c*sec(b*x+a))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((d*csc(b*x + a))^(7/2)*(c*sec(b*x + a))^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\sqrt{d \csc (bx+a)}\sqrt{c \sec (bx+a)}cd^3 \csc (bx+a)^3 \sec (bx+a)\,, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(b*x+a))^(7/2)*(c*sec(b*x+a))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*csc(b*x + a))*sqrt(c*sec(b*x + a))*c*d^3*csc(b*x + a)^3*sec
(b*x + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(b*x+a))**(7/2)*(c*sec(b*x+a))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (d\csc (bx+a))^{\frac{7}{2}} (c\sec (bx+a))^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(b*x+a))^(7/2)*(c*sec(b*x+a))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((d*csc(b*x + a))^(7/2)*(c*sec(b*x + a))^(3/2), x)
```

3.240 $\int (d\csc(a+bx))^{5/2}(c\sec(a+bx))^{3/2}\,dx$

Optimal. Leaf size=69

$$\frac{8cd^3\sqrt{c\sec(a+bx)}}{3b\sqrt{d\csc(a+bx)}}-\frac{2cd\sqrt{c\sec(a+bx)}(d\csc(a+bx))^{3/2}}{3b}$$

```
[Out] (8*c*d^3*Sqrt[c*Sec[a + b*x]])/(3*b*Sqrt[d*Csc[a + b*x]]) - (2*c*d*(d*Csc[a
 + b*x])^(3/2)*Sqrt[c*Sec[a + b*x]])/(3*b)
```

Rubi [A] time = 0.102838, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.08, Rules used = {2625, 2619}

$$\frac{8cd^3\sqrt{c\sec(a+bx)}}{3b\sqrt{d\csc(a+bx)}}-\frac{2cd\sqrt{c\sec(a+bx)}(d\csc(a+bx))^{3/2}}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Csc[a + b*x])^(5/2)*(c*Sec[a + b*x])^(3/2),x]
```

```
[Out] (8*c*d^3*Sqrt[c*Sec[a + b*x]])/(3*b*Sqrt[d*Csc[a + b*x]]) - (2*c*d*(d*Csc[a
 + b*x])^(3/2)*Sqrt[c*Sec[a + b*x]])/(3*b)
```

Rule 2625

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> -Simp[(a*b*(a*Csc[e + f*x])^(m - 1)*(b*Sec[e + f*x])^(n -
 1))/(f*(m - 1)), x] + Dist[(a^2*(m + n - 2))/(m - 1), Int[(a*Csc[e + f*x])
^(m - 2)*(b*Sec[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m,
1] && IntegersQ[2*m, 2*n] &&  !GtQ[n, m]
```

Rule 2619

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(a*b*(a*Csc[e + f*x])^(m - 1)*(b*Sec[e + f*x])^(n - 1
))/(f*(n - 1)), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n - 2, 0] &&
NeQ[n, 1]
```

Rubi steps

$$\begin{aligned}\int (d\csc(a+bx))^{5/2}(c\sec(a+bx))^{3/2}\,dx &= -\frac{2cd(d\csc(a+bx))^{3/2}\sqrt{c\sec(a+bx)}}{3b}+\frac{1}{3}\left(4d^2\right)\int \sqrt{d\csc(a+bx)}(c\sec(a+b\\ &= \frac{8cd^3\sqrt{c\sec(a+bx)}}{3b\sqrt{d\csc(a+bx)}}-\frac{2cd(d\csc(a+bx))^{3/2}\sqrt{c\sec(a+bx)}}{3b}\end{aligned}$$

Mathematica [A] time = 0.135484, size = 45, normalized size = 0.65

$$-\frac{2cd^3\left(\csc^2(a+bx)-4\right)\sqrt{c\sec(a+bx)}}{3b\sqrt{d\csc(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Csc[a + b*x])^(5/2)*(c*Sec[a + b*x])^(3/2),x]
```

```
[Out] (-2*c*d^3*(-4 + Csc[a + b*x]^2)*Sqrt[c*Sec[a + b*x]])/(3*b*Sqrt[d*Csc[a + b
*x]])
```

Maple [A] time = 0.162, size = 54, normalized size = 0.8

$$-\frac{\left(8\,(\cos(bx+a))^2-6\right)\cos(bx+a)\sin(bx+a)}{3\,b}\left(\frac{d}{\sin(bx+a)}\right)^{\frac{5}{2}}\left(\frac{c}{\cos(bx+a)}\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*csc(b*x+a))^(5/2)*(c*sec(b*x+a))^(3/2),x)
```

```
[Out] -2/3/b*(4*cos(b*x+a)^2-3)*cos(b*x+a)*(d/sin(b*x+a))^(5/2)*(c/cos(b*x+a))^(3
/2)*sin(b*x+a)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (d\csc(bx+a))^{\frac{5}{2}}(c\sec(bx+a))^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(b*x+a))^(5/2)*(c*sec(b*x+a))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((d*csc(b*x + a))^(5/2)*(c*sec(b*x + a))^(3/2), x)
```

Fricas [A] time = 2.06235, size = 135, normalized size = 1.96

$$-\frac{2\left(4\,cd^2\cos(bx+a)^2-3\,cd^2\right)\sqrt{\frac{c}{\cos(bx+a)}}\sqrt{\frac{d}{\sin(bx+a)}}}{3\,b\sin(bx+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(b*x+a))^(5/2)*(c*sec(b*x+a))^(3/2),x, algorithm="fricas")
```

```
[Out] -2/3*(4*c*d^2*cos(b*x + a)^2 - 3*c*d^2)*sqrt(c/cos(b*x + a))*sqrt(d/sin(b*x
 + a))/(b*sin(b*x + a))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(b*x+a))**(5/2)*(c*sec(b*x+a))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (d\csc(bx+a))^{\frac{5}{2}}(c\sec(bx+a))^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(b*x+a))^(5/2)*(c*sec(b*x+a))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((d*csc(b*x + a))^(5/2)*(c*sec(b*x + a))^(3/2), x)
```

3.241 $\int (d\csc(a+bx))^{3/2}(c\sec(a+bx))^{3/2}\,dx$

Optimal. Leaf size=125

$$-\frac{4c^2d^2E\left(a+bx-\frac{\pi}{4}\middle|2\right)}{b\sqrt{\sin(2a+2bx)}\sqrt{c\sec(a+bx)}\sqrt{d\csc(a+bx)}}+\frac{4cd^3\sqrt{c\sec(a+bx)}}{b(d\csc(a+bx))^{3/2}}-\frac{2cd\sqrt{c\sec(a+bx)}\sqrt{d\csc(a+bx)}}{b}$$

```
[Out] (4*c*d^3*Sqrt[c*Sec[a + b*x]])/(b*(d*Csc[a + b*x])^(3/2)) - (2*c*d*Sqrt[d*C
sc[a + b*x]]*Sqrt[c*Sec[a + b*x]])/b - (4*c^2*d^2*EllipticE[a - Pi/4 + b*x,
 2])/(b*Sqrt[d*Csc[a + b*x]]*Sqrt[c*Sec[a + b*x]]*Sqrt[Sin[2*a + 2*b*x]])
```

Rubi [A] time = 0.204733, antiderivative size = 125, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2625, 2626, 2630, 2572, 2639}

$$-\frac{4c^2d^2E\left(a+bx-\frac{\pi}{4}\middle|2\right)}{b\sqrt{\sin(2a+2bx)}\sqrt{c\sec(a+bx)}\sqrt{d\csc(a+bx)}}+\frac{4cd^3\sqrt{c\sec(a+bx)}}{b(d\csc(a+bx))^{3/2}}-\frac{2cd\sqrt{c\sec(a+bx)}\sqrt{d\csc(a+bx)}}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Csc[a + b*x])^(3/2)*(c*Sec[a + b*x])^(3/2),x]
```


```
[Out] (4*c*d^3*Sqrt[c*Sec[a + b*x]])/(b*(d*Csc[a + b*x])^(3/2)) - (2*c*d*Sqrt[d*C
sc[a + b*x]]*Sqrt[c*Sec[a + b*x]])/b - (4*c^2*d^2*EllipticE[a - Pi/4 + b*x,
 2])/(b*Sqrt[d*Csc[a + b*x]]*Sqrt[c*Sec[a + b*x]]*Sqrt[Sin[2*a + 2*b*x]])
```


Rule 2625

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> -Simp[(a*b*(a*Csc[e + f*x])^(m - 1)*(b*Sec[e + f*x])^(n -
 1))/(f*(m - 1)), x] + Dist[(a^2*(m + n - 2))/(m - 1), Int[(a*Csc[e + f*x])
^(m - 2)*(b*Sec[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m,
1] && IntegersQ[2*m, 2*n] &&  !GtQ[n, m]
```

Rule 2626

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*b*(a*Csc[e + f*x])^(m - 1)*(b*Sec[e + f*x])^(n -
1))/(f*(n - 1)), x] + Dist[(b^2*(m + n - 2))/(n - 1), Int[(a*Csc[e + f*x])^
m*(b*Sec[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1
] && IntegersQ[2*m, 2*n]
```

Rule 2630

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Dist[(a*Csc[e + f*x])^m*(b*Sec[e + f*x])^n*(a*Sin[e + f*x]
)^m*(b*Cos[e + f*x])^n, Int[1/((a*Sin[e + f*x])^m*(b*Cos[e + f*x])^n), x],
x] /; FreeQ[{a, b, e, f, m, n}, x] && IntegerQ[m - 1/2] && IntegerQ[n - 1/2
]
```

Rule 2572

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[(Sqrt[a*Sin[e + f*x]]*Sqrt[b*Cos[e + f*x]])/Sqrt[Sin[2*
e + 2*f*x]], Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (d\csc(a+bx))^{3/2}(c\sec(a+bx))^{3/2}\,dx = -\frac{2cd\sqrt{d\csc(a+bx)}\sqrt{c\sec(a+bx)}}{b} + \left(2d^2\right)\int \frac{(c\sec(a+bx))^{3/2}}{\sqrt{d\csc(a+bx)}}\,dx$$

$$= \frac{4cd^3\sqrt{c\sec(a+bx)}}{b(d\csc(a+bx))^{3/2}} - \frac{2cd\sqrt{d\csc(a+bx)}\sqrt{c\sec(a+bx)}}{b} - \left(4c^2d^2\right)\int \frac{}{\sqrt{d\csc(a}}$$

$$= \frac{4cd^3\sqrt{c\sec(a+bx)}}{b(d\csc(a+bx))^{3/2}} - \frac{2cd\sqrt{d\csc(a+bx)}\sqrt{c\sec(a+bx)}}{b} - \frac{\left(4c^2d^2\right)\int \sqrt{}}{\sqrt{c\cos(a+bx)}\sqrt{d\,c}}$$

$$= \frac{4cd^3\sqrt{c\sec(a+bx)}}{b(d\csc(a+bx))^{3/2}} - \frac{2cd\sqrt{d\csc(a+bx)}\sqrt{c\sec(a+bx)}}{b} - \frac{\left(4c^2d^2\right)\int \sqrt{}}{\sqrt{d\csc(a+bx)}\sqrt{c\,s}}$$

$$= \frac{4cd^3\sqrt{c\sec(a+bx)}}{b(d\csc(a+bx))^{3/2}} - \frac{2cd\sqrt{d\csc(a+bx)}\sqrt{c\sec(a+bx)}}{b} - \frac{4c^2d^2E}{b\sqrt{d\csc(a+bx)}\sqrt{c}}$$

Mathematica [C] time = 0.511562, size = 99, normalized size = 0.79

$$-\frac{2cd\tan^2(a+bx)\sqrt{c\sec(a+bx)}\sqrt{d\csc(a+bx)}\left(2\cos^2(a+bx)\sqrt[4]{-\cot^2(a+bx)}\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{4},\frac{1}{2},\csc^2(a\right.\right.}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Csc[a + b*x])^(3/2)*(c*Sec[a + b*x])^(3/2),x]
```

```
[Out] (-2*c*d*Sqrt[d*Csc[a + b*x]]*(Cos[2*(a + b*x)]*Cot[a + b*x]^2 + 2*Cos[a + b
*x]^2*(-Cot[a + b*x]^2)^(1/4)*Hypergeometric2F1[-1/2, 1/4, 1/2, Csc[a + b*x
]^2])*Sqrt[c*Sec[a + b*x]]*Tan[a + b*x]^2)/b
```

Maple [B] time = 0.187, size = 503, normalized size = 4.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*csc(b*x+a))^(3/2)*(c*sec(b*x+a))^(3/2),x)
```

```
[Out] 1/b*2^(1/2)*(4*cos(b*x+a)*(-(-1+cos(b*x+a)-sin(b*x+a))/sin(b*x+a))^(1/2)*((
-1+cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1
/2)*EllipticE((-(-1+cos(b*x+a)-sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))-2
*cos(b*x+a)*(-(-1+cos(b*x+a)-sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a)+
sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*EllipticF(
(-(-1+cos(b*x+a)-sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))+4*(-(-1+cos(b*x
+a)-sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(
1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*EllipticE((-(-1+cos(b*x+a)-sin(b*x+
a))/sin(b*x+a))^(1/2),1/2*2^(1/2))-2*(-(-1+cos(b*x+a)-sin(b*x+a))/sin(b*x+a
))^(1/2)*((-1+cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin
(b*x+a))^(1/2)*EllipticF((-(-1+cos(b*x+a)-sin(b*x+a))/sin(b*x+a))^(1/2),1/2
*2^(1/2))-2*cos(b*x+a)*2^(1/2)+2^(1/2))*cos(b*x+a)*(d/sin(b*x+a))^(3/2)*(c/
```

```
cos(b*x+a))^(3/2)*sin(b*x+a)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (d\csc(bx+a))^{\frac{3}{2}}(c\sec(bx+a))^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(b*x+a))^(3/2)*(c*sec(b*x+a))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((d*csc(b*x + a))^(3/2)*(c*sec(b*x + a))^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\sqrt{d\csc(bx+a)}\sqrt{c\sec(bx+a)}cd\csc(bx+a)\sec(bx+a)\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(b*x+a))^(3/2)*(c*sec(b*x+a))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*csc(b*x + a))*sqrt(c*sec(b*x + a))*c*d*csc(b*x + a)*sec(b*x
 + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(b*x+a))**(3/2)*(c*sec(b*x+a))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (d\csc(bx+a))^{\frac{3}{2}}(c\sec(bx+a))^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(b*x+a))^(3/2)*(c*sec(b*x+a))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((d*csc(b*x + a))^(3/2)*(c*sec(b*x + a))^(3/2), x)
```

3.242 $\int \sqrt{d\csc(a+bx)}(c\sec(a+bx))^{3/2}\,dx$

Optimal. Leaf size=31

$$\frac{2cd\sqrt{c\sec(a+bx)}}{b\sqrt{d\csc(a+bx)}}$$

```
[Out] (2*c*d*Sqrt[c*Sec[a + b*x]])/(b*Sqrt[d*Csc[a + b*x]])
```

Rubi [A] time = 0.0476698, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.04, Rules used = {2619}

$$\frac{2cd\sqrt{c\sec(a+bx)}}{b\sqrt{d\csc(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d*Csc[a + b*x]]*(c*Sec[a + b*x])^(3/2),x]
```

```
[Out] (2*c*d*Sqrt[c*Sec[a + b*x]])/(b*Sqrt[d*Csc[a + b*x]])
```

Rule 2619

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(a*b*(a*Csc[e + f*x])^(m - 1)*(b*Sec[e + f*x])^(n - 1
))/(f*(n - 1)), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n - 2, 0] &&
NeQ[n, 1]
```

Rubi steps

$$\int \sqrt{d\csc(a+bx)}(c\sec(a+bx))^{3/2}\,dx = \frac{2cd\sqrt{c\sec(a+bx)}}{b\sqrt{d\csc(a+bx)}}$$

Mathematica [A] time = 0.0613883, size = 31, normalized size = 1.

$$\frac{2cd\sqrt{c\sec(a+bx)}}{b\sqrt{d\csc(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d*Csc[a + b*x]]*(c*Sec[a + b*x])^(3/2),x]
```

```
[Out] (2*c*d*Sqrt[c*Sec[a + b*x]])/(b*Sqrt[d*Csc[a + b*x]])
```

Maple [A] time = 0.169, size = 42, normalized size = 1.4

$$2\,\frac{\cos(bx+a)\sin(bx+a)}{b}\sqrt{\frac{d}{\sin(bx+a)}}\left(\frac{c}{\cos(bx+a)}\right)^{3/2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*csc(b*x+a))^(1/2)*(c*sec(b*x+a))^(3/2),x)
```

```
[Out] 2/b*cos(b*x+a)*sin(b*x+a)*(d/sin(b*x+a))^(1/2)*(c/cos(b*x+a))^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{d \csc (bx + a)} (c \sec (bx + a))^{\frac{3}{2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(b*x+a))^(1/2)*(c*sec(b*x+a))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*csc(b*x + a))*(c*sec(b*x + a))^(3/2), x)
```

Fricas [A] time = 1.65792, size = 84, normalized size = 2.71

$$\frac{2 \, c \sqrt{\frac{c}{\cos(bx+a)}} \sqrt{\frac{d}{\sin(bx+a)}} \sin (bx + a)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(b*x+a))^(1/2)*(c*sec(b*x+a))^(3/2),x, algorithm="fricas")
```

```
[Out] 2*c*sqrt(c/cos(b*x + a))*sqrt(d/sin(b*x + a))*sin(b*x + a)/b
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(b*x+a))**(1/2)*(c*sec(b*x+a))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{d \csc (bx + a)} (c \sec (bx + a))^{\frac{3}{2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(b*x+a))^(1/2)*(c*sec(b*x+a))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*csc(b*x + a))*(c*sec(b*x + a))^(3/2), x)
```

3.243 $\int \frac{(c\sec(a+bx))^{3/2}}{\sqrt{d\csc(a+bx)}}\,dx$

Optimal. Leaf size=89

$$\frac{2cd\sqrt{c\sec(a+bx)}}{b(d\csc(a+bx))^{3/2}}-\frac{2c^2E\left(a+bx-\frac{\pi}{4}\middle|2\right)}{b\sqrt{\sin(2a+2bx)}\sqrt{c\sec(a+bx)}\sqrt{d\csc(a+bx)}}$$

```
[Out] (2*c*d*Sqrt[c*Sec[a + b*x]])/(b*(d*Csc[a + b*x])^(3/2)) - (2*c^2*EllipticE[
a - Pi/4 + b*x, 2])/(b*Sqrt[d*Csc[a + b*x]]*Sqrt[c*Sec[a + b*x]]*Sqrt[Sin[2
*a + 2*b*x]])
```

Rubi [A] time = 0.143064, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {2626, 2630, 2572, 2639}

$$\frac{2cd\sqrt{c\sec(a+bx)}}{b(d\csc(a+bx))^{3/2}}-\frac{2c^2E\left(a+bx-\frac{\pi}{4}\middle|2\right)}{b\sqrt{\sin(2a+2bx)}\sqrt{c\sec(a+bx)}\sqrt{d\csc(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*Sec[a + b*x])^(3/2)/Sqrt[d*Csc[a + b*x]],x]
```

```
[Out] (2*c*d*Sqrt[c*Sec[a + b*x]])/(b*(d*Csc[a + b*x])^(3/2)) - (2*c^2*EllipticE[
a - Pi/4 + b*x, 2])/(b*Sqrt[d*Csc[a + b*x]]*Sqrt[c*Sec[a + b*x]]*Sqrt[Sin[2
*a + 2*b*x]])
```

Rule 2626

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*b*(a*Csc[e + f*x])^(m - 1)*(b*Sec[e + f*x])^(n -
1))/(f*(n - 1)), x] + Dist[(b^2*(m + n - 2))/(n - 1), Int[(a*Csc[e + f*x])^
m*(b*Sec[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1
] && IntegersQ[2*m, 2*n]
```

Rule 2630

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Dist[(a*Csc[e + f*x])^m*(b*Sec[e + f*x])^n*(a*Sin[e + f*x]
)^m*(b*Cos[e + f*x])^n, Int[1/((a*Sin[e + f*x])^m*(b*Cos[e + f*x])^n), x],
x] /; FreeQ[{a, b, e, f, m, n}, x] && IntegerQ[m - 1/2] && IntegerQ[n - 1/2
]
```

Rule 2572

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[(Sqrt[a*Sin[e + f*x]]*Sqrt[b*Cos[e + f*x]])/Sqrt[Sin[2*
e + 2*f*x]], Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(c\sec(a+bx))^{3/2}}{\sqrt{d\csc(a+bx)}}\,dx = \frac{2cd\sqrt{c\sec(a+bx)}}{b(d\csc(a+bx))^{3/2}} - \left(2c^2\right)\int \frac{1}{\sqrt{d\csc(a+bx)}\sqrt{c\sec(a+bx)}}\,dx$$
$$= \frac{2cd\sqrt{c\sec(a+bx)}}{b(d\csc(a+bx))^{3/2}} - \frac{\left(2c^2\right)\int \sqrt{c\cos(a+bx)}\sqrt{d\sin(a+bx)}\,dx}{\sqrt{c\cos(a+bx)}\sqrt{d\csc(a+bx)}\sqrt{c\sec(a+bx)}\sqrt{d\sin(a+bx)}}$$
$$= \frac{2cd\sqrt{c\sec(a+bx)}}{b(d\csc(a+bx))^{3/2}} - \frac{\left(2c^2\right)\int \sqrt{\sin(2a+2bx)}\,dx}{\sqrt{d\csc(a+bx)}\sqrt{c\sec(a+bx)}\sqrt{\sin(2a+2bx)}}$$
$$= \frac{2cd\sqrt{c\sec(a+bx)}}{b(d\csc(a+bx))^{3/2}} - \frac{2c^2E\left(a-\frac{\pi}{4}+bx\middle|2\right)}{b\sqrt{d\csc(a+bx)}\sqrt{c\sec(a+bx)}\sqrt{\sin(2a+2bx)}}$$

Mathematica [C] time = 0.366759, size = 66, normalized size = 0.74

$$-\frac{2cd\sqrt{c\sec(a+bx)}\left(\sqrt[4]{-\cot^2(a+bx)}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{4},\frac{1}{2},\csc^2(a+bx)\right)-1\right)}{b(d\csc(a+bx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*Sec[a + b*x])^(3/2)/Sqrt[d*Csc[a + b*x]],x]
```

```
[Out] (-2*c*d*(-1 + (-Cot[a + b*x]^2)^(1/4)*Hypergeometric2F1[-1/2, 1/4, 1/2, Csc
[a + b*x]^2])*Sqrt[c*Sec[a + b*x]])/(b*(d*Csc[a + b*x])^(3/2))
```

Maple [B] time = 0.191, size = 505, normalized size = 5.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*sec(b*x+a))^(3/2)/(d*csc(b*x+a))^(1/2),x)
```

```
[Out] 1/b*2^(1/2)*(2*cos(b*x+a)*(-(-1+cos(b*x+a)-sin(b*x+a))/sin(b*x+a))^(1/2)*((
-1+cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1
/2)*EllipticE((-(-1+cos(b*x+a)-sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))-c
os(b*x+a)*(-(-1+cos(b*x+a)-sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a)+si
n(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*EllipticF((-
(-1+cos(b*x+a)-sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))+2*(-(-1+cos(b*x+a
)-sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/
2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*EllipticE((-(-1+cos(b*x+a)-sin(b*x+a)
)/sin(b*x+a))^(1/2),1/2*2^(1/2))-(-(-1+cos(b*x+a)-sin(b*x+a))/sin(b*x+a))^(
1/2)*((-1+cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x
+a))^(1/2)*EllipticF((-(-1+cos(b*x+a)-sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(
1/2))-cos(b*x+a)*2^(1/2)+2^(1/2))*cos(b*x+a)*(c/cos(b*x+a))^(3/2)/(d/sin(b*
x+a))^(1/2)/sin(b*x+a)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(c\sec(bx+a))^{\frac{3}{2}}}{\sqrt{d\csc(bx+a)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sec(b*x+a))^(3/2)/(d*csc(b*x+a))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((c*sec(b*x + a))^(3/2)/sqrt(d*csc(b*x + a)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{d \csc(bx+a)}\sqrt{c \sec(bx+a)}c \sec(bx+a)}{d \csc(bx+a)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sec(b*x+a))^(3/2)/(d*csc(b*x+a))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*csc(b*x + a))*sqrt(c*sec(b*x + a))*c*sec(b*x + a)/(d*csc(b*
x + a)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sec(b*x+a))**(3/2)/(d*csc(b*x+a))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(c \sec(bx+a))^{\frac{3}{2}}}{\sqrt{d \csc(bx+a)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sec(b*x+a))^(3/2)/(d*csc(b*x+a))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((c*sec(b*x + a))^(3/2)/sqrt(d*csc(b*x + a)), x)
```

3.244 $\int \frac{(c\sec(a+bx))^{3/2}}{(d\csc(a+bx))^{3/2}}\,dx$

Optimal. Leaf size=327

$$\frac{c^2\tan^{-1}\left(1-\sqrt{2}\sqrt{\tan(a+bx)}\right)\sqrt{\tan(a+bx)}\sqrt{d\csc(a+bx)}}{\sqrt{2}bd^2\sqrt{c\sec(a+bx)}}-\frac{c^2\tan^{-1}\left(\sqrt{2}\sqrt{\tan(a+bx)}+1\right)\sqrt{\tan(a+bx)}\sqrt{d\csc(a}}{\sqrt{2}bd^2\sqrt{c\sec(a+bx)}}$$

```
[Out] (2*c*Sqrt[c*Sec[a + b*x]])/(b*d*Sqrt[d*Csc[a + b*x]]) + (c^2*ArcTan[1 - Sqr
t[2]*Sqrt[Tan[a + b*x]]]*Sqrt[d*Csc[a + b*x]]*Sqrt[Tan[a + b*x]])/(Sqrt[2]*
b*d^2*Sqrt[c*Sec[a + b*x]]) - (c^2*ArcTan[1 + Sqrt[2]*Sqrt[Tan[a + b*x]]]*S
qrt[d*Csc[a + b*x]]*Sqrt[Tan[a + b*x]])/(Sqrt[2]*b*d^2*Sqrt[c*Sec[a + b*x]]
) + (c^2*Sqrt[d*Csc[a + b*x]]*Log[1 - Sqrt[2]*Sqrt[Tan[a + b*x]] + Tan[a +
b*x]]*Sqrt[Tan[a + b*x]])/(2*Sqrt[2]*b*d^2*Sqrt[c*Sec[a + b*x]]) - (c^2*Sqr
t[d*Csc[a + b*x]]*Log[1 + Sqrt[2]*Sqrt[Tan[a + b*x]] + Tan[a + b*x]]*Sqrt[T
an[a + b*x]])/(2*Sqrt[2]*b*d^2*Sqrt[c*Sec[a + b*x]])
```

Rubi [A] time = 0.21939, antiderivative size = 327, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 10, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {2624, 2629, 3476, 329, 211, 1165, 628, 1162, 617, 204}

$$\frac{c^2\tan^{-1}\left(1-\sqrt{2}\sqrt{\tan(a+bx)}\right)\sqrt{\tan(a+bx)}\sqrt{d\csc(a+bx)}}{\sqrt{2}bd^2\sqrt{c\sec(a+bx)}}-\frac{c^2\tan^{-1}\left(\sqrt{2}\sqrt{\tan(a+bx)}+1\right)\sqrt{\tan(a+bx)}\sqrt{d\csc(a}}{\sqrt{2}bd^2\sqrt{c\sec(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*Sec[a + b*x])^(3/2)/(d*Csc[a + b*x])^(3/2),x]
```

```
[Out] (2*c*Sqrt[c*Sec[a + b*x]])/(b*d*Sqrt[d*Csc[a + b*x]]) + (c^2*ArcTan[1 - Sqr
t[2]*Sqrt[Tan[a + b*x]]]*Sqrt[d*Csc[a + b*x]]*Sqrt[Tan[a + b*x]])/(Sqrt[2]*
b*d^2*Sqrt[c*Sec[a + b*x]]) - (c^2*ArcTan[1 + Sqrt[2]*Sqrt[Tan[a + b*x]]]*S
qrt[d*Csc[a + b*x]]*Sqrt[Tan[a + b*x]])/(Sqrt[2]*b*d^2*Sqrt[c*Sec[a + b*x]]
) + (c^2*Sqrt[d*Csc[a + b*x]]*Log[1 - Sqrt[2]*Sqrt[Tan[a + b*x]] + Tan[a +
b*x]]*Sqrt[Tan[a + b*x]])/(2*Sqrt[2]*b*d^2*Sqrt[c*Sec[a + b*x]]) - (c^2*Sqr
t[d*Csc[a + b*x]]*Log[1 + Sqrt[2]*Sqrt[Tan[a + b*x]] + Tan[a + b*x]]*Sqrt[T
an[a + b*x]])/(2*Sqrt[2]*b*d^2*Sqrt[c*Sec[a + b*x]])
```

Rule 2624

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(b*(a*Csc[e + f*x])^(m + 1)*(b*Sec[e + f*x])^(n - 1))
/(f*a*(n - 1)), x] + Dist[(b^2*(m + 1))/(a^2*(n - 1)), Int[(a*Csc[e + f*x])
^(m + 2)*(b*Sec[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ[
n, 1] && LtQ[m, -1] && IntegersQ[2*m, 2*n]
```

Rule 2629

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Dist[((a*Csc[e + f*x])^m*(b*Sec[e + f*x])^n)/Tan[e + f*x]^
n, Int[Tan[e + f*x]^n, x], x] /; FreeQ[{a, b, e, f, m, n}, x] &&  !IntegerQ
[n] && EqQ[m + n, 0]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
```

```
IntegerQ[n]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{(c\sec(a+bx))^{3/2}}{(d\csc(a+bx))^{3/2}}\,dx = \frac{2c\sqrt{c\sec(a+bx)}}{bd\sqrt{d\csc(a+bx)}} - \frac{c^2\int \frac{\sqrt{d\csc(a+bx)}}{\sqrt{c\sec(a+bx)}}\,dx}{d^2}$$

$$= \frac{2c\sqrt{c\sec(a+bx)}}{bd\sqrt{d\csc(a+bx)}} - \frac{\left(c^2\sqrt{d\csc(a+bx)}\sqrt{\tan(a+bx)}\right)\int \frac{1}{\sqrt{\tan(a+bx)}}\,dx}{d^2\sqrt{c\sec(a+bx)}}$$

$$= \frac{2c\sqrt{c\sec(a+bx)}}{bd\sqrt{d\csc(a+bx)}} - \frac{\left(c^2\sqrt{d\csc(a+bx)}\sqrt{\tan(a+bx)}\right)\text{Subst}\left(\int \frac{1}{\sqrt{x}(1+x^2)}\,dx, x, \tan(a+bx)\right)}{bd^2\sqrt{c\sec(a+bx)}}$$

$$= \frac{2c\sqrt{c\sec(a+bx)}}{bd\sqrt{d\csc(a+bx)}} - \frac{\left(2c^2\sqrt{d\csc(a+bx)}\sqrt{\tan(a+bx)}\right)\text{Subst}\left(\int \frac{1}{1+x^4}\,dx, x, \sqrt{\tan(a+bx)}\right)}{bd^2\sqrt{c\sec(a+bx)}}$$

$$= \frac{2c\sqrt{c\sec(a+bx)}}{bd\sqrt{d\csc(a+bx)}} - \frac{\left(c^2\sqrt{d\csc(a+bx)}\sqrt{\tan(a+bx)}\right)\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \sqrt{\tan(a+bx)}\right)}{bd^2\sqrt{c\sec(a+bx)}} -$$

$$= \frac{2c\sqrt{c\sec(a+bx)}}{bd\sqrt{d\csc(a+bx)}} - \frac{\left(c^2\sqrt{d\csc(a+bx)}\sqrt{\tan(a+bx)}\right)\text{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}\,dx, x, \sqrt{\tan(a+bx)}\right)}{2bd^2\sqrt{c\sec(a+bx)}}$$

$$= \frac{2c\sqrt{c\sec(a+bx)}}{bd\sqrt{d\csc(a+bx)}} + \frac{c^2\sqrt{d\csc(a+bx)}\log\left(1-\sqrt{2}\sqrt{\tan(a+bx)}+\tan(a+bx)\right)\sqrt{\tan(a+b}}{2\sqrt{2}bd^2\sqrt{c\sec(a+bx)}}$$

$$= \frac{2c\sqrt{c\sec(a+bx)}}{bd\sqrt{d\csc(a+bx)}} + \frac{c^2\tan^{-1}\left(1-\sqrt{2}\sqrt{\tan(a+bx)}\right)\sqrt{d\csc(a+bx)}\sqrt{\tan(a+bx)}}{\sqrt{2}bd^2\sqrt{c\sec(a+bx)}} - \frac{c^2\tan}{}$$

Mathematica [C] time = 0.242222, size = 64, normalized size = 0.2

$$\frac{2c\sqrt{c\sec(a+bx)}\left(\cot^2(a+bx)\text{Hypergeometric2F1}\left(\frac{3}{4},1,\frac{7}{4},-\cot^2(a+bx)\right)+3\right)}{3bd\sqrt{d\csc(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*Sec[a + b*x])^(3/2)/(d*Csc[a + b*x])^(3/2),x]
```

```
[Out] (2*c*(3 + Cot[a + b*x]^2*Hypergeometric2F1[3/4, 1, 7/4, -Cot[a + b*x]^2])*S
qrt[c*Sec[a + b*x]])/(3*b*d*Sqrt[d*Csc[a + b*x]])
```

Maple [C] time = 0.171, size = 656, normalized size = 2.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*sec(b*x+a))^(3/2)/(d*csc(b*x+a))^(3/2),x)
```

```
[Out] 1/2/b*2^(1/2)*(I*sin(b*x+a)*EllipticPi((-(-1+cos(b*x+a)-sin(b*x+a))/sin(b*x
+a))^(1/2),1/2-1/2*I,1/2*2^(1/2))*(-(-1+cos(b*x+a)-sin(b*x+a))/sin(b*x+a))^
(1/2)*((-1+cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*
x+a))^(1/2)-I*sin(b*x+a)*(-(-1+cos(b*x+a)-sin(b*x+a))/sin(b*x+a))^(1/2)*((-
1+cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/
2)*EllipticPi((-(-1+cos(b*x+a)-sin(b*x+a))/sin(b*x+a))^(1/2),1/2+1/2*I,1/2*
2^(1/2))+sin(b*x+a)*EllipticPi((-(-1+cos(b*x+a)-sin(b*x+a))/sin(b*x+a))^(1/
2),1/2-1/2*I,1/2*2^(1/2))*(-(-1+cos(b*x+a)-sin(b*x+a))/sin(b*x+a))^(1/2)*((
```

```
-1+cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1
/2)+sin(b*x+a)*(-(-1+cos(b*x+a)-sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+
a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*Ellipti
cPi((-(-1+cos(b*x+a)-sin(b*x+a))/sin(b*x+a))^(1/2),1/2+1/2*I,1/2*2^(1/2))-2
*sin(b*x+a)*(-(-1+cos(b*x+a)-sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a)+
sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*EllipticF(
(-(-1+cos(b*x+a)-sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))+2*cos(b*x+a)*2^
(1/2)-2*2^(1/2))*cos(b*x+a)*(c/cos(b*x+a))^(3/2)/(-1+cos(b*x+a))/(d/sin(b*x
+a))^(3/2)/sin(b*x+a)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(c\sec(bx+a))^{\frac{3}{2}}}{(d\csc(bx+a))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sec(b*x+a))^(3/2)/(d*csc(b*x+a))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((c*sec(b*x + a))^(3/2)/(d*csc(b*x + a))^(3/2), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sec(b*x+a))^(3/2)/(d*csc(b*x+a))^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sec(b*x+a))**(3/2)/(d*csc(b*x+a))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(c\sec(bx+a))^{\frac{3}{2}}}{(d\csc(bx+a))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sec(b*x+a))^(3/2)/(d*csc(b*x+a))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((c*sec(b*x + a))^(3/2)/(d*csc(b*x + a))^(3/2), x)
```

3.245 $\int \frac{(c\sec(a+bx))^{3/2}}{(d\csc(a+bx))^{5/2}}\,dx$

Optimal. Leaf size=94

$$\frac{2c\sqrt{c\sec(a+bx)}}{bd(d\csc(a+bx))^{3/2}}-\frac{3c^2E\left(a+bx-\frac{\pi}{4}\middle|2\right)}{bd^2\sqrt{\sin(2a+2bx)}\sqrt{c\sec(a+bx)}\sqrt{d\csc(a+bx)}}$$

```
[Out] (2*c*Sqrt[c*Sec[a + b*x]])/(b*d*(d*Csc[a + b*x])^(3/2)) - (3*c^2*EllipticE[
a - Pi/4 + b*x, 2])/(b*d^2*Sqrt[d*Csc[a + b*x]]*Sqrt[c*Sec[a + b*x]]*Sqrt[S
in[2*a + 2*b*x]])
```

Rubi [A] time = 0.152684, antiderivative size = 94, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {2624, 2630, 2572, 2639}

$$\frac{2c\sqrt{c\sec(a+bx)}}{bd(d\csc(a+bx))^{3/2}}-\frac{3c^2E\left(a+bx-\frac{\pi}{4}\middle|2\right)}{bd^2\sqrt{\sin(2a+2bx)}\sqrt{c\sec(a+bx)}\sqrt{d\csc(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*Sec[a + b*x])^(3/2)/(d*Csc[a + b*x])^(5/2),x]
```

```
[Out] (2*c*Sqrt[c*Sec[a + b*x]])/(b*d*(d*Csc[a + b*x])^(3/2)) - (3*c^2*EllipticE[
a - Pi/4 + b*x, 2])/(b*d^2*Sqrt[d*Csc[a + b*x]]*Sqrt[c*Sec[a + b*x]]*Sqrt[S
in[2*a + 2*b*x]])
```

Rule 2624

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(b*(a*Csc[e + f*x])^(m + 1)*(b*Sec[e + f*x])^(n - 1))
/(f*a*(n - 1)), x] + Dist[(b^2*(m + 1))/(a^2*(n - 1)), Int[(a*Csc[e + f*x])
^(m + 2)*(b*Sec[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ[
n, 1] && LtQ[m, -1] && IntegersQ[2*m, 2*n]
```

Rule 2630

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Dist[(a*Csc[e + f*x])^m*(b*Sec[e + f*x])^n*(a*Sin[e + f*x]
)^m*(b*Cos[e + f*x])^n, Int[1/((a*Sin[e + f*x])^m*(b*Cos[e + f*x])^n), x],
x] /; FreeQ[{a, b, e, f, m, n}, x] && IntegerQ[m - 1/2] && IntegerQ[n - 1/2
]
```

Rule 2572

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[(Sqrt[a*Sin[e + f*x]]*Sqrt[b*Cos[e + f*x]])/Sqrt[Sin[2*
e + 2*f*x]], Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{(c\sec(a+bx))^{3/2}}{(d\csc(a+bx))^{5/2}}\,dx &= \frac{2c\sqrt{c\sec(a+bx)}}{bd(d\csc(a+bx))^{3/2}} - \frac{(3c^2)\int \frac{1}{\sqrt{d\csc(a+bx)}\sqrt{c\sec(a+bx)}}\,dx}{d^2} \\ &= \frac{2c\sqrt{c\sec(a+bx)}}{bd(d\csc(a+bx))^{3/2}} - \frac{(3c^2)\int \sqrt{c\cos(a+bx)}\sqrt{d\sin(a+bx)}\,dx}{d^2\sqrt{c\cos(a+bx)}\sqrt{d\csc(a+bx)}\sqrt{c\sec(a+bx)}\sqrt{d\sin(a+bx)}} \\ &= \frac{2c\sqrt{c\sec(a+bx)}}{bd(d\csc(a+bx))^{3/2}} - \frac{(3c^2)\int \sqrt{\sin(2a+2bx)}\,dx}{d^2\sqrt{d\csc(a+bx)}\sqrt{c\sec(a+bx)}\sqrt{\sin(2a+2bx)}} \\ &= \frac{2c\sqrt{c\sec(a+bx)}}{bd(d\csc(a+bx))^{3/2}} - \frac{3c^2E\left(a-\frac{\pi}{4}+bx\middle|2\right)}{bd^2\sqrt{d\csc(a+bx)}\sqrt{c\sec(a+bx)}\sqrt{\sin(2a+2bx)}}\end{aligned}$$

Mathematica [C] time = 0.406607, size = 69, normalized size = 0.73

$$-\frac{c\sqrt{c\sec(a+bx)}\left(3\sqrt[4]{-\cot^2(a+bx)}\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{4},\frac{1}{2},\csc^2(a+bx)\right)-2\right)}{bd(d\csc(a+bx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*Sec[a + b*x])^(3/2)/(d*Csc[a + b*x])^(5/2),x]
```

```
[Out] -((c*(-2 + 3*(-Cot[a + b*x]^2)^(1/4)*Hypergeometric2F1[-1/2, 1/4, 1/2, Csc[
a + b*x]^2])*Sqrt[c*Sec[a + b*x]])/(b*d*(d*Csc[a + b*x])^(3/2)))
```

Maple [B] time = 0.17, size = 520, normalized size = 5.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*sec(b*x+a))^(3/2)/(d*csc(b*x+a))^(5/2),x)
```

```
[Out] 1/2/b*2^(1/2)*(6*cos(b*x+a)*(-(-1+cos(b*x+a)-sin(b*x+a))/sin(b*x+a))^(1/2)*
((-1+cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^
(1/2)*EllipticE((-(-1+cos(b*x+a)-sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))
-3*cos(b*x+a)*(-(-1+cos(b*x+a)-sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a
)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*Elliptic
F((-(-1+cos(b*x+a)-sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))+6*(-(-1+cos(b
*x+a)-sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a)+sin(b*x+a))/sin(b*x+a))
^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*EllipticE((-(-1+cos(b*x+a)-sin(b*
x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))-3*(-(-1+cos(b*x+a)-sin(b*x+a))/sin(b*x
+a))^(1/2)*((-1+cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/s
in(b*x+a))^(1/2)*EllipticF((-(-1+cos(b*x+a)-sin(b*x+a))/sin(b*x+a))^(1/2),1
/2*2^(1/2))+cos(b*x+a)^2*2^(1/2)-3*cos(b*x+a)*2^(1/2)+2*2^(1/2))*cos(b*x+a)
*(c/cos(b*x+a))^(3/2)/sin(b*x+a)^3/(d/sin(b*x+a))^(5/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(c\sec(bx+a))^{\frac{3}{2}}}{(d\csc(bx+a))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sec(b*x+a))^(3/2)/(d*csc(b*x+a))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((c*sec(b*x + a))^(3/2)/(d*csc(b*x + a))^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{d\csc\left(bx+a\right)}\sqrt{c\sec\left(bx+a\right)}c\sec\left(bx+a\right)}{d^3\csc\left(bx+a\right)^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sec(b*x+a))^(3/2)/(d*csc(b*x+a))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*csc(b*x + a))*sqrt(c*sec(b*x + a))*c*sec(b*x + a)/(d^3*csc(
b*x + a)^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sec(b*x+a))**(3/2)/(d*csc(b*x+a))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(c\sec\left(bx+a\right)\right)^{\frac{3}{2}}}{\left(d\csc\left(bx+a\right)\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sec(b*x+a))^(3/2)/(d*csc(b*x+a))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((c*sec(b*x + a))^(3/2)/(d*csc(b*x + a))^(5/2), x)
```

3.246 $\int (d\csc(a+bx))^{9/2}(c\sec(a+bx))^{5/2}\,dx$

Optimal. Leaf size=166

$$\frac{40c^2d^4\sqrt{\sin(2a+2bx)}\,\text{EllipticF}\left(a+bx-\frac{\pi}{4},2\right)\sqrt{c\sec(a+bx)}\sqrt{d\csc(a+bx)}}{21b}+\frac{40cd^5(c\sec(a+bx))^{3/2}}{21b\sqrt{d\csc(a+bx)}}-\frac{20cd^3(c\sec}{}$$

```
[Out] (40*c*d^5*(c*Sec[a + b*x])^(3/2))/(21*b*Sqrt[d*Csc[a + b*x]]) - (20*c*d^3*(
d*Csc[a + b*x])^(3/2)*(c*Sec[a + b*x])^(3/2))/(21*b) - (2*c*d*(d*Csc[a + b*
x])^(7/2)*(c*Sec[a + b*x])^(3/2))/(7*b) + (40*c^2*d^4*Sqrt[d*Csc[a + b*x]]*
EllipticF[a - Pi/4 + b*x, 2]*Sqrt[c*Sec[a + b*x]]*Sqrt[Sin[2*a + 2*b*x]])/(
21*b)
```

Rubi [A] time = 0.264753, antiderivative size = 166, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2625, 2626, 2630, 2573, 2641}

$$\frac{40c^2d^4\sqrt{\sin(2a+2bx)}F\left(a+bx-\frac{\pi}{4}\middle|2\right)\sqrt{c\sec(a+bx)}\sqrt{d\csc(a+bx)}}{21b}+\frac{40cd^5(c\sec(a+bx))^{3/2}}{21b\sqrt{d\csc(a+bx)}}-\frac{20cd^3(c\sec(a+bx}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Csc[a + b*x])^(9/2)*(c*Sec[a + b*x])^(5/2),x]
```

```
[Out] (40*c*d^5*(c*Sec[a + b*x])^(3/2))/(21*b*Sqrt[d*Csc[a + b*x]]) - (20*c*d^3*(
d*Csc[a + b*x])^(3/2)*(c*Sec[a + b*x])^(3/2))/(21*b) - (2*c*d*(d*Csc[a + b*
x])^(7/2)*(c*Sec[a + b*x])^(3/2))/(7*b) + (40*c^2*d^4*Sqrt[d*Csc[a + b*x]]*
EllipticF[a - Pi/4 + b*x, 2]*Sqrt[c*Sec[a + b*x]]*Sqrt[Sin[2*a + 2*b*x]])/(
21*b)
```

Rule 2625

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> -Simp[(a*b*(a*Csc[e + f*x])^(m - 1)*(b*Sec[e + f*x])^(n -
 1))/(f*(m - 1)), x] + Dist[(a^2*(m + n - 2))/(m - 1), Int[(a*Csc[e + f*x])
^(m - 2)*(b*Sec[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m,
1] && IntegersQ[2*m, 2*n] &&  !GtQ[n, m]
```

Rule 2626

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*b*(a*Csc[e + f*x])^(m - 1)*(b*Sec[e + f*x])^(n -
1))/(f*(n - 1)), x] + Dist[(b^2*(m + n - 2))/(n - 1), Int[(a*Csc[e + f*x])^
m*(b*Sec[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1
] && IntegersQ[2*m, 2*n]
```

Rule 2630

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Dist[(a*Csc[e + f*x])^m*(b*Sec[e + f*x])^n*(a*Sin[e + f*x]
)^m*(b*Cos[e + f*x])^n, Int[1/((a*Sin[e + f*x])^m*(b*Cos[e + f*x])^n), x],
x] /; FreeQ[{a, b, e, f, m, n}, x] && IntegerQ[m - 1/2] && IntegerQ[n - 1/2
]
```

Rule 2573

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (d\csc(a+bx))^{9/2}(c\sec(a+bx))^{5/2}\,dx = -\frac{2cd(d\csc(a+bx))^{7/2}(c\sec(a+bx))^{3/2}}{7b} + \frac{1}{7}\left(10d^2\right)\int (d\csc(a+bx))^{5/2}(c\sec($$

$$= -\frac{20cd^3(d\csc(a+bx))^{3/2}(c\sec(a+bx))^{3/2}}{21b} - \frac{2cd(d\csc(a+bx))^{7/2}(c\sec(a+bx$$

$$= \frac{40cd^5(c\sec(a+bx))^{3/2}}{21b\sqrt{d\csc(a+bx)}} - \frac{20cd^3(d\csc(a+bx))^{3/2}(c\sec(a+bx))^{3/2}}{21b} - \frac{2cd(d\csc$$

$$= \frac{40cd^5(c\sec(a+bx))^{3/2}}{21b\sqrt{d\csc(a+bx)}} - \frac{20cd^3(d\csc(a+bx))^{3/2}(c\sec(a+bx))^{3/2}}{21b} - \frac{2cd(d\csc$$

$$= \frac{40cd^5(c\sec(a+bx))^{3/2}}{21b\sqrt{d\csc(a+bx)}} - \frac{20cd^3(d\csc(a+bx))^{3/2}(c\sec(a+bx))^{3/2}}{21b} - \frac{2cd(d\csc$$

$$= \frac{40cd^5(c\sec(a+bx))^{3/2}}{21b\sqrt{d\csc(a+bx)}} - \frac{20cd^3(d\csc(a+bx))^{3/2}(c\sec(a+bx))^{3/2}}{21b} - \frac{2cd(d\csc$$

Mathematica [C] time = 1.42326, size = 92, normalized size = 0.55

$$-\frac{2cd^5(c\sec(a+bx))^{3/2}\left(20\left(-\cot^2(a+bx)\right)^{3/4}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{3}{2},\csc^2(a+bx)\right)+\cot^2(a+bx)\left(3\csc^2(a+b\right.\right.}{21b\sqrt{d\csc(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Csc[a + b*x])^(9/2)*(c*Sec[a + b*x])^(5/2),x]
```

```
[Out] (-2*c*d^5*(-7 + Cot[a + b*x]^2*(13 + 3*Csc[a + b*x]^2) + 20*(-Cot[a + b*x]^
2)^(3/4)*Hypergeometric2F1[1/2, 3/4, 3/2, Csc[a + b*x]^2])*(c*Sec[a + b*x])
^(3/2))/(21*b*Sqrt[d*Csc[a + b*x]])
```

Maple [B] time = 0.232, size = 563, normalized size = 3.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*csc(b*x+a))^(9/2)*(c*sec(b*x+a))^(5/2),x)
```

```
[Out] -1/21/b*2^(1/2)*(40*(-(-1+cos(b*x+a)-sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos
(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*El
lipticF((-(-1+cos(b*x+a)-sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))*cos(b*x
+a)^4*sin(b*x+a)+40*cos(b*x+a)^3*sin(b*x+a)*(-(-1+cos(b*x+a)-sin(b*x+a))/si
n(b*x+a))^(1/2)*((-1+cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+
```

```
a))/sin(b*x+a))^(1/2)*EllipticF((-(-1+cos(b*x+a)-sin(b*x+a))/sin(b*x+a))^(1
/2),1/2*2^(1/2))-40*cos(b*x+a)^2*sin(b*x+a)*(-(-1+cos(b*x+a)-sin(b*x+a))/si
n(b*x+a))^(1/2)*((-1+cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+
a))/sin(b*x+a))^(1/2)*EllipticF((-(-1+cos(b*x+a)-sin(b*x+a))/sin(b*x+a))^(1
/2),1/2*2^(1/2))-40*cos(b*x+a)*sin(b*x+a)*(-(-1+cos(b*x+a)-sin(b*x+a))/sin(
b*x+a))^(1/2)*((-1+cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a)
)/sin(b*x+a))^(1/2)*EllipticF((-(-1+cos(b*x+a)-sin(b*x+a))/sin(b*x+a))^(1/2
),1/2*2^(1/2))-20*2^(1/2)*cos(b*x+a)^4+30*cos(b*x+a)^2*2^(1/2)-7*2^(1/2))*c
os(b*x+a)*(d/sin(b*x+a))^(9/2)*(c/cos(b*x+a))^(5/2)*sin(b*x+a)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (d\csc(bx+a))^{\frac{9}{2}}(c\sec(bx+a))^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(b*x+a))^(9/2)*(c*sec(b*x+a))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((d*csc(b*x + a))^(9/2)*(c*sec(b*x + a))^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\sqrt{d\csc(bx+a)}\sqrt{c\sec(bx+a)}c^2d^4\csc(bx+a)^4\sec(bx+a)^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(b*x+a))^(9/2)*(c*sec(b*x+a))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*csc(b*x + a))*sqrt(c*sec(b*x + a))*c^2*d^4*csc(b*x + a)^4*s
ec(b*x + a)^2, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(b*x+a))**(9/2)*(c*sec(b*x+a))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (d\csc(bx+a))^{\frac{9}{2}}(c\sec(bx+a))^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(b*x+a))^(9/2)*(c*sec(b*x+a))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((d*csc(b*x + a))^(9/2)*(c*sec(b*x + a))^(5/2), x)
```

3.247 $\int (d\csc(a+bx))^{7/2}(c\sec(a+bx))^{5/2}\,dx$

Optimal. Leaf size=106

$$-\frac{64c^3d^3\sqrt{d\csc(a+bx)}}{15b\sqrt{c\sec(a+bx)}}+\frac{16cd^3(c\sec(a+bx))^{3/2}\sqrt{d\csc(a+bx)}}{15b}-\frac{2cd(c\sec(a+bx))^{3/2}(d\csc(a+bx))^{5/2}}{5b}$$

```
[Out] (-64*c^3*d^3*Sqrt[d*Csc[a + b*x]])/(15*b*Sqrt[c*Sec[a + b*x]]) + (16*c*d^3*
Sqrt[d*Csc[a + b*x]]*(c*Sec[a + b*x])^(3/2))/(15*b) - (2*c*d*(d*Csc[a + b*x
])^(5/2)*(c*Sec[a + b*x])^(3/2))/(5*b)
```

Rubi [A] time = 0.162101, antiderivative size = 106, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {2625, 2626, 2619}

$$-\frac{64c^3d^3\sqrt{d\csc(a+bx)}}{15b\sqrt{c\sec(a+bx)}}+\frac{16cd^3(c\sec(a+bx))^{3/2}\sqrt{d\csc(a+bx)}}{15b}-\frac{2cd(c\sec(a+bx))^{3/2}(d\csc(a+bx))^{5/2}}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Csc[a + b*x])^(7/2)*(c*Sec[a + b*x])^(5/2),x]
```

```
[Out] (-64*c^3*d^3*Sqrt[d*Csc[a + b*x]])/(15*b*Sqrt[c*Sec[a + b*x]]) + (16*c*d^3*
Sqrt[d*Csc[a + b*x]]*(c*Sec[a + b*x])^(3/2))/(15*b) - (2*c*d*(d*Csc[a + b*x
])^(5/2)*(c*Sec[a + b*x])^(3/2))/(5*b)
```

Rule 2625

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> -Simp[(a*b*(a*Csc[e + f*x])^(m - 1)*(b*Sec[e + f*x])^(n -
 1))/(f*(m - 1)), x] + Dist[(a^2*(m + n - 2))/(m - 1), Int[(a*Csc[e + f*x])
^(m - 2)*(b*Sec[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m,
1] && IntegersQ[2*m, 2*n] &&  !GtQ[n, m]
```

Rule 2626

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*b*(a*Csc[e + f*x])^(m - 1)*(b*Sec[e + f*x])^(n -
1))/(f*(n - 1)), x] + Dist[(b^2*(m + n - 2))/(n - 1), Int[(a*Csc[e + f*x])^
m*(b*Sec[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1
] && IntegersQ[2*m, 2*n]
```

Rule 2619

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(a*b*(a*Csc[e + f*x])^(m - 1)*(b*Sec[e + f*x])^(n - 1
))/(f*(n - 1)), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n - 2, 0] &&
NeQ[n, 1]
```

Rubi steps

$$\int (d\csc(a+bx))^{7/2}(c\sec(a+bx))^{5/2}\,dx = -\frac{2cd(d\csc(a+bx))^{5/2}(c\sec(a+bx))^{3/2}}{5b} + \frac{1}{5}\left(8d^2\right)\int (d\csc(a+bx))^{3/2}(c\sec$$
$$= \frac{16cd^3\sqrt{d\csc(a+bx)}(c\sec(a+bx))^{3/2}}{15b} - \frac{2cd(d\csc(a+bx))^{5/2}(c\sec(a+bx)}{5b}$$
$$= -\frac{64c^3d^3\sqrt{d\csc(a+bx)}}{15b\sqrt{c\sec(a+bx)}} + \frac{16cd^3\sqrt{d\csc(a+bx)}(c\sec(a+bx))^{3/2}}{15b} - \frac{2cd(d\,cs}{}$$

Mathematica [A] time = 0.193343, size = 57, normalized size = 0.54

$$-\frac{2cd^3(c\sec(a+bx))^{3/2}\sqrt{d\csc(a+bx)}\left(32\cos^2(a+bx)+3\cot^2(a+bx)-5\right)}{15b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Csc[a + b*x])^(7/2)*(c*Sec[a + b*x])^(5/2),x]
```

```
[Out] (-2*c*d^3*(-5 + 32*Cos[a + b*x]^2 + 3*Cot[a + b*x]^2)*Sqrt[d*Csc[a + b*x]]*
(c*Sec[a + b*x])^(3/2))/(15*b)
```

Maple [A] time = 0.165, size = 64, normalized size = 0.6

$$\frac{\left(64\left(\cos(bx+a)\right)^4-80\left(\cos(bx+a)\right)^2+10\right)\cos(bx+a)\sin(bx+a)}{15\,b}\left(\frac{d}{\sin(bx+a)}\right)^{\frac{7}{2}}\left(\frac{c}{\cos(bx+a)}\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*csc(b*x+a))^(7/2)*(c*sec(b*x+a))^(5/2),x)
```

```
[Out] 2/15/b*(32*cos(b*x+a)^4-40*cos(b*x+a)^2+5)*cos(b*x+a)*(d/sin(b*x+a))^(7/2)*
(c/cos(b*x+a))^(5/2)*sin(b*x+a)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (d\csc(bx+a))^{\frac{7}{2}}(c\sec(bx+a))^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(b*x+a))^(7/2)*(c*sec(b*x+a))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((d*csc(b*x + a))^(7/2)*(c*sec(b*x + a))^(5/2), x)
```

Fricas [A] time = 1.87092, size = 207, normalized size = 1.95

$$-\frac{2\left(32\,c^2d^3\cos(bx+a)^4-40\,c^2d^3\cos(bx+a)^2+5\,c^2d^3\right)\sqrt{\frac{c}{\cos(bx+a)}}\sqrt{\frac{d}{\sin(bx+a)}}}{15\left(b\cos(bx+a)^3-b\cos(bx+a)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(b*x+a))^(7/2)*(c*sec(b*x+a))^(5/2),x, algorithm="fricas")
```

```
[Out] -2/15*(32*c^2*d^3*cos(b*x + a)^4 - 40*c^2*d^3*cos(b*x + a)^2 + 5*c^2*d^3)*s
qrt(c/cos(b*x + a))*sqrt(d/sin(b*x + a))/(b*cos(b*x + a)^3 - b*cos(b*x + a)
)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(b*x+a))**(7/2)*(c*sec(b*x+a))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (d \csc (bx + a))^{\frac{7}{2}} (c \sec (bx + a))^{\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(b*x+a))^(7/2)*(c*sec(b*x+a))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((d*csc(b*x + a))^(7/2)*(c*sec(b*x + a))^(5/2), x)
```

3.248 $\int (d\csc(a+bx))^{5/2}(c\sec(a+bx))^{5/2}\,dx$

Optimal. Leaf size=131

$$\frac{4c^2d^2\sqrt{\sin(2a+2bx)}\text{EllipticF}\left(a+bx-\frac{\pi}{4},2\right)\sqrt{c\sec(a+bx)}\sqrt{d\csc(a+bx)}}{3b}+\frac{4cd^3(c\sec(a+bx))^{3/2}}{3b\sqrt{d\csc(a+bx)}}-\frac{2cd(c\sec(a+}{}$$

```
[Out] (4*c*d^3*(c*Sec[a + b*x])^(3/2))/(3*b*Sqrt[d*Csc[a + b*x]]) - (2*c*d*(d*Csc
[a + b*x])^(3/2)*(c*Sec[a + b*x])^(3/2))/(3*b) + (4*c^2*d^2*Sqrt[d*Csc[a +
b*x]]*EllipticF[a - Pi/4 + b*x, 2]*Sqrt[c*Sec[a + b*x]]*Sqrt[Sin[2*a + 2*b*
x]])/(3*b)
```

Rubi [A] time = 0.2044, antiderivative size = 131, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2625, 2626, 2630, 2573, 2641}

$$\frac{4c^2d^2\sqrt{\sin(2a+2bx)}F\left(a+bx-\frac{\pi}{4}\middle|2\right)\sqrt{c\sec(a+bx)}\sqrt{d\csc(a+bx)}}{3b}+\frac{4cd^3(c\sec(a+bx))^{3/2}}{3b\sqrt{d\csc(a+bx)}}-\frac{2cd(c\sec(a+bx))^{3/2}(}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Csc[a + b*x])^(5/2)*(c*Sec[a + b*x])^(5/2),x]
```

```
[Out] (4*c*d^3*(c*Sec[a + b*x])^(3/2))/(3*b*Sqrt[d*Csc[a + b*x]]) - (2*c*d*(d*Csc
[a + b*x])^(3/2)*(c*Sec[a + b*x])^(3/2))/(3*b) + (4*c^2*d^2*Sqrt[d*Csc[a +
b*x]]*EllipticF[a - Pi/4 + b*x, 2]*Sqrt[c*Sec[a + b*x]]*Sqrt[Sin[2*a + 2*b*
x]])/(3*b)
```

Rule 2625

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> -Simp[(a*b*(a*Csc[e + f*x])^(m - 1)*(b*Sec[e + f*x])^(n -
 1))/(f*(m - 1)), x] + Dist[(a^2*(m + n - 2))/(m - 1), Int[(a*Csc[e + f*x])
^(m - 2)*(b*Sec[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m,
1] && IntegersQ[2*m, 2*n] &&  !GtQ[n, m]
```

Rule 2626

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*b*(a*Csc[e + f*x])^(m - 1)*(b*Sec[e + f*x])^(n -
1))/(f*(n - 1)), x] + Dist[(b^2*(m + n - 2))/(n - 1), Int[(a*Csc[e + f*x])^
m*(b*Sec[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1
] && IntegersQ[2*m, 2*n]
```

Rule 2630

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Dist[(a*Csc[e + f*x])^m*(b*Sec[e + f*x])^n*(a*Sin[e + f*x]
)^m*(b*Cos[e + f*x])^n, Int[1/((a*Sin[e + f*x])^m*(b*Cos[e + f*x])^n), x],
x] /; FreeQ[{a, b, e, f, m, n}, x] && IntegerQ[m - 1/2] && IntegerQ[n - 1/2
]
```

Rule 2573

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
```

```
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (d\csc(a+bx))^{5/2}(c\sec(a+bx))^{5/2}\,dx = -\frac{2cd(d\csc(a+bx))^{3/2}(c\sec(a+bx))^{3/2}}{3b} + \left(2d^2\right)\int \sqrt{d\csc(a+bx)}(c\sec(a+b$$

$$= \frac{4cd^3(c\sec(a+bx))^{3/2}}{3b\sqrt{d\csc(a+bx)}} - \frac{2cd(d\csc(a+bx))^{3/2}(c\sec(a+bx))^{3/2}}{3b} + \frac{1}{3}\left(4c^2d^2\right)\int$$

$$= \frac{4cd^3(c\sec(a+bx))^{3/2}}{3b\sqrt{d\csc(a+bx)}} - \frac{2cd(d\csc(a+bx))^{3/2}(c\sec(a+bx))^{3/2}}{3b} + \frac{1}{3}\left(4c^2d^2\sqrt{c\,c}\right.$$

$$= \frac{4cd^3(c\sec(a+bx))^{3/2}}{3b\sqrt{d\csc(a+bx)}} - \frac{2cd(d\csc(a+bx))^{3/2}(c\sec(a+bx))^{3/2}}{3b} + \frac{1}{3}\left(4c^2d^2\sqrt{d\,c}\right.$$

$$= \frac{4cd^3(c\sec(a+bx))^{3/2}}{3b\sqrt{d\csc(a+bx)}} - \frac{2cd(d\csc(a+bx))^{3/2}(c\sec(a+bx))^{3/2}}{3b} + \frac{4c^2d^2\sqrt{d\csc}}{}$$

Mathematica [C] time = 0.65213, size = 87, normalized size = 0.66

$$-\frac{2c^3d\tan^2(a+bx)(d\csc(a+bx))^{3/2}\left(2\left(-\cot^2(a+bx)\right)^{3/4}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{3}{2},\csc^2(a+bx)\right)+\cot^2(a+bx)-\right.}{3b\sqrt{c\sec(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Csc[a + b*x])^(5/2)*(c*Sec[a + b*x])^(5/2),x]
```

```
[Out] (-2*c^3*d*(d*Csc[a + b*x])^(3/2)*(-1 + Cot[a + b*x]^2 + 2*(-Cot[a + b*x]^2)
^(3/4)*Hypergeometric2F1[1/2, 3/4, 3/2, Csc[a + b*x]^2])*Tan[a + b*x]^2)/(3
*b*Sqrt[c*Sec[a + b*x]])
```

Maple [B] time = 0.195, size = 304, normalized size = 2.3

$$\frac{\cos(bx+a)\sqrt{2}\sin(bx+a)}{3b}\left(4(\cos(bx+a))^2\sin(bx+a)\sqrt{-\frac{-1+\cos(bx+a)-\sin(bx+a)}{\sin(bx+a)}}\sqrt{\frac{-1+\cos(bx+a)+\sin}{\sin(bx+a)}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*csc(b*x+a))^(5/2)*(c*sec(b*x+a))^(5/2),x)
```

```
[Out] 1/3/b*2^(1/2)*(4*cos(b*x+a)^2*sin(b*x+a)*(-(-1+cos(b*x+a)-sin(b*x+a))/sin(b
*x+a))^(1/2)*((-1+cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))
/sin(b*x+a))^(1/2)*EllipticF((-(-1+cos(b*x+a)-sin(b*x+a))/sin(b*x+a))^(1/2)
,1/2*2^(1/2))+4*cos(b*x+a)*sin(b*x+a)*(-(-1+cos(b*x+a)-sin(b*x+a))/sin(b*x+
a))^(1/2)*((-1+cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/si
n(b*x+a))^(1/2)*EllipticF((-(-1+cos(b*x+a)-sin(b*x+a))/sin(b*x+a))^(1/2),1/
2*2^(1/2))-2*cos(b*x+a)^2*2^(1/2)+2^(1/2))*cos(b*x+a)*(d/sin(b*x+a))^(5/2)*
```

```
(c/cos(b*x+a))^(5/2)*sin(b*x+a)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (d\csc(bx+a))^{\frac{5}{2}}(c\sec(bx+a))^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(b*x+a))^(5/2)*(c*sec(b*x+a))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((d*csc(b*x + a))^(5/2)*(c*sec(b*x + a))^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{d\csc(bx+a)}\sqrt{c\sec(bx+a)}c^2d^2\csc(bx+a)^2\sec(bx+a)^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(b*x+a))^(5/2)*(c*sec(b*x+a))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*csc(b*x + a))*sqrt(c*sec(b*x + a))*c^2*d^2*csc(b*x + a)^2*s
ec(b*x + a)^2, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(b*x+a))**(5/2)*(c*sec(b*x+a))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (d\csc(bx+a))^{\frac{5}{2}}(c\sec(bx+a))^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(b*x+a))^(5/2)*(c*sec(b*x+a))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((d*csc(b*x + a))^(5/2)*(c*sec(b*x + a))^(5/2), x)
```

3.249 $\int (d\csc(a+bx))^{3/2}(c\sec(a+bx))^{5/2}\,dx$

Optimal. Leaf size=69

$$\frac{2cd(c\sec(a+bx))^{3/2}\sqrt{d\csc(a+bx)}}{3b}-\frac{8c^3d\sqrt{d\csc(a+bx)}}{3b\sqrt{c\sec(a+bx)}}$$

```
[Out] (-8*c^3*d*Sqrt[d*Csc[a + b*x]])/(3*b*Sqrt[c*Sec[a + b*x]]) + (2*c*d*Sqrt[d*
Csc[a + b*x]]*(c*Sec[a + b*x])^(3/2))/(3*b)
```

Rubi [A] time = 0.102423, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.08, Rules used = {2626, 2619}

$$\frac{2cd(c\sec(a+bx))^{3/2}\sqrt{d\csc(a+bx)}}{3b}-\frac{8c^3d\sqrt{d\csc(a+bx)}}{3b\sqrt{c\sec(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Csc[a + b*x])^(3/2)*(c*Sec[a + b*x])^(5/2),x]
```

```
[Out] (-8*c^3*d*Sqrt[d*Csc[a + b*x]])/(3*b*Sqrt[c*Sec[a + b*x]]) + (2*c*d*Sqrt[d*
Csc[a + b*x]]*(c*Sec[a + b*x])^(3/2))/(3*b)
```

Rule 2626

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*b*(a*Csc[e + f*x])^(m - 1)*(b*Sec[e + f*x])^(n -
1))/(f*(n - 1)), x] + Dist[(b^2*(m + n - 2))/(n - 1), Int[(a*Csc[e + f*x])^
m*(b*Sec[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1
] && IntegersQ[2*m, 2*n]
```

Rule 2619

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(a*b*(a*Csc[e + f*x])^(m - 1)*(b*Sec[e + f*x])^(n - 1
))/(f*(n - 1)), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n - 2, 0] &&
NeQ[n, 1]
```

Rubi steps

$$\begin{aligned}\int (d\csc(a+bx))^{3/2}(c\sec(a+bx))^{5/2}\,dx &= \frac{2cd\sqrt{d\csc(a+bx)}(c\sec(a+bx))^{3/2}}{3b}+\frac{1}{3}\left(4c^2\right)\int (d\csc(a+bx))^{3/2}\sqrt{c\sec(a+bx)}\,dx\\ &= -\frac{8c^3d\sqrt{d\csc(a+bx)}}{3b\sqrt{c\sec(a+bx)}}+\frac{2cd\sqrt{d\csc(a+bx)}(c\sec(a+bx))^{3/2}}{3b}\end{aligned}$$

Mathematica [A] time = 0.207171, size = 45, normalized size = 0.65

$$-\frac{2cd(2\cos(2(a+bx))+1)(c\sec(a+bx))^{3/2}\sqrt{d\csc(a+bx)}}{3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Csc[a + b*x])^(3/2)*(c*Sec[a + b*x])^(5/2),x]
```

```
[Out] (-2*c*d*(1 + 2*Cos[2*(a + b*x)])*Sqrt[d*Csc[a + b*x]]*(c*Sec[a + b*x])^(3/2
))/(3*b)
```

Maple [A] time = 0.161, size = 54, normalized size = 0.8

$$-\frac{\left(8\,(\cos(bx+a))^2-2\right)\cos(bx+a)\sin(bx+a)}{3\,b}\left(\frac{d}{\sin(bx+a)}\right)^{\frac{3}{2}}\left(\frac{c}{\cos(bx+a)}\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*csc(b*x+a))^(3/2)*(c*sec(b*x+a))^(5/2),x)
```

```
[Out] -2/3/b*(4*cos(b*x+a)^2-1)*cos(b*x+a)*(d/sin(b*x+a))^(3/2)*(c/cos(b*x+a))^(5
/2)*sin(b*x+a)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (d\csc(bx+a))^{\frac{3}{2}}(c\sec(bx+a))^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(b*x+a))^(3/2)*(c*sec(b*x+a))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((d*csc(b*x + a))^(3/2)*(c*sec(b*x + a))^(5/2), x)
```

Fricas [A] time = 1.54961, size = 132, normalized size = 1.91

$$-\frac{2\left(4\,c^2d\cos(bx+a)^2-c^2d\right)\sqrt{\frac{c}{\cos(bx+a)}}\sqrt{\frac{d}{\sin(bx+a)}}}{3\,b\cos(bx+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(b*x+a))^(3/2)*(c*sec(b*x+a))^(5/2),x, algorithm="fricas")
```

```
[Out] -2/3*(4*c^2*d*cos(b*x + a)^2 - c^2*d)*sqrt(c/cos(b*x + a))*sqrt(d/sin(b*x +
 a))/(b*cos(b*x + a))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(b*x+a))**(3/2)*(c*sec(b*x+a))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (d\csc(bx+a))^{\frac{3}{2}}(c\sec(bx+a))^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(b*x+a))^(3/2)*(c*sec(b*x+a))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((d*csc(b*x + a))^(3/2)*(c*sec(b*x + a))^(5/2), x)
```

3.250 $\int \sqrt{d\csc(a+bx)}(c\sec(a+bx))^{5/2}\,dx$

Optimal. Leaf size=93

$$\frac{2c^2\sqrt{\sin(2a+2bx)}\text{EllipticF}\left(a+bx-\frac{\pi}{4},2\right)\sqrt{c\sec(a+bx)}\sqrt{d\csc(a+bx)}}{3b}+\frac{2cd(c\sec(a+bx))^{3/2}}{3b\sqrt{d\csc(a+bx)}}$$

```
[Out] (2*c*d*(c*Sec[a + b*x])^(3/2))/(3*b*Sqrt[d*Csc[a + b*x]]) + (2*c^2*Sqrt[d*C
sc[a + b*x]]*EllipticF[a - Pi/4 + b*x, 2]*Sqrt[c*Sec[a + b*x]]*Sqrt[Sin[2*a
 + 2*b*x]])/(3*b)
```

Rubi [A] time = 0.140581, antiderivative size = 93, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {2626, 2630, 2573, 2641}

$$\frac{2c^2\sqrt{\sin(2a+2bx)}F\left(a+bx-\frac{\pi}{4}\middle|2\right)\sqrt{c\sec(a+bx)}\sqrt{d\csc(a+bx)}}{3b}+\frac{2cd(c\sec(a+bx))^{3/2}}{3b\sqrt{d\csc(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d*Csc[a + b*x]]*(c*Sec[a + b*x])^(5/2),x]
```


```
[Out] (2*c*d*(c*Sec[a + b*x])^(3/2))/(3*b*Sqrt[d*Csc[a + b*x]]) + (2*c^2*Sqrt[d*C
sc[a + b*x]]*EllipticF[a - Pi/4 + b*x, 2]*Sqrt[c*Sec[a + b*x]]*Sqrt[Sin[2*a
 + 2*b*x]])/(3*b)
```


Rule 2626

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*b*(a*Csc[e + f*x])^(m - 1)*(b*Sec[e + f*x])^(n -
1))/(f*(n - 1)), x] + Dist[(b^2*(m + n - 2))/(n - 1), Int[(a*Csc[e + f*x])^
m*(b*Sec[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1
] && IntegersQ[2*m, 2*n]
```

Rule 2630

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Dist[(a*Csc[e + f*x])^m*(b*Sec[e + f*x])^n*(a*Sin[e + f*x]
)^m*(b*Cos[e + f*x])^n, Int[1/((a*Sin[e + f*x])^m*(b*Cos[e + f*x])^n), x],
x] /; FreeQ[{a, b, e, f, m, n}, x] && IntegerQ[m - 1/2] && IntegerQ[n - 1/2
]
```

Rule 2573

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sqrt{d\csc(a+bx)}(c\sec(a+bx))^{5/2}\,dx = \frac{2cd(c\sec(a+bx))^{3/2}}{3b\sqrt{d\csc(a+bx)}} + \frac{1}{3}\left(2c^2\right)\int \sqrt{d\csc(a+bx)}\sqrt{c\sec(a+bx)}\,dx$$
$$= \frac{2cd(c\sec(a+bx))^{3/2}}{3b\sqrt{d\csc(a+bx)}} + \frac{1}{3}\left(2c^2\sqrt{c\cos(a+bx)}\sqrt{d\csc(a+bx)}\sqrt{c\sec(a+bx)}\sqrt{d\,\mathrm{si}$$
$$= \frac{2cd(c\sec(a+bx))^{3/2}}{3b\sqrt{d\csc(a+bx)}} + \frac{1}{3}\left(2c^2\sqrt{d\csc(a+bx)}\sqrt{c\sec(a+bx)}\sqrt{\sin(2a+2bx)}\right)\int$$
$$= \frac{2cd(c\sec(a+bx))^{3/2}}{3b\sqrt{d\csc(a+bx)}} + \frac{2c^2\sqrt{d\csc(a+bx)}F\left(a-\frac{\pi}{4}+bx\middle|2\right)\sqrt{c\sec(a+bx)}\sqrt{\sin(}}{3b}$$

Mathematica [C] time = 0.57948, size = 68, normalized size = 0.73

$$-\frac{2cd(c\sec(a+bx))^{3/2}\left(\left(-\cot^2(a+bx)\right)^{3/4}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{3}{2},\csc^2(a+bx)\right)-1\right)}{3b\sqrt{d\csc(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d*Csc[a + b*x]]*(c*Sec[a + b*x])^(5/2),x]
```

```
[Out] (-2*c*d*(-1 + (-Cot[a + b*x]^2)^(3/4)*Hypergeometric2F1[1/2, 3/4, 3/2, Csc[
a + b*x]^2])*(c*Sec[a + b*x])^(3/2))/(3*b*Sqrt[d*Csc[a + b*x]])
```

Maple [A] time = 0.206, size = 190, normalized size = 2.

$$-\frac{\cos(bx+a)\sqrt{2}\sin(bx+a)}{3b(-1+\cos(bx+a))}\left(2\cos(bx+a)\sin(bx+a)\sqrt{-\frac{-1+\cos(bx+a)-\sin(bx+a)}{\sin(bx+a)}}\sqrt{\frac{-1+\cos(bx+a)+\sin(}{\sin(bx+a)}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*csc(b*x+a))^(1/2)*(c*sec(b*x+a))^(5/2),x)
```

```
[Out] -1/3/b*2^(1/2)*(2*cos(b*x+a)*sin(b*x+a)*(-(-1+cos(b*x+a)-sin(b*x+a))/sin(b*
x+a))^(1/2)*((-1+cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/
sin(b*x+a))^(1/2)*EllipticF((-(-1+cos(b*x+a)-sin(b*x+a))/sin(b*x+a))^(1/2),
1/2*2^(1/2))-cos(b*x+a)*2^(1/2)+2^(1/2))*cos(b*x+a)*(d/sin(b*x+a))^(1/2)*(c
/cos(b*x+a))^(5/2)*sin(b*x+a)/(-1+cos(b*x+a))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{d\csc(bx+a)}\,(c\sec(bx+a))^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(b*x+a))^(1/2)*(c*sec(b*x+a))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*csc(b*x + a))*(c*sec(b*x + a))^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{d\csc(bx+a)}\sqrt{c\sec(bx+a)}c^2\sec(bx+a)^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(b*x+a))^(1/2)*(c*sec(b*x+a))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*csc(b*x + a))*sqrt(c*sec(b*x + a))*c^2*sec(b*x + a)^2, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(b*x+a))**(1/2)*(c*sec(b*x+a))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{d\csc(bx+a)}\,(c\sec(bx+a))^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(b*x+a))^(1/2)*(c*sec(b*x+a))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*csc(b*x + a))*(c*sec(b*x + a))^(5/2), x)
```

3.251 $\int \frac{(c\sec(a+bx))^{5/2}}{\sqrt{d\csc(a+bx)}}\,dx$

Optimal. Leaf size=33

$$\frac{2cd(c\sec(a+bx))^{3/2}}{3b(d\csc(a+bx))^{3/2}}$$

```
[Out] (2*c*d*(c*Sec[a + b*x])^(3/2))/(3*b*(d*Csc[a + b*x])^(3/2))
```

Rubi [A] time = 0.0494305, antiderivative size = 33, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.04, Rules used = {2619}

$$\frac{2cd(c\sec(a+bx))^{3/2}}{3b(d\csc(a+bx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*Sec[a + b*x])^(5/2)/Sqrt[d*Csc[a + b*x]],x]
```

```
[Out] (2*c*d*(c*Sec[a + b*x])^(3/2))/(3*b*(d*Csc[a + b*x])^(3/2))
```

Rule 2619

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(a*b*(a*Csc[e + f*x])^(m - 1)*(b*Sec[e + f*x])^(n - 1
))/(f*(n - 1)), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n - 2, 0] &&
NeQ[n, 1]
```

Rubi steps

$$\int \frac{(c\sec(a+bx))^{5/2}}{\sqrt{d\csc(a+bx)}}\,dx = \frac{2cd(c\sec(a+bx))^{3/2}}{3b(d\csc(a+bx))^{3/2}}$$

Mathematica [A] time = 0.111973, size = 33, normalized size = 1.

$$\frac{2cd(c\sec(a+bx))^{3/2}}{3b(d\csc(a+bx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*Sec[a + b*x])^(5/2)/Sqrt[d*Csc[a + b*x]],x]
```

```
[Out] (2*c*d*(c*Sec[a + b*x])^(3/2))/(3*b*(d*Csc[a + b*x])^(3/2))
```

Maple [A] time = 0.149, size = 42, normalized size = 1.3

$$\frac{2\cos(bx+a)\sin(bx+a)}{3b}\left(\frac{c}{\cos(bx+a)}\right)^{\frac{5}{2}}\frac{1}{\sqrt{\frac{d}{\sin(bx+a)}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*sec(b*x+a))^(5/2)/(d*csc(b*x+a))^(1/2),x)
```

```
[Out] 2/3/b*cos(b*x+a)*sin(b*x+a)*(c/cos(b*x+a))^(5/2)/(d/sin(b*x+a))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(c\sec(bx+a))^{\frac{5}{2}}}{\sqrt{d\csc(bx+a)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sec(b*x+a))^(5/2)/(d*csc(b*x+a))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((c*sec(b*x + a))^(5/2)/sqrt(d*csc(b*x + a)), x)
```

Fricas [B] time = 1.36281, size = 127, normalized size = 3.85

$$-\frac{2\left(c^2\cos(bx+a)^2-c^2\right)\sqrt{\frac{c}{\cos(bx+a)}}\sqrt{\frac{d}{\sin(bx+a)}}}{3\,bd\cos(bx+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sec(b*x+a))^(5/2)/(d*csc(b*x+a))^(1/2),x, algorithm="fricas")
```

```
[Out] -2/3*(c^2*cos(b*x + a)^2 - c^2)*sqrt(c/cos(b*x + a))*sqrt(d/sin(b*x + a))/(
b*d*cos(b*x + a))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sec(b*x+a))**(5/2)/(d*csc(b*x+a))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(c\sec(bx+a))^{\frac{5}{2}}}{\sqrt{d\csc(bx+a)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sec(b*x+a))^(5/2)/(d*csc(b*x+a))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((c*sec(b*x + a))^(5/2)/sqrt(d*csc(b*x + a)), x)
```

3.252 $\int \frac{(c\sec(a+bx))^{5/2}}{(d\csc(a+bx))^{3/2}}\,dx$

Optimal. Leaf size=98

$$\frac{2c(c\sec(a+bx))^{3/2}}{3bd\sqrt{d\csc(a+bx)}}-\frac{c^2\sqrt{\sin(2a+2bx)}\text{EllipticF}\left(a+bx-\frac{\pi}{4},2\right)\sqrt{c\sec(a+bx)}\sqrt{d\csc(a+bx)}}{3bd^2}$$

```
[Out] (2*c*(c*Sec[a + b*x])^(3/2))/(3*b*d*Sqrt[d*Csc[a + b*x]]) - (c^2*Sqrt[d*Csc
[a + b*x]]*EllipticF[a - Pi/4 + b*x, 2]*Sqrt[c*Sec[a + b*x]]*Sqrt[Sin[2*a +
 2*b*x]])/(3*b*d^2)
```

Rubi [A] time = 0.149594, antiderivative size = 98, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {2624, 2630, 2573, 2641}

$$\frac{2c(c\sec(a+bx))^{3/2}}{3bd\sqrt{d\csc(a+bx)}}-\frac{c^2\sqrt{\sin(2a+2bx)}F\left(a+bx-\frac{\pi}{4}\middle|2\right)\sqrt{c\sec(a+bx)}\sqrt{d\csc(a+bx)}}{3bd^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c*Sec[a + b*x])^(5/2)/(d*Csc[a + b*x])^(3/2),x]
```

```
[Out] (2*c*(c*Sec[a + b*x])^(3/2))/(3*b*d*Sqrt[d*Csc[a + b*x]]) - (c^2*Sqrt[d*Csc
[a + b*x]]*EllipticF[a - Pi/4 + b*x, 2]*Sqrt[c*Sec[a + b*x]]*Sqrt[Sin[2*a +
 2*b*x]])/(3*b*d^2)
```

Rule 2624

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(b*(a*Csc[e + f*x])^(m + 1)*(b*Sec[e + f*x])^(n - 1))
/(f*a*(n - 1)), x] + Dist[(b^2*(m + 1))/(a^2*(n - 1)), Int[(a*Csc[e + f*x])
^(m + 2)*(b*Sec[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ[
n, 1] && LtQ[m, -1] && IntegersQ[2*m, 2*n]
```

Rule 2630

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Dist[(a*Csc[e + f*x])^m*(b*Sec[e + f*x])^n*(a*Sin[e + f*x]
)^m*(b*Cos[e + f*x])^n, Int[1/((a*Sin[e + f*x])^m*(b*Cos[e + f*x])^n), x],
x] /; FreeQ[{a, b, e, f, m, n}, x] && IntegerQ[m - 1/2] && IntegerQ[n - 1/2
]
```

Rule 2573

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(c\sec(a+bx))^{5/2}}{(d\csc(a+bx))^{3/2}}\,dx = \frac{2c(c\sec(a+bx))^{3/2}}{3bd\sqrt{d\csc(a+bx)}} - \frac{c^2\int\sqrt{d\csc(a+bx)}\sqrt{c\sec(a+bx)}\,dx}{3d^2}$$
$$= \frac{2c(c\sec(a+bx))^{3/2}}{3bd\sqrt{d\csc(a+bx)}} - \frac{\left(c^2\sqrt{c\cos(a+bx)}\sqrt{d\csc(a+bx)}\sqrt{c\sec(a+bx)}\sqrt{d\sin(a+bx)}\right)\int\frac{}{\sqrt{c}}}{3d^2}$$
$$= \frac{2c(c\sec(a+bx))^{3/2}}{3bd\sqrt{d\csc(a+bx)}} - \frac{\left(c^2\sqrt{d\csc(a+bx)}\sqrt{c\sec(a+bx)}\sqrt{\sin(2a+2bx)}\right)\int\frac{1}{\sqrt{\sin(2a+2bx)}}\,dx}{3d^2}$$
$$= \frac{2c(c\sec(a+bx))^{3/2}}{3bd\sqrt{d\csc(a+bx)}} - \frac{c^2\sqrt{d\csc(a+bx)}F\left(a-\frac{\pi}{4}+bx\middle|2\right)\sqrt{c\sec(a+bx)}\sqrt{\sin(2a+2bx)}}{3bd^2}$$

Mathematica [C] time = 0.527651, size = 70, normalized size = 0.71

$$\frac{c(c\sec(a+bx))^{3/2}\left(\left(-\cot^2(a+bx)\right)^{3/4}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{3}{2},\csc^2(a+bx)\right)+2\right)}{3bd\sqrt{d\csc(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*Sec[a + b*x])^(5/2)/(d*Csc[a + b*x])^(3/2),x]
```

```
[Out] (c*(2 + (-Cot[a + b*x]^2)^(3/4)*Hypergeometric2F1[1/2, 3/4, 3/2, Csc[a + b*
x]^2])*(c*Sec[a + b*x])^(3/2))/(3*b*d*Sqrt[d*Csc[a + b*x]])
```

Maple [A] time = 0.176, size = 192, normalized size = 2.

$$\frac{\cos(bx+a)\sqrt{2}}{3\,b\,(-1+\cos(bx+a))\sin(bx+a)}\left(\cos(bx+a)\sin(bx+a)\sqrt{-\frac{-1+\cos(bx+a)-\sin(bx+a)}{\sin(bx+a)}}\sqrt{\frac{-1+\cos(bx+a)}{\sin(bx}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*sec(b*x+a))^(5/2)/(d*csc(b*x+a))^(3/2),x)
```

```
[Out] 1/3/b*2^(1/2)*(cos(b*x+a)*sin(b*x+a)*(-(-1+cos(b*x+a)-sin(b*x+a))/sin(b*x+a
))^(1/2)*((-1+cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin
(b*x+a))^(1/2)*EllipticF((-(-1+cos(b*x+a)-sin(b*x+a))/sin(b*x+a))^(1/2),1/2
*2^(1/2))+cos(b*x+a)*2^(1/2)-2^(1/2))*cos(b*x+a)*(c/cos(b*x+a))^(5/2)/(-1+c
os(b*x+a))/(d/sin(b*x+a))^(3/2)/sin(b*x+a)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(c\sec(bx+a))^{\frac{5}{2}}}{(d\csc(bx+a))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sec(b*x+a))^(5/2)/(d*csc(b*x+a))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((c*sec(b*x + a))^(5/2)/(d*csc(b*x + a))^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{d\csc\left(bx+a\right)}\sqrt{c\sec\left(bx+a\right)}c^{2}\sec\left(bx+a\right)^{2}}{d^{2}\csc\left(bx+a\right)^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sec(b*x+a))^(5/2)/(d*csc(b*x+a))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*csc(b*x + a))*sqrt(c*sec(b*x + a))*c^2*sec(b*x + a)^2/(d^2*
csc(b*x + a)^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sec(b*x+a))**(5/2)/(d*csc(b*x+a))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(c\sec\left(bx+a\right)\right)^{\frac{5}{2}}}{\left(d\csc\left(bx+a\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sec(b*x+a))^(5/2)/(d*csc(b*x+a))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((c*sec(b*x + a))^(5/2)/(d*csc(b*x + a))^(3/2), x)
```

3.253 $\int \frac{(c\sec(a+bx))^{5/2}}{(d\csc(a+bx))^{5/2}}\,dx$

Optimal. Leaf size=329

$$\frac{c^2\tan^{-1}\left(1-\sqrt{2}\sqrt{\tan(a+bx)}\right)\sqrt{c\sec(a+bx)}}{\sqrt{2}bd^2\sqrt{\tan(a+bx)}\sqrt{d\csc(a+bx)}}-\frac{c^2\tan^{-1}\left(\sqrt{2}\sqrt{\tan(a+bx)}+1\right)\sqrt{c\sec(a+bx)}}{\sqrt{2}bd^2\sqrt{\tan(a+bx)}\sqrt{d\csc(a+bx)}}-\frac{c^2\sqrt{c\sec(a+bx)}\,\mathrm{l}}{2\sqrt{2}bd^2}$$

```
[Out] (2*c*(c*Sec[a + b*x])^(3/2))/(3*b*d*(d*Csc[a + b*x])^(3/2)) + (c^2*ArcTan[1
 - Sqrt[2]*Sqrt[Tan[a + b*x]]]*Sqrt[c*Sec[a + b*x]])/(Sqrt[2]*b*d^2*Sqrt[d*
Csc[a + b*x]]*Sqrt[Tan[a + b*x]]) - (c^2*ArcTan[1 + Sqrt[2]*Sqrt[Tan[a + b*
x]]]*Sqrt[c*Sec[a + b*x]])/(Sqrt[2]*b*d^2*Sqrt[d*Csc[a + b*x]]*Sqrt[Tan[a +
 b*x]]) - (c^2*Log[1 - Sqrt[2]*Sqrt[Tan[a + b*x]] + Tan[a + b*x]]*Sqrt[c*Se
c[a + b*x]])/(2*Sqrt[2]*b*d^2*Sqrt[d*Csc[a + b*x]]*Sqrt[Tan[a + b*x]]) + (c
^2*Log[1 + Sqrt[2]*Sqrt[Tan[a + b*x]] + Tan[a + b*x]]*Sqrt[c*Sec[a + b*x]])
/(2*Sqrt[2]*b*d^2*Sqrt[d*Csc[a + b*x]]*Sqrt[Tan[a + b*x]])
```

Rubi [A] time = 0.216921, antiderivative size = 329, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 10, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {2624, 2629, 3476, 329, 297, 1162, 617, 204, 1165, 628}

$$\frac{c^2\tan^{-1}\left(1-\sqrt{2}\sqrt{\tan(a+bx)}\right)\sqrt{c\sec(a+bx)}}{\sqrt{2}bd^2\sqrt{\tan(a+bx)}\sqrt{d\csc(a+bx)}}-\frac{c^2\tan^{-1}\left(\sqrt{2}\sqrt{\tan(a+bx)}+1\right)\sqrt{c\sec(a+bx)}}{\sqrt{2}bd^2\sqrt{\tan(a+bx)}\sqrt{d\csc(a+bx)}}-\frac{c^2\sqrt{c\sec(a+bx)}\,\mathrm{l}}{2\sqrt{2}bd^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c*Sec[a + b*x])^(5/2)/(d*Csc[a + b*x])^(5/2),x]
```

```
[Out] (2*c*(c*Sec[a + b*x])^(3/2))/(3*b*d*(d*Csc[a + b*x])^(3/2)) + (c^2*ArcTan[1
 - Sqrt[2]*Sqrt[Tan[a + b*x]]]*Sqrt[c*Sec[a + b*x]])/(Sqrt[2]*b*d^2*Sqrt[d*
Csc[a + b*x]]*Sqrt[Tan[a + b*x]]) - (c^2*ArcTan[1 + Sqrt[2]*Sqrt[Tan[a + b*
x]]]*Sqrt[c*Sec[a + b*x]])/(Sqrt[2]*b*d^2*Sqrt[d*Csc[a + b*x]]*Sqrt[Tan[a +
 b*x]]) - (c^2*Log[1 - Sqrt[2]*Sqrt[Tan[a + b*x]] + Tan[a + b*x]]*Sqrt[c*Se
c[a + b*x]])/(2*Sqrt[2]*b*d^2*Sqrt[d*Csc[a + b*x]]*Sqrt[Tan[a + b*x]]) + (c
^2*Log[1 + Sqrt[2]*Sqrt[Tan[a + b*x]] + Tan[a + b*x]]*Sqrt[c*Sec[a + b*x]])
/(2*Sqrt[2]*b*d^2*Sqrt[d*Csc[a + b*x]]*Sqrt[Tan[a + b*x]])
```

Rule 2624

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(b*(a*Csc[e + f*x])^(m + 1)*(b*Sec[e + f*x])^(n - 1))
/(f*a*(n - 1)), x] + Dist[(b^2*(m + 1))/(a^2*(n - 1)), Int[(a*Csc[e + f*x])
^(m + 2)*(b*Sec[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ[
n, 1] && LtQ[m, -1] && IntegersQ[2*m, 2*n]
```

Rule 2629

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Dist[((a*Csc[e + f*x])^m*(b*Sec[e + f*x])^n)/Tan[e + f*x]^
n, Int[Tan[e + f*x]^n, x], x] /; FreeQ[{a, b, e, f, m, n}, x] && !IntegerQ
[n] && EqQ[m + n, 0]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] && !
```

```
IntegerQ[n]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{(c\sec(a+bx))^{5/2}}{(d\csc(a+bx))^{5/2}}\,dx = \frac{2c(c\sec(a+bx))^{3/2}}{3bd(d\csc(a+bx))^{3/2}} - \frac{c^2\int \frac{\sqrt{c\sec(a+bx)}}{\sqrt{d\csc(a+bx)}}\,dx}{d^2}$$

$$= \frac{2c(c\sec(a+bx))^{3/2}}{3bd(d\csc(a+bx))^{3/2}} - \frac{\left(c^2\sqrt{c\sec(a+bx)}\right)\int \sqrt{\tan(a+bx)}\,dx}{d^2\sqrt{d\csc(a+bx)}\sqrt{\tan(a+bx)}}$$

$$= \frac{2c(c\sec(a+bx))^{3/2}}{3bd(d\csc(a+bx))^{3/2}} - \frac{\left(c^2\sqrt{c\sec(a+bx)}\right)\text{Subst}\left(\int \frac{\sqrt{x}}{1+x^2}\,dx, x, \tan(a+bx)\right)}{bd^2\sqrt{d\csc(a+bx)}\sqrt{\tan(a+bx)}}$$

$$= \frac{2c(c\sec(a+bx))^{3/2}}{3bd(d\csc(a+bx))^{3/2}} - \frac{\left(2c^2\sqrt{c\sec(a+bx)}\right)\text{Subst}\left(\int \frac{x^2}{1+x^4}\,dx, x, \sqrt{\tan(a+bx)}\right)}{bd^2\sqrt{d\csc(a+bx)}\sqrt{\tan(a+bx)}}$$

$$= \frac{2c(c\sec(a+bx))^{3/2}}{3bd(d\csc(a+bx))^{3/2}} + \frac{\left(c^2\sqrt{c\sec(a+bx)}\right)\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \sqrt{\tan(a+bx)}\right)}{bd^2\sqrt{d\csc(a+bx)}\sqrt{\tan(a+bx)}} - \frac{\left(c^2\sqrt{c\sec\right.}{}$$

$$= \frac{2c(c\sec(a+bx))^{3/2}}{3bd(d\csc(a+bx))^{3/2}} - \frac{\left(c^2\sqrt{c\sec(a+bx)}\right)\text{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}\,dx, x, \sqrt{\tan(a+bx)}\right)}{2bd^2\sqrt{d\csc(a+bx)}\sqrt{\tan(a+bx)}} - \frac{\left(c^2\sqrt{\right.}{}$$

$$= \frac{2c(c\sec(a+bx))^{3/2}}{3bd(d\csc(a+bx))^{3/2}} - \frac{c^2\log\left(1-\sqrt{2}\sqrt{\tan(a+bx)}+\tan(a+bx)\right)\sqrt{c\sec(a+bx)}}{2\sqrt{2}bd^2\sqrt{d\csc(a+bx)}\sqrt{\tan(a+bx)}} + \frac{c^2\log}{}$$

$$= \frac{2c(c\sec(a+bx))^{3/2}}{3bd(d\csc(a+bx))^{3/2}} + \frac{c^2\tan^{-1}\left(1-\sqrt{2}\sqrt{\tan(a+bx)}\right)\sqrt{c\sec(a+bx)}}{\sqrt{2}bd^2\sqrt{d\csc(a+bx)}\sqrt{\tan(a+bx)}} - \frac{c^2\tan^{-1}\left(1+\sqrt{2}\right.}{\sqrt{2}bd^2\sqrt{d\,c}}$$

Mathematica [A] time = 1.73026, size = 280, normalized size = 0.85

$$-\frac{c(c\sec(a+bx))^{3/2}\left(8\cos(a+bx)\sqrt[4]{\cot^2(a+bx)}\cot(a+bx)-8\sqrt[4]{\cot^2(a+bx)}\csc(a+bx)+3\sqrt{2}\cos(a+bx)\cot(a\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*Sec[a + b*x])^(5/2)/(d*Csc[a + b*x])^(5/2),x]
```

```
[Out] -(c*(6*Sqrt[2]*ArcTan[1 - Sqrt[2]*(Cot[a + b*x]^2)^(1/4)]*Cos[a + b*x]*Cot[
a + b*x] - 6*Sqrt[2]*ArcTan[1 + Sqrt[2]*(Cot[a + b*x]^2)^(1/4)]*Cos[a + b*x
]*Cot[a + b*x] + 8*Cos[a + b*x]*Cot[a + b*x]*(Cot[a + b*x]^2)^(1/4) - 8*(Co
t[a + b*x]^2)^(1/4)*Csc[a + b*x] + 3*Sqrt[2]*Cos[a + b*x]*Cot[a + b*x]*Log[
1 - Sqrt[2]*(Cot[a + b*x]^2)^(1/4) + Sqrt[Cot[a + b*x]^2]] - 3*Sqrt[2]*Cos[
a + b*x]*Cot[a + b*x]*Log[1 + Sqrt[2]*(Cot[a + b*x]^2)^(1/4) + Sqrt[Cot[a +
 b*x]^2]])*(c*Sec[a + b*x])^(3/2))/(12*b*d^2*(Cot[a + b*x]^2)^(1/4)*Sqrt[d*
Csc[a + b*x]])
```

Maple [C] time = 0.172, size = 544, normalized size = 1.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*sec(b*x+a))^(5/2)/(d*csc(b*x+a))^(5/2),x)
```

```
[Out] 1/6/b*2^(1/2)*(3*I*cos(b*x+a)*(-(-1+cos(b*x+a)-sin(b*x+a))/sin(b*x+a))^(1/2
)*((-1+cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a)
```

```
)^(1/2)*EllipticPi((-(-1+cos(b*x+a)-sin(b*x+a))/sin(b*x+a))^(1/2),1/2-1/2*I
,1/2*2^(1/2))-3*I*cos(b*x+a)*(-(-1+cos(b*x+a)-sin(b*x+a))/sin(b*x+a))^(1/2)
*((-1+cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))
^(1/2)*EllipticPi((-(-1+cos(b*x+a)-sin(b*x+a))/sin(b*x+a))^(1/2),1/2+1/2*I,
1/2*2^(1/2))-3*cos(b*x+a)*(-(-1+cos(b*x+a)-sin(b*x+a))/sin(b*x+a))^(1/2)*((
-1+cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1
/2)*EllipticPi((-(-1+cos(b*x+a)-sin(b*x+a))/sin(b*x+a))^(1/2),1/2-1/2*I,1/2
*2^(1/2))-3*cos(b*x+a)*(-(-1+cos(b*x+a)-sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+
cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)
*EllipticPi((-(-1+cos(b*x+a)-sin(b*x+a))/sin(b*x+a))^(1/2),1/2+1/2*I,1/2*2^
(1/2))+2*cos(b*x+a)*2^(1/2)-2*2^(1/2))*cos(b*x+a)*(c/cos(b*x+a))^(5/2)/(-1+
cos(b*x+a))/(d/sin(b*x+a))^(5/2)/sin(b*x+a)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(c\sec(bx+a))^{\frac{5}{2}}}{(d\csc(bx+a))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sec(b*x+a))^(5/2)/(d*csc(b*x+a))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((c*sec(b*x + a))^(5/2)/(d*csc(b*x + a))^(5/2), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sec(b*x+a))^(5/2)/(d*csc(b*x+a))^(5/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sec(b*x+a))**(5/2)/(d*csc(b*x+a))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(c\sec(bx+a))^{\frac{5}{2}}}{(d\csc(bx+a))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sec(b*x+a))^(5/2)/(d*csc(b*x+a))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((c*sec(b*x + a))^(5/2)/(d*csc(b*x + a))^(5/2), x)
```

3.254 $\int \frac{(d\csc(a+bx))^{9/2}}{\sqrt{c\sec(a+bx)}}\,dx$

Optimal. Leaf size=69

$$-\frac{8cd^3(d\csc(a+bx))^{3/2}}{21b(c\sec(a+bx))^{3/2}}-\frac{2cd(d\csc(a+bx))^{7/2}}{7b(c\sec(a+bx))^{3/2}}$$

```
[Out] (-8*c*d^3*(d*Csc[a + b*x])^(3/2))/(21*b*(c*Sec[a + b*x])^(3/2)) - (2*c*d*(d
*Csc[a + b*x])^(7/2))/(7*b*(c*Sec[a + b*x])^(3/2))
```

Rubi [A] time = 0.0996172, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.08, Rules used = {2625, 2619}

$$-\frac{8cd^3(d\csc(a+bx))^{3/2}}{21b(c\sec(a+bx))^{3/2}}-\frac{2cd(d\csc(a+bx))^{7/2}}{7b(c\sec(a+bx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Csc[a + b*x])^(9/2)/Sqrt[c*Sec[a + b*x]],x]
```

```
[Out] (-8*c*d^3*(d*Csc[a + b*x])^(3/2))/(21*b*(c*Sec[a + b*x])^(3/2)) - (2*c*d*(d
*Csc[a + b*x])^(7/2))/(7*b*(c*Sec[a + b*x])^(3/2))
```

Rule 2625

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> -Simp[(a*b*(a*Csc[e + f*x])^(m - 1)*(b*Sec[e + f*x])^(n -
 1))/(f*(m - 1)), x] + Dist[(a^2*(m + n - 2))/(m - 1), Int[(a*Csc[e + f*x])
^(m - 2)*(b*Sec[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m,
1] && IntegersQ[2*m, 2*n] &&  !GtQ[n, m]
```

Rule 2619

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(a*b*(a*Csc[e + f*x])^(m - 1)*(b*Sec[e + f*x])^(n - 1
))/(f*(n - 1)), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n - 2, 0] &&
NeQ[n, 1]
```

Rubi steps

$$\begin{aligned}\int \frac{(d\csc(a+bx))^{9/2}}{\sqrt{c\sec(a+bx)}}\,dx &= -\frac{2cd(d\csc(a+bx))^{7/2}}{7b(c\sec(a+bx))^{3/2}}+\frac{1}{7}\left(4d^2\right)\int \frac{(d\csc(a+bx))^{5/2}}{\sqrt{c\sec(a+bx)}}\,dx\\ &= -\frac{8cd^3(d\csc(a+bx))^{3/2}}{21b(c\sec(a+bx))^{3/2}}-\frac{2cd(d\csc(a+bx))^{7/2}}{7b(c\sec(a+bx))^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.25782, size = 45, normalized size = 0.65

$$\frac{2cd(2\cos(2(a+bx))-5)(d\csc(a+bx))^{7/2}}{21b(c\sec(a+bx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Csc[a + b*x])^(9/2)/Sqrt[c*Sec[a + b*x]],x]
```

```
[Out] (2*c*d*(-5 + 2*Cos[2*(a + b*x)])*(d*Csc[a + b*x])^(7/2))/(21*b*(c*Sec[a + b
*x])^(3/2))
```

Maple [A] time = 0.164, size = 54, normalized size = 0.8

$$\frac{\left(8\left(\cos\left(bx+a\right)\right)^{2}-14\right)\cos\left(bx+a\right)\sin\left(bx+a\right)}{21\,b}\left(\frac{d}{\sin\left(bx+a\right)}\right)^{\frac{9}{2}}\frac{1}{\sqrt{\frac{c}{\cos(bx+a)}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*csc(b*x+a))^(9/2)/(c*sec(b*x+a))^(1/2),x)
```

```
[Out] 2/21/b*(4*cos(b*x+a)^2-7)*(d/sin(b*x+a))^(9/2)*cos(b*x+a)*sin(b*x+a)/(c/cos
(b*x+a))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(d\csc\left(bx+a\right)\right)^{\frac{9}{2}}}{\sqrt{c\sec\left(bx+a\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(b*x+a))^(9/2)/(c*sec(b*x+a))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((d*csc(b*x + a))^(9/2)/sqrt(c*sec(b*x + a)), x)
```

Fricas [A] time = 2.32804, size = 185, normalized size = 2.68

$$-\frac{2\left(4\,d^{4}\cos\left(bx+a\right)^{4}-7\,d^{4}\cos\left(bx+a\right)^{2}\right)\sqrt{\frac{c}{\cos(bx+a)}}\sqrt{\frac{d}{\sin(bx+a)}}}{21\left(bc\cos\left(bx+a\right)^{2}-bc\right)\sin\left(bx+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(b*x+a))^(9/2)/(c*sec(b*x+a))^(1/2),x, algorithm="fricas")
```

```
[Out] -2/21*(4*d^4*cos(b*x + a)^4 - 7*d^4*cos(b*x + a)^2)*sqrt(c/cos(b*x + a))*sq
rt(d/sin(b*x + a))/((b*c*cos(b*x + a)^2 - b*c)*sin(b*x + a))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(b*x+a))**(9/2)/(c*sec(b*x+a))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(d\csc(bx+a))^{\frac{9}{2}}}{\sqrt{c\sec(bx+a)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(b*x+a))^(9/2)/(c*sec(b*x+a))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((d*csc(b*x + a))^(9/2)/sqrt(c*sec(b*x + a)), x)
```

3.255 $\int \frac{(d\csc(a+bx))^{7/2}}{\sqrt{c\sec(a+bx)}}\,dx$

Optimal. Leaf size=128

$$-\frac{4cd^3\sqrt{d\csc(a+bx)}}{5b(c\sec(a+bx))^{3/2}}-\frac{4d^4E\left(a+bx-\frac{\pi}{4}\middle|2\right)}{5b\sqrt{\sin(2a+2bx)}\sqrt{c\sec(a+bx)}\sqrt{d\csc(a+bx)}}-\frac{2cd(d\csc(a+bx))^{5/2}}{5b(c\sec(a+bx))^{3/2}}$$

```
[Out] (-4*c*d^3*Sqrt[d*Csc[a + b*x]])/(5*b*(c*Sec[a + b*x])^(3/2)) - (2*c*d*(d*Cs
c[a + b*x])^(5/2))/(5*b*(c*Sec[a + b*x])^(3/2)) - (4*d^4*EllipticE[a - Pi/4
 + b*x, 2])/(5*b*Sqrt[d*Csc[a + b*x]]*Sqrt[c*Sec[a + b*x]]*Sqrt[Sin[2*a + 2
*b*x]])
```

Rubi [A] time = 0.192731, antiderivative size = 128, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {2625, 2630, 2572, 2639}

$$-\frac{4cd^3\sqrt{d\csc(a+bx)}}{5b(c\sec(a+bx))^{3/2}}-\frac{4d^4E\left(a+bx-\frac{\pi}{4}\middle|2\right)}{5b\sqrt{\sin(2a+2bx)}\sqrt{c\sec(a+bx)}\sqrt{d\csc(a+bx)}}-\frac{2cd(d\csc(a+bx))^{5/2}}{5b(c\sec(a+bx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Csc[a + b*x])^(7/2)/Sqrt[c*Sec[a + b*x]],x]
```


```
[Out] (-4*c*d^3*Sqrt[d*Csc[a + b*x]])/(5*b*(c*Sec[a + b*x])^(3/2)) - (2*c*d*(d*Cs
c[a + b*x])^(5/2))/(5*b*(c*Sec[a + b*x])^(3/2)) - (4*d^4*EllipticE[a - Pi/4
 + b*x, 2])/(5*b*Sqrt[d*Csc[a + b*x]]*Sqrt[c*Sec[a + b*x]]*Sqrt[Sin[2*a + 2
*b*x]])
```


Rule 2625

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> -Simp[(a*b*(a*Csc[e + f*x])^(m - 1)*(b*Sec[e + f*x])^(n -
 1))/(f*(m - 1)), x] + Dist[(a^2*(m + n - 2))/(m - 1), Int[(a*Csc[e + f*x])
^(m - 2)*(b*Sec[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m,
1] && IntegersQ[2*m, 2*n] &&  !GtQ[n, m]
```

Rule 2630

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Dist[(a*Csc[e + f*x])^m*(b*Sec[e + f*x])^n*(a*Sin[e + f*x]
)^m*(b*Cos[e + f*x])^n, Int[1/((a*Sin[e + f*x])^m*(b*Cos[e + f*x])^n), x],
x] /; FreeQ[{a, b, e, f, m, n}, x] && IntegerQ[m - 1/2] && IntegerQ[n - 1/2
]
```

Rule 2572

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[(Sqrt[a*Sin[e + f*x]]*Sqrt[b*Cos[e + f*x]])/Sqrt[Sin[2*
e + 2*f*x]], Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(d\csc(a+bx))^{7/2}}{\sqrt{c\sec(a+bx)}}dx = -\frac{2cd(d\csc(a+bx))^{5/2}}{5b(c\sec(a+bx))^{3/2}} + \frac{1}{5}\left(2d^2\right)\int \frac{(d\csc(a+bx))^{3/2}}{\sqrt{c\sec(a+bx)}}dx$$
$$= -\frac{4cd^3\sqrt{d\csc(a+bx)}}{5b(c\sec(a+bx))^{3/2}} - \frac{2cd(d\csc(a+bx))^{5/2}}{5b(c\sec(a+bx))^{3/2}} - \frac{1}{5}\left(4d^4\right)\int \frac{1}{\sqrt{d\csc(a+bx)}\sqrt{c\sec(a+bx)}}dx$$
$$= -\frac{4cd^3\sqrt{d\csc(a+bx)}}{5b(c\sec(a+bx))^{3/2}} - \frac{2cd(d\csc(a+bx))^{5/2}}{5b(c\sec(a+bx))^{3/2}} - \frac{\left(4d^4\right)\int \sqrt{c\cos(a+bx)}\sqrt{d\sin(a+b}{5\sqrt{c\cos(a+bx)}\sqrt{d\csc(a+bx)}\sqrt{c\sec(a+bx)}}$$
$$= -\frac{4cd^3\sqrt{d\csc(a+bx)}}{5b(c\sec(a+bx))^{3/2}} - \frac{2cd(d\csc(a+bx))^{5/2}}{5b(c\sec(a+bx))^{3/2}} - \frac{\left(4d^4\right)\int \sqrt{\sin(2a+2bx)}\,dx}{5\sqrt{d\csc(a+bx)}\sqrt{c\sec(a+bx)}\sqrt{\sin(2a+2bx)}}$$
$$= -\frac{4cd^3\sqrt{d\csc(a+bx)}}{5b(c\sec(a+bx))^{3/2}} - \frac{2cd(d\csc(a+bx))^{5/2}}{5b(c\sec(a+bx))^{3/2}} - \frac{4d^4E\left(a-\frac{\pi}{4}+bx\middle|2\right)}{5b\sqrt{d\csc(a+bx)}\sqrt{c\sec(a+bx)}\sqrt{\sin(2a+2b}}$$

Mathematica [C] time = 1.04791, size = 104, normalized size = 0.81

$$-\frac{2d^2\tan^2(a+bx)(d\csc(a+bx))^{3/2}\left(\sin(2(a+bx))\sqrt[4]{-\cot^2(a+bx)}\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{4},\frac{1}{2},\csc^2(a+bx)\right)-(\cos}{5b\sqrt{c\sec(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Csc[a + b*x])^(7/2)/Sqrt[c*Sec[a + b*x]],x]
```

```
[Out] (-2*d^2*(d*Csc[a + b*x])^(3/2)*(-((-2 + Cos[2*(a + b*x)])*Cot[a + b*x]^3) +
 (-Cot[a + b*x]^2)^(1/4)*Hypergeometric2F1[-1/2, 1/4, 1/2, Csc[a + b*x]^2]*
Sin[2*(a + b*x)])*Tan[a + b*x]^2)/(5*b*Sqrt[c*Sec[a + b*x]])
```

Maple [B] time = 0.207, size = 992, normalized size = 7.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*csc(b*x+a))^(7/2)/(c*sec(b*x+a))^(1/2),x)
```

```
[Out] -1/5/b*2^(1/2)*(4*cos(b*x+a)^3*(-(-1+cos(b*x+a)-sin(b*x+a))/sin(b*x+a))^(1/
2)*((-1+cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a
))^(1/2)*EllipticE((-(-1+cos(b*x+a)-sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/
2))-2*cos(b*x+a)^3*(-(-1+cos(b*x+a)-sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(
b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*Ell
ipticF((-(-1+cos(b*x+a)-sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))+4*cos(b*
x+a)^2*(-(-1+cos(b*x+a)-sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a)+sin(b
*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*EllipticE((-(-1
+cos(b*x+a)-sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))-2*cos(b*x+a)^2*(-(-1
+cos(b*x+a)-sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a)+sin(b*x+a))/sin(b
*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*EllipticF((-(-1+cos(b*x+a)-
sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))-4*cos(b*x+a)*(-(-1+cos(b*x+a)-si
n(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*(
(-1+cos(b*x+a))/sin(b*x+a))^(1/2)*EllipticE((-(-1+cos(b*x+a)-sin(b*x+a))/si
n(b*x+a))^(1/2),1/2*2^(1/2))+2*cos(b*x+a)*(-(-1+cos(b*x+a)-sin(b*x+a))/sin(
b*x+a))^(1/2)*((-1+cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a)
```

```
)/sin(b*x+a))^(1/2)*EllipticF((-(-1+cos(b*x+a)-sin(b*x+a))/sin(b*x+a))^(1/2
),1/2*2^(1/2))-2*cos(b*x+a)^3*2^(1/2)-4*(-(-1+cos(b*x+a)-sin(b*x+a))/sin(b*
x+a))^(1/2)*((-1+cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/
sin(b*x+a))^(1/2)*EllipticE((-(-1+cos(b*x+a)-sin(b*x+a))/sin(b*x+a))^(1/2),
1/2*2^(1/2))+2*(-(-1+cos(b*x+a)-sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+
a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*Ellipti
cF((-(-1+cos(b*x+a)-sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))+cos(b*x+a)^2
*2^(1/2)+2*cos(b*x+a)*2^(1/2))*(d/sin(b*x+a))^(7/2)*sin(b*x+a)/(c/cos(b*x+a
))^(1/2)/cos(b*x+a)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(d\csc(bx+a))^{\frac{7}{2}}}{\sqrt{c\sec(bx+a)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(b*x+a))^(7/2)/(c*sec(b*x+a))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((d*csc(b*x + a))^(7/2)/sqrt(c*sec(b*x + a)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\sqrt{d\csc(bx+a)}\sqrt{c\sec(bx+a)}d^{3}\csc(bx+a)^{3}}{c\sec(bx+a)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(b*x+a))^(7/2)/(c*sec(b*x+a))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*csc(b*x + a))*sqrt(c*sec(b*x + a))*d^3*csc(b*x + a)^3/(c*se
c(b*x + a)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(b*x+a))**(7/2)/(c*sec(b*x+a))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(d\csc(bx+a))^{\frac{7}{2}}}{\sqrt{c\sec(bx+a)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(b*x+a))^(7/2)/(c*sec(b*x+a))^(1/2),x, algorithm="giac")

[Out] integrate((d*csc(b*x + a))^(7/2)/sqrt(c*sec(b*x + a)), x)
```

3.256 $\int \frac{(d\csc(a+bx))^{5/2}}{\sqrt{c\sec(a+bx)}}\,dx$

Optimal. Leaf size=33

$$-\frac{2cd(d\csc(a+bx))^{3/2}}{3b(c\sec(a+bx))^{3/2}}$$

```
[Out] (-2*c*d*(d*Csc[a + b*x])^(3/2))/(3*b*(c*Sec[a + b*x])^(3/2))
```

Rubi [A] time = 0.0480831, antiderivative size = 33, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.04, Rules used = {2619}

$$-\frac{2cd(d\csc(a+bx))^{3/2}}{3b(c\sec(a+bx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Csc[a + b*x])^(5/2)/Sqrt[c*Sec[a + b*x]],x]
```

```
[Out] (-2*c*d*(d*Csc[a + b*x])^(3/2))/(3*b*(c*Sec[a + b*x])^(3/2))
```

Rule 2619

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(a*b*(a*Csc[e + f*x])^(m - 1)*(b*Sec[e + f*x])^(n - 1
))/(f*(n - 1)), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n - 2, 0] &&
NeQ[n, 1]
```

Rubi steps

$$\int \frac{(d\csc(a+bx))^{5/2}}{\sqrt{c\sec(a+bx)}}\,dx = -\frac{2cd(d\csc(a+bx))^{3/2}}{3b(c\sec(a+bx))^{3/2}}$$

Mathematica [A] time = 0.111938, size = 33, normalized size = 1.

$$-\frac{2cd(d\csc(a+bx))^{3/2}}{3b(c\sec(a+bx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Csc[a + b*x])^(5/2)/Sqrt[c*Sec[a + b*x]],x]
```

```
[Out] (-2*c*d*(d*Csc[a + b*x])^(3/2))/(3*b*(c*Sec[a + b*x])^(3/2))
```

Maple [A] time = 0.153, size = 42, normalized size = 1.3

$$-\frac{2\cos(bx+a)\sin(bx+a)}{3b}\left(\frac{d}{\sin(bx+a)}\right)^{\frac{5}{2}}\frac{1}{\sqrt{\frac{c}{\cos(bx+a)}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*csc(b*x+a))^(5/2)/(c*sec(b*x+a))^(1/2),x)
```

```
[Out] -2/3/b*cos(b*x+a)*sin(b*x+a)*(d/sin(b*x+a))^(5/2)/(c/cos(b*x+a))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(d\csc(bx+a))^{\frac{5}{2}}}{\sqrt{c\sec(bx+a)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(b*x+a))^(5/2)/(c*sec(b*x+a))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((d*csc(b*x + a))^(5/2)/sqrt(c*sec(b*x + a)), x)
```

Fricas [A] time = 2.06628, size = 116, normalized size = 3.52

$$-\frac{2\,d^2\sqrt{\frac{c}{\cos(bx+a)}}\sqrt{\frac{d}{\sin(bx+a)}}\cos(bx+a)^2}{3\,bc\sin(bx+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(b*x+a))^(5/2)/(c*sec(b*x+a))^(1/2),x, algorithm="fricas")
```

```
[Out] -2/3*d^2*sqrt(c/cos(b*x + a))*sqrt(d/sin(b*x + a))*cos(b*x + a)^2/(b*c*sin(
b*x + a))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(b*x+a))**(5/2)/(c*sec(b*x+a))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(d\csc(bx+a))^{\frac{5}{2}}}{\sqrt{c\sec(bx+a)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(b*x+a))^(5/2)/(c*sec(b*x+a))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((d*csc(b*x + a))^(5/2)/sqrt(c*sec(b*x + a)), x)
```

3.257 $\int \frac{(d\csc(a+bx))^{3/2}}{\sqrt{c\sec(a+bx)}}\,dx$

Optimal. Leaf size=89

$$-\frac{2d^2E\left(a+bx-\frac{\pi}{4}\middle|2\right)}{b\sqrt{\sin(2a+2bx)}\sqrt{c\sec(a+bx)}\sqrt{d\csc(a+bx)}}-\frac{2cd\sqrt{d\csc(a+bx)}}{b(c\sec(a+bx))^{3/2}}$$

```
[Out] (-2*c*d*Sqrt[d*Csc[a + b*x]])/(b*(c*Sec[a + b*x])^(3/2)) - (2*d^2*EllipticE
[a - Pi/4 + b*x, 2])/(b*Sqrt[d*Csc[a + b*x]]*Sqrt[c*Sec[a + b*x]]*Sqrt[Sin[
2*a + 2*b*x]])
```

Rubi [A] time = 0.139264, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {2625, 2630, 2572, 2639}

$$-\frac{2d^2E\left(a+bx-\frac{\pi}{4}\middle|2\right)}{b\sqrt{\sin(2a+2bx)}\sqrt{c\sec(a+bx)}\sqrt{d\csc(a+bx)}}-\frac{2cd\sqrt{d\csc(a+bx)}}{b(c\sec(a+bx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Csc[a + b*x])^(3/2)/Sqrt[c*Sec[a + b*x]],x]
```

```
[Out] (-2*c*d*Sqrt[d*Csc[a + b*x]])/(b*(c*Sec[a + b*x])^(3/2)) - (2*d^2*EllipticE
[a - Pi/4 + b*x, 2])/(b*Sqrt[d*Csc[a + b*x]]*Sqrt[c*Sec[a + b*x]]*Sqrt[Sin[
2*a + 2*b*x]])
```

Rule 2625

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> -Simp[(a*b*(a*Csc[e + f*x])^(m - 1)*(b*Sec[e + f*x])^(n -
 1))/(f*(m - 1)), x] + Dist[(a^2*(m + n - 2))/(m - 1), Int[(a*Csc[e + f*x])
^(m - 2)*(b*Sec[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m,
1] && IntegersQ[2*m, 2*n] &&  !GtQ[n, m]
```

Rule 2630

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Dist[(a*Csc[e + f*x])^m*(b*Sec[e + f*x])^n*(a*Sin[e + f*x]
)^m*(b*Cos[e + f*x])^n, Int[1/((a*Sin[e + f*x])^m*(b*Cos[e + f*x])^n), x],
x] /; FreeQ[{a, b, e, f, m, n}, x] && IntegerQ[m - 1/2] && IntegerQ[n - 1/2
]
```

Rule 2572

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[(Sqrt[a*Sin[e + f*x]]*Sqrt[b*Cos[e + f*x]])/Sqrt[Sin[2*
e + 2*f*x]], Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(d\csc(a+bx))^{3/2}}{\sqrt{c\sec(a+bx)}}\,dx = -\frac{2cd\sqrt{d\csc(a+bx)}}{b(c\sec(a+bx))^{3/2}} - \left(2d^2\right)\int \frac{1}{\sqrt{d\csc(a+bx)}\sqrt{c\sec(a+bx)}}\,dx$$
$$= -\frac{2cd\sqrt{d\csc(a+bx)}}{b(c\sec(a+bx))^{3/2}} - \frac{\left(2d^2\right)\int \sqrt{c\cos(a+bx)}\sqrt{d\sin(a+bx)}\,dx}{\sqrt{c\cos(a+bx)}\sqrt{d\csc(a+bx)}\sqrt{c\sec(a+bx)}\sqrt{d\sin(a+bx)}}$$
$$= -\frac{2cd\sqrt{d\csc(a+bx)}}{b(c\sec(a+bx))^{3/2}} - \frac{\left(2d^2\right)\int \sqrt{\sin(2a+2bx)}\,dx}{\sqrt{d\csc(a+bx)}\sqrt{c\sec(a+bx)}\sqrt{\sin(2a+2bx)}}$$
$$= -\frac{2cd\sqrt{d\csc(a+bx)}}{b(c\sec(a+bx))^{3/2}} - \frac{2d^2E\left(a-\frac{\pi}{4}+bx\middle|2\right)}{b\sqrt{d\csc(a+bx)}\sqrt{c\sec(a+bx)}\sqrt{\sin(2a+2bx)}}$$

Mathematica [C] time = 0.466088, size = 80, normalized size = 0.9

$$-\frac{2d^2\tan(a+bx)\left(\sqrt[4]{-\cot^2(a+bx)}\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{4},\frac{1}{2},\csc^2(a+bx)\right)+\cot^2(a+bx)\right)}{b\sqrt{c\sec(a+bx)}\sqrt{d\csc(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Csc[a + b*x])^(3/2)/Sqrt[c*Sec[a + b*x]],x]
```

```
[Out] (-2*d^2*(Cot[a + b*x]^2 + (-Cot[a + b*x]^2)^(1/4)*Hypergeometric2F1[-1/2, 1
/4, 1/2, Csc[a + b*x]^2])*Tan[a + b*x])/(b*Sqrt[d*Csc[a + b*x]]*Sqrt[c*Sec[
a + b*x]])
```

Maple [B] time = 0.185, size = 502, normalized size = 5.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*csc(b*x+a))^(3/2)/(c*sec(b*x+a))^(1/2),x)
```

```
[Out] 1/b*2^(1/2)*(2*cos(b*x+a)*(-(-1+cos(b*x+a)-sin(b*x+a))/sin(b*x+a))^(1/2)*((
-1+cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1
/2)*EllipticE((-(-1+cos(b*x+a)-sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))-c
os(b*x+a)*(-(-1+cos(b*x+a)-sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a)+si
n(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*EllipticF((-
(-1+cos(b*x+a)-sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))+2*(-(-1+cos(b*x+a
)-sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/
2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*EllipticE((-(-1+cos(b*x+a)-sin(b*x+a)
)/sin(b*x+a))^(1/2),1/2*2^(1/2))-(-(-1+cos(b*x+a)-sin(b*x+a))/sin(b*x+a))^(
1/2)*((-1+cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x
+a))^(1/2)*EllipticF((-(-1+cos(b*x+a)-sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(
1/2))-cos(b*x+a)*2^(1/2))*(d/sin(b*x+a))^(3/2)*sin(b*x+a)/(c/cos(b*x+a))^(1
/2)/cos(b*x+a)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(d\csc(bx+a))^{\frac{3}{2}}}{\sqrt{c\sec(bx+a)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(b*x+a))^(3/2)/(c*sec(b*x+a))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((d*csc(b*x + a))^(3/2)/sqrt(c*sec(b*x + a)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{d\csc\left(bx+a\right)}\sqrt{c\sec\left(bx+a\right)}d\csc\left(bx+a\right)}{c\sec\left(bx+a\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(b*x+a))^(3/2)/(c*sec(b*x+a))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*csc(b*x + a))*sqrt(c*sec(b*x + a))*d*csc(b*x + a)/(c*sec(b*
x + a)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(b*x+a))**(3/2)/(c*sec(b*x+a))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(d\csc\left(bx+a\right)\right)^{\frac{3}{2}}}{\sqrt{c\sec\left(bx+a\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(b*x+a))^(3/2)/(c*sec(b*x+a))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((d*csc(b*x + a))^(3/2)/sqrt(c*sec(b*x + a)), x)
```

3.258 $\int \frac{\sqrt{d\csc(a+bx)}}{\sqrt{c\sec(a+bx)}}\,dx$

Optimal. Leaf size=270

$$-\frac{\sqrt{\tan(a+bx)}\tan^{-1}\left(1-\sqrt{2}\sqrt{\tan(a+bx)}\right)\sqrt{d\csc(a+bx)}}{\sqrt{2}b\sqrt{c\sec(a+bx)}}+\frac{\tan^{-1}\left(\sqrt{2}\sqrt{\tan(a+bx)}+1\right)\sqrt{\tan(a+bx)}\sqrt{d\csc(a+bx}}{\sqrt{2}b\sqrt{c\sec(a+bx)}}$$

```
[Out] -((ArcTan[1 - Sqrt[2]*Sqrt[Tan[a + b*x]]]*Sqrt[d*Csc[a + b*x]]*Sqrt[Tan[a +
 b*x]])/(Sqrt[2]*b*Sqrt[c*Sec[a + b*x]])) + (ArcTan[1 + Sqrt[2]*Sqrt[Tan[a
+ b*x]]]*Sqrt[d*Csc[a + b*x]]*Sqrt[Tan[a + b*x]])/(Sqrt[2]*b*Sqrt[c*Sec[a +
 b*x]]) - (Sqrt[d*Csc[a + b*x]]*Log[1 - Sqrt[2]*Sqrt[Tan[a + b*x]] + Tan[a
+ b*x]]*Sqrt[Tan[a + b*x]])/(2*Sqrt[2]*b*Sqrt[c*Sec[a + b*x]]) + (Sqrt[d*Cs
c[a + b*x]]*Log[1 + Sqrt[2]*Sqrt[Tan[a + b*x]] + Tan[a + b*x]]*Sqrt[Tan[a +
 b*x]])/(2*Sqrt[2]*b*Sqrt[c*Sec[a + b*x]])
```

Rubi [A] time = 0.130926, antiderivative size = 270, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.36, Rules used = {2629, 3476, 329, 211, 1165, 628, 1162, 617, 204}

$$-\frac{\sqrt{\tan(a+bx)}\tan^{-1}\left(1-\sqrt{2}\sqrt{\tan(a+bx)}\right)\sqrt{d\csc(a+bx)}}{\sqrt{2}b\sqrt{c\sec(a+bx)}}+\frac{\tan^{-1}\left(\sqrt{2}\sqrt{\tan(a+bx)}+1\right)\sqrt{\tan(a+bx)}\sqrt{d\csc(a+bx}}{\sqrt{2}b\sqrt{c\sec(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d*Csc[a + b*x]]/Sqrt[c*Sec[a + b*x]],x]
```

```
[Out] -((ArcTan[1 - Sqrt[2]*Sqrt[Tan[a + b*x]]]*Sqrt[d*Csc[a + b*x]]*Sqrt[Tan[a +
 b*x]])/(Sqrt[2]*b*Sqrt[c*Sec[a + b*x]])) + (ArcTan[1 + Sqrt[2]*Sqrt[Tan[a
+ b*x]]]*Sqrt[d*Csc[a + b*x]]*Sqrt[Tan[a + b*x]])/(Sqrt[2]*b*Sqrt[c*Sec[a +
 b*x]]) - (Sqrt[d*Csc[a + b*x]]*Log[1 - Sqrt[2]*Sqrt[Tan[a + b*x]] + Tan[a
+ b*x]]*Sqrt[Tan[a + b*x]])/(2*Sqrt[2]*b*Sqrt[c*Sec[a + b*x]]) + (Sqrt[d*Cs
c[a + b*x]]*Log[1 + Sqrt[2]*Sqrt[Tan[a + b*x]] + Tan[a + b*x]]*Sqrt[Tan[a +
 b*x]])/(2*Sqrt[2]*b*Sqrt[c*Sec[a + b*x]])
```

Rule 2629

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Dist[((a*Csc[e + f*x])^m*(b*Sec[e + f*x])^n)/Tan[e + f*x]^
n, Int[Tan[e + f*x]^n, x], x] /; FreeQ[{a, b, e, f, m, n}, x] &&  !IntegerQ
[n] && EqQ[m + n, 0]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\sqrt{d\csc(a+bx)}}{\sqrt{c\sec(a+bx)}}\,dx = \frac{\left(\sqrt{d\csc(a+bx)}\sqrt{\tan(a+bx)}\right)\int \frac{1}{\sqrt{\tan(a+bx)}}\,dx}{\sqrt{c\sec(a+bx)}}$$

$$= \frac{\left(\sqrt{d\csc(a+bx)}\sqrt{\tan(a+bx)}\right)\text{Subst}\left(\int \frac{1}{\sqrt{x}(1+x^2)}\,dx, x, \tan(a+bx)\right)}{b\sqrt{c\sec(a+bx)}}$$

$$= \frac{\left(2\sqrt{d\csc(a+bx)}\sqrt{\tan(a+bx)}\right)\text{Subst}\left(\int \frac{1}{1+x^4}\,dx, x, \sqrt{\tan(a+bx)}\right)}{b\sqrt{c\sec(a+bx)}}$$

$$= \frac{\left(\sqrt{d\csc(a+bx)}\sqrt{\tan(a+bx)}\right)\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \sqrt{\tan(a+bx)}\right)}{b\sqrt{c\sec(a+bx)}} + \frac{\left(\sqrt{d\csc(a+bx)}\sqrt{\tan(a+\right.}{b}$$

$$= \frac{\left(\sqrt{d\csc(a+bx)}\sqrt{\tan(a+bx)}\right)\text{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}\,dx, x, \sqrt{\tan(a+bx)}\right)}{2b\sqrt{c\sec(a+bx)}} + \frac{\left(\sqrt{d\csc(a+bx)}\sqrt{\tan(\right.}{}$$

$$= -\frac{\sqrt{d\csc(a+bx)}\log\left(1-\sqrt{2}\sqrt{\tan(a+bx)}+\tan(a+bx)\right)\sqrt{\tan(a+bx)}}{2\sqrt{2}b\sqrt{c\sec(a+bx)}} + \frac{\sqrt{d\csc(a+bx)}\log\left(1\right.}{}$$

$$= -\frac{\tan^{-1}\left(1-\sqrt{2}\sqrt{\tan(a+bx)}\right)\sqrt{d\csc(a+bx)}\sqrt{\tan(a+bx)}}{\sqrt{2}b\sqrt{c\sec(a+bx)}} + \frac{\tan^{-1}\left(1+\sqrt{2}\sqrt{\tan(a+bx)}\right)\sqrt{d\,c}}{\sqrt{2}b\sqrt{c\sec(a}}$$

Mathematica [C] time = 0.125277, size = 55, normalized size = 0.2

$$-\frac{2\cot(a+bx)\sqrt{d\csc(a+bx)}\text{Hypergeometric2F1}\left(\frac{3}{4},1,\frac{7}{4},-\cot^2(a+bx)\right)}{3b\sqrt{c\sec(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d*Csc[a + b*x]]/Sqrt[c*Sec[a + b*x]],x]
```

```
[Out] (-2*Cot[a + b*x]*Sqrt[d*Csc[a + b*x]]*Hypergeometric2F1[3/4, 1, 7/4, -Cot[a
 + b*x]^2])/(3*b*Sqrt[c*Sec[a + b*x]])
```

Maple [C] time = 0.184, size = 322, normalized size = 1.2

$$-\frac{\sqrt{2}(\sin(bx+a))^2}{2b\cos(bx+a)(-1+\cos(bx+a))}\sqrt{\frac{d}{\sin(bx+a)}}\sqrt{-\frac{-1+\cos(bx+a)-\sin(bx+a)}{\sin(bx+a)}}\sqrt{\frac{-1+\cos(bx+a)+\sin(bx+a}{\sin(bx+a)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*csc(b*x+a))^(1/2)/(c*sec(b*x+a))^(1/2),x)
```

```
[Out] -1/2/b*2^(1/2)*(d/sin(b*x+a))^(1/2)*(-(-1+cos(b*x+a)-sin(b*x+a))/sin(b*x+a)
)^(1/2)*((-1+cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(
b*x+a))^(1/2)*(I*EllipticPi((-(-1+cos(b*x+a)-sin(b*x+a))/sin(b*x+a))^(1/2),
1/2-1/2*I,1/2*2^(1/2))-I*EllipticPi((-(-1+cos(b*x+a)-sin(b*x+a))/sin(b*x+a)
)^(1/2),1/2+1/2*I,1/2*2^(1/2))-2*EllipticF((-(-1+cos(b*x+a)-sin(b*x+a))/sin
(b*x+a))^(1/2),1/2*2^(1/2))+EllipticPi((-(-1+cos(b*x+a)-sin(b*x+a))/sin(b*x
+a))^(1/2),1/2-1/2*I,1/2*2^(1/2))+EllipticPi((-(-1+cos(b*x+a)-sin(b*x+a))/s
in(b*x+a))^(1/2),1/2+1/2*I,1/2*2^(1/2)))*sin(b*x+a)^2/(c/cos(b*x+a))^(1/2)/
```

```
cos(b*x+a)/(-1+cos(b*x+a))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{d \csc(bx+a)}}{\sqrt{c \sec(bx+a)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(b*x+a))^(1/2)/(c*sec(b*x+a))^(1/2),x, algorithm="maxima")

[Out] integrate(sqrt(d*csc(b*x + a))/sqrt(c*sec(b*x + a)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(b*x+a))^(1/2)/(c*sec(b*x+a))^(1/2),x, algorithm="fricas")

[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{d \csc(a+bx)}}{\sqrt{c \sec(a+bx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(b*x+a))**(1/2)/(c*sec(b*x+a))**(1/2),x)

[Out] Integral(sqrt(d*csc(a + b*x))/sqrt(c*sec(a + b*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{d \csc(bx+a)}}{\sqrt{c \sec(bx+a)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(b*x+a))^(1/2)/(c*sec(b*x+a))^(1/2),x, algorithm="giac")

[Out] integrate(sqrt(d*csc(b*x + a))/sqrt(c*sec(b*x + a)), x)
```

3.259 $\int \frac{1}{\sqrt{d\csc(a+bx)}\sqrt{c\sec(a+bx)}}\,dx$

Optimal. Leaf size=53

$$\frac{E\left(a+bx-\frac{\pi}{4}\middle|2\right)}{b\sqrt{\sin(2a+2bx)}\sqrt{c\sec(a+bx)}\sqrt{d\csc(a+bx)}}$$

```
[Out] EllipticE[a - Pi/4 + b*x, 2]/(b*Sqrt[d*Csc[a + b*x]]*Sqrt[c*Sec[a + b*x]]*S
qrt[Sin[2*a + 2*b*x]])
```

Rubi [A] time = 0.0884838, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {2630, 2572, 2639}

$$\frac{E\left(a+bx-\frac{\pi}{4}\middle|2\right)}{b\sqrt{\sin(2a+2bx)}\sqrt{c\sec(a+bx)}\sqrt{d\csc(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[d*Csc[a + b*x]]*Sqrt[c*Sec[a + b*x]]),x]
```

```
[Out] EllipticE[a - Pi/4 + b*x, 2]/(b*Sqrt[d*Csc[a + b*x]]*Sqrt[c*Sec[a + b*x]]*S
qrt[Sin[2*a + 2*b*x]])
```

Rule 2630

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Dist[(a*Csc[e + f*x])^m*(b*Sec[e + f*x])^n*(a*Sin[e + f*x]
)^m*(b*Cos[e + f*x])^n, Int[1/((a*Sin[e + f*x])^m*(b*Cos[e + f*x])^n), x],
x] /; FreeQ[{a, b, e, f, m, n}, x] && IntegerQ[m - 1/2] && IntegerQ[n - 1/2
]
```

Rule 2572

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[(Sqrt[a*Sin[e + f*x]]*Sqrt[b*Cos[e + f*x]])/Sqrt[Sin[2*
e + 2*f*x]], Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt{d\csc(a+bx)}\sqrt{c\sec(a+bx)}}\,dx &= \frac{\int \sqrt{c\cos(a+bx)}\sqrt{d\sin(a+bx)}\,dx}{\sqrt{c\cos(a+bx)}\sqrt{d\csc(a+bx)}\sqrt{c\sec(a+bx)}\sqrt{d\sin(a+bx)}}\\
&= \frac{\int \sqrt{\sin(2a+2bx)}\,dx}{\sqrt{d\csc(a+bx)}\sqrt{c\sec(a+bx)}\sqrt{\sin(2a+2bx)}}\\
&= \frac{E\left(a-\frac{\pi}{4}+bx\middle|2\right)}{b\sqrt{d\csc(a+bx)}\sqrt{c\sec(a+bx)}\sqrt{\sin(2a+2bx)}}
\end{aligned}$$

Mathematica [C] time = 0.218229, size = 66, normalized size = 1.25

$$\frac{\tan(a+bx)\sqrt[4]{-\cot^2(a+bx)}\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{4},\frac{1}{2},\csc^2(a+bx)\right)}{b\sqrt{c\sec(a+bx)}\sqrt{d\csc(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[d*Csc[a + b*x]]*Sqrt[c*Sec[a + b*x]]),x]
```

```
[Out] ((-Cot[a + b*x]^2)^(1/4)*Hypergeometric2F1[-1/2, 1/4, 1/2, Csc[a + b*x]^2]*
Tan[a + b*x])/(b*Sqrt[d*Csc[a + b*x]]*Sqrt[c*Sec[a + b*x]])
```

Maple [B] time = 0.172, size = 517, normalized size = 9.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*csc(b*x+a))^(1/2)/(c*sec(b*x+a))^(1/2),x)
```

```
[Out] -1/2/b*2^(1/2)*(2*cos(b*x+a)*(-(-1+cos(b*x+a)-sin(b*x+a))/sin(b*x+a))^(1/2)
*((-1+cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))
^(1/2)*EllipticE((-(-1+cos(b*x+a)-sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2)
)-cos(b*x+a)*(-(-1+cos(b*x+a)-sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a)
+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*EllipticF
((-(-1+cos(b*x+a)-sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))+2*(-(-1+cos(b*
x+a)-sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^
(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*EllipticE((-(-1+cos(b*x+a)-sin(b*x
+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))-(-(-1+cos(b*x+a)-sin(b*x+a))/sin(b*x+a)
)^(1/2)*((-1+cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(
b*x+a))^(1/2)*EllipticF((-(-1+cos(b*x+a)-sin(b*x+a))/sin(b*x+a))^(1/2),1/2*
2^(1/2))+cos(b*x+a)^2*2^(1/2)-cos(b*x+a)*2^(1/2))/cos(b*x+a)/sin(b*x+a)/(d/
sin(b*x+a))^(1/2)/(c/cos(b*x+a))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{d\csc(bx+a)}\sqrt{c\sec(bx+a)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*csc(b*x+a))^(1/2)/(c*sec(b*x+a))^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate(1/(sqrt(d*csc(b*x + a))*sqrt(c*sec(b*x + a))), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{d\csc(bx+a)}\sqrt{c\sec(bx+a)}}{cd\csc(bx+a)\sec(bx+a)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*csc(b*x+a))^(1/2)/(c*sec(b*x+a))^(1/2),x, algorithm="fricas"
)
```

```
[Out] integral(sqrt(d*csc(b*x + a))*sqrt(c*sec(b*x + a))/(c*d*csc(b*x + a)*sec(b*
x + a)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{c\sec{(a+bx)}}\sqrt{d\csc{(a+bx)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*csc(b*x+a))**(1/2)/(c*sec(b*x+a))**(1/2),x)
```

```
[Out] Integral(1/(sqrt(c*sec(a + b*x))*sqrt(d*csc(a + b*x))), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{d\csc{(bx+a)}}\sqrt{c\sec{(bx+a)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*csc(b*x+a))^(1/2)/(c*sec(b*x+a))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(d*csc(b*x + a))*sqrt(c*sec(b*x + a))), x)
```

3.260 $\int \frac{1}{(d\csc(a+bx))^{3/2}\sqrt{c\sec(a+bx)}}\,dx$

Optimal. Leaf size=322

$$-\frac{\tan^{-1}\left(1-\sqrt{2}\sqrt{\tan(a+bx)}\right)\sqrt{\tan(a+bx)}\sqrt{d\csc(a+bx)}}{4\sqrt{2}bd^2\sqrt{c\sec(a+bx)}}+\frac{\tan^{-1}\left(\sqrt{2}\sqrt{\tan(a+bx)}+1\right)\sqrt{\tan(a+bx)}\sqrt{d\csc(a+}}{4\sqrt{2}bd^2\sqrt{c\sec(a+bx)}}$$

```
[Out] -c/(2*b*d*Sqrt[d*Csc[a + b*x]]*(c*Sec[a + b*x])^(3/2)) - (ArcTan[1 - Sqrt[2
]*Sqrt[Tan[a + b*x]]]*Sqrt[d*Csc[a + b*x]]*Sqrt[Tan[a + b*x]])/(4*Sqrt[2]*b
*d^2*Sqrt[c*Sec[a + b*x]]) + (ArcTan[1 + Sqrt[2]*Sqrt[Tan[a + b*x]]]*Sqrt[d
*Csc[a + b*x]]*Sqrt[Tan[a + b*x]])/(4*Sqrt[2]*b*d^2*Sqrt[c*Sec[a + b*x]]) -
 (Sqrt[d*Csc[a + b*x]]*Log[1 - Sqrt[2]*Sqrt[Tan[a + b*x]] + Tan[a + b*x]]*S
qrt[Tan[a + b*x]])/(8*Sqrt[2]*b*d^2*Sqrt[c*Sec[a + b*x]]) + (Sqrt[d*Csc[a +
 b*x]]*Log[1 + Sqrt[2]*Sqrt[Tan[a + b*x]] + Tan[a + b*x]]*Sqrt[Tan[a + b*x]
])/(8*Sqrt[2]*b*d^2*Sqrt[c*Sec[a + b*x]])
```

Rubi [A] time = 0.204094, antiderivative size = 322, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 10, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {2627, 2629, 3476, 329, 211, 1165, 628, 1162, 617, 204}

$$-\frac{\tan^{-1}\left(1-\sqrt{2}\sqrt{\tan(a+bx)}\right)\sqrt{\tan(a+bx)}\sqrt{d\csc(a+bx)}}{4\sqrt{2}bd^2\sqrt{c\sec(a+bx)}}+\frac{\tan^{-1}\left(\sqrt{2}\sqrt{\tan(a+bx)}+1\right)\sqrt{\tan(a+bx)}\sqrt{d\csc(a+}}{4\sqrt{2}bd^2\sqrt{c\sec(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d*Csc[a + b*x])^(3/2)*Sqrt[c*Sec[a + b*x]]),x]
```


```
[Out] -c/(2*b*d*Sqrt[d*Csc[a + b*x]]*(c*Sec[a + b*x])^(3/2)) - (ArcTan[1 - Sqrt[2
]*Sqrt[Tan[a + b*x]]]*Sqrt[d*Csc[a + b*x]]*Sqrt[Tan[a + b*x]])/(4*Sqrt[2]*b
*d^2*Sqrt[c*Sec[a + b*x]]) + (ArcTan[1 + Sqrt[2]*Sqrt[Tan[a + b*x]]]*Sqrt[d
*Csc[a + b*x]]*Sqrt[Tan[a + b*x]])/(4*Sqrt[2]*b*d^2*Sqrt[c*Sec[a + b*x]]) -
 (Sqrt[d*Csc[a + b*x]]*Log[1 - Sqrt[2]*Sqrt[Tan[a + b*x]] + Tan[a + b*x]]*S
qrt[Tan[a + b*x]])/(8*Sqrt[2]*b*d^2*Sqrt[c*Sec[a + b*x]]) + (Sqrt[d*Csc[a +
 b*x]]*Log[1 + Sqrt[2]*Sqrt[Tan[a + b*x]] + Tan[a + b*x]]*Sqrt[Tan[a + b*x]
])/(8*Sqrt[2]*b*d^2*Sqrt[c*Sec[a + b*x]])
```


Rule 2627

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Simp[(b*(a*Csc[e + f*x])^(m + 1)*(b*Sec[e + f*x])^(n - 1)
)/(a*f*(m + n)), x] + Dist[(m + 1)/(a^2*(m + n)), Int[(a*Csc[e + f*x])^(m +
 2)*(b*Sec[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -1] &
& NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2629

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Dist[((a*Csc[e + f*x])^m*(b*Sec[e + f*x])^n)/Tan[e + f*x]^
n, Int[Tan[e + f*x]^n, x], x] /; FreeQ[{a, b, e, f, m, n}, x] &&  !IntegerQ
[n] && EqQ[m + n, 0]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
```

```
IntegerQ[n]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{(d\csc(a+bx))^{3/2}\sqrt{c\sec(a+bx)}}dx = -\frac{c}{2bd\sqrt{d\csc(a+bx)}(c\sec(a+bx))^{3/2}} + \frac{\int \frac{\sqrt{d\csc(a+bx)}}{\sqrt{c\sec(a+bx)}}dx}{4d^2}$$

$$= -\frac{c}{2bd\sqrt{d\csc(a+bx)}(c\sec(a+bx))^{3/2}} + \frac{\left(\sqrt{d\csc(a+bx)}\sqrt{\tan(a+bx)}\right)\int \frac{}{\sqrt{\mathrm{t}}}}{4d^2\sqrt{c\sec(a+bx)}}$$

$$= -\frac{c}{2bd\sqrt{d\csc(a+bx)}(c\sec(a+bx))^{3/2}} + \frac{\left(\sqrt{d\csc(a+bx)}\sqrt{\tan(a+bx)}\right)\text{Subs}}{4bd^2\sqrt{c\sec}}$$

$$= -\frac{c}{2bd\sqrt{d\csc(a+bx)}(c\sec(a+bx))^{3/2}} + \frac{\left(\sqrt{d\csc(a+bx)}\sqrt{\tan(a+bx)}\right)\text{Subs}}{2bd^2\sqrt{c\sec(}}$$

$$= -\frac{c}{2bd\sqrt{d\csc(a+bx)}(c\sec(a+bx))^{3/2}} + \frac{\left(\sqrt{d\csc(a+bx)}\sqrt{\tan(a+bx)}\right)\text{Subs}}{4bd^2\sqrt{c\sec(}}$$

$$= -\frac{c}{2bd\sqrt{d\csc(a+bx)}(c\sec(a+bx))^{3/2}} + \frac{\left(\sqrt{d\csc(a+bx)}\sqrt{\tan(a+bx)}\right)\text{Subs}}{8bd^2\sqrt{c\,\mathrm{se}}}$$

$$= -\frac{c}{2bd\sqrt{d\csc(a+bx)}(c\sec(a+bx))^{3/2}} - \frac{\sqrt{d\csc(a+bx)}\log\left(1-\sqrt{2}\sqrt{\tan(a-}\right.}{8\sqrt{2}bd^2\sqrt{c\,\mathrm{s}}}$$

$$= -\frac{c}{2bd\sqrt{d\csc(a+bx)}(c\sec(a+bx))^{3/2}} - \frac{\tan^{-1}\left(1-\sqrt{2}\sqrt{\tan(a+bx)}\right)\sqrt{d\csc}}{4\sqrt{2}bd^2\sqrt{c\sec(a+}}$$

Mathematica [C] time = 0.255619, size = 66, normalized size = 0.2

$$-\frac{\cot(a+bx)\left(\csc^2(a+bx)\text{Hypergeometric2F1}\left(\frac{3}{4},1,\frac{7}{4},-\cot^2(a+bx)\right)+3\right)}{6b\sqrt{c\sec(a+bx)}(d\csc(a+bx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d*Csc[a + b*x])^(3/2)*Sqrt[c*Sec[a + b*x]]),x]
```

```
[Out] -(Cot[a + b*x]*(3 + Csc[a + b*x]^2*Hypergeometric2F1[3/4, 1, 7/4, -Cot[a +
b*x]^2]))/(6*b*(d*Csc[a + b*x])^(3/2)*Sqrt[c*Sec[a + b*x]])
```

Maple [C] time = 0.17, size = 668, normalized size = 2.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*csc(b*x+a))^(3/2)/(c*sec(b*x+a))^(1/2),x)
```

```
[Out] -1/8/b*2^(1/2)*(I*sin(b*x+a)*EllipticPi((-(-1+cos(b*x+a)-sin(b*x+a))/sin(b*
x+a))^(1/2),1/2-1/2*I,1/2*2^(1/2))*(-(-1+cos(b*x+a)-sin(b*x+a))/sin(b*x+a))
^(1/2)*((-1+cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b
*x+a))^(1/2)-I*sin(b*x+a)*(-(-1+cos(b*x+a)-sin(b*x+a))/sin(b*x+a))^(1/2)*((
-1+cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1
/2)*EllipticPi((-(-1+cos(b*x+a)-sin(b*x+a))/sin(b*x+a))^(1/2),1/2+1/2*I,1/2
*2^(1/2))+sin(b*x+a)*EllipticPi((-(-1+cos(b*x+a)-sin(b*x+a))/sin(b*x+a))^(1
/2),1/2-1/2*I,1/2*2^(1/2))*(-(-1+cos(b*x+a)-sin(b*x+a))/sin(b*x+a))^(1/2)*(
```

```
(-1+cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(
1/2)-2*sin(b*x+a)*(-(-1+cos(b*x+a)-sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b
*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*Elli
pticF((-(-1+cos(b*x+a)-sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))+sin(b*x+a
)*(-(-1+cos(b*x+a)-sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a)+sin(b*x+a)
)/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*EllipticPi((-(-1+cos
(b*x+a)-sin(b*x+a))/sin(b*x+a))^(1/2),1/2+1/2*I,1/2*2^(1/2))+2*cos(b*x+a)^3
*2^(1/2)-2*cos(b*x+a)^2*2^(1/2))/(-1+cos(b*x+a))/cos(b*x+a)/sin(b*x+a)/(d/s
in(b*x+a))^(3/2)/(c/cos(b*x+a))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(d\csc(bx+a))^{\frac{3}{2}}\sqrt{c\sec(bx+a)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*csc(b*x+a))^(3/2)/(c*sec(b*x+a))^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate(1/((d*csc(b*x + a))^(3/2)*sqrt(c*sec(b*x + a))), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*csc(b*x+a))^(3/2)/(c*sec(b*x+a))^(1/2),x, algorithm="fricas"
)
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*csc(b*x+a))**(3/2)/(c*sec(b*x+a))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(d\csc(bx+a))^{\frac{3}{2}}\sqrt{c\sec(bx+a)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*csc(b*x+a))^(3/2)/(c*sec(b*x+a))^(1/2),x, algorithm="giac")

[Out] integrate(1/((d*csc(b*x + a))^(3/2)*sqrt(c*sec(b*x + a))), x)
```

3.261 $\int \frac{1}{(d\csc(a+bx))^{5/2}\sqrt{c\sec(a+bx)}}\,dx$

Optimal. Leaf size=95

$$\frac{E\left(a+bx-\frac{\pi}{4}\middle|2\right)}{2bd^2\sqrt{\sin(2a+2bx)}\sqrt{c\sec(a+bx)}\sqrt{d\csc(a+bx)}}-\frac{c}{3bd(c\sec(a+bx))^{3/2}(d\csc(a+bx))^{3/2}}$$

```
[Out] -c/(3*b*d*(d*Csc[a + b*x])^(3/2)*(c*Sec[a + b*x])^(3/2)) + EllipticE[a - Pi
/4 + b*x, 2]/(2*b*d^2*Sqrt[d*Csc[a + b*x]]*Sqrt[c*Sec[a + b*x]]*Sqrt[Sin[2*
a + 2*b*x]])
```

Rubi [A] time = 0.141121, antiderivative size = 95, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {2627, 2630, 2572, 2639}

$$\frac{E\left(a+bx-\frac{\pi}{4}\middle|2\right)}{2bd^2\sqrt{\sin(2a+2bx)}\sqrt{c\sec(a+bx)}\sqrt{d\csc(a+bx)}}-\frac{c}{3bd(c\sec(a+bx))^{3/2}(d\csc(a+bx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d*Csc[a + b*x])^(5/2)*Sqrt[c*Sec[a + b*x]]),x]
```

```
[Out] -c/(3*b*d*(d*Csc[a + b*x])^(3/2)*(c*Sec[a + b*x])^(3/2)) + EllipticE[a - Pi
/4 + b*x, 2]/(2*b*d^2*Sqrt[d*Csc[a + b*x]]*Sqrt[c*Sec[a + b*x]]*Sqrt[Sin[2*
a + 2*b*x]])
```

Rule 2627

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Simp[(b*(a*Csc[e + f*x])^(m + 1)*(b*Sec[e + f*x])^(n - 1)
)/(a*f*(m + n)), x] + Dist[(m + 1)/(a^2*(m + n)), Int[(a*Csc[e + f*x])^(m +
 2)*(b*Sec[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -1] &
& NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2630

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Dist[(a*Csc[e + f*x])^m*(b*Sec[e + f*x])^n*(a*Sin[e + f*x]
)^m*(b*Cos[e + f*x])^n, Int[1/((a*Sin[e + f*x])^m*(b*Cos[e + f*x])^n), x],
x] /; FreeQ[{a, b, e, f, m, n}, x] && IntegerQ[m - 1/2] && IntegerQ[n - 1/2
]
```

Rule 2572

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[(Sqrt[a*Sin[e + f*x]]*Sqrt[b*Cos[e + f*x]])/Sqrt[Sin[2*
e + 2*f*x]], Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{1}{(d\csc(a+bx))^{5/2}\sqrt{c\sec(a+bx)}}\,dx = -\frac{c}{3bd(d\csc(a+bx))^{3/2}(c\sec(a+bx))^{3/2}} + \frac{\int \frac{1}{\sqrt{d\csc(a+bx)}\sqrt{c\sec(a+bx)}}\,dx}{2d^2}$$
$$= -\frac{c}{3bd(d\csc(a+bx))^{3/2}(c\sec(a+bx))^{3/2}} + \frac{\int \sqrt{c\cos(a+bx)}\sqrt{c}}{2d^2\sqrt{c\cos(a+bx)}\sqrt{d\csc(a+bx)}}$$
$$= -\frac{c}{3bd(d\csc(a+bx))^{3/2}(c\sec(a+bx))^{3/2}} + \frac{\int \sqrt{\sin(2a+2bx)}\,d}{2d^2\sqrt{d\csc(a+bx)}\sqrt{c\sec(a+bx)}}$$
$$= -\frac{c}{3bd(d\csc(a+bx))^{3/2}(c\sec(a+bx))^{3/2}} + \frac{E\left(a-\frac{\pi}{4}+bx\middle|2\right)}{2bd^2\sqrt{d\csc(a+bx)}\sqrt{c\sec(a+bx}}$$

Mathematica [C] time = 0.354372, size = 84, normalized size = 0.88

$$-\frac{\tan(a+bx)\left(-3\sqrt[4]{-\cot^2(a+bx)}\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{4},\frac{1}{2},\csc^2(a+bx)\right)+\cos(2(a+bx))+1\right)}{6bd^2\sqrt{c\sec(a+bx)}\sqrt{d\csc(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d*Csc[a + b*x])^(5/2)*Sqrt[c*Sec[a + b*x]]),x]
```

```
[Out] -((1 + Cos[2*(a + b*x)] - 3*(-Cot[a + b*x]^2)^(1/4)*Hypergeometric2F1[-1/2,
 1/4, 1/2, Csc[a + b*x]^2])*Tan[a + b*x])/(6*b*d^2*Sqrt[d*Csc[a + b*x]]*Sqr
t[c*Sec[a + b*x]])
```

Maple [B] time = 0.213, size = 531, normalized size = 5.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*csc(b*x+a))^(5/2)/(c*sec(b*x+a))^(1/2),x)
```

```
[Out] 1/12/b*2^(1/2)*(2*2^(1/2)*cos(b*x+a)^4-6*cos(b*x+a)*(-(-1+cos(b*x+a)-sin(b*
x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+
cos(b*x+a))/sin(b*x+a))^(1/2)*EllipticE((-(-1+cos(b*x+a)-sin(b*x+a))/sin(b*
x+a))^(1/2),1/2*2^(1/2))+3*cos(b*x+a)*(-(-1+cos(b*x+a)-sin(b*x+a))/sin(b*x+
a))^(1/2)*((-1+cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/si
n(b*x+a))^(1/2)*EllipticF((-(-1+cos(b*x+a)-sin(b*x+a))/sin(b*x+a))^(1/2),1/
2*2^(1/2))-6*(-(-1+cos(b*x+a)-sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a)
+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*EllipticE
((-(-1+cos(b*x+a)-sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))+3*(-(-1+cos(b*
x+a)-sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^
(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*EllipticF((-(-1+cos(b*x+a)-sin(b*x
+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))-5*cos(b*x+a)^2*2^(1/2)+3*cos(b*x+a)*2^(
1/2))/(d/sin(b*x+a))^(5/2)/(c/cos(b*x+a))^(1/2)/cos(b*x+a)/sin(b*x+a)^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(d\csc(bx+a))^{\frac{5}{2}}\sqrt{c\sec(bx+a)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*csc(b*x+a))^(5/2)/(c*sec(b*x+a))^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate(1/((d*csc(b*x + a))^(5/2)*sqrt(c*sec(b*x + a))), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{d\csc(bx+a)}\sqrt{c\sec(bx+a)}}{cd^3\csc(bx+a)^3\sec(bx+a)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*csc(b*x+a))^(5/2)/(c*sec(b*x+a))^(1/2),x, algorithm="fricas"
)
```

```
[Out] integral(sqrt(d*csc(b*x + a))*sqrt(c*sec(b*x + a))/(c*d^3*csc(b*x + a)^3*se
c(b*x + a)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*csc(b*x+a))**(5/2)/(c*sec(b*x+a))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(d\csc(bx+a))^{\frac{5}{2}}\sqrt{c\sec(bx+a)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*csc(b*x+a))^(5/2)/(c*sec(b*x+a))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/((d*csc(b*x + a))^(5/2)*sqrt(c*sec(b*x + a))), x)
```

3.262 $\int \frac{(d \csc(a+bx))^{11/2}}{(c \sec(a+bx))^{3/2}} dx$

Optimal. Leaf size=110

$$\frac{8d^5\sqrt{d\csc(a+bx)}}{45bc\sqrt{c\sec(a+bx)}}+\frac{2d^3(d\csc(a+bx))^{5/2}}{45bc\sqrt{c\sec(a+bx)}}-\frac{2d(d\csc(a+bx))^{9/2}}{9bc\sqrt{c\sec(a+bx)}}$$

```
[Out] (8*d^5*Sqrt[d*Csc[a + b*x]])/(45*b*c*Sqrt[c*Sec[a + b*x]]) + (2*d^3*(d*Csc[
a + b*x])^(5/2))/(45*b*c*Sqrt[c*Sec[a + b*x]]) - (2*d*(d*Csc[a + b*x])^(9/2
))/(9*b*c*Sqrt[c*Sec[a + b*x]])
```

Rubi [A] time = 0.153996, antiderivative size = 110, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {2623, 2625, 2619}

$$\frac{8d^5\sqrt{d\csc(a+bx)}}{45bc\sqrt{c\sec(a+bx)}}+\frac{2d^3(d\csc(a+bx))^{5/2}}{45bc\sqrt{c\sec(a+bx)}}-\frac{2d(d\csc(a+bx))^{9/2}}{9bc\sqrt{c\sec(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Csc[a + b*x])^(11/2)/(c*Sec[a + b*x])^(3/2),x]
```

```
[Out] (8*d^5*Sqrt[d*Csc[a + b*x]])/(45*b*c*Sqrt[c*Sec[a + b*x]]) + (2*d^3*(d*Csc[
a + b*x])^(5/2))/(45*b*c*Sqrt[c*Sec[a + b*x]]) - (2*d*(d*Csc[a + b*x])^(9/2
))/(9*b*c*Sqrt[c*Sec[a + b*x]])
```

Rule 2623

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> -Simp[(a*(a*Csc[e + f*x])^(m - 1)*(b*Sec[e + f*x])^(n + 1)
)/(f*b*(m - 1)), x] + Dist[(a^2*(n + 1))/(b^2*(m - 1)), Int[(a*Csc[e + f*x]
)^(m - 2)*(b*Sec[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ
[m, 1] && LtQ[n, -1] && IntegersQ[2*m, 2*n]
```

Rule 2625

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> -Simp[(a*b*(a*Csc[e + f*x])^(m - 1)*(b*Sec[e + f*x])^(n -
 1))/(f*(m - 1)), x] + Dist[(a^2*(m + n - 2))/(m - 1), Int[(a*Csc[e + f*x])
^(m - 2)*(b*Sec[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m,
1] && IntegersQ[2*m, 2*n] &&  !GtQ[n, m]
```

Rule 2619

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(a*b*(a*Csc[e + f*x])^(m - 1)*(b*Sec[e + f*x])^(n - 1
))/(f*(n - 1)), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n - 2, 0] &&
NeQ[n, 1]
```

Rubi steps

$$\int \frac{(d\csc(a+bx))^{11/2}}{(c\sec(a+bx))^{3/2}}\,dx = -\frac{2d(d\csc(a+bx))^{9/2}}{9bc\sqrt{c\sec(a+bx)}} - \frac{d^2\int (d\csc(a+bx))^{7/2}\sqrt{c\sec(a+bx)}\,dx}{9c^2}$$
$$= \frac{2d^3(d\csc(a+bx))^{5/2}}{45bc\sqrt{c\sec(a+bx)}} - \frac{2d(d\csc(a+bx))^{9/2}}{9bc\sqrt{c\sec(a+bx)}} - \frac{(4d^4)\int (d\csc(a+bx))^{3/2}\sqrt{c\sec(a+bx)}\,dx}{45c^2}$$
$$= \frac{8d^5\sqrt{d\csc(a+bx)}}{45bc\sqrt{c\sec(a+bx)}} + \frac{2d^3(d\csc(a+bx))^{5/2}}{45bc\sqrt{c\sec(a+bx)}} - \frac{2d(d\csc(a+bx))^{9/2}}{9bc\sqrt{c\sec(a+bx)}}$$

Mathematica [A] time = 0.280943, size = 57, normalized size = 0.52

$$\frac{2d^3(2\cos(2(a+bx))-7)\cot^2(a+bx)(d\csc(a+bx))^{5/2}}{45bc\sqrt{c\sec(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Csc[a + b*x])^(11/2)/(c*Sec[a + b*x])^(3/2),x]
```

```
[Out] (2*d^3*(-7 + 2*Cos[2*(a + b*x)])*Cot[a + b*x]^2*(d*Csc[a + b*x])^(5/2))/(45
*b*c*Sqrt[c*Sec[a + b*x]])
```

Maple [A] time = 0.158, size = 54, normalized size = 0.5

$$\frac{\left(8\,(\cos(bx+a))^2-18\right)\cos(bx+a)\sin(bx+a)}{45\,b}\left(\frac{d}{\sin(bx+a)}\right)^{\frac{11}{2}}\left(\frac{c}{\cos(bx+a)}\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*csc(b*x+a))^(11/2)/(c*sec(b*x+a))^(3/2),x)
```

```
[Out] 2/45/b*(4*cos(b*x+a)^2-9)*(d/sin(b*x+a))^(11/2)*cos(b*x+a)*sin(b*x+a)/(c/co
s(b*x+a))^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(d\csc(bx+a))^{\frac{11}{2}}}{(c\sec(bx+a))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(b*x+a))^(11/2)/(c*sec(b*x+a))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((d*csc(b*x + a))^(11/2)/(c*sec(b*x + a))^(3/2), x)
```

Fricas [A] time = 2.21246, size = 203, normalized size = 1.85

$$\frac{2\left(4\,d^5\cos(bx+a)^5-9\,d^5\cos(bx+a)^3\right)\sqrt{\frac{c}{\cos(bx+a)}}\sqrt{\frac{d}{\sin(bx+a)}}}{45\left(bc^2\cos(bx+a)^4-2\,bc^2\cos(bx+a)^2+bc^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(b*x+a))^(11/2)/(c*sec(b*x+a))^(3/2),x, algorithm="fricas")
```

```
[Out] 2/45*(4*d^5*cos(b*x + a)^5 - 9*d^5*cos(b*x + a)^3)*sqrt(c/cos(b*x + a))*sqr
t(d/sin(b*x + a))/(b*c^2*cos(b*x + a)^4 - 2*b*c^2*cos(b*x + a)^2 + b*c^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(b*x+a))**(11/2)/(c*sec(b*x+a))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(d\csc(bx+a))^{\frac{11}{2}}}{(c\sec(bx+a))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(b*x+a))^(11/2)/(c*sec(b*x+a))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((d*csc(b*x + a))^(11/2)/(c*sec(b*x + a))^(3/2), x)
```

3.263 $\int \frac{(d\csc(a+bx))^{9/2}}{(c\sec(a+bx))^{3/2}}\,dx$

Optimal. Leaf size=135

$$-\frac{2d^4\sqrt{\sin(2a+2bx)}\text{EllipticF}\left(a+bx-\frac{\pi}{4},2\right)\sqrt{c\sec(a+bx)}\sqrt{d\csc(a+bx)}}{21bc^2}+\frac{2d^3(d\csc(a+bx))^{3/2}}{21bc\sqrt{c\sec(a+bx)}}-\frac{2d(d\csc(a+bx))^{7}}{7bc\sqrt{c\sec(a+bx}}$$

```
[Out] (2*d^3*(d*Csc[a + b*x])^(3/2))/(21*b*c*Sqrt[c*Sec[a + b*x]]) - (2*d*(d*Csc[
a + b*x])^(7/2))/(7*b*c*Sqrt[c*Sec[a + b*x]]) - (2*d^4*Sqrt[d*Csc[a + b*x]]
*EllipticF[a - Pi/4 + b*x, 2]*Sqrt[c*Sec[a + b*x]]*Sqrt[Sin[2*a + 2*b*x]])/
(21*b*c^2)
```

Rubi [A] time = 0.199619, antiderivative size = 135, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2623, 2625, 2630, 2573, 2641}

$$-\frac{2d^4\sqrt{\sin(2a+2bx)}F\left(a+bx-\frac{\pi}{4}\middle|2\right)\sqrt{c\sec(a+bx)}\sqrt{d\csc(a+bx)}}{21bc^2}+\frac{2d^3(d\csc(a+bx))^{3/2}}{21bc\sqrt{c\sec(a+bx)}}-\frac{2d(d\csc(a+bx))^{7/2}}{7bc\sqrt{c\sec(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Csc[a + b*x])^(9/2)/(c*Sec[a + b*x])^(3/2),x]
```

```
[Out] (2*d^3*(d*Csc[a + b*x])^(3/2))/(21*b*c*Sqrt[c*Sec[a + b*x]]) - (2*d*(d*Csc[
a + b*x])^(7/2))/(7*b*c*Sqrt[c*Sec[a + b*x]]) - (2*d^4*Sqrt[d*Csc[a + b*x]]
*EllipticF[a - Pi/4 + b*x, 2]*Sqrt[c*Sec[a + b*x]]*Sqrt[Sin[2*a + 2*b*x]])/
(21*b*c^2)
```

Rule 2623

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> -Simp[(a*(a*Csc[e + f*x])^(m - 1)*(b*Sec[e + f*x])^(n + 1)
)/(f*b*(m - 1)), x] + Dist[(a^2*(n + 1))/(b^2*(m - 1)), Int[(a*Csc[e + f*x]
)^(m - 2)*(b*Sec[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ
[m, 1] && LtQ[n, -1] && IntegersQ[2*m, 2*n]
```

Rule 2625

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> -Simp[(a*b*(a*Csc[e + f*x])^(m - 1)*(b*Sec[e + f*x])^(n -
 1))/(f*(m - 1)), x] + Dist[(a^2*(m + n - 2))/(m - 1), Int[(a*Csc[e + f*x])
^(m - 2)*(b*Sec[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m,
1] && IntegersQ[2*m, 2*n] &&  !GtQ[n, m]
```

Rule 2630

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Dist[(a*Csc[e + f*x])^m*(b*Sec[e + f*x])^n*(a*Sin[e + f*x]
)^m*(b*Cos[e + f*x])^n, Int[1/((a*Sin[e + f*x])^m*(b*Cos[e + f*x])^n), x],
x] /; FreeQ[{a, b, e, f, m, n}, x] && IntegerQ[m - 1/2] && IntegerQ[n - 1/2
]
```

Rule 2573

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
```

```
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(d\csc(a+bx))^{9/2}}{(c\sec(a+bx))^{3/2}}\,dx = -\frac{2d(d\csc(a+bx))^{7/2}}{7bc\sqrt{c\sec(a+bx)}} - \frac{d^2\int (d\csc(a+bx))^{5/2}\sqrt{c\sec(a+bx)}\,dx}{7c^2}$$

$$= \frac{2d^3(d\csc(a+bx))^{3/2}}{21bc\sqrt{c\sec(a+bx)}} - \frac{2d(d\csc(a+bx))^{7/2}}{7bc\sqrt{c\sec(a+bx)}} - \frac{\left(2d^4\right)\int \sqrt{d\csc(a+bx)}\sqrt{c\sec(a+bx)}\,dx}{21c^2}$$

$$= \frac{2d^3(d\csc(a+bx))^{3/2}}{21bc\sqrt{c\sec(a+bx)}} - \frac{2d(d\csc(a+bx))^{7/2}}{7bc\sqrt{c\sec(a+bx)}} - \frac{\left(2d^4\sqrt{c\cos(a+bx)}\sqrt{d\csc(a+bx)}\sqrt{c\sec(a-}{2}$$

$$= \frac{2d^3(d\csc(a+bx))^{3/2}}{21bc\sqrt{c\sec(a+bx)}} - \frac{2d(d\csc(a+bx))^{7/2}}{7bc\sqrt{c\sec(a+bx)}} - \frac{\left(2d^4\sqrt{d\csc(a+bx)}\sqrt{c\sec(a+bx)}\sqrt{\sin(2a+}{21c^2}$$

$$= \frac{2d^3(d\csc(a+bx))^{3/2}}{21bc\sqrt{c\sec(a+bx)}} - \frac{2d(d\csc(a+bx))^{7/2}}{7bc\sqrt{c\sec(a+bx)}} - \frac{2d^4\sqrt{d\csc(a+bx)}F\left(a-\frac{\pi}{4}+bx\middle|2\right)\sqrt{c\sec(a}{21bc^2}$$

Mathematica [C] time = 1.42184, size = 119, normalized size = 0.88

$$-\frac{d^3\cos(2(a+bx))(d\csc(a+bx))^{3/2}\left((\cos(2(a+bx))+5)\csc^4(a+bx)-2\left(-\cot^2(a+bx)\right)^{3/4}\sec^2(a+bx)\text{Hypergeo}}{21bc\left(\csc^2(a+bx)-2\right)\sqrt{c\sec(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Csc[a + b*x])^(9/2)/(c*Sec[a + b*x])^(3/2),x]
```

```
[Out] -(d^3*Cos[2*(a + b*x)]*(d*Csc[a + b*x])^(3/2)*((5 + Cos[2*(a + b*x)])*Csc[a
 + b*x]^4 - 2*(-Cot[a + b*x]^2)^(3/4)*Hypergeometric2F1[1/2, 3/4, 3/2, Csc[
a + b*x]^2]*Sec[a + b*x]^2))/(21*b*c*(-2 + Csc[a + b*x]^2)*Sqrt[c*Sec[a + b
*x]])
```

Maple [B] time = 0.193, size = 550, normalized size = 4.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*csc(b*x+a))^(9/2)/(c*sec(b*x+a))^(3/2),x)
```

```
[Out] 1/21/b*2^(1/2)*(2*cos(b*x+a)^3*sin(b*x+a)*(-(-1+cos(b*x+a)-sin(b*x+a))/sin(
b*x+a))^(1/2)*((-1+cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a)
)/sin(b*x+a))^(1/2)*EllipticF((-(-1+cos(b*x+a)-sin(b*x+a))/sin(b*x+a))^(1/2
),1/2*2^(1/2))+2*cos(b*x+a)^2*sin(b*x+a)*(-(-1+cos(b*x+a)-sin(b*x+a))/sin(b
*x+a))^(1/2)*((-1+cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))
/sin(b*x+a))^(1/2)*EllipticF((-(-1+cos(b*x+a)-sin(b*x+a))/sin(b*x+a))^(1/2)
```

```
,1/2*2^(1/2))-2*cos(b*x+a)*sin(b*x+a)*(-(-1+cos(b*x+a)-sin(b*x+a))/sin(b*x+
a))^(1/2)*((-1+cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/si
n(b*x+a))^(1/2)*EllipticF((-(-1+cos(b*x+a)-sin(b*x+a))/sin(b*x+a))^(1/2),1/
2*2^(1/2))-2*sin(b*x+a)*(-(-1+cos(b*x+a)-sin(b*x+a))/sin(b*x+a))^(1/2)*((-1
+cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2
)*EllipticF((-(-1+cos(b*x+a)-sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))-cos
(b*x+a)^3*2^(1/2)-2*cos(b*x+a)*2^(1/2))*(d/sin(b*x+a))^(9/2)*sin(b*x+a)/cos
(b*x+a)^2/(c/cos(b*x+a))^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(d\csc(bx+a))^{\frac{9}{2}}}{(c\sec(bx+a))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(b*x+a))^(9/2)/(c*sec(b*x+a))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((d*csc(b*x + a))^(9/2)/(c*sec(b*x + a))^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{d\csc(bx+a)}\sqrt{c\sec(bx+a)}d^4\csc(bx+a)^4}{c^2\sec(bx+a)^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(b*x+a))^(9/2)/(c*sec(b*x+a))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*csc(b*x + a))*sqrt(c*sec(b*x + a))*d^4*csc(b*x + a)^4/(c^2*
sec(b*x + a)^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(b*x+a))**(9/2)/(c*sec(b*x+a))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(d\csc(bx+a))^{\frac{9}{2}}}{(c\sec(bx+a))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(b*x+a))^(9/2)/(c*sec(b*x+a))^(3/2),x, algorithm="giac")

[Out] integrate((d*csc(b*x + a))^(9/2)/(c*sec(b*x + a))^(3/2), x)
```

3.264 $\int \frac{(d\csc(a+bx))^{7/2}}{(c\sec(a+bx))^{3/2}}\,dx$

Optimal. Leaf size=33

$$-\frac{2cd(d\csc(a+bx))^{5/2}}{5b(c\sec(a+bx))^{5/2}}$$

```
[Out] (-2*c*d*(d*Csc[a + b*x])^(5/2))/(5*b*(c*Sec[a + b*x])^(5/2))
```

Rubi [A] time = 0.0533232, antiderivative size = 33, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.04, Rules used = {2619}

$$-\frac{2cd(d\csc(a+bx))^{5/2}}{5b(c\sec(a+bx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Csc[a + b*x])^(7/2)/(c*Sec[a + b*x])^(3/2),x]
```

```
[Out] (-2*c*d*(d*Csc[a + b*x])^(5/2))/(5*b*(c*Sec[a + b*x])^(5/2))
```

Rule 2619

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(a*b*(a*Csc[e + f*x])^(m - 1)*(b*Sec[e + f*x])^(n - 1
))/(f*(n - 1)), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n - 2, 0] &&
NeQ[n, 1]
```

Rubi steps

$$\int \frac{(d\csc(a+bx))^{7/2}}{(c\sec(a+bx))^{3/2}}\,dx = -\frac{2cd(d\csc(a+bx))^{5/2}}{5b(c\sec(a+bx))^{5/2}}$$

Mathematica [A] time = 0.133327, size = 45, normalized size = 1.36

$$-\frac{2d^3\cot^2(a+bx)\sqrt{d\csc(a+bx)}}{5bc\sqrt{c\sec(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Csc[a + b*x])^(7/2)/(c*Sec[a + b*x])^(3/2),x]
```

```
[Out] (-2*d^3*Cot[a + b*x]^2*Sqrt[d*Csc[a + b*x]])/(5*b*c*Sqrt[c*Sec[a + b*x]])
```

Maple [A] time = 0.142, size = 42, normalized size = 1.3

$$-\frac{2\cos(bx+a)\sin(bx+a)}{5b}\left(\frac{d}{\sin(bx+a)}\right)^{\frac{7}{2}}\left(\frac{c}{\cos(bx+a)}\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*csc(b*x+a))^(7/2)/(c*sec(b*x+a))^(3/2),x)
```

```
[Out] -2/5/b*cos(b*x+a)*sin(b*x+a)*(d/sin(b*x+a))^(7/2)/(c/cos(b*x+a))^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(d\csc(bx+a))^{\frac{7}{2}}}{(c\sec(bx+a))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(b*x+a))^(7/2)/(c*sec(b*x+a))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((d*csc(b*x + a))^(7/2)/(c*sec(b*x + a))^(3/2), x)
```

Fricas [B] time = 1.81826, size = 131, normalized size = 3.97

$$\frac{2\,d^3\sqrt{\frac{c}{\cos(bx+a)}}\sqrt{\frac{d}{\sin(bx+a)}}\cos(bx+a)^3}{5\left(bc^2\cos(bx+a)^2-bc^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(b*x+a))^(7/2)/(c*sec(b*x+a))^(3/2),x, algorithm="fricas")
```

```
[Out] 2/5*d^3*sqrt(c/cos(b*x + a))*sqrt(d/sin(b*x + a))*cos(b*x + a)^3/(b*c^2*cos
(b*x + a)^2 - b*c^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(b*x+a))**(7/2)/(c*sec(b*x+a))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(d\csc(bx+a))^{\frac{7}{2}}}{(c\sec(bx+a))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(b*x+a))^(7/2)/(c*sec(b*x+a))^(3/2),x, algorithm="giac")

[Out] integrate((d*csc(b*x + a))^(7/2)/(c*sec(b*x + a))^(3/2), x)
```

3.265 $\int \frac{(d\csc(a+bx))^{5/2}}{(c\sec(a+bx))^{3/2}}\,dx$

Optimal. Leaf size=98

$$-\frac{d^2\sqrt{\sin(2a+2bx)}\text{EllipticF}\left(a+bx-\frac{\pi}{4},2\right)\sqrt{c\sec(a+bx)}\sqrt{d\csc(a+bx)}}{3bc^2}-\frac{2d(d\csc(a+bx))^{3/2}}{3bc\sqrt{c\sec(a+bx)}}$$

```
[Out] (-2*d*(d*Csc[a + b*x])^(3/2))/(3*b*c*Sqrt[c*Sec[a + b*x]]) - (d^2*Sqrt[d*Cs
c[a + b*x]]*EllipticF[a - Pi/4 + b*x, 2]*Sqrt[c*Sec[a + b*x]]*Sqrt[Sin[2*a
+ 2*b*x]])/(3*b*c^2)
```

Rubi [A] time = 0.147519, antiderivative size = 98, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {2623, 2630, 2573, 2641}

$$-\frac{d^2\sqrt{\sin(2a+2bx)}F\left(a+bx-\frac{\pi}{4}\middle|2\right)\sqrt{c\sec(a+bx)}\sqrt{d\csc(a+bx)}}{3bc^2}-\frac{2d(d\csc(a+bx))^{3/2}}{3bc\sqrt{c\sec(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Csc[a + b*x])^(5/2)/(c*Sec[a + b*x])^(3/2),x]
```

```
[Out] (-2*d*(d*Csc[a + b*x])^(3/2))/(3*b*c*Sqrt[c*Sec[a + b*x]]) - (d^2*Sqrt[d*Cs
c[a + b*x]]*EllipticF[a - Pi/4 + b*x, 2]*Sqrt[c*Sec[a + b*x]]*Sqrt[Sin[2*a
+ 2*b*x]])/(3*b*c^2)
```

Rule 2623

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> -Simp[(a*(a*Csc[e + f*x])^(m - 1)*(b*Sec[e + f*x])^(n + 1)
)/(f*b*(m - 1)), x] + Dist[(a^2*(n + 1))/(b^2*(m - 1)), Int[(a*Csc[e + f*x]
)^(m - 2)*(b*Sec[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ
[m, 1] && LtQ[n, -1] && IntegersQ[2*m, 2*n]
```

Rule 2630

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Dist[(a*Csc[e + f*x])^m*(b*Sec[e + f*x])^n*(a*Sin[e + f*x]
)^m*(b*Cos[e + f*x])^n, Int[1/((a*Sin[e + f*x])^m*(b*Cos[e + f*x])^n), x],
x] /; FreeQ[{a, b, e, f, m, n}, x] && IntegerQ[m - 1/2] && IntegerQ[n - 1/2
]
```

Rule 2573

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(d\csc(a+bx))^{5/2}}{(c\sec(a+bx))^{3/2}}\,dx = -\frac{2d(d\csc(a+bx))^{3/2}}{3bc\sqrt{c\sec(a+bx)}} - \frac{d^2\int \sqrt{d\csc(a+bx)}\sqrt{c\sec(a+bx)}\,dx}{3c^2}$$
$$= -\frac{2d(d\csc(a+bx))^{3/2}}{3bc\sqrt{c\sec(a+bx)}} - \frac{\left(d^2\sqrt{c\cos(a+bx)}\sqrt{d\csc(a+bx)}\sqrt{c\sec(a+bx)}\sqrt{d\sin(a+bx)}\right)\int \frac{}{\sqrt{c\,c}}}{3c^2}$$
$$= -\frac{2d(d\csc(a+bx))^{3/2}}{3bc\sqrt{c\sec(a+bx)}} - \frac{\left(d^2\sqrt{d\csc(a+bx)}\sqrt{c\sec(a+bx)}\sqrt{\sin(2a+2bx)}\right)\int \frac{1}{\sqrt{\sin(2a+2bx)}}\,dx}{3c^2}$$
$$= -\frac{2d(d\csc(a+bx))^{3/2}}{3bc\sqrt{c\sec(a+bx)}} - \frac{d^2\sqrt{d\csc(a+bx)}F\left(a-\frac{\pi}{4}+bx\middle|2\right)\sqrt{c\sec(a+bx)}\sqrt{\sin(2a+2bx)}}{3bc^2}$$

Mathematica [C] time = 0.798222, size = 105, normalized size = 1.07

$$-\frac{d\cos(2(a+bx))\sec^3(a+bx)(d\csc(a+bx))^{3/2}\left(2\cot^2(a+bx)-\left(-\cot^2(a+bx)\right)^{3/4}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{3}{2},\csc^2\right.\right.}{3b\left(\csc^2(a+bx)-2\right)(c\sec(a+bx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Csc[a + b*x])^(5/2)/(c*Sec[a + b*x])^(3/2),x]
```

```
[Out] -(d*Cos[2*(a + b*x)]*(d*Csc[a + b*x])^(3/2)*(2*Cot[a + b*x]^2 - (-Cot[a + b
*x]^2)^(3/4)*Hypergeometric2F1[1/2, 3/4, 3/2, Csc[a + b*x]^2])*Sec[a + b*x]
^3)/(3*b*(-2 + Csc[a + b*x]^2)*(c*Sec[a + b*x])^(3/2))
```

Maple [B] time = 0.173, size = 290, normalized size = 3.

$$-\frac{\sqrt{2}\sin(bx+a)}{3b(\cos(bx+a))^2}\left(\cos(bx+a)\sin(bx+a)\sqrt{-\frac{-1+\cos(bx+a)-\sin(bx+a)}{\sin(bx+a)}}\sqrt{\frac{-1+\cos(bx+a)+\sin(bx+a)}{\sin(bx+a)}}\sqrt{-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*csc(b*x+a))^(5/2)/(c*sec(b*x+a))^(3/2),x)
```

```
[Out] -1/3/b*2^(1/2)*(cos(b*x+a)*sin(b*x+a)*(-(-1+cos(b*x+a)-sin(b*x+a))/sin(b*x+
a))^(1/2)*((-1+cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/si
n(b*x+a))^(1/2)*EllipticF((-(-1+cos(b*x+a)-sin(b*x+a))/sin(b*x+a))^(1/2),1/
2*2^(1/2))+sin(b*x+a)*(-(-1+cos(b*x+a)-sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+c
os(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*
EllipticF((-(-1+cos(b*x+a)-sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))+cos(b
*x+a)*2^(1/2))*(d/sin(b*x+a))^(5/2)*sin(b*x+a)/cos(b*x+a)^2/(c/cos(b*x+a))^
(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(d\csc(bx+a))^{\frac{5}{2}}}{(c\sec(bx+a))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(b*x+a))^(5/2)/(c*sec(b*x+a))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((d*csc(b*x + a))^(5/2)/(c*sec(b*x + a))^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{d\csc(bx+a)}\sqrt{c\sec(bx+a)}d^2\csc(bx+a)^2}{c^2\sec(bx+a)^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(b*x+a))^(5/2)/(c*sec(b*x+a))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*csc(b*x + a))*sqrt(c*sec(b*x + a))*d^2*csc(b*x + a)^2/(c^2*
sec(b*x + a)^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(b*x+a))**(5/2)/(c*sec(b*x+a))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(d\csc(bx+a))^{\frac{5}{2}}}{(c\sec(bx+a))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(b*x+a))^(5/2)/(c*sec(b*x+a))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((d*csc(b*x + a))^(5/2)/(c*sec(b*x + a))^(3/2), x)
```

3.266 $\int \frac{(d \csc(a+bx))^{3/2}}{(c \sec(a+bx))^{3/2}} \, dx$

Optimal. Leaf size=327

$$\frac{d^2 \tan^{-1}\left(1-\sqrt{2}\sqrt{\tan(a+bx)}\right)\sqrt{c\sec(a+bx)}}{\sqrt{2}bc^2\sqrt{\tan(a+bx)}\sqrt{d\csc(a+bx)}}-\frac{d^2 \tan^{-1}\left(\sqrt{2}\sqrt{\tan(a+bx)}+1\right)\sqrt{c\sec(a+bx)}}{\sqrt{2}bc^2\sqrt{\tan(a+bx)}\sqrt{d\csc(a+bx)}}-\frac{d^2\sqrt{c\sec(a+bx)}\log}{2\sqrt{2}bc^2\sqrt{}}$$

```
[Out] (-2*d*Sqrt[d*Csc[a + b*x]])/(b*c*Sqrt[c*Sec[a + b*x]]) + (d^2*ArcTan[1 - Sq
rt[2]*Sqrt[Tan[a + b*x]]]*Sqrt[c*Sec[a + b*x]])/(Sqrt[2]*b*c^2*Sqrt[d*Csc[a
 + b*x]]*Sqrt[Tan[a + b*x]]) - (d^2*ArcTan[1 + Sqrt[2]*Sqrt[Tan[a + b*x]]]*
Sqrt[c*Sec[a + b*x]])/(Sqrt[2]*b*c^2*Sqrt[d*Csc[a + b*x]]*Sqrt[Tan[a + b*x]
]) - (d^2*Log[1 - Sqrt[2]*Sqrt[Tan[a + b*x]] + Tan[a + b*x]]*Sqrt[c*Sec[a +
 b*x]])/(2*Sqrt[2]*b*c^2*Sqrt[d*Csc[a + b*x]]*Sqrt[Tan[a + b*x]]) + (d^2*Lo
g[1 + Sqrt[2]*Sqrt[Tan[a + b*x]] + Tan[a + b*x]]*Sqrt[c*Sec[a + b*x]])/(2*S
qrt[2]*b*c^2*Sqrt[d*Csc[a + b*x]]*Sqrt[Tan[a + b*x]])
```

Rubi [A] time = 0.215005, antiderivative size = 327, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 10, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {2623, 2629, 3476, 329, 297, 1162, 617, 204, 1165, 628}

$$\frac{d^2 \tan^{-1}\left(1-\sqrt{2}\sqrt{\tan(a+bx)}\right)\sqrt{c\sec(a+bx)}}{\sqrt{2}bc^2\sqrt{\tan(a+bx)}\sqrt{d\csc(a+bx)}}-\frac{d^2 \tan^{-1}\left(\sqrt{2}\sqrt{\tan(a+bx)}+1\right)\sqrt{c\sec(a+bx)}}{\sqrt{2}bc^2\sqrt{\tan(a+bx)}\sqrt{d\csc(a+bx)}}-\frac{d^2\sqrt{c\sec(a+bx)}\log}{2\sqrt{2}bc^2\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Csc[a + b*x])^(3/2)/(c*Sec[a + b*x])^(3/2),x]
```


```
[Out] (-2*d*Sqrt[d*Csc[a + b*x]])/(b*c*Sqrt[c*Sec[a + b*x]]) + (d^2*ArcTan[1 - Sq
rt[2]*Sqrt[Tan[a + b*x]]]*Sqrt[c*Sec[a + b*x]])/(Sqrt[2]*b*c^2*Sqrt[d*Csc[a
 + b*x]]*Sqrt[Tan[a + b*x]]) - (d^2*ArcTan[1 + Sqrt[2]*Sqrt[Tan[a + b*x]]]*
Sqrt[c*Sec[a + b*x]])/(Sqrt[2]*b*c^2*Sqrt[d*Csc[a + b*x]]*Sqrt[Tan[a + b*x]
]) - (d^2*Log[1 - Sqrt[2]*Sqrt[Tan[a + b*x]] + Tan[a + b*x]]*Sqrt[c*Sec[a +
 b*x]])/(2*Sqrt[2]*b*c^2*Sqrt[d*Csc[a + b*x]]*Sqrt[Tan[a + b*x]]) + (d^2*Lo
g[1 + Sqrt[2]*Sqrt[Tan[a + b*x]] + Tan[a + b*x]]*Sqrt[c*Sec[a + b*x]])/(2*S
qrt[2]*b*c^2*Sqrt[d*Csc[a + b*x]]*Sqrt[Tan[a + b*x]])
```


Rule 2623

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> -Simp[(a*(a*Csc[e + f*x])^(m - 1)*(b*Sec[e + f*x])^(n + 1)
)/(f*b*(m - 1)), x] + Dist[(a^2*(n + 1))/(b^2*(m - 1)), Int[(a*Csc[e + f*x]
)^(m - 2)*(b*Sec[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ
[m, 1] && LtQ[n, -1] && IntegersQ[2*m, 2*n]
```

Rule 2629

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Dist[((a*Csc[e + f*x])^m*(b*Sec[e + f*x])^n)/Tan[e + f*x]^
n, Int[Tan[e + f*x]^n, x], x] /; FreeQ[{a, b, e, f, m, n}, x] &&  !IntegerQ
[n] && EqQ[m + n, 0]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
```

```
IntegerQ[n]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{(d\csc(a+bx))^{3/2}}{(c\sec(a+bx))^{3/2}}\,dx = -\frac{2d\sqrt{d\csc(a+bx)}}{bc\sqrt{c\sec(a+bx)}} - \frac{d^2\int \frac{\sqrt{c\sec(a+bx)}}{\sqrt{d\csc(a+bx)}}\,dx}{c^2}$$

$$= -\frac{2d\sqrt{d\csc(a+bx)}}{bc\sqrt{c\sec(a+bx)}} - \frac{\left(d^2\sqrt{c\sec(a+bx)}\right)\int \sqrt{\tan(a+bx)}\,dx}{c^2\sqrt{d\csc(a+bx)}\sqrt{\tan(a+bx)}}$$

$$= -\frac{2d\sqrt{d\csc(a+bx)}}{bc\sqrt{c\sec(a+bx)}} - \frac{\left(d^2\sqrt{c\sec(a+bx)}\right)\text{Subst}\left(\int \frac{\sqrt{x}}{1+x^2}\,dx, x, \tan(a+bx)\right)}{bc^2\sqrt{d\csc(a+bx)}\sqrt{\tan(a+bx)}}$$

$$= -\frac{2d\sqrt{d\csc(a+bx)}}{bc\sqrt{c\sec(a+bx)}} - \frac{\left(2d^2\sqrt{c\sec(a+bx)}\right)\text{Subst}\left(\int \frac{x^2}{1+x^4}\,dx, x, \sqrt{\tan(a+bx)}\right)}{bc^2\sqrt{d\csc(a+bx)}\sqrt{\tan(a+bx)}}$$

$$= -\frac{2d\sqrt{d\csc(a+bx)}}{bc\sqrt{c\sec(a+bx)}} + \frac{\left(d^2\sqrt{c\sec(a+bx)}\right)\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \sqrt{\tan(a+bx)}\right)}{bc^2\sqrt{d\csc(a+bx)}\sqrt{\tan(a+bx)}} - \frac{\left(d^2\sqrt{c\sec(a+\right.}{bc}$$

$$= -\frac{2d\sqrt{d\csc(a+bx)}}{bc\sqrt{c\sec(a+bx)}} - \frac{\left(d^2\sqrt{c\sec(a+bx)}\right)\text{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}\,dx, x, \sqrt{\tan(a+bx)}\right)}{2bc^2\sqrt{d\csc(a+bx)}\sqrt{\tan(a+bx)}} - \frac{\left(d^2\sqrt{c\sec\right.}{}$$

$$= -\frac{2d\sqrt{d\csc(a+bx)}}{bc\sqrt{c\sec(a+bx)}} - \frac{d^2\log\left(1-\sqrt{2}\sqrt{\tan(a+bx)}+\tan(a+bx)\right)\sqrt{c\sec(a+bx)}}{2\sqrt{2}bc^2\sqrt{d\csc(a+bx)}\sqrt{\tan(a+bx)}} + \frac{d^2\log\left(1+\right.}{}$$

$$= -\frac{2d\sqrt{d\csc(a+bx)}}{bc\sqrt{c\sec(a+bx)}} + \frac{d^2\tan^{-1}\left(1-\sqrt{2}\sqrt{\tan(a+bx)}\right)\sqrt{c\sec(a+bx)}}{\sqrt{2}bc^2\sqrt{d\csc(a+bx)}\sqrt{\tan(a+bx)}} - \frac{d^2\tan^{-1}\left(1+\sqrt{2}\sqrt{\tan\right.}{\sqrt{2}bc^2\sqrt{d\csc(a}}$$

Mathematica [A] time = 0.634882, size = 199, normalized size = 0.61

$$-\frac{d\sqrt{d\csc(a+bx)}\left(8\sqrt[4]{\cot^2(a+bx)}+\sqrt{2}\log\left(\sqrt{\cot^2(a+bx)}-\sqrt{2}\sqrt[4]{\cot^2(a+bx)}+1\right)-\sqrt{2}\log\left(\sqrt{\cot^2(a+bx)}+\sqrt{2}\sqrt[4]{}\right.\right.}{4bc\sqrt[4]{\cot^2(a+bx)}\sqrt{c\sec(a+}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Csc[a + b*x])^(3/2)/(c*Sec[a + b*x])^(3/2),x]
```

```
[Out] -(d*Sqrt[d*Csc[a + b*x]]*(2*Sqrt[2]*ArcTan[1 - Sqrt[2]*(Cot[a + b*x]^2)^(1/
4)] - 2*Sqrt[2]*ArcTan[1 + Sqrt[2]*(Cot[a + b*x]^2)^(1/4)] + 8*(Cot[a + b*x
]^2)^(1/4) + Sqrt[2]*Log[1 - Sqrt[2]*(Cot[a + b*x]^2)^(1/4) + Sqrt[Cot[a +
b*x]^2]] - Sqrt[2]*Log[1 + Sqrt[2]*(Cot[a + b*x]^2)^(1/4) + Sqrt[Cot[a + b*
x]^2]]))/(4*b*c*(Cot[a + b*x]^2)^(1/4)*Sqrt[c*Sec[a + b*x]])
```

Maple [C] time = 0.178, size = 975, normalized size = 3.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*csc(b*x+a))^(3/2)/(c*sec(b*x+a))^(3/2),x)
```

```
[Out] -1/2/b*2^(1/2)*(I*cos(b*x+a)*EllipticPi((-(-1+cos(b*x+a)-sin(b*x+a))/sin(b*
x+a))^(1/2),1/2-1/2*I,1/2*2^(1/2))*(-(-1+cos(b*x+a)-sin(b*x+a))/sin(b*x+a))
^(1/2)*((-1+cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b
*x+a))^(1/2)-I*cos(b*x+a)*(-(-1+cos(b*x+a)-sin(b*x+a))/sin(b*x+a))^(1/2)*((
```

```
-1+cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1
/2)*EllipticPi((-(-1+cos(b*x+a)-sin(b*x+a))/sin(b*x+a))^(1/2),1/2+1/2*I,1/2
*2^(1/2))-cos(b*x+a)*(-(-1+cos(b*x+a)-sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+co
s(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*E
llipticPi((-(-1+cos(b*x+a)-sin(b*x+a))/sin(b*x+a))^(1/2),1/2-1/2*I,1/2*2^(1
/2))-cos(b*x+a)*(-(-1+cos(b*x+a)-sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x
+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*Ellipt
icPi((-(-1+cos(b*x+a)-sin(b*x+a))/sin(b*x+a))^(1/2),1/2+1/2*I,1/2*2^(1/2))+
I*EllipticPi((-(-1+cos(b*x+a)-sin(b*x+a))/sin(b*x+a))^(1/2),1/2-1/2*I,1/2*2
^(1/2))*(-(-1+cos(b*x+a)-sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a)+sin(
b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)-I*(-(-1+cos(b*
x+a)-sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^
(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*EllipticPi((-(-1+cos(b*x+a)-sin(b*
x+a))/sin(b*x+a))^(1/2),1/2+1/2*I,1/2*2^(1/2))-EllipticPi((-(-1+cos(b*x+a)-
sin(b*x+a))/sin(b*x+a))^(1/2),1/2-1/2*I,1/2*2^(1/2))*(-(-1+cos(b*x+a)-sin(b
*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1
+cos(b*x+a))/sin(b*x+a))^(1/2)-(-(-1+cos(b*x+a)-sin(b*x+a))/sin(b*x+a))^(1/
2)*((-1+cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a
))^(1/2)*EllipticPi((-(-1+cos(b*x+a)-sin(b*x+a))/sin(b*x+a))^(1/2),1/2+1/2*
I,1/2*2^(1/2))+2*cos(b*x+a)*2^(1/2))*(d/sin(b*x+a))^(3/2)*sin(b*x+a)/cos(b*
x+a)^2/(c/cos(b*x+a))^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(d\csc(bx+a))^{\frac{3}{2}}}{(c\sec(bx+a))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(b*x+a))^(3/2)/(c*sec(b*x+a))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((d*csc(b*x + a))^(3/2)/(c*sec(b*x + a))^(3/2), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(b*x+a))^(3/2)/(c*sec(b*x+a))^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(b*x+a))**(3/2)/(c*sec(b*x+a))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(d\csc(bx+a))^{\frac{3}{2}}}{(c\sec(bx+a))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(b*x+a))^(3/2)/(c*sec(b*x+a))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((d*csc(b*x + a))^(3/2)/(c*sec(b*x + a))^(3/2), x)
```

3.267 $\int \frac{\sqrt{d \csc(a+bx)}}{(c \sec(a+bx))^{3/2}} \, dx$

Optimal. Leaf size=92

$$\frac{\sqrt{\sin(2a+2bx)}\text{EllipticF}\left(a+bx-\frac{\pi}{4},2\right)\sqrt{c\sec(a+bx)}\sqrt{d\csc(a+bx)}}{2bc^2}+\frac{d}{bc\sqrt{c\sec(a+bx)}\sqrt{d\csc(a+bx)}}$$

```
[Out] d/(b*c*Sqrt[d*Csc[a + b*x]]*Sqrt[c*Sec[a + b*x]]) + (Sqrt[d*Csc[a + b*x]]*E
llipticF[a - Pi/4 + b*x, 2]*Sqrt[c*Sec[a + b*x]]*Sqrt[Sin[2*a + 2*b*x]])/(2
*b*c^2)
```

Rubi [A] time = 0.142954, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {2628, 2630, 2573, 2641}

$$\frac{\sqrt{\sin(2a+2bx)}F\left(a+bx-\frac{\pi}{4}\middle|2\right)\sqrt{c\sec(a+bx)}\sqrt{d\csc(a+bx)}}{2bc^2}+\frac{d}{bc\sqrt{c\sec(a+bx)}\sqrt{d\csc(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d*Csc[a + b*x]]/(c*Sec[a + b*x])^(3/2),x]
```

```
[Out] d/(b*c*Sqrt[d*Csc[a + b*x]]*Sqrt[c*Sec[a + b*x]]) + (Sqrt[d*Csc[a + b*x]]*E
llipticF[a - Pi/4 + b*x, 2]*Sqrt[c*Sec[a + b*x]]*Sqrt[Sin[2*a + 2*b*x]])/(2
*b*c^2)
```

Rule 2628

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> -Simp[(a*(a*Csc[e + f*x])^(m - 1)*(b*Sec[e + f*x])^(n + 1
))/(b*f*(m + n)), x] + Dist[(n + 1)/(b^2*(m + n)), Int[(a*Csc[e + f*x])^m*(
b*Sec[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && LtQ[n, -1]
&& NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2630

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Dist[(a*Csc[e + f*x])^m*(b*Sec[e + f*x])^n*(a*Sin[e + f*x]
)^m*(b*Cos[e + f*x])^n, Int[1/((a*Sin[e + f*x])^m*(b*Cos[e + f*x])^n), x],
x] /; FreeQ[{a, b, e, f, m, n}, x] && IntegerQ[m - 1/2] && IntegerQ[n - 1/2
]
```

Rule 2573

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sqrt{d\csc(a+bx)}}{(c\sec(a+bx))^{3/2}}\,dx = \frac{d}{bc\sqrt{d\csc(a+bx)}\sqrt{c\sec(a+bx)}} + \frac{\int \sqrt{d\csc(a+bx)}\sqrt{c\sec(a+bx)}\,dx}{2c^2}$$
$$= \frac{d}{bc\sqrt{d\csc(a+bx)}\sqrt{c\sec(a+bx)}} + \frac{\left(\sqrt{c\cos(a+bx)}\sqrt{d\csc(a+bx)}\sqrt{c\sec(a+bx)}\sqrt{d\sin(a+bx}\right.}{2c^2}$$
$$= \frac{d}{bc\sqrt{d\csc(a+bx)}\sqrt{c\sec(a+bx)}} + \frac{\left(\sqrt{d\csc(a+bx)}\sqrt{c\sec(a+bx)}\sqrt{\sin(2a+2bx)}\right)\int \frac{1}{\sqrt{\sin(2a+2b}}}{2c^2}$$
$$= \frac{d}{bc\sqrt{d\csc(a+bx)}\sqrt{c\sec(a+bx)}} + \frac{\sqrt{d\csc(a+bx)}F\left(a-\frac{\pi}{4}+bx\middle|2\right)\sqrt{c\sec(a+bx)}\sqrt{\sin(2a+2}}{2bc^2}$$

Mathematica [C] time = 0.641334, size = 84, normalized size = 0.91

$$\frac{d\sec^3(a+bx)\left(-\left(-\cot^2(a+bx)\right)^{3/4}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{3}{2},\csc^2(a+bx)\right)+\cos(2(a+bx))+1\right)}{2b(c\sec(a+bx))^{3/2}\sqrt{d\csc(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d*Csc[a + b*x]]/(c*Sec[a + b*x])^(3/2),x]
```

```
[Out] (d*(1 + Cos[2*(a + b*x)] - (-Cot[a + b*x]^2)^(3/4)*Hypergeometric2F1[1/2, 3
/4, 3/2, Csc[a + b*x]^2])*Sec[a + b*x]^3)/(2*b*Sqrt[d*Csc[a + b*x]]*(c*Sec[
a + b*x])^(3/2))
```

Maple [A] time = 0.19, size = 195, normalized size = 2.1

$$\frac{\sqrt{2}\sin(bx+a)}{2b(-1+\cos(bx+a))(\cos(bx+a))^2}\left(-\sin(bx+a)\sqrt{-\frac{-1+\cos(bx+a)-\sin(bx+a)}{\sin(bx+a)}}\sqrt{\frac{-1+\cos(bx+a)+\sin(bx+}{\sin(bx+a)}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*csc(b*x+a))^(1/2)/(c*sec(b*x+a))^(3/2),x)
```

```
[Out] 1/2/b*2^(1/2)*(-sin(b*x+a)*(-(-1+cos(b*x+a)-sin(b*x+a))/sin(b*x+a))^(1/2)*(
(-1+cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(
1/2)*EllipticF((-(-1+cos(b*x+a)-sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))+
cos(b*x+a)^2*2^(1/2)-cos(b*x+a)*2^(1/2))*(d/sin(b*x+a))^(1/2)*sin(b*x+a)/(-
1+cos(b*x+a))/cos(b*x+a)^2/(c/cos(b*x+a))^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{d\csc(bx+a)}}{(c\sec(bx+a))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(b*x+a))^(1/2)/(c*sec(b*x+a))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*csc(b*x + a))/(c*sec(b*x + a))^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{d\csc(bx+a)}\sqrt{c\sec(bx+a)}}{c^2\sec(bx+a)^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(b*x+a))^(1/2)/(c*sec(b*x+a))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*csc(b*x + a))*sqrt(c*sec(b*x + a))/(c^2*sec(b*x + a)^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{d\csc(a+bx)}}{(c\sec(a+bx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(b*x+a))**(1/2)/(c*sec(b*x+a))**(3/2),x)
```

```
[Out] Integral(sqrt(d*csc(a + b*x))/(c*sec(a + b*x))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{d\csc(bx+a)}}{(c\sec(bx+a))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(b*x+a))^(1/2)/(c*sec(b*x+a))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*csc(b*x + a))/(c*sec(b*x + a))^(3/2), x)
```

3.268 $\int \frac{1}{\sqrt{d\csc(a+bx)}(c\sec(a+bx))^{3/2}}\,dx$

Optimal. Leaf size=322

$$-\frac{\tan^{-1}\left(1-\sqrt{2}\sqrt{\tan(a+bx)}\right)\sqrt{c\sec(a+bx)}}{4\sqrt{2}bc^2\sqrt{\tan(a+bx)}\sqrt{d\csc(a+bx)}}+\frac{\tan^{-1}\left(\sqrt{2}\sqrt{\tan(a+bx)}+1\right)\sqrt{c\sec(a+bx)}}{4\sqrt{2}bc^2\sqrt{\tan(a+bx)}\sqrt{d\csc(a+bx)}}+\frac{\sqrt{c\sec(a+bx)}\log\left(\tan(\right.}{8\sqrt{2}bc^2\sqrt{\tan(}$$

```
[Out] d/(2*b*c*(d*Csc[a + b*x])^(3/2)*Sqrt[c*Sec[a + b*x]]) - (ArcTan[1 - Sqrt[2]
*Sqrt[Tan[a + b*x]]]*Sqrt[c*Sec[a + b*x]])/(4*Sqrt[2]*b*c^2*Sqrt[d*Csc[a +
b*x]]*Sqrt[Tan[a + b*x]]) + (ArcTan[1 + Sqrt[2]*Sqrt[Tan[a + b*x]]]*Sqrt[c*
Sec[a + b*x]])/(4*Sqrt[2]*b*c^2*Sqrt[d*Csc[a + b*x]]*Sqrt[Tan[a + b*x]]) +
(Log[1 - Sqrt[2]*Sqrt[Tan[a + b*x]] + Tan[a + b*x]]*Sqrt[c*Sec[a + b*x]])/(
8*Sqrt[2]*b*c^2*Sqrt[d*Csc[a + b*x]]*Sqrt[Tan[a + b*x]]) - (Log[1 + Sqrt[2]
*Sqrt[Tan[a + b*x]] + Tan[a + b*x]]*Sqrt[c*Sec[a + b*x]])/(8*Sqrt[2]*b*c^2*
Sqrt[d*Csc[a + b*x]]*Sqrt[Tan[a + b*x]])
```

Rubi [A] time = 0.213415, antiderivative size = 322, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 10, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {2628, 2629, 3476, 329, 297, 1162, 617, 204, 1165, 628}

$$-\frac{\tan^{-1}\left(1-\sqrt{2}\sqrt{\tan(a+bx)}\right)\sqrt{c\sec(a+bx)}}{4\sqrt{2}bc^2\sqrt{\tan(a+bx)}\sqrt{d\csc(a+bx)}}+\frac{\tan^{-1}\left(\sqrt{2}\sqrt{\tan(a+bx)}+1\right)\sqrt{c\sec(a+bx)}}{4\sqrt{2}bc^2\sqrt{\tan(a+bx)}\sqrt{d\csc(a+bx)}}+\frac{\sqrt{c\sec(a+bx)}\log\left(\tan(\right.}{8\sqrt{2}bc^2\sqrt{\tan(}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[d*Csc[a + b*x]]*(c*Sec[a + b*x])^(3/2)),x]
```

```
[Out] d/(2*b*c*(d*Csc[a + b*x])^(3/2)*Sqrt[c*Sec[a + b*x]]) - (ArcTan[1 - Sqrt[2]
*Sqrt[Tan[a + b*x]]]*Sqrt[c*Sec[a + b*x]])/(4*Sqrt[2]*b*c^2*Sqrt[d*Csc[a +
b*x]]*Sqrt[Tan[a + b*x]]) + (ArcTan[1 + Sqrt[2]*Sqrt[Tan[a + b*x]]]*Sqrt[c*
Sec[a + b*x]])/(4*Sqrt[2]*b*c^2*Sqrt[d*Csc[a + b*x]]*Sqrt[Tan[a + b*x]]) +
(Log[1 - Sqrt[2]*Sqrt[Tan[a + b*x]] + Tan[a + b*x]]*Sqrt[c*Sec[a + b*x]])/(
8*Sqrt[2]*b*c^2*Sqrt[d*Csc[a + b*x]]*Sqrt[Tan[a + b*x]]) - (Log[1 + Sqrt[2]
*Sqrt[Tan[a + b*x]] + Tan[a + b*x]]*Sqrt[c*Sec[a + b*x]])/(8*Sqrt[2]*b*c^2*
Sqrt[d*Csc[a + b*x]]*Sqrt[Tan[a + b*x]])
```

Rule 2628

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> -Simp[(a*(a*Csc[e + f*x])^(m - 1)*(b*Sec[e + f*x])^(n + 1
))/(b*f*(m + n)), x] + Dist[(n + 1)/(b^2*(m + n)), Int[(a*Csc[e + f*x])^m*(
b*Sec[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && LtQ[n, -1]
&& NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2629

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Dist[((a*Csc[e + f*x])^m*(b*Sec[e + f*x])^n)/Tan[e + f*x]^
n, Int[Tan[e + f*x]^n, x], x] /; FreeQ[{a, b, e, f, m, n}, x] &&  !IntegerQ
[n] && EqQ[m + n, 0]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
```

```
IntegerQ[n]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{\sqrt{d\csc(a+bx)}(c\sec(a+bx))^{3/2}}\,dx &= \frac{d}{2bc(d\csc(a+bx))^{3/2}\sqrt{c\sec(a+bx)}} + \frac{\int \frac{\sqrt{c\sec(a+bx)}}{\sqrt{d\csc(a+bx)}}\,dx}{4c^2} \\
&= \frac{d}{2bc(d\csc(a+bx))^{3/2}\sqrt{c\sec(a+bx)}} + \frac{\sqrt{c\sec(a+bx)}\int \sqrt{\tan(a+bx)}\,dx}{4c^2\sqrt{d\csc(a+bx)}\sqrt{\tan(a+bx)}} \\
&= \frac{d}{2bc(d\csc(a+bx))^{3/2}\sqrt{c\sec(a+bx)}} + \frac{\sqrt{c\sec(a+bx)}\,\text{Subst}\left(\int \frac{\sqrt{x}}{1+x^2}\,dx, x, \tan(\right.}{4bc^2\sqrt{d\csc(a+bx)}\sqrt{\tan(a+bx)}} \\
&= \frac{d}{2bc(d\csc(a+bx))^{3/2}\sqrt{c\sec(a+bx)}} + \frac{\sqrt{c\sec(a+bx)}\,\text{Subst}\left(\int \frac{x^2}{1+x^4}\,dx, x, \sqrt{\tan}\right.}{2bc^2\sqrt{d\csc(a+bx)}\sqrt{\tan(a+bx}} \\
&= \frac{d}{2bc(d\csc(a+bx))^{3/2}\sqrt{c\sec(a+bx)}} - \frac{\sqrt{c\sec(a+bx)}\,\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \sqrt{\tan}\right.}{4bc^2\sqrt{d\csc(a+bx)}\sqrt{\tan(a+bx}} \\
&= \frac{d}{2bc(d\csc(a+bx))^{3/2}\sqrt{c\sec(a+bx)}} + \frac{\sqrt{c\sec(a+bx)}\,\text{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}\,dx, x,\right.}{8bc^2\sqrt{d\csc(a+bx)}\sqrt{\tan(a+}} \\
&= \frac{d}{2bc(d\csc(a+bx))^{3/2}\sqrt{c\sec(a+bx)}} + \frac{\log\left(1-\sqrt{2}\sqrt{\tan(a+bx)}+\tan(a+bx)\right)}{8\sqrt{2}bc^2\sqrt{d\csc(a+bx)}\sqrt{\tan(a}} \\
&= \frac{d}{2bc(d\csc(a+bx))^{3/2}\sqrt{c\sec(a+bx)}} - \frac{\tan^{-1}\left(1-\sqrt{2}\sqrt{\tan(a+bx)}\right)\sqrt{c\sec(a+}}{4\sqrt{2}bc^2\sqrt{d\csc(a+bx)}\sqrt{\tan(a+bx)}}
\end{aligned}
$$

Mathematica [A] time = 1.97279, size = 223, normalized size = 0.69

$$
\frac{\sqrt{d\csc(a+bx)}\left(4\sqrt[4]{\cot^2(a+bx)} - 4\cos(2(a+bx))\sqrt[4]{\cot^2(a+bx)} + \sqrt{2}\log\left(\sqrt{\cot^2(a+bx)} - \sqrt{2}\sqrt[4]{\cot^2(a+bx)} + 1\right) - \right.}{16bcd\sqrt[4]{\cot^2(a}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[d*Csc[a + b*x]]*(c*Sec[a + b*x])^(3/2)),x]
```

```
[Out] (Sqrt[d*Csc[a + b*x]]*(2*Sqrt[2]*ArcTan[1 - Sqrt[2]*(Cot[a + b*x]^2)^(1/4)]
 - 2*Sqrt[2]*ArcTan[1 + Sqrt[2]*(Cot[a + b*x]^2)^(1/4)] + 4*(Cot[a + b*x]^2
)^(1/4) - 4*Cos[2*(a + b*x)]*(Cot[a + b*x]^2)^(1/4) + Sqrt[2]*Log[1 - Sqrt[
2]*(Cot[a + b*x]^2)^(1/4) + Sqrt[Cot[a + b*x]^2]] - Sqrt[2]*Log[1 + Sqrt[2]
*(Cot[a + b*x]^2)^(1/4) + Sqrt[Cot[a + b*x]^2]]))/(16*b*c*d*(Cot[a + b*x]^2
)^(1/4)*Sqrt[c*Sec[a + b*x]])
```

Maple [C] time = 0.19, size = 526, normalized size = 1.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*csc(b*x+a))^(1/2)/(c*sec(b*x+a))^(3/2),x)
```

```
[Out] 1/8/b*2^(1/2)*(-I*EllipticPi((-(-1+cos(b*x+a)-sin(b*x+a))/sin(b*x+a))^(1/2)
,1/2-1/2*I,1/2*2^(1/2))*(-(-1+cos(b*x+a)-sin(b*x+a))/sin(b*x+a))^(1/2)*((-1
+cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2
```

```
)+I*(-(-1+cos(b*x+a)-sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a)+sin(b*x+
a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*EllipticPi((-(-1+c
os(b*x+a)-sin(b*x+a))/sin(b*x+a))^(1/2),1/2+1/2*I,1/2*2^(1/2))+EllipticPi((
-(-1+cos(b*x+a)-sin(b*x+a))/sin(b*x+a))^(1/2),1/2-1/2*I,1/2*2^(1/2))*(-(-1+
cos(b*x+a)-sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a)+sin(b*x+a))/sin(b*
x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)+(-(-1+cos(b*x+a)-sin(b*x+a))
/sin(b*x+a))^(1/2)*((-1+cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b
*x+a))/sin(b*x+a))^(1/2)*EllipticPi((-(-1+cos(b*x+a)-sin(b*x+a))/sin(b*x+a)
)^(1/2),1/2+1/2*I,1/2*2^(1/2))+2*cos(b*x+a)^2*2^(1/2)-2*cos(b*x+a)*2^(1/2))
*sin(b*x+a)/(-1+cos(b*x+a))/cos(b*x+a)^2/(d/sin(b*x+a))^(1/2)/(c/cos(b*x+a)
)^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{d\csc(bx+a)}\,(c\sec(bx+a))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*csc(b*x+a))^(1/2)/(c*sec(b*x+a))^(3/2),x, algorithm="maxima"
)
```

```
[Out] integrate(1/(sqrt(d*csc(b*x + a))*(c*sec(b*x + a))^(3/2)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*csc(b*x+a))^(1/2)/(c*sec(b*x+a))^(3/2),x, algorithm="fricas"
)
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*csc(b*x+a))**(1/2)/(c*sec(b*x+a))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{d\csc(bx+a)}\,(c\sec(bx+a))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*csc(b*x+a))^(1/2)/(c*sec(b*x+a))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(d*csc(b*x + a))*(c*sec(b*x + a))^(3/2)), x)
```

3.269 $\int \frac{1}{(d\csc(a+bx))^{3/2}(c\sec(a+bx))^{3/2}}\,dx$

Optimal. Leaf size=135

$$\frac{\sqrt{\sin(2a+2bx)}\,\text{EllipticF}\left(a+bx-\frac{\pi}{4},2\right)\sqrt{c\sec(a+bx)}\sqrt{d\csc(a+bx)}}{12bc^2d^2}-\frac{c}{3bd(c\sec(a+bx))^{5/2}\sqrt{d\csc(a+bx)}}+\frac{}{6bcd\cdots}$$

```
[Out] -c/(3*b*d*Sqrt[d*Csc[a + b*x]]*(c*Sec[a + b*x])^(5/2)) + 1/(6*b*c*d*Sqrt[d*
Csc[a + b*x]]*Sqrt[c*Sec[a + b*x]]) + (Sqrt[d*Csc[a + b*x]]*EllipticF[a - P
i/4 + b*x, 2]*Sqrt[c*Sec[a + b*x]]*Sqrt[Sin[2*a + 2*b*x]])/(12*b*c^2*d^2)
```

Rubi [A] time = 0.207116, antiderivative size = 135, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2627, 2628, 2630, 2573, 2641}

$$\frac{\sqrt{\sin(2a+2bx)}F\left(a+bx-\frac{\pi}{4}\middle|2\right)\sqrt{c\sec(a+bx)}\sqrt{d\csc(a+bx)}}{12bc^2d^2}-\frac{c}{3bd(c\sec(a+bx))^{5/2}\sqrt{d\csc(a+bx)}}+\frac{}{6bcd\sqrt{c\sec(\cdots}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d*Csc[a + b*x])^(3/2)*(c*Sec[a + b*x])^(3/2)),x]
```

```
[Out] -c/(3*b*d*Sqrt[d*Csc[a + b*x]]*(c*Sec[a + b*x])^(5/2)) + 1/(6*b*c*d*Sqrt[d*
Csc[a + b*x]]*Sqrt[c*Sec[a + b*x]]) + (Sqrt[d*Csc[a + b*x]]*EllipticF[a - P
i/4 + b*x, 2]*Sqrt[c*Sec[a + b*x]]*Sqrt[Sin[2*a + 2*b*x]])/(12*b*c^2*d^2)
```

Rule 2627

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Simp[(b*(a*Csc[e + f*x])^(m + 1)*(b*Sec[e + f*x])^(n - 1)
)/(a*f*(m + n)), x] + Dist[(m + 1)/(a^2*(m + n)), Int[(a*Csc[e + f*x])^(m +
 2)*(b*Sec[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -1] &
& NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2628

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> -Simp[(a*(a*Csc[e + f*x])^(m - 1)*(b*Sec[e + f*x])^(n + 1
))/(b*f*(m + n)), x] + Dist[(n + 1)/(b^2*(m + n)), Int[(a*Csc[e + f*x])^m*(
b*Sec[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && LtQ[n, -1]
&& NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2630

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Dist[(a*Csc[e + f*x])^m*(b*Sec[e + f*x])^n*(a*Sin[e + f*x]
)^m*(b*Cos[e + f*x])^n, Int[1/((a*Sin[e + f*x])^m*(b*Cos[e + f*x])^n), x],
x] /; FreeQ[{a, b, e, f, m, n}, x] && IntegerQ[m - 1/2] && IntegerQ[n - 1/2
]
```

Rule 2573

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
```

```
, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{1}{(d\csc(a+bx))^{3/2}(c\sec(a+bx))^{3/2}}\,dx = -\frac{c}{3bd\sqrt{d\csc(a+bx)}(c\sec(a+bx))^{5/2}} + \frac{\int \frac{\sqrt{d\csc(a+bx)}}{(c\sec(a+bx))^{3/2}}\,dx}{6d^2}$$

$$= -\frac{c}{3bd\sqrt{d\csc(a+bx)}(c\sec(a+bx))^{5/2}} + \frac{1}{6bcd\sqrt{d\csc(a+bx)}\sqrt{c\sec(a+bx)}} +$$

$$= -\frac{c}{3bd\sqrt{d\csc(a+bx)}(c\sec(a+bx))^{5/2}} + \frac{1}{6bcd\sqrt{d\csc(a+bx)}\sqrt{c\sec(a+bx)}} +$$

$$= -\frac{c}{3bd\sqrt{d\csc(a+bx)}(c\sec(a+bx))^{5/2}} + \frac{1}{6bcd\sqrt{d\csc(a+bx)}\sqrt{c\sec(a+bx)}} +$$

$$= -\frac{c}{3bd\sqrt{d\csc(a+bx)}(c\sec(a+bx))^{5/2}} + \frac{1}{6bcd\sqrt{d\csc(a+bx)}\sqrt{c\sec(a+bx)}} +$$

Mathematica [C] time = 0.554515, size = 89, normalized size = 0.66

$$\frac{\frac{\csc^2(a+bx)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{3}{2},\csc^2(a+bx)\right)}{\sqrt[4]{-\cot^2(a+bx)}} - 2\cos(2(a+bx))}{12bcd\sqrt{c\sec(a+bx)}\sqrt{d\csc(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d*Csc[a + b*x])^(3/2)*(c*Sec[a + b*x])^(3/2)),x]
```

```
[Out] (-2*Cos[2*(a + b*x)] + (Csc[a + b*x]^2*Hypergeometric2F1[1/2, 3/4, 3/2, Csc
[a + b*x]^2])/(-Cot[a + b*x]^2)^(1/4))/(12*b*c*d*Sqrt[d*Csc[a + b*x]]*Sqrt[
c*Sec[a + b*x]])
```

Maple [A] time = 0.181, size = 222, normalized size = 1.6

$$-\frac{\sqrt{2}}{12\,b\,(-1+\cos(bx+a))\,(\cos(bx+a))^2\sin(bx+a)}\left(2\sqrt{2}\,(\cos(bx+a))^4+\sin(bx+a)\sqrt{-\frac{-1+\cos(bx+a)-\sin(bx}{\sin(bx+a)}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*csc(b*x+a))^(3/2)/(c*sec(b*x+a))^(3/2),x)
```

```
[Out] -1/12/b*2^(1/2)*(2*2^(1/2)*cos(b*x+a)^4+sin(b*x+a)*(-(-1+cos(b*x+a)-sin(b*x
+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+c
os(b*x+a))/sin(b*x+a))^(1/2)*EllipticF((-(-1+cos(b*x+a)-sin(b*x+a))/sin(b*x
+a))^(1/2),1/2*2^(1/2))-2*cos(b*x+a)^3*2^(1/2)-cos(b*x+a)^2*2^(1/2)+cos(b*x
+a)*2^(1/2))/(-1+cos(b*x+a))/cos(b*x+a)^2/sin(b*x+a)/(c/cos(b*x+a))^(3/2)/(
```

```
d/sin(b*x+a))^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(d\csc(bx+a))^{\frac{3}{2}}(c\sec(bx+a))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*csc(b*x+a))^(3/2)/(c*sec(b*x+a))^(3/2),x, algorithm="maxima"
)
```

```
[Out] integrate(1/((d*csc(b*x + a))^(3/2)*(c*sec(b*x + a))^(3/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{d\csc(bx+a)}\sqrt{c\sec(bx+a)}}{c^2d^2\csc(bx+a)^2\sec(bx+a)^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*csc(b*x+a))^(3/2)/(c*sec(b*x+a))^(3/2),x, algorithm="fricas"
)
```

```
[Out] integral(sqrt(d*csc(b*x + a))*sqrt(c*sec(b*x + a))/(c^2*d^2*csc(b*x + a)^2*
sec(b*x + a)^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*csc(b*x+a))**(3/2)/(c*sec(b*x+a))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(d\csc(bx+a))^{\frac{3}{2}}(c\sec(bx+a))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*csc(b*x+a))^(3/2)/(c*sec(b*x+a))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((d*csc(b*x + a))^(3/2)*(c*sec(b*x + a))^(3/2)), x)
```

3.270 $\int \frac{1}{(d\csc(a+bx))^{5/2}(c\sec(a+bx))^{3/2}}\,dx$

Optimal. Leaf size=371

$$-\frac{3\tan^{-1}\left(1-\sqrt{2}\sqrt{\tan(a+bx)}\right)\sqrt{c\sec(a+bx)}}{32\sqrt{2}bc^2d^2\sqrt{\tan(a+bx)}\sqrt{d\csc(a+bx)}}+\frac{3\tan^{-1}\left(\sqrt{2}\sqrt{\tan(a+bx)}+1\right)\sqrt{c\sec(a+bx)}}{32\sqrt{2}bc^2d^2\sqrt{\tan(a+bx)}\sqrt{d\csc(a+bx)}}+\frac{3\sqrt{c\sec(a+bx)}\log\left(\right.}{64\sqrt{2}bc^2d^2\sqrt{}}$$

```
[Out] -c/(4*b*d*(d*Csc[a + b*x])^(3/2)*(c*Sec[a + b*x])^(5/2)) + 3/(16*b*c*d*(d*C
sc[a + b*x])^(3/2)*Sqrt[c*Sec[a + b*x]]) - (3*ArcTan[1 - Sqrt[2]*Sqrt[Tan[a
 + b*x]]]*Sqrt[c*Sec[a + b*x]])/(32*Sqrt[2]*b*c^2*d^2*Sqrt[d*Csc[a + b*x]]*
Sqrt[Tan[a + b*x]]) + (3*ArcTan[1 + Sqrt[2]*Sqrt[Tan[a + b*x]]]*Sqrt[c*Sec[
a + b*x]])/(32*Sqrt[2]*b*c^2*d^2*Sqrt[d*Csc[a + b*x]]*Sqrt[Tan[a + b*x]]) +
 (3*Log[1 - Sqrt[2]*Sqrt[Tan[a + b*x]] + Tan[a + b*x]]*Sqrt[c*Sec[a + b*x]]
)/(64*Sqrt[2]*b*c^2*d^2*Sqrt[d*Csc[a + b*x]]*Sqrt[Tan[a + b*x]]) - (3*Log[1
 + Sqrt[2]*Sqrt[Tan[a + b*x]] + Tan[a + b*x]]*Sqrt[c*Sec[a + b*x]])/(64*Sqr
t[2]*b*c^2*d^2*Sqrt[d*Csc[a + b*x]]*Sqrt[Tan[a + b*x]])
```

Rubi [A] time = 0.297432, antiderivative size = 371, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 11, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.44, Rules used = {2627, 2628, 2629, 3476, 329, 297, 1162, 617, 204, 1165, 628}

$$-\frac{3\tan^{-1}\left(1-\sqrt{2}\sqrt{\tan(a+bx)}\right)\sqrt{c\sec(a+bx)}}{32\sqrt{2}bc^2d^2\sqrt{\tan(a+bx)}\sqrt{d\csc(a+bx)}}+\frac{3\tan^{-1}\left(\sqrt{2}\sqrt{\tan(a+bx)}+1\right)\sqrt{c\sec(a+bx)}}{32\sqrt{2}bc^2d^2\sqrt{\tan(a+bx)}\sqrt{d\csc(a+bx)}}+\frac{3\sqrt{c\sec(a+bx)}\log\left(\right.}{64\sqrt{2}bc^2d^2\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d*Csc[a + b*x])^(5/2)*(c*Sec[a + b*x])^(3/2)),x]
```


```
[Out] -c/(4*b*d*(d*Csc[a + b*x])^(3/2)*(c*Sec[a + b*x])^(5/2)) + 3/(16*b*c*d*(d*C
sc[a + b*x])^(3/2)*Sqrt[c*Sec[a + b*x]]) - (3*ArcTan[1 - Sqrt[2]*Sqrt[Tan[a
 + b*x]]]*Sqrt[c*Sec[a + b*x]])/(32*Sqrt[2]*b*c^2*d^2*Sqrt[d*Csc[a + b*x]]*
Sqrt[Tan[a + b*x]]) + (3*ArcTan[1 + Sqrt[2]*Sqrt[Tan[a + b*x]]]*Sqrt[c*Sec[
a + b*x]])/(32*Sqrt[2]*b*c^2*d^2*Sqrt[d*Csc[a + b*x]]*Sqrt[Tan[a + b*x]]) +
 (3*Log[1 - Sqrt[2]*Sqrt[Tan[a + b*x]] + Tan[a + b*x]]*Sqrt[c*Sec[a + b*x]]
)/(64*Sqrt[2]*b*c^2*d^2*Sqrt[d*Csc[a + b*x]]*Sqrt[Tan[a + b*x]]) - (3*Log[1
 + Sqrt[2]*Sqrt[Tan[a + b*x]] + Tan[a + b*x]]*Sqrt[c*Sec[a + b*x]])/(64*Sqr
t[2]*b*c^2*d^2*Sqrt[d*Csc[a + b*x]]*Sqrt[Tan[a + b*x]])
```


Rule 2627

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Simp[(b*(a*Csc[e + f*x])^(m + 1)*(b*Sec[e + f*x])^(n - 1)
)/(a*f*(m + n)), x] + Dist[(m + 1)/(a^2*(m + n)), Int[(a*Csc[e + f*x])^(m +
 2)*(b*Sec[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -1] &
& NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2628

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> -Simp[(a*(a*Csc[e + f*x])^(m - 1)*(b*Sec[e + f*x])^(n + 1
))/(b*f*(m + n)), x] + Dist[(n + 1)/(b^2*(m + n)), Int[(a*Csc[e + f*x])^m*(
b*Sec[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && LtQ[n, -1]
&& NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2629

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Dist[((a*Csc[e + f*x])^m*(b*Sec[e + f*x])^n)/Tan[e + f*x]^
n, Int[Tan[e + f*x]^n, x], x] /; FreeQ[{a, b, e, f, m, n}, x] &&  !IntegerQ
[n] && EqQ[m + n, 0]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{1}{(d\csc(a+bx))^{5/2}(c\sec(a+bx))^{3/2}}\,dx = -\frac{c}{4bd(d\csc(a+bx))^{3/2}(c\sec(a+bx))^{5/2}} + \frac{3\int \frac{1}{\sqrt{d\csc(a+bx)}(c\sec(a+bx))^{3/2}}\,dx}{8d^2}$$

$$= -\frac{c}{4bd(d\csc(a+bx))^{3/2}(c\sec(a+bx))^{5/2}} + \frac{3}{16bcd(d\csc(a+bx))^{3/2}\sqrt{c\sec(a+}}$$

$$= -\frac{c}{4bd(d\csc(a+bx))^{3/2}(c\sec(a+bx))^{5/2}} + \frac{3}{16bcd(d\csc(a+bx))^{3/2}\sqrt{c\sec(a+}}$$

$$= -\frac{c}{4bd(d\csc(a+bx))^{3/2}(c\sec(a+bx))^{5/2}} + \frac{3}{16bcd(d\csc(a+bx))^{3/2}\sqrt{c\sec(a+}}$$

$$= -\frac{c}{4bd(d\csc(a+bx))^{3/2}(c\sec(a+bx))^{5/2}} + \frac{3}{16bcd(d\csc(a+bx))^{3/2}\sqrt{c\sec(a+}}$$

$$= -\frac{c}{4bd(d\csc(a+bx))^{3/2}(c\sec(a+bx))^{5/2}} + \frac{3}{16bcd(d\csc(a+bx))^{3/2}\sqrt{c\sec(a+}}$$

$$= -\frac{c}{4bd(d\csc(a+bx))^{3/2}(c\sec(a+bx))^{5/2}} + \frac{3}{16bcd(d\csc(a+bx))^{3/2}\sqrt{c\sec(a+}}$$

$$= -\frac{c}{4bd(d\csc(a+bx))^{3/2}(c\sec(a+bx))^{5/2}} + \frac{3}{16bcd(d\csc(a+bx))^{3/2}\sqrt{c\sec(a+}}$$

$$= -\frac{c}{4bd(d\csc(a+bx))^{3/2}(c\sec(a+bx))^{5/2}} + \frac{3}{16bcd(d\csc(a+bx))^{3/2}\sqrt{c\sec(a+}}$$

Mathematica [A] time = 2.30413, size = 246, normalized size = 0.66

$$\frac{\sqrt{d\csc(a+bx)}\left(8\sqrt[4]{\cot^2(a+bx)} - 12\cos(2(a+bx))\sqrt[4]{\cot^2(a+bx)} + 4\cos(4(a+bx))\sqrt[4]{\cot^2(a+bx)} + 3\sqrt{2}\log\left(\sqrt{\cot^2(\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d*Csc[a + b*x])^(5/2)*(c*Sec[a + b*x])^(3/2)),x]
```

```
[Out] (Sqrt[d*Csc[a + b*x]]*(6*Sqrt[2]*ArcTan[1 - Sqrt[2]*(Cot[a + b*x]^2)^(1/4)]
 - 6*Sqrt[2]*ArcTan[1 + Sqrt[2]*(Cot[a + b*x]^2)^(1/4)] + 8*(Cot[a + b*x]^2
)^(1/4) - 12*Cos[2*(a + b*x)]*(Cot[a + b*x]^2)^(1/4) + 4*Cos[4*(a + b*x)]*(
Cot[a + b*x]^2)^(1/4) + 3*Sqrt[2]*Log[1 - Sqrt[2]*(Cot[a + b*x]^2)^(1/4) +
Sqrt[Cot[a + b*x]^2]] - 3*Sqrt[2]*Log[1 + Sqrt[2]*(Cot[a + b*x]^2)^(1/4) +
Sqrt[Cot[a + b*x]^2]]))/(128*b*c*d^3*(Cot[a + b*x]^2)^(1/4)*Sqrt[c*Sec[a +
b*x]])
```

Maple [C] time = 0.174, size = 556, normalized size = 1.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*csc(b*x+a))^(5/2)/(c*sec(b*x+a))^(3/2),x)
```

```
[Out] -1/64/b*2^(1/2)*(8*2^(1/2)*cos(b*x+a)^4-3*I*(-(-1+cos(b*x+a)-sin(b*x+a))/si
n(b*x+a))^(1/2)*((-1+cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+
a))/sin(b*x+a))^(1/2)*EllipticPi((-(-1+cos(b*x+a)-sin(b*x+a))/sin(b*x+a))^(
1/2),1/2+1/2*I,1/2*2^(1/2))+3*I*EllipticPi((-(-1+cos(b*x+a)-sin(b*x+a))/sin
(b*x+a))^(1/2),1/2-1/2*I,1/2*2^(1/2))*(-(-1+cos(b*x+a)-sin(b*x+a))/sin(b*x+
a))^(1/2)*((-1+cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/si
n(b*x+a))^(1/2)-8*cos(b*x+a)^3*2^(1/2)-3*(-(-1+cos(b*x+a)-sin(b*x+a))/sin(b
*x+a))^(1/2)*((-1+cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))
/sin(b*x+a))^(1/2)*EllipticPi((-(-1+cos(b*x+a)-sin(b*x+a))/sin(b*x+a))^(1/2
),1/2+1/2*I,1/2*2^(1/2))-3*EllipticPi((-(-1+cos(b*x+a)-sin(b*x+a))/sin(b*x+
a))^(1/2),1/2-1/2*I,1/2*2^(1/2))*(-(-1+cos(b*x+a)-sin(b*x+a))/sin(b*x+a))^(
1/2)*((-1+cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x
+a))^(1/2)-6*cos(b*x+a)^2*2^(1/2)+6*cos(b*x+a)*2^(1/2))/(-1+cos(b*x+a))/cos
(b*x+a)^2/sin(b*x+a)/(d/sin(b*x+a))^(5/2)/(c/cos(b*x+a))^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(d\csc(bx+a))^{\frac{5}{2}}(c\sec(bx+a))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*csc(b*x+a))^(5/2)/(c*sec(b*x+a))^(3/2),x, algorithm="maxima"
)
```

```
[Out] integrate(1/((d*csc(b*x + a))^(5/2)*(c*sec(b*x + a))^(3/2)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*csc(b*x+a))^(5/2)/(c*sec(b*x+a))^(3/2),x, algorithm="fricas"
)
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*csc(b*x+a))**(5/2)/(c*sec(b*x+a))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(d\csc\left(bx+a\right)\right)^{\frac{5}{2}}\left(c\sec\left(bx+a\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*csc(b*x+a))^(5/2)/(c*sec(b*x+a))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((d*csc(b*x + a))^(5/2)*(c*sec(b*x + a))^(3/2)), x)
```

3.271 $\int \frac{(d\csc(a+bx))^{9/2}}{(c\sec(a+bx))^{5/2}}\,dx$

Optimal. Leaf size=33

$$-\frac{2cd(d\csc(a+bx))^{7/2}}{7b(c\sec(a+bx))^{7/2}}$$

```
[Out] (-2*c*d*(d*Csc[a + b*x])^(7/2))/(7*b*(c*Sec[a + b*x])^(7/2))
```

Rubi [A] time = 0.0540944, antiderivative size = 33, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.04, Rules used = {2619}

$$-\frac{2cd(d\csc(a+bx))^{7/2}}{7b(c\sec(a+bx))^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Csc[a + b*x])^(9/2)/(c*Sec[a + b*x])^(5/2),x]
```

```
[Out] (-2*c*d*(d*Csc[a + b*x])^(7/2))/(7*b*(c*Sec[a + b*x])^(7/2))
```

Rule 2619

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(a*b*(a*Csc[e + f*x])^(m - 1)*(b*Sec[e + f*x])^(n - 1
))/(f*(n - 1)), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n - 2, 0] &&
NeQ[n, 1]
```

Rubi steps

$$\int \frac{(d\csc(a+bx))^{9/2}}{(c\sec(a+bx))^{5/2}}\,dx = -\frac{2cd(d\csc(a+bx))^{7/2}}{7b(c\sec(a+bx))^{7/2}}$$

Mathematica [A] time = 0.158198, size = 45, normalized size = 1.36

$$-\frac{2d^4\cot^3(a+bx)\sqrt{d\csc(a+bx)}}{7bc^2\sqrt{c\sec(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Csc[a + b*x])^(9/2)/(c*Sec[a + b*x])^(5/2),x]
```

```
[Out] (-2*d^4*Cot[a + b*x]^3*Sqrt[d*Csc[a + b*x]])/(7*b*c^2*Sqrt[c*Sec[a + b*x]])
```

Maple [A] time = 0.138, size = 42, normalized size = 1.3

$$-\frac{2\cos(bx+a)\sin(bx+a)}{7b}\left(\frac{d}{\sin(bx+a)}\right)^{\frac{9}{2}}\left(\frac{c}{\cos(bx+a)}\right)^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*csc(b*x+a))^(9/2)/(c*sec(b*x+a))^(5/2),x)
```

```
[Out] -2/7/b*cos(b*x+a)*sin(b*x+a)*(d/sin(b*x+a))^(9/2)/(c/cos(b*x+a))^(5/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(d\csc(bx+a))^{\frac{9}{2}}}{(c\sec(bx+a))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(b*x+a))^(9/2)/(c*sec(b*x+a))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((d*csc(b*x + a))^(9/2)/(c*sec(b*x + a))^(5/2), x)
```

Fricas [B] time = 2.30939, size = 151, normalized size = 4.58

$$\frac{2\,d^4\sqrt{\frac{c}{\cos(bx+a)}}\sqrt{\frac{d}{\sin(bx+a)}}\cos(bx+a)^4}{7\left(bc^3\cos(bx+a)^2-bc^3\right)\sin(bx+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(b*x+a))^(9/2)/(c*sec(b*x+a))^(5/2),x, algorithm="fricas")
```

```
[Out] 2/7*d^4*sqrt(c/cos(b*x + a))*sqrt(d/sin(b*x + a))*cos(b*x + a)^4/((b*c^3*co
s(b*x + a)^2 - b*c^3)*sin(b*x + a))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(b*x+a))**(9/2)/(c*sec(b*x+a))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(d\csc(bx+a))^{\frac{9}{2}}}{(c\sec(bx+a))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(b*x+a))^(9/2)/(c*sec(b*x+a))^(5/2),x, algorithm="giac")

[Out] integrate((d*csc(b*x + a))^(9/2)/(c*sec(b*x + a))^(5/2), x)
```

3.272 $\int \frac{(d \csc(a+bx))^{7/2}}{(c \sec(a+bx))^{5/2}} dx$

Optimal. Leaf size=135

$$\frac{6d^4E\left(a+bx-\frac{\pi}{4}\middle|2\right)}{5bc^2\sqrt{\sin(2a+2bx)}\sqrt{c\sec(a+bx)}\sqrt{d\csc(a+bx)}}+\frac{6d^3\sqrt{d\csc(a+bx)}}{5bc(c\sec(a+bx))^{3/2}}-\frac{2d(d\csc(a+bx))^{5/2}}{5bc(c\sec(a+bx))^{3/2}}$$

```
[Out] (6*d^3*Sqrt[d*Csc[a + b*x]])/(5*b*c*(c*Sec[a + b*x])^(3/2)) - (2*d*(d*Csc[a
 + b*x])^(5/2))/(5*b*c*(c*Sec[a + b*x])^(3/2)) + (6*d^4*EllipticE[a - Pi/4
+ b*x, 2])/(5*b*c^2*Sqrt[d*Csc[a + b*x]]*Sqrt[c*Sec[a + b*x]]*Sqrt[Sin[2*a
+ 2*b*x]])
```

Rubi [A] time = 0.202232, antiderivative size = 135, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2623, 2625, 2630, 2572, 2639}

$$\frac{6d^4E\left(a+bx-\frac{\pi}{4}\middle|2\right)}{5bc^2\sqrt{\sin(2a+2bx)}\sqrt{c\sec(a+bx)}\sqrt{d\csc(a+bx)}}+\frac{6d^3\sqrt{d\csc(a+bx)}}{5bc(c\sec(a+bx))^{3/2}}-\frac{2d(d\csc(a+bx))^{5/2}}{5bc(c\sec(a+bx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Csc[a + b*x])^(7/2)/(c*Sec[a + b*x])^(5/2),x]
```

```
[Out] (6*d^3*Sqrt[d*Csc[a + b*x]])/(5*b*c*(c*Sec[a + b*x])^(3/2)) - (2*d*(d*Csc[a
 + b*x])^(5/2))/(5*b*c*(c*Sec[a + b*x])^(3/2)) + (6*d^4*EllipticE[a - Pi/4
+ b*x, 2])/(5*b*c^2*Sqrt[d*Csc[a + b*x]]*Sqrt[c*Sec[a + b*x]]*Sqrt[Sin[2*a
+ 2*b*x]])
```

Rule 2623

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> -Simp[(a*(a*Csc[e + f*x])^(m - 1)*(b*Sec[e + f*x])^(n + 1)
)/(f*b*(m - 1)), x] + Dist[(a^2*(n + 1))/(b^2*(m - 1)), Int[(a*Csc[e + f*x]
)^(m - 2)*(b*Sec[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ
[m, 1] && LtQ[n, -1] && IntegersQ[2*m, 2*n]
```

Rule 2625

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> -Simp[(a*b*(a*Csc[e + f*x])^(m - 1)*(b*Sec[e + f*x])^(n -
 1))/(f*(m - 1)), x] + Dist[(a^2*(m + n - 2))/(m - 1), Int[(a*Csc[e + f*x])
^(m - 2)*(b*Sec[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m,
1] && IntegersQ[2*m, 2*n] &&  !GtQ[n, m]
```

Rule 2630

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Dist[(a*Csc[e + f*x])^m*(b*Sec[e + f*x])^n*(a*Sin[e + f*x]
)^m*(b*Cos[e + f*x])^n, Int[1/((a*Sin[e + f*x])^m*(b*Cos[e + f*x])^n), x],
x] /; FreeQ[{a, b, e, f, m, n}, x] && IntegerQ[m - 1/2] && IntegerQ[n - 1/2
]
```

Rule 2572

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[(Sqrt[a*Sin[e + f*x]]*Sqrt[b*Cos[e + f*x]])/Sqrt[Sin[2*
```

```
e + 2*f*x]], Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{(d\csc(a+bx))^{7/2}}{(c\sec(a+bx))^{5/2}}\,dx &= -\frac{2d(d\csc(a+bx))^{5/2}}{5bc(c\sec(a+bx))^{3/2}} - \frac{\left(3d^2\right)\int \frac{(d\csc(a+bx))^{3/2}}{\sqrt{c\sec(a+bx)}}\,dx}{5c^2} \\
&= \frac{6d^3\sqrt{d\csc(a+bx)}}{5bc(c\sec(a+bx))^{3/2}} - \frac{2d(d\csc(a+bx))^{5/2}}{5bc(c\sec(a+bx))^{3/2}} + \frac{\left(6d^4\right)\int \frac{1}{\sqrt{d\csc(a+bx)}\sqrt{c\sec(a+bx)}}\,dx}{5c^2} \\
&= \frac{6d^3\sqrt{d\csc(a+bx)}}{5bc(c\sec(a+bx))^{3/2}} - \frac{2d(d\csc(a+bx))^{5/2}}{5bc(c\sec(a+bx))^{3/2}} + \frac{\left(6d^4\right)\int \sqrt{c\cos(a+bx)}\sqrt{d\sin(a}{5c^2\sqrt{c\cos(a+bx)}\sqrt{d\csc(a+bx)}\sqrt{c\sec(a+} \\
&= \frac{6d^3\sqrt{d\csc(a+bx)}}{5bc(c\sec(a+bx))^{3/2}} - \frac{2d(d\csc(a+bx))^{5/2}}{5bc(c\sec(a+bx))^{3/2}} + \frac{\left(6d^4\right)\int \sqrt{\sin(2a+2bx)}\,dx}{5c^2\sqrt{d\csc(a+bx)}\sqrt{c\sec(a+bx)}\sqrt{\sin(2a+} \\
&= \frac{6d^3\sqrt{d\csc(a+bx)}}{5bc(c\sec(a+bx))^{3/2}} - \frac{2d(d\csc(a+bx))^{5/2}}{5bc(c\sec(a+bx))^{3/2}} + \frac{6d^4E\left(a-\frac{\pi}{4}+bx\middle|2\right)}{5bc^2\sqrt{d\csc(a+bx)}\sqrt{c\sec(a+bx)}\sqrt{\sin(2a}
\end{aligned}$$

Mathematica [C] time = 1.80213, size = 101, normalized size = 0.75

$$\frac{d^5\sqrt{c\sec(a+bx)}\left(6\sqrt[4]{-\cot^2(a+bx)}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{4},\frac{1}{2},\csc^2(a+bx)\right)+(1-3\cos(2(a+bx)))\cot^2(a+b}{5bc^3(d\csc(a+bx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Csc[a + b*x])^(7/2)/(c*Sec[a + b*x])^(5/2),x]
```

```
[Out] (d^5*((1 - 3*Cos[2*(a + b*x)])*Cot[a + b*x]^2*Csc[a + b*x]^2 + 6*(-Cot[a +
b*x]^2)^(1/4)*Hypergeometric2F1[-1/2, 1/4, 1/2, Csc[a + b*x]^2])*Sqrt[c*Sec
[a + b*x]])/(5*b*c^3*(d*Csc[a + b*x])^(3/2))
```

Maple [B] time = 0.196, size = 993, normalized size = 7.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*csc(b*x+a))^(7/2)/(c*sec(b*x+a))^(5/2),x)
```

```
[Out] 1/5/b*2^(1/2)*(6*cos(b*x+a)^3*(-(-1+cos(b*x+a)-sin(b*x+a))/sin(b*x+a))^(1/2
)*((-1+cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a)
)^(1/2)*EllipticE((-(-1+cos(b*x+a)-sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2
))-3*cos(b*x+a)^3*(-(-1+cos(b*x+a)-sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b
*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*Elli
pticF((-(-1+cos(b*x+a)-sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))+6*cos(b*x
+a)^2*(-(-1+cos(b*x+a)-sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a)+sin(b*
x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*EllipticE((-(-1+
```

```
cos(b*x+a)-sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))-3*cos(b*x+a)^2*(-(-1+
cos(b*x+a)-sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a)+sin(b*x+a))/sin(b*
x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*EllipticF((-(-1+cos(b*x+a)-s
in(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))-6*cos(b*x+a)*(-(-1+cos(b*x+a)-sin
(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((
-1+cos(b*x+a))/sin(b*x+a))^(1/2)*EllipticE((-(-1+cos(b*x+a)-sin(b*x+a))/sin
(b*x+a))^(1/2),1/2*2^(1/2))+3*cos(b*x+a)*(-(-1+cos(b*x+a)-sin(b*x+a))/sin(b
*x+a))^(1/2)*((-1+cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))
/sin(b*x+a))^(1/2)*EllipticF((-(-1+cos(b*x+a)-sin(b*x+a))/sin(b*x+a))^(1/2)
,1/2*2^(1/2))-3*cos(b*x+a)^3*2^(1/2)-6*(-(-1+cos(b*x+a)-sin(b*x+a))/sin(b*x
+a))^(1/2)*((-1+cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/s
in(b*x+a))^(1/2)*EllipticE((-(-1+cos(b*x+a)-sin(b*x+a))/sin(b*x+a))^(1/2),1
/2*2^(1/2))+3*(-(-1+cos(b*x+a)-sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a
)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*Elliptic
F((-(-1+cos(b*x+a)-sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))-cos(b*x+a)^2*
2^(1/2)+3*cos(b*x+a)*2^(1/2))*(d/sin(b*x+a))^(7/2)*sin(b*x+a)/cos(b*x+a)^3/
(c/cos(b*x+a))^(5/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(d\csc(bx+a))^{\frac{7}{2}}}{(c\sec(bx+a))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(b*x+a))^(7/2)/(c*sec(b*x+a))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((d*csc(b*x + a))^(7/2)/(c*sec(b*x + a))^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{d\csc(bx+a)}\sqrt{c\sec(bx+a)}d^3\csc(bx+a)^3}{c^3\sec(bx+a)^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(b*x+a))^(7/2)/(c*sec(b*x+a))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*csc(b*x + a))*sqrt(c*sec(b*x + a))*d^3*csc(b*x + a)^3/(c^3*
sec(b*x + a)^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(b*x+a))**(7/2)/(c*sec(b*x+a))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(d\csc(bx+a))^{\frac{7}{2}}}{(c\sec(bx+a))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(b*x+a))^(7/2)/(c*sec(b*x+a))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((d*csc(b*x + a))^(7/2)/(c*sec(b*x + a))^(5/2), x)
```

3.273 $\int \frac{(d\csc(a+bx))^{5/2}}{(c\sec(a+bx))^{5/2}}\,dx$

Optimal. Leaf size=329

$$\frac{d^2\tan^{-1}\left(1-\sqrt{2}\sqrt{\tan(a+bx)}\right)\sqrt{\tan(a+bx)}\sqrt{d\csc(a+bx)}}{\sqrt{2}bc^2\sqrt{c\sec(a+bx)}}-\frac{d^2\tan^{-1}\left(\sqrt{2}\sqrt{\tan(a+bx)}+1\right)\sqrt{\tan(a+bx)}\sqrt{d\csc(a+}}{\sqrt{2}bc^2\sqrt{c\sec(a+bx)}}$$

```
[Out] (-2*d*(d*Csc[a + b*x])^(3/2))/(3*b*c*(c*Sec[a + b*x])^(3/2)) + (d^2*ArcTan[
1 - Sqrt[2]*Sqrt[Tan[a + b*x]]]*Sqrt[d*Csc[a + b*x]]*Sqrt[Tan[a + b*x]])/(S
qrt[2]*b*c^2*Sqrt[c*Sec[a + b*x]]) - (d^2*ArcTan[1 + Sqrt[2]*Sqrt[Tan[a + b
*x]]]*Sqrt[d*Csc[a + b*x]]*Sqrt[Tan[a + b*x]])/(Sqrt[2]*b*c^2*Sqrt[c*Sec[a
+ b*x]]) + (d^2*Sqrt[d*Csc[a + b*x]]*Log[1 - Sqrt[2]*Sqrt[Tan[a + b*x]] + T
an[a + b*x]]*Sqrt[Tan[a + b*x]])/(2*Sqrt[2]*b*c^2*Sqrt[c*Sec[a + b*x]]) - (
d^2*Sqrt[d*Csc[a + b*x]]*Log[1 + Sqrt[2]*Sqrt[Tan[a + b*x]] + Tan[a + b*x]]
*Sqrt[Tan[a + b*x]])/(2*Sqrt[2]*b*c^2*Sqrt[c*Sec[a + b*x]])
```

Rubi [A] time = 0.214465, antiderivative size = 329, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 10, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {2623, 2629, 3476, 329, 211, 1165, 628, 1162, 617, 204}

$$\frac{d^2\tan^{-1}\left(1-\sqrt{2}\sqrt{\tan(a+bx)}\right)\sqrt{\tan(a+bx)}\sqrt{d\csc(a+bx)}}{\sqrt{2}bc^2\sqrt{c\sec(a+bx)}}-\frac{d^2\tan^{-1}\left(\sqrt{2}\sqrt{\tan(a+bx)}+1\right)\sqrt{\tan(a+bx)}\sqrt{d\csc(a+}}{\sqrt{2}bc^2\sqrt{c\sec(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Csc[a + b*x])^(5/2)/(c*Sec[a + b*x])^(5/2),x]
```


```
[Out] (-2*d*(d*Csc[a + b*x])^(3/2))/(3*b*c*(c*Sec[a + b*x])^(3/2)) + (d^2*ArcTan[
1 - Sqrt[2]*Sqrt[Tan[a + b*x]]]*Sqrt[d*Csc[a + b*x]]*Sqrt[Tan[a + b*x]])/(S
qrt[2]*b*c^2*Sqrt[c*Sec[a + b*x]]) - (d^2*ArcTan[1 + Sqrt[2]*Sqrt[Tan[a + b
*x]]]*Sqrt[d*Csc[a + b*x]]*Sqrt[Tan[a + b*x]])/(Sqrt[2]*b*c^2*Sqrt[c*Sec[a
+ b*x]]) + (d^2*Sqrt[d*Csc[a + b*x]]*Log[1 - Sqrt[2]*Sqrt[Tan[a + b*x]] + T
an[a + b*x]]*Sqrt[Tan[a + b*x]])/(2*Sqrt[2]*b*c^2*Sqrt[c*Sec[a + b*x]]) - (
d^2*Sqrt[d*Csc[a + b*x]]*Log[1 + Sqrt[2]*Sqrt[Tan[a + b*x]] + Tan[a + b*x]]
*Sqrt[Tan[a + b*x]])/(2*Sqrt[2]*b*c^2*Sqrt[c*Sec[a + b*x]])
```


Rule 2623

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> -Simp[(a*(a*Csc[e + f*x])^(m - 1)*(b*Sec[e + f*x])^(n + 1)
)/(f*b*(m - 1)), x] + Dist[(a^2*(n + 1))/(b^2*(m - 1)), Int[(a*Csc[e + f*x]
)^(m - 2)*(b*Sec[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ
[m, 1] && LtQ[n, -1] && IntegersQ[2*m, 2*n]
```

Rule 2629

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Dist[((a*Csc[e + f*x])^m*(b*Sec[e + f*x])^n)/Tan[e + f*x]^
n, Int[Tan[e + f*x]^n, x], x] /; FreeQ[{a, b, e, f, m, n}, x] &&  !IntegerQ
[n] && EqQ[m + n, 0]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
```

```
IntegerQ[n]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{(d\csc(a+bx))^{5/2}}{(c\sec(a+bx))^{5/2}}\,dx &= -\frac{2d(d\csc(a+bx))^{3/2}}{3bc(c\sec(a+bx))^{3/2}} - \frac{d^2\int \frac{\sqrt{d\csc(a+bx)}}{\sqrt{c\sec(a+bx)}}\,dx}{c^2} \\
&= -\frac{2d(d\csc(a+bx))^{3/2}}{3bc(c\sec(a+bx))^{3/2}} - \frac{\left(d^2\sqrt{d\csc(a+bx)}\sqrt{\tan(a+bx)}\right)\int \frac{1}{\sqrt{\tan(a+bx)}}\,dx}{c^2\sqrt{c\sec(a+bx)}} \\
&= -\frac{2d(d\csc(a+bx))^{3/2}}{3bc(c\sec(a+bx))^{3/2}} - \frac{\left(d^2\sqrt{d\csc(a+bx)}\sqrt{\tan(a+bx)}\right)\text{Subst}\left(\int \frac{1}{\sqrt{x}(1+x^2)}\,dx, x, \tan(a+bx\right.}{bc^2\sqrt{c\sec(a+bx)}} \\
&= -\frac{2d(d\csc(a+bx))^{3/2}}{3bc(c\sec(a+bx))^{3/2}} - \frac{\left(2d^2\sqrt{d\csc(a+bx)}\sqrt{\tan(a+bx)}\right)\text{Subst}\left(\int \frac{1}{1+x^4}\,dx, x, \sqrt{\tan(a+bx)}\right.}{bc^2\sqrt{c\sec(a+bx)}} \\
&= -\frac{2d(d\csc(a+bx))^{3/2}}{3bc(c\sec(a+bx))^{3/2}} - \frac{\left(d^2\sqrt{d\csc(a+bx)}\sqrt{\tan(a+bx)}\right)\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \sqrt{\tan(a+bx)}\right)}{bc^2\sqrt{c\sec(a+bx)}} \\
&= -\frac{2d(d\csc(a+bx))^{3/2}}{3bc(c\sec(a+bx))^{3/2}} - \frac{\left(d^2\sqrt{d\csc(a+bx)}\sqrt{\tan(a+bx)}\right)\text{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}\,dx, x, \sqrt{\tan(a+\right.}{2bc^2\sqrt{c\sec(a+bx)}} \\
&= -\frac{2d(d\csc(a+bx))^{3/2}}{3bc(c\sec(a+bx))^{3/2}} + \frac{d^2\sqrt{d\csc(a+bx)}\log\left(1-\sqrt{2}\sqrt{\tan(a+bx)}+\tan(a+bx)\right)\sqrt{\tan(a+}}{2\sqrt{2}bc^2\sqrt{c\sec(a+bx)}} \\
&= -\frac{2d(d\csc(a+bx))^{3/2}}{3bc(c\sec(a+bx))^{3/2}} + \frac{d^2\tan^{-1}\left(1-\sqrt{2}\sqrt{\tan(a+bx)}\right)\sqrt{d\csc(a+bx)}\sqrt{\tan(a+bx)}}{\sqrt{2}bc^2\sqrt{c\sec(a+bx)}} - \frac{d^2\,\text{ta}}{}
\end{aligned}
$$

Mathematica [C] time = 0.200299, size = 55, normalized size = 0.17

$$
\frac{2d(d\csc(a+bx))^{3/2}\left(\text{Hypergeometric2F1}\left(\frac{3}{4},1,\frac{7}{4},-\cot^2(a+bx)\right)-1\right)}{3bc(c\sec(a+bx))^{3/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Csc[a + b*x])^(5/2)/(c*Sec[a + b*x])^(5/2),x]
```

```
[Out] (2*d*(d*Csc[a + b*x])^(3/2)*(-1 + Hypergeometric2F1[3/4, 1, 7/4, -Cot[a + b
*x]^2]))/(3*b*c*(c*Sec[a + b*x])^(3/2))
```

Maple [C] time = 0.179, size = 1259, normalized size = 3.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*csc(b*x+a))^(5/2)/(c*sec(b*x+a))^(5/2),x)
```

```
[Out] -1/6/b*2^(1/2)*(-3*I*cos(b*x+a)*sin(b*x+a)*(-(-1+cos(b*x+a)-sin(b*x+a))/sin
(b*x+a))^(1/2)*((-1+cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a
))/sin(b*x+a))^(1/2)*EllipticPi((-(-1+cos(b*x+a)-sin(b*x+a))/sin(b*x+a))^(1
/2),1/2+1/2*I,1/2*2^(1/2))+3*I*cos(b*x+a)*sin(b*x+a)*(-(-1+cos(b*x+a)-sin(b
*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1
+cos(b*x+a))/sin(b*x+a))^(1/2)*EllipticPi((-(-1+cos(b*x+a)-sin(b*x+a))/sin(
b*x+a))^(1/2),1/2-1/2*I,1/2*2^(1/2))+3*cos(b*x+a)*sin(b*x+a)*(-(-1+cos(b*x+
a)-sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1
```

```
/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*EllipticPi((-(-1+cos(b*x+a)-sin(b*x+
a))/sin(b*x+a))^(1/2),1/2+1/2*I,1/2*2^(1/2))+3*cos(b*x+a)*sin(b*x+a)*(-(-1+
cos(b*x+a)-sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a)+sin(b*x+a))/sin(b*
x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*EllipticPi((-(-1+cos(b*x+a)-
sin(b*x+a))/sin(b*x+a))^(1/2),1/2-1/2*I,1/2*2^(1/2))-6*cos(b*x+a)*sin(b*x+a
)*(-(-1+cos(b*x+a)-sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a)+sin(b*x+a)
)/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*EllipticF((-(-1+cos(
b*x+a)-sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))-3*I*sin(b*x+a)*(-(-1+cos(
b*x+a)-sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a)+sin(b*x+a))/sin(b*x+a)
)^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*EllipticPi((-(-1+cos(b*x+a)-sin(
b*x+a))/sin(b*x+a))^(1/2),1/2+1/2*I,1/2*2^(1/2))+3*I*sin(b*x+a)*(-(-1+cos(b
*x+a)-sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a)+sin(b*x+a))/sin(b*x+a))
^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*EllipticPi((-(-1+cos(b*x+a)-sin(b
*x+a))/sin(b*x+a))^(1/2),1/2-1/2*I,1/2*2^(1/2))+3*sin(b*x+a)*(-(-1+cos(b*x+
a)-sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1
/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*EllipticPi((-(-1+cos(b*x+a)-sin(b*x+
a))/sin(b*x+a))^(1/2),1/2+1/2*I,1/2*2^(1/2))+3*sin(b*x+a)*EllipticPi((-(-1+
cos(b*x+a)-sin(b*x+a))/sin(b*x+a))^(1/2),1/2-1/2*I,1/2*2^(1/2))*(-(-1+cos(b
*x+a)-sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a)+sin(b*x+a))/sin(b*x+a))
^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)-6*sin(b*x+a)*(-(-1+cos(b*x+a)-sin
(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((
-1+cos(b*x+a))/sin(b*x+a))^(1/2)*EllipticF((-(-1+cos(b*x+a)-sin(b*x+a))/sin
(b*x+a))^(1/2),1/2*2^(1/2))+2*cos(b*x+a)^2*2^(1/2))*(d/sin(b*x+a))^(5/2)*si
n(b*x+a)/cos(b*x+a)^3/(c/cos(b*x+a))^(5/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(d\csc(bx+a))^{\frac{5}{2}}}{(c\sec(bx+a))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(b*x+a))^(5/2)/(c*sec(b*x+a))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((d*csc(b*x + a))^(5/2)/(c*sec(b*x + a))^(5/2), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(b*x+a))^(5/2)/(c*sec(b*x+a))^(5/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(b*x+a))**(5/2)/(c*sec(b*x+a))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(d \csc (bx + a))^{\frac{5}{2}}}{(c \sec (bx + a))^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(b*x+a))^(5/2)/(c*sec(b*x+a))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((d*csc(b*x + a))^(5/2)/(c*sec(b*x + a))^(5/2), x)
```

3.274 $\int \frac{(d\csc(a+bx))^{3/2}}{(c\sec(a+bx))^{5/2}}\,dx$

Optimal. Leaf size=94

$$-\frac{3d^2E\left(a+bx-\frac{\pi}{4}\middle|2\right)}{bc^2\sqrt{\sin(2a+2bx)}\sqrt{c\sec(a+bx)}\sqrt{d\csc(a+bx)}}-\frac{2d\sqrt{d\csc(a+bx)}}{bc(c\sec(a+bx))^{3/2}}$$

```
[Out] (-2*d*Sqrt[d*Csc[a + b*x]])/(b*c*(c*Sec[a + b*x])^(3/2)) - (3*d^2*EllipticE
[a - Pi/4 + b*x, 2])/(b*c^2*Sqrt[d*Csc[a + b*x]]*Sqrt[c*Sec[a + b*x]]*Sqrt[
Sin[2*a + 2*b*x]])
```

Rubi [A] time = 0.146961, antiderivative size = 94, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {2623, 2630, 2572, 2639}

$$-\frac{3d^2E\left(a+bx-\frac{\pi}{4}\middle|2\right)}{bc^2\sqrt{\sin(2a+2bx)}\sqrt{c\sec(a+bx)}\sqrt{d\csc(a+bx)}}-\frac{2d\sqrt{d\csc(a+bx)}}{bc(c\sec(a+bx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Csc[a + b*x])^(3/2)/(c*Sec[a + b*x])^(5/2),x]
```

```
[Out] (-2*d*Sqrt[d*Csc[a + b*x]])/(b*c*(c*Sec[a + b*x])^(3/2)) - (3*d^2*EllipticE
[a - Pi/4 + b*x, 2])/(b*c^2*Sqrt[d*Csc[a + b*x]]*Sqrt[c*Sec[a + b*x]]*Sqrt[
Sin[2*a + 2*b*x]])
```

Rule 2623

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> -Simp[(a*(a*Csc[e + f*x])^(m - 1)*(b*Sec[e + f*x])^(n + 1)
)/(f*b*(m - 1)), x] + Dist[(a^2*(n + 1))/(b^2*(m - 1)), Int[(a*Csc[e + f*x]
)^(m - 2)*(b*Sec[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ
[m, 1] && LtQ[n, -1] && IntegersQ[2*m, 2*n]
```

Rule 2630

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Dist[(a*Csc[e + f*x])^m*(b*Sec[e + f*x])^n*(a*Sin[e + f*x]
)^m*(b*Cos[e + f*x])^n, Int[1/((a*Sin[e + f*x])^m*(b*Cos[e + f*x])^n), x],
x] /; FreeQ[{a, b, e, f, m, n}, x] && IntegerQ[m - 1/2] && IntegerQ[n - 1/2
]
```

Rule 2572

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[(Sqrt[a*Sin[e + f*x]]*Sqrt[b*Cos[e + f*x]])/Sqrt[Sin[2*
e + 2*f*x]], Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(d\csc(a+bx))^{3/2}}{(c\sec(a+bx))^{5/2}}\,dx = -\frac{2d\sqrt{d\csc(a+bx)}}{bc(c\sec(a+bx))^{3/2}} - \frac{(3d^2)\int \frac{1}{\sqrt{d\csc(a+bx)}\sqrt{c\sec(a+bx)}}\,dx}{c^2}$$
$$= -\frac{2d\sqrt{d\csc(a+bx)}}{bc(c\sec(a+bx))^{3/2}} - \frac{(3d^2)\int \sqrt{c\cos(a+bx)}\sqrt{d\sin(a+bx)}\,dx}{c^2\sqrt{c\cos(a+bx)}\sqrt{d\csc(a+bx)}\sqrt{c\sec(a+bx)}\sqrt{d\sin(a+bx)}}$$
$$= -\frac{2d\sqrt{d\csc(a+bx)}}{bc(c\sec(a+bx))^{3/2}} - \frac{(3d^2)\int \sqrt{\sin(2a+2bx)}\,dx}{c^2\sqrt{d\csc(a+bx)}\sqrt{c\sec(a+bx)}\sqrt{\sin(2a+2bx)}}$$
$$= -\frac{2d\sqrt{d\csc(a+bx)}}{bc(c\sec(a+bx))^{3/2}} - \frac{3d^2E\left(a-\frac{\pi}{4}+bx\middle|2\right)}{bc^2\sqrt{d\csc(a+bx)}\sqrt{c\sec(a+bx)}\sqrt{\sin(2a+2bx)}}$$

Mathematica [C] time = 0.660887, size = 80, normalized size = 0.85

$$-\frac{d^3\sqrt{c\sec(a+bx)}\left(3\sqrt[4]{-\cot^2(a+bx)}\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{4},\frac{1}{2},\csc^2(a+bx)\right)+2\cot^2(a+bx)\right)}{bc^3(d\csc(a+bx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Csc[a + b*x])^(3/2)/(c*Sec[a + b*x])^(5/2),x]
```

```
[Out] -((d^3*(2*Cot[a + b*x]^2 + 3*(-Cot[a + b*x]^2)^(1/4)*Hypergeometric2F1[-1/2
, 1/4, 1/2, Csc[a + b*x]^2])*Sqrt[c*Sec[a + b*x]])/(b*c^3*(d*Csc[a + b*x])^
(3/2)))
```

Maple [B] time = 0.169, size = 515, normalized size = 5.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*csc(b*x+a))^(3/2)/(c*sec(b*x+a))^(5/2),x)
```

```
[Out] 1/2/b*2^(1/2)*(6*cos(b*x+a)*(-(-1+cos(b*x+a)-sin(b*x+a))/sin(b*x+a))^(1/2)*
((-1+cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^
(1/2)*EllipticE((-(-1+cos(b*x+a)-sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))
-3*cos(b*x+a)*(-(-1+cos(b*x+a)-sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a
)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*Elliptic
F((-(-1+cos(b*x+a)-sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))+6*(-(-1+cos(b
*x+a)-sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a)+sin(b*x+a))/sin(b*x+a))
^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*EllipticE((-(-1+cos(b*x+a)-sin(b*
x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))-3*(-(-1+cos(b*x+a)-sin(b*x+a))/sin(b*x
+a))^(1/2)*((-1+cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/s
in(b*x+a))^(1/2)*EllipticF((-(-1+cos(b*x+a)-sin(b*x+a))/sin(b*x+a))^(1/2),1
/2*2^(1/2))+cos(b*x+a)^2*2^(1/2)-3*cos(b*x+a)*2^(1/2))*(d/sin(b*x+a))^(3/2)
*sin(b*x+a)/cos(b*x+a)^3/(c/cos(b*x+a))^(5/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(d\csc(bx+a))^{\frac{3}{2}}}{(c\sec(bx+a))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(b*x+a))^(3/2)/(c*sec(b*x+a))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((d*csc(b*x + a))^(3/2)/(c*sec(b*x + a))^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{d\csc(bx+a)}\sqrt{c\sec(bx+a)}d\csc(bx+a)}{c^3\sec(bx+a)^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(b*x+a))^(3/2)/(c*sec(b*x+a))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*csc(b*x + a))*sqrt(c*sec(b*x + a))*d*csc(b*x + a)/(c^3*sec(
b*x + a)^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(b*x+a))**(3/2)/(c*sec(b*x+a))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(d\csc(bx+a))^{\frac{3}{2}}}{(c\sec(bx+a))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(b*x+a))^(3/2)/(c*sec(b*x+a))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((d*csc(b*x + a))^(3/2)/(c*sec(b*x + a))^(5/2), x)
```

3.275 $\int \frac{\sqrt{d\csc(a+bx)}}{(c\sec(a+bx))^{5/2}}\,dx$

Optimal. Leaf size=322

$$-\frac{3\tan^{-1}\left(1-\sqrt{2}\sqrt{\tan(a+bx)}\right)\sqrt{\tan(a+bx)}\sqrt{d\csc(a+bx)}}{4\sqrt{2}bc^2\sqrt{c\sec(a+bx)}}+\frac{3\tan^{-1}\left(\sqrt{2}\sqrt{\tan(a+bx)}+1\right)\sqrt{\tan(a+bx)}\sqrt{d\csc(a+}}{4\sqrt{2}bc^2\sqrt{c\sec(a+bx)}}$$

```
[Out] d/(2*b*c*Sqrt[d*Csc[a + b*x]]*(c*Sec[a + b*x])^(3/2)) - (3*ArcTan[1 - Sqrt[
2]*Sqrt[Tan[a + b*x]]]*Sqrt[d*Csc[a + b*x]]*Sqrt[Tan[a + b*x]])/(4*Sqrt[2]*
b*c^2*Sqrt[c*Sec[a + b*x]]) + (3*ArcTan[1 + Sqrt[2]*Sqrt[Tan[a + b*x]]]*Sqr
t[d*Csc[a + b*x]]*Sqrt[Tan[a + b*x]])/(4*Sqrt[2]*b*c^2*Sqrt[c*Sec[a + b*x]]
) - (3*Sqrt[d*Csc[a + b*x]]*Log[1 - Sqrt[2]*Sqrt[Tan[a + b*x]] + Tan[a + b*
x]]*Sqrt[Tan[a + b*x]])/(8*Sqrt[2]*b*c^2*Sqrt[c*Sec[a + b*x]]) + (3*Sqrt[d*
Csc[a + b*x]]*Log[1 + Sqrt[2]*Sqrt[Tan[a + b*x]] + Tan[a + b*x]]*Sqrt[Tan[a
 + b*x]])/(8*Sqrt[2]*b*c^2*Sqrt[c*Sec[a + b*x]])
```

Rubi [A] time = 0.203437, antiderivative size = 322, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 10, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {2628, 2629, 3476, 329, 211, 1165, 628, 1162, 617, 204}

$$-\frac{3\tan^{-1}\left(1-\sqrt{2}\sqrt{\tan(a+bx)}\right)\sqrt{\tan(a+bx)}\sqrt{d\csc(a+bx)}}{4\sqrt{2}bc^2\sqrt{c\sec(a+bx)}}+\frac{3\tan^{-1}\left(\sqrt{2}\sqrt{\tan(a+bx)}+1\right)\sqrt{\tan(a+bx)}\sqrt{d\csc(a+}}{4\sqrt{2}bc^2\sqrt{c\sec(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d*Csc[a + b*x]]/(c*Sec[a + b*x])^(5/2),x]
```

```
[Out] d/(2*b*c*Sqrt[d*Csc[a + b*x]]*(c*Sec[a + b*x])^(3/2)) - (3*ArcTan[1 - Sqrt[
2]*Sqrt[Tan[a + b*x]]]*Sqrt[d*Csc[a + b*x]]*Sqrt[Tan[a + b*x]])/(4*Sqrt[2]*
b*c^2*Sqrt[c*Sec[a + b*x]]) + (3*ArcTan[1 + Sqrt[2]*Sqrt[Tan[a + b*x]]]*Sqr
t[d*Csc[a + b*x]]*Sqrt[Tan[a + b*x]])/(4*Sqrt[2]*b*c^2*Sqrt[c*Sec[a + b*x]]
) - (3*Sqrt[d*Csc[a + b*x]]*Log[1 - Sqrt[2]*Sqrt[Tan[a + b*x]] + Tan[a + b*
x]]*Sqrt[Tan[a + b*x]])/(8*Sqrt[2]*b*c^2*Sqrt[c*Sec[a + b*x]]) + (3*Sqrt[d*
Csc[a + b*x]]*Log[1 + Sqrt[2]*Sqrt[Tan[a + b*x]] + Tan[a + b*x]]*Sqrt[Tan[a
 + b*x]])/(8*Sqrt[2]*b*c^2*Sqrt[c*Sec[a + b*x]])
```

Rule 2628

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> -Simp[(a*(a*Csc[e + f*x])^(m - 1)*(b*Sec[e + f*x])^(n + 1
))/(b*f*(m + n)), x] + Dist[(n + 1)/(b^2*(m + n)), Int[(a*Csc[e + f*x])^m*(
b*Sec[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && LtQ[n, -1]
&& NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2629

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Dist[((a*Csc[e + f*x])^m*(b*Sec[e + f*x])^n)/Tan[e + f*x]^
n, Int[Tan[e + f*x]^n, x], x] /; FreeQ[{a, b, e, f, m, n}, x] &&  !IntegerQ
[n] && EqQ[m + n, 0]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
```

```
IntegerQ[n]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\sqrt{d\csc(a+bx)}}{(c\sec(a+bx))^{5/2}}\,dx = \frac{d}{2bc\sqrt{d\csc(a+bx)}(c\sec(a+bx))^{3/2}} + \frac{3\int \frac{\sqrt{d\csc(a+bx)}}{\sqrt{c\sec(a+bx)}}\,dx}{4c^2}$$

$$= \frac{d}{2bc\sqrt{d\csc(a+bx)}(c\sec(a+bx))^{3/2}} + \frac{\left(3\sqrt{d\csc(a+bx)}\sqrt{\tan(a+bx)}\right)\int \frac{1}{\sqrt{\tan(a+bx)}}\,dx}{4c^2\sqrt{c\sec(a+bx)}}$$

$$= \frac{d}{2bc\sqrt{d\csc(a+bx)}(c\sec(a+bx))^{3/2}} + \frac{\left(3\sqrt{d\csc(a+bx)}\sqrt{\tan(a+bx)}\right)\text{Subst}\left(\int \frac{1}{\sqrt{x}(1+x^2)}\,dx, x\right.}{4bc^2\sqrt{c\sec(a+bx)}}$$

$$= \frac{d}{2bc\sqrt{d\csc(a+bx)}(c\sec(a+bx))^{3/2}} + \frac{\left(3\sqrt{d\csc(a+bx)}\sqrt{\tan(a+bx)}\right)\text{Subst}\left(\int \frac{1}{1+x^4}\,dx, x, \sqrt{t}\right.}{2bc^2\sqrt{c\sec(a+bx)}}$$

$$= \frac{d}{2bc\sqrt{d\csc(a+bx)}(c\sec(a+bx))^{3/2}} + \frac{\left(3\sqrt{d\csc(a+bx)}\sqrt{\tan(a+bx)}\right)\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \sqrt{t}\right.}{4bc^2\sqrt{c\sec(a+bx)}}$$

$$= \frac{d}{2bc\sqrt{d\csc(a+bx)}(c\sec(a+bx))^{3/2}} + \frac{\left(3\sqrt{d\csc(a+bx)}\sqrt{\tan(a+bx)}\right)\text{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}\,dx, x\right.}{8bc^2\sqrt{c\sec(a+bx)}}$$

$$= \frac{d}{2bc\sqrt{d\csc(a+bx)}(c\sec(a+bx))^{3/2}} - \frac{3\sqrt{d\csc(a+bx)}\log\left(1-\sqrt{2}\sqrt{\tan(a+bx)}+\tan(a+bx\right.}{8\sqrt{2}bc^2\sqrt{c\sec(a+bx)}}$$

$$= \frac{d}{2bc\sqrt{d\csc(a+bx)}(c\sec(a+bx))^{3/2}} - \frac{3\tan^{-1}\left(1-\sqrt{2}\sqrt{\tan(a+bx)}\right)\sqrt{d\csc(a+bx)}\sqrt{\tan(a+}}{4\sqrt{2}bc^2\sqrt{c\sec(a+bx)}}$$

Mathematica [C] time = 0.220849, size = 70, normalized size = 0.22

$$-\frac{\cot(a+bx)\sqrt{d\csc(a+bx)}\left(2\text{Hypergeometric2F1}\left(\frac{3}{4},1,\frac{7}{4},-\cot^2(a+bx)\right)+\cos(2(a+bx))-1\right)}{4bc^2\sqrt{c\sec(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d*Csc[a + b*x]]/(c*Sec[a + b*x])^(5/2),x]
```

```
[Out] -(Cot[a + b*x]*Sqrt[d*Csc[a + b*x]]*(-1 + Cos[2*(a + b*x)] + 2*Hypergeometr
ic2F1[3/4, 1, 7/4, -Cot[a + b*x]^2]))/(4*b*c^2*Sqrt[c*Sec[a + b*x]])
```

Maple [C] time = 0.175, size = 668, normalized size = 2.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*csc(b*x+a))^(1/2)/(c*sec(b*x+a))^(5/2),x)
```

```
[Out] 1/8/b*2^(1/2)*(3*I*sin(b*x+a)*(-(-1+cos(b*x+a)-sin(b*x+a))/sin(b*x+a))^(1/2
)*((-1+cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a)
)^(1/2)*EllipticPi((-(-1+cos(b*x+a)-sin(b*x+a))/sin(b*x+a))^(1/2),1/2+1/2*I
,1/2*2^(1/2))-3*I*sin(b*x+a)*(-(-1+cos(b*x+a)-sin(b*x+a))/sin(b*x+a))^(1/2)
*((-1+cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))
^(1/2)*EllipticPi((-(-1+cos(b*x+a)-sin(b*x+a))/sin(b*x+a))^(1/2),1/2-1/2*I,
1/2*2^(1/2))-3*sin(b*x+a)*(-(-1+cos(b*x+a)-sin(b*x+a))/sin(b*x+a))^(1/2)*((
-1+cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1
```

```
/2)*EllipticPi((-(-1+cos(b*x+a)-sin(b*x+a))/sin(b*x+a))^(1/2),1/2+1/2*I,1/2
*2^(1/2))+6*sin(b*x+a)*(-(-1+cos(b*x+a)-sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+
cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)
*EllipticF((-(-1+cos(b*x+a)-sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))-3*si
n(b*x+a)*EllipticPi((-(-1+cos(b*x+a)-sin(b*x+a))/sin(b*x+a))^(1/2),1/2-1/2*
I,1/2*2^(1/2))*(-(-1+cos(b*x+a)-sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+
a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)+2*cos(b
*x+a)^3*2^(1/2)-2*cos(b*x+a)^2*2^(1/2))*(d/sin(b*x+a))^(1/2)*sin(b*x+a)/(-1
+cos(b*x+a))/cos(b*x+a)^3/(c/cos(b*x+a))^(5/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{d\csc(bx+a)}}{(c\sec(bx+a))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(b*x+a))^(1/2)/(c*sec(b*x+a))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*csc(b*x + a))/(c*sec(b*x + a))^(5/2), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(b*x+a))^(1/2)/(c*sec(b*x+a))^(5/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(b*x+a))**(1/2)/(c*sec(b*x+a))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{d\csc(bx+a)}}{(c\sec(bx+a))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(b*x+a))^(1/2)/(c*sec(b*x+a))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*csc(b*x + a))/(c*sec(b*x + a))^(5/2), x)
```

3.276 $\int \frac{1}{\sqrt{d\csc(a+bx)}(c\sec(a+bx))^{5/2}}\,dx$

Optimal. Leaf size=95

$$\frac{E\left(a+bx-\frac{\pi}{4}\middle|2\right)}{2bc^2\sqrt{\sin(2a+2bx)}\sqrt{c\sec(a+bx)}\sqrt{d\csc(a+bx)}}+\frac{d}{3bc(c\sec(a+bx))^{3/2}(d\csc(a+bx))^{3/2}}$$

```
[Out] d/(3*b*c*(d*Csc[a + b*x])^(3/2)*(c*Sec[a + b*x])^(3/2)) + EllipticE[a - Pi/
4 + b*x, 2]/(2*b*c^2*Sqrt[d*Csc[a + b*x]]*Sqrt[c*Sec[a + b*x]]*Sqrt[Sin[2*a
 + 2*b*x]])
```

Rubi [A] time = 0.141849, antiderivative size = 95, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {2628, 2630, 2572, 2639}

$$\frac{E\left(a+bx-\frac{\pi}{4}\middle|2\right)}{2bc^2\sqrt{\sin(2a+2bx)}\sqrt{c\sec(a+bx)}\sqrt{d\csc(a+bx)}}+\frac{d}{3bc(c\sec(a+bx))^{3/2}(d\csc(a+bx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[d*Csc[a + b*x]]*(c*Sec[a + b*x])^(5/2)),x]
```


```
[Out] d/(3*b*c*(d*Csc[a + b*x])^(3/2)*(c*Sec[a + b*x])^(3/2)) + EllipticE[a - Pi/
4 + b*x, 2]/(2*b*c^2*Sqrt[d*Csc[a + b*x]]*Sqrt[c*Sec[a + b*x]]*Sqrt[Sin[2*a
 + 2*b*x]])
```


Rule 2628

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> -Simp[(a*(a*Csc[e + f*x])^(m - 1)*(b*Sec[e + f*x])^(n + 1
))/(b*f*(m + n)), x] + Dist[(n + 1)/(b^2*(m + n)), Int[(a*Csc[e + f*x])^m*(
b*Sec[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && LtQ[n, -1]
&& NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2630

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Dist[(a*Csc[e + f*x])^m*(b*Sec[e + f*x])^n*(a*Sin[e + f*x]
)^m*(b*Cos[e + f*x])^n, Int[1/((a*Sin[e + f*x])^m*(b*Cos[e + f*x])^n), x],
x] /; FreeQ[{a, b, e, f, m, n}, x] && IntegerQ[m - 1/2] && IntegerQ[n - 1/2
]
```

Rule 2572

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[(Sqrt[a*Sin[e + f*x]]*Sqrt[b*Cos[e + f*x]])/Sqrt[Sin[2*
e + 2*f*x]], Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{1}{\sqrt{d\csc(a+bx)}(c\sec(a+bx))^{5/2}}\,dx = \frac{d}{3bc(d\csc(a+bx))^{3/2}(c\sec(a+bx))^{3/2}} + \frac{\int \frac{1}{\sqrt{d\csc(a+bx)}\sqrt{c\sec(a+bx)}}\,dx}{2c^2}$$
$$= \frac{d}{3bc(d\csc(a+bx))^{3/2}(c\sec(a+bx))^{3/2}} + \frac{\int \sqrt{c\cos(a+bx)}\sqrt{d\,s}}{2c^2\sqrt{c\cos(a+bx)}\sqrt{d\csc(a+bx)}\sqrt{c}}$$
$$= \frac{d}{3bc(d\csc(a+bx))^{3/2}(c\sec(a+bx))^{3/2}} + \frac{\int \sqrt{\sin(2a+2bx)}\,dx}{2c^2\sqrt{d\csc(a+bx)}\sqrt{c\sec(a+bx)}\sqrt{s}}$$
$$= \frac{d}{3bc(d\csc(a+bx))^{3/2}(c\sec(a+bx))^{3/2}} + \frac{E\left(a-\frac{\pi}{4}+bx\middle|2\right)}{2bc^2\sqrt{d\csc(a+bx)}\sqrt{c\sec(a+bx)}\sqrt{}}$$

Mathematica [C] time = 0.363158, size = 79, normalized size = 0.83

$$\frac{d\sqrt{c\sec(a+bx)}\left(3\sqrt[4]{-\cot^2(a+bx)}\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{4},\frac{1}{2},\csc^2(a+bx)\right)+\cos(2(a+bx))+1\right)}{6bc^3(d\csc(a+bx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[d*Csc[a + b*x]]*(c*Sec[a + b*x])^(5/2)),x]
```

```
[Out] (d*(1 + Cos[2*(a + b*x)] + 3*(-Cot[a + b*x]^2)^(1/4)*Hypergeometric2F1[-1/2
, 1/4, 1/2, Csc[a + b*x]^2])*Sqrt[c*Sec[a + b*x]])/(6*b*c^3*(d*Csc[a + b*x]
)^(3/2))
```

Maple [B] time = 0.192, size = 530, normalized size = 5.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*csc(b*x+a))^(1/2)/(c*sec(b*x+a))^(5/2),x)
```

```
[Out] -1/12/b*2^(1/2)*(2*2^(1/2)*cos(b*x+a)^4+6*cos(b*x+a)*(-(-1+cos(b*x+a)-sin(b
*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1
+cos(b*x+a))/sin(b*x+a))^(1/2)*EllipticE((-(-1+cos(b*x+a)-sin(b*x+a))/sin(b
*x+a))^(1/2),1/2*2^(1/2))-3*cos(b*x+a)*(-(-1+cos(b*x+a)-sin(b*x+a))/sin(b*x
+a))^(1/2)*((-1+cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/s
in(b*x+a))^(1/2)*EllipticF((-(-1+cos(b*x+a)-sin(b*x+a))/sin(b*x+a))^(1/2),1
/2*2^(1/2))+6*(-(-1+cos(b*x+a)-sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a
)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*Elliptic
E((-(-1+cos(b*x+a)-sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))-3*(-(-1+cos(b
*x+a)-sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a)+sin(b*x+a))/sin(b*x+a))
^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*EllipticF((-(-1+cos(b*x+a)-sin(b*
x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))+cos(b*x+a)^2*2^(1/2)-3*cos(b*x+a)*2^(1
/2))/cos(b*x+a)^3/sin(b*x+a)/(d/sin(b*x+a))^(1/2)/(c/cos(b*x+a))^(5/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{d\csc(bx+a)}\,(c\sec(bx+a))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*csc(b*x+a))^(1/2)/(c*sec(b*x+a))^(5/2),x, algorithm="maxima"
)
```

```
[Out] integrate(1/(sqrt(d*csc(b*x + a))*(c*sec(b*x + a))^(5/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{d\csc\left(bx+a\right)}\sqrt{c\sec\left(bx+a\right)}}{c^{3}d\csc\left(bx+a\right)\sec\left(bx+a\right)^{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*csc(b*x+a))^(1/2)/(c*sec(b*x+a))^(5/2),x, algorithm="fricas"
)
```

```
[Out] integral(sqrt(d*csc(b*x + a))*sqrt(c*sec(b*x + a))/(c^3*d*csc(b*x + a)*sec(
b*x + a)^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*csc(b*x+a))**(1/2)/(c*sec(b*x+a))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{d\csc\left(bx+a\right)}\left(c\sec\left(bx+a\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*csc(b*x+a))^(1/2)/(c*sec(b*x+a))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(d*csc(b*x + a))*(c*sec(b*x + a))^(5/2)), x)
```

3.277 $\int \frac{1}{(d\csc(a+bx))^{3/2}(c\sec(a+bx))^{5/2}}\,dx$

Optimal. Leaf size=371

$$-\frac{3\tan^{-1}\left(1-\sqrt{2}\sqrt{\tan(a+bx)}\right)\sqrt{\tan(a+bx)}\sqrt{d\csc(a+bx)}}{32\sqrt{2}bc^2d^2\sqrt{c\sec(a+bx)}}+\frac{3\tan^{-1}\left(\sqrt{2}\sqrt{\tan(a+bx)}+1\right)\sqrt{\tan(a+bx)}\sqrt{d\csc(}}{32\sqrt{2}bc^2d^2\sqrt{c\sec(a+bx)}}$$

```
[Out] -c/(4*b*d*Sqrt[d*Csc[a + b*x]]*(c*Sec[a + b*x])^(7/2)) + 1/(16*b*c*d*Sqrt[d
*Csc[a + b*x]]*(c*Sec[a + b*x])^(3/2)) - (3*ArcTan[1 - Sqrt[2]*Sqrt[Tan[a +
 b*x]]]*Sqrt[d*Csc[a + b*x]]*Sqrt[Tan[a + b*x]])/(32*Sqrt[2]*b*c^2*d^2*Sqrt
[c*Sec[a + b*x]]) + (3*ArcTan[1 + Sqrt[2]*Sqrt[Tan[a + b*x]]]*Sqrt[d*Csc[a
+ b*x]]*Sqrt[Tan[a + b*x]])/(32*Sqrt[2]*b*c^2*d^2*Sqrt[c*Sec[a + b*x]]) - (
3*Sqrt[d*Csc[a + b*x]]*Log[1 - Sqrt[2]*Sqrt[Tan[a + b*x]] + Tan[a + b*x]]*S
qrt[Tan[a + b*x]])/(64*Sqrt[2]*b*c^2*d^2*Sqrt[c*Sec[a + b*x]]) + (3*Sqrt[d*
Csc[a + b*x]]*Log[1 + Sqrt[2]*Sqrt[Tan[a + b*x]] + Tan[a + b*x]]*Sqrt[Tan[a
 + b*x]])/(64*Sqrt[2]*b*c^2*d^2*Sqrt[c*Sec[a + b*x]])
```

Rubi [A] time = 0.290564, antiderivative size = 371, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 11, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.44, Rules used = {2627, 2628, 2629, 3476, 329, 211, 1165, 628, 1162, 617, 204}

$$-\frac{3\tan^{-1}\left(1-\sqrt{2}\sqrt{\tan(a+bx)}\right)\sqrt{\tan(a+bx)}\sqrt{d\csc(a+bx)}}{32\sqrt{2}bc^2d^2\sqrt{c\sec(a+bx)}}+\frac{3\tan^{-1}\left(\sqrt{2}\sqrt{\tan(a+bx)}+1\right)\sqrt{\tan(a+bx)}\sqrt{d\csc(}}{32\sqrt{2}bc^2d^2\sqrt{c\sec(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d*Csc[a + b*x])^(3/2)*(c*Sec[a + b*x])^(5/2)),x]
```

```
[Out] -c/(4*b*d*Sqrt[d*Csc[a + b*x]]*(c*Sec[a + b*x])^(7/2)) + 1/(16*b*c*d*Sqrt[d
*Csc[a + b*x]]*(c*Sec[a + b*x])^(3/2)) - (3*ArcTan[1 - Sqrt[2]*Sqrt[Tan[a +
 b*x]]]*Sqrt[d*Csc[a + b*x]]*Sqrt[Tan[a + b*x]])/(32*Sqrt[2]*b*c^2*d^2*Sqrt
[c*Sec[a + b*x]]) + (3*ArcTan[1 + Sqrt[2]*Sqrt[Tan[a + b*x]]]*Sqrt[d*Csc[a
+ b*x]]*Sqrt[Tan[a + b*x]])/(32*Sqrt[2]*b*c^2*d^2*Sqrt[c*Sec[a + b*x]]) - (
3*Sqrt[d*Csc[a + b*x]]*Log[1 - Sqrt[2]*Sqrt[Tan[a + b*x]] + Tan[a + b*x]]*S
qrt[Tan[a + b*x]])/(64*Sqrt[2]*b*c^2*d^2*Sqrt[c*Sec[a + b*x]]) + (3*Sqrt[d*
Csc[a + b*x]]*Log[1 + Sqrt[2]*Sqrt[Tan[a + b*x]] + Tan[a + b*x]]*Sqrt[Tan[a
 + b*x]])/(64*Sqrt[2]*b*c^2*d^2*Sqrt[c*Sec[a + b*x]])
```

Rule 2627

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Simp[(b*(a*Csc[e + f*x])^(m + 1)*(b*Sec[e + f*x])^(n - 1)
)/(a*f*(m + n)), x] + Dist[(m + 1)/(a^2*(m + n)), Int[(a*Csc[e + f*x])^(m +
 2)*(b*Sec[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -1] &
& NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2628

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> -Simp[(a*(a*Csc[e + f*x])^(m - 1)*(b*Sec[e + f*x])^(n + 1
))/(b*f*(m + n)), x] + Dist[(n + 1)/(b^2*(m + n)), Int[(a*Csc[e + f*x])^m*(
b*Sec[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && LtQ[n, -1]
&& NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2629

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Dist[((a*Csc[e + f*x])^m*(b*Sec[e + f*x])^n)/Tan[e + f*x]^
n, Int[Tan[e + f*x]^n, x], x] /; FreeQ[{a, b, e, f, m, n}, x] && !IntegerQ
[n] && EqQ[m + n, 0]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] && !
IntegerQ[n]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{(d\csc(a+bx))^{3/2}(c\sec(a+bx))^{5/2}}\,dx = -\frac{c}{4bd\sqrt{d\csc(a+bx)}(c\sec(a+bx))^{7/2}} + \frac{\int \frac{\sqrt{d\csc(a+bx)}}{(c\sec(a+bx))^{5/2}}\,dx}{8d^2}$$

$$= -\frac{c}{4bd\sqrt{d\csc(a+bx)}(c\sec(a+bx))^{7/2}} + \frac{1}{16bcd\sqrt{d\csc(a+bx)}(c\sec(a+bx}$$

$$= -\frac{c}{4bd\sqrt{d\csc(a+bx)}(c\sec(a+bx))^{7/2}} + \frac{1}{16bcd\sqrt{d\csc(a+bx)}(c\sec(a+bx}$$

$$= -\frac{c}{4bd\sqrt{d\csc(a+bx)}(c\sec(a+bx))^{7/2}} + \frac{1}{16bcd\sqrt{d\csc(a+bx)}(c\sec(a+bx}$$

$$= -\frac{c}{4bd\sqrt{d\csc(a+bx)}(c\sec(a+bx))^{7/2}} + \frac{1}{16bcd\sqrt{d\csc(a+bx)}(c\sec(a+bx}$$

$$= -\frac{c}{4bd\sqrt{d\csc(a+bx)}(c\sec(a+bx))^{7/2}} + \frac{1}{16bcd\sqrt{d\csc(a+bx)}(c\sec(a+bx}$$

$$= -\frac{c}{4bd\sqrt{d\csc(a+bx)}(c\sec(a+bx))^{7/2}} + \frac{1}{16bcd\sqrt{d\csc(a+bx)}(c\sec(a+bx}$$

$$= -\frac{c}{4bd\sqrt{d\csc(a+bx)}(c\sec(a+bx))^{7/2}} + \frac{1}{16bcd\sqrt{d\csc(a+bx)}(c\sec(a+bx}$$

$$= -\frac{c}{4bd\sqrt{d\csc(a+bx)}(c\sec(a+bx))^{7/2}} + \frac{1}{16bcd\sqrt{d\csc(a+bx)}(c\sec(a+bx}$$

Mathematica [C] time = 0.378894, size = 81, normalized size = 0.22

$$-\frac{\csc^3(a+bx)\left(2\text{Hypergeometric2F1}\left(\frac{3}{4},1,\frac{7}{4},-\cot^2(a+bx)\right)+\cos(2(a+bx))-\cos(4(a+bx))\right)}{32bc(c\sec(a+bx))^{3/2}(d\csc(a+bx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d*Csc[a + b*x])^(3/2)*(c*Sec[a + b*x])^(5/2)),x]
```

```
[Out] -(Csc[a + b*x]^3*(Cos[2*(a + b*x)] - Cos[4*(a + b*x)] + 2*Hypergeometric2F1
[3/4, 1, 7/4, -Cot[a + b*x]^2]))/(32*b*c*(d*Csc[a + b*x])^(3/2)*(c*Sec[a +
b*x])^(3/2))
```

Maple [C] time = 0.183, size = 696, normalized size = 1.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*csc(b*x+a))^(3/2)/(c*sec(b*x+a))^(5/2),x)
```

```
[Out] 1/64/b*2^(1/2)*(3*I*sin(b*x+a)*(-(-1+cos(b*x+a)-sin(b*x+a))/sin(b*x+a))^(1/
2)*((-1+cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a
```

```
))^(1/2)*EllipticPi((-(-1+cos(b*x+a)-sin(b*x+a))/sin(b*x+a))^(1/2),1/2+1/2*
I,1/2*2^(1/2))-3*I*sin(b*x+a)*(-(-1+cos(b*x+a)-sin(b*x+a))/sin(b*x+a))^(1/2
)*((-1+cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a)
)^(1/2)*EllipticPi((-(-1+cos(b*x+a)-sin(b*x+a))/sin(b*x+a))^(1/2),1/2-1/2*I
,1/2*2^(1/2))-8*2^(1/2)*cos(b*x+a)^5-3*sin(b*x+a)*(-(-1+cos(b*x+a)-sin(b*x+
a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+co
s(b*x+a))/sin(b*x+a))^(1/2)*EllipticPi((-(-1+cos(b*x+a)-sin(b*x+a))/sin(b*x
+a))^(1/2),1/2+1/2*I,1/2*2^(1/2))+6*sin(b*x+a)*(-(-1+cos(b*x+a)-sin(b*x+a))
/sin(b*x+a))^(1/2)*((-1+cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b
*x+a))/sin(b*x+a))^(1/2)*EllipticF((-(-1+cos(b*x+a)-sin(b*x+a))/sin(b*x+a))
^(1/2),1/2*2^(1/2))-3*sin(b*x+a)*EllipticPi((-(-1+cos(b*x+a)-sin(b*x+a))/si
n(b*x+a))^(1/2),1/2-1/2*I,1/2*2^(1/2))*(-(-1+cos(b*x+a)-sin(b*x+a))/sin(b*x
+a))^(1/2)*((-1+cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/s
in(b*x+a))^(1/2)+8*2^(1/2)*cos(b*x+a)^4+2*cos(b*x+a)^3*2^(1/2)-2*cos(b*x+a)
^2*2^(1/2))/(-1+cos(b*x+a))/cos(b*x+a)^3/sin(b*x+a)/(d/sin(b*x+a))^(3/2)/(c
/cos(b*x+a))^(5/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(d\csc(bx+a))^{\frac{3}{2}}(c\sec(bx+a))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*csc(b*x+a))^(3/2)/(c*sec(b*x+a))^(5/2),x, algorithm="maxima"
)
```

```
[Out] integrate(1/((d*csc(b*x + a))^(3/2)*(c*sec(b*x + a))^(5/2)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*csc(b*x+a))^(3/2)/(c*sec(b*x+a))^(5/2),x, algorithm="fricas"
)
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*csc(b*x+a))**(3/2)/(c*sec(b*x+a))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(d\csc(bx+a))^{\frac{3}{2}}(c\sec(bx+a))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*csc(b*x+a))^(3/2)/(c*sec(b*x+a))^(5/2),x, algorithm="giac")

[Out] integrate(1/((d*csc(b*x + a))^(3/2)*(c*sec(b*x + a))^(5/2)), x)
```

3.278 $\int \frac{1}{(d\csc(a+bx))^{5/2}(c\sec(a+bx))^{5/2}}\,dx$

Optimal. Leaf size=135

$$\frac{3E\left(a+bx-\frac{\pi}{4}\middle|2\right)}{20bc^2d^2\sqrt{\sin(2a+2bx)}\sqrt{c\sec(a+bx)}\sqrt{d\csc(a+bx)}}-\frac{c}{5bd(c\sec(a+bx))^{7/2}(d\csc(a+bx))^{3/2}}+\frac{1}{10bcd(c\sec(a+bx))^{3/2}(}$$

```
[Out] -c/(5*b*d*(d*Csc[a + b*x])^(3/2)*(c*Sec[a + b*x])^(7/2)) + 1/(10*b*c*d*(d*C
sc[a + b*x])^(3/2)*(c*Sec[a + b*x])^(3/2)) + (3*EllipticE[a - Pi/4 + b*x, 2
])/(20*b*c^2*d^2*Sqrt[d*Csc[a + b*x]]*Sqrt[c*Sec[a + b*x]]*Sqrt[Sin[2*a + 2
*b*x]])
```

Rubi [A] time = 0.205576, antiderivative size = 135, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2627, 2628, 2630, 2572, 2639}

$$\frac{3E\left(a+bx-\frac{\pi}{4}\middle|2\right)}{20bc^2d^2\sqrt{\sin(2a+2bx)}\sqrt{c\sec(a+bx)}\sqrt{d\csc(a+bx)}}-\frac{c}{5bd(c\sec(a+bx))^{7/2}(d\csc(a+bx))^{3/2}}+\frac{1}{10bcd(c\sec(a+bx))^{3/2}(}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d*Csc[a + b*x])^(5/2)*(c*Sec[a + b*x])^(5/2)),x]
```


```
[Out] -c/(5*b*d*(d*Csc[a + b*x])^(3/2)*(c*Sec[a + b*x])^(7/2)) + 1/(10*b*c*d*(d*C
sc[a + b*x])^(3/2)*(c*Sec[a + b*x])^(3/2)) + (3*EllipticE[a - Pi/4 + b*x, 2
])/(20*b*c^2*d^2*Sqrt[d*Csc[a + b*x]]*Sqrt[c*Sec[a + b*x]]*Sqrt[Sin[2*a + 2
*b*x]])
```


Rule 2627

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Simp[(b*(a*Csc[e + f*x])^(m + 1)*(b*Sec[e + f*x])^(n - 1)
)/(a*f*(m + n)), x] + Dist[(m + 1)/(a^2*(m + n)), Int[(a*Csc[e + f*x])^(m +
 2)*(b*Sec[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -1] &
& NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2628

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> -Simp[(a*(a*Csc[e + f*x])^(m - 1)*(b*Sec[e + f*x])^(n + 1
))/(b*f*(m + n)), x] + Dist[(n + 1)/(b^2*(m + n)), Int[(a*Csc[e + f*x])^m*(
b*Sec[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && LtQ[n, -1]
&& NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2630

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Dist[(a*Csc[e + f*x])^m*(b*Sec[e + f*x])^n*(a*Sin[e + f*x]
)^m*(b*Cos[e + f*x])^n, Int[1/((a*Sin[e + f*x])^m*(b*Cos[e + f*x])^n), x],
x] /; FreeQ[{a, b, e, f, m, n}, x] && IntegerQ[m - 1/2] && IntegerQ[n - 1/2
]
```

Rule 2572

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[(Sqrt[a*Sin[e + f*x]]*Sqrt[b*Cos[e + f*x]])/Sqrt[Sin[2*
```

```
e + 2*f*x]], Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{1}{(d\csc(a+bx))^{5/2}(c\sec(a+bx))^{5/2}}\,dx = -\frac{c}{5bd(d\csc(a+bx))^{3/2}(c\sec(a+bx))^{7/2}} + \frac{3\int \frac{1}{\sqrt{d\csc(a+bx)}(c\sec(a+bx))^{5/2}}\,dx}{10d^2}$$

$$= -\frac{c}{5bd(d\csc(a+bx))^{3/2}(c\sec(a+bx))^{7/2}} + \frac{1}{10bcd(d\csc(a+bx))^{3/2}(c\sec(a\cdots}$$

$$= -\frac{c}{5bd(d\csc(a+bx))^{3/2}(c\sec(a+bx))^{7/2}} + \frac{1}{10bcd(d\csc(a+bx))^{3/2}(c\sec(a\cdots}$$

$$= -\frac{c}{5bd(d\csc(a+bx))^{3/2}(c\sec(a+bx))^{7/2}} + \frac{1}{10bcd(d\csc(a+bx))^{3/2}(c\sec(a\cdots}$$

$$= -\frac{c}{5bd(d\csc(a+bx))^{3/2}(c\sec(a+bx))^{7/2}} + \frac{1}{10bcd(d\csc(a+bx))^{3/2}(c\sec(a\cdots}$$

Mathematica [C] time = 0.657982, size = 90, normalized size = 0.67

$$\frac{\sqrt{c\sec(a+bx)}\left(3\sqrt[4]{-\cot^2(a+bx)}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{4},\frac{1}{2},\csc^2(a+bx)\right)-2\cos^2(a+bx)\cos(2(a+bx))\right)}{20bc^3d(d\csc(a+bx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d*Csc[a + b*x])^(5/2)*(c*Sec[a + b*x])^(5/2)),x]
```

```
[Out] ((-2*Cos[a + b*x]^2*Cos[2*(a + b*x)] + 3*(-Cot[a + b*x]^2)^(1/4)*Hypergeome
tric2F1[-1/2, 1/4, 1/2, Csc[a + b*x]^2])*Sqrt[c*Sec[a + b*x]])/(20*b*c^3*d*
(d*Csc[a + b*x])^(3/2))
```

Maple [B] time = 0.165, size = 544, normalized size = 4.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*csc(b*x+a))^(5/2)/(c*sec(b*x+a))^(5/2),x)
```

```
[Out] 1/40/b*2^(1/2)*(4*2^(1/2)*cos(b*x+a)^6+3*cos(b*x+a)*(-(-1+cos(b*x+a)-sin(b*
x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+
cos(b*x+a))/sin(b*x+a))^(1/2)*EllipticF((-(-1+cos(b*x+a)-sin(b*x+a))/sin(b*
x+a))^(1/2),1/2*2^(1/2))-6*cos(b*x+a)*(-(-1+cos(b*x+a)-sin(b*x+a))/sin(b*x+
a))^(1/2)*((-1+cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/si
n(b*x+a))^(1/2)*EllipticE((-(-1+cos(b*x+a)-sin(b*x+a))/sin(b*x+a))^(1/2),1/
2*2^(1/2))-6*2^(1/2)*cos(b*x+a)^4+3*(-(-1+cos(b*x+a)-sin(b*x+a))/sin(b*x+a)
)^(1/2)*((-1+cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(
```

```
b*x+a))^(1/2)*EllipticF((-(-1+cos(b*x+a)-sin(b*x+a))/sin(b*x+a))^(1/2),1/2*
2^(1/2))-6*(-(-1+cos(b*x+a)-sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a)+s
in(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*EllipticE((
-(-1+cos(b*x+a)-sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))-cos(b*x+a)^2*2^(
1/2)+3*cos(b*x+a)*2^(1/2))/cos(b*x+a)^3/sin(b*x+a)^3/(d/sin(b*x+a))^(5/2)/(
c/cos(b*x+a))^(5/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(d\csc\left(bx+a\right)\right)^{\frac{5}{2}}\left(c\sec\left(bx+a\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*csc(b*x+a))^(5/2)/(c*sec(b*x+a))^(5/2),x, algorithm="maxima"
)
```

```
[Out] integrate(1/((d*csc(b*x + a))^(5/2)*(c*sec(b*x + a))^(5/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{d\csc\left(bx+a\right)}\sqrt{c\sec\left(bx+a\right)}}{c^{3}d^{3}\csc\left(bx+a\right)^{3}\sec\left(bx+a\right)^{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*csc(b*x+a))^(5/2)/(c*sec(b*x+a))^(5/2),x, algorithm="fricas"
)
```

```
[Out] integral(sqrt(d*csc(b*x + a))*sqrt(c*sec(b*x + a))/(c^3*d^3*csc(b*x + a)^3*
sec(b*x + a)^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*csc(b*x+a))**(5/2)/(c*sec(b*x+a))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(d\csc\left(bx+a\right)\right)^{\frac{5}{2}}\left(c\sec\left(bx+a\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*csc(b*x+a))^(5/2)/(c*sec(b*x+a))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(1/((d*csc(b*x + a))^(5/2)*(c*sec(b*x + a))^(5/2)), x)
```

3.279 $\int \frac{1}{(d\csc(a+bx))^{7/2}(c\sec(a+bx))^{5/2}}\,dx$

Optimal. Leaf size=406

$$-\frac{5\tan^{-1}\left(1-\sqrt{2}\sqrt{\tan(a+bx)}\right)\sqrt{\tan(a+bx)}\sqrt{d\csc(a+bx)}}{128\sqrt{2}bc^2d^4\sqrt{c\sec(a+bx)}}+\frac{5\tan^{-1}\left(\sqrt{2}\sqrt{\tan(a+bx)}+1\right)\sqrt{\tan(a+bx)}\sqrt{d\csc(a+bx)}}{128\sqrt{2}bc^2d^4\sqrt{c\sec(a+bx)}}$$

```
[Out] -c/(6*b*d*(d*Csc[a + b*x])^(5/2)*(c*Sec[a + b*x])^(7/2)) - (5*c)/(48*b*d^3*
Sqrt[d*Csc[a + b*x]]*(c*Sec[a + b*x])^(7/2)) + 5/(192*b*c*d^3*Sqrt[d*Csc[a
+ b*x]]*(c*Sec[a + b*x])^(3/2)) - (5*ArcTan[1 - Sqrt[2]*Sqrt[Tan[a + b*x]]]
*Sqrt[d*Csc[a + b*x]]*Sqrt[Tan[a + b*x]])/(128*Sqrt[2]*b*c^2*d^4*Sqrt[c*Sec
[a + b*x]]) + (5*ArcTan[1 + Sqrt[2]*Sqrt[Tan[a + b*x]]]*Sqrt[d*Csc[a + b*x]
]*Sqrt[Tan[a + b*x]])/(128*Sqrt[2]*b*c^2*d^4*Sqrt[c*Sec[a + b*x]]) - (5*Sqr
t[d*Csc[a + b*x]]*Log[1 - Sqrt[2]*Sqrt[Tan[a + b*x]] + Tan[a + b*x]]*Sqrt[T
an[a + b*x]])/(256*Sqrt[2]*b*c^2*d^4*Sqrt[c*Sec[a + b*x]]) + (5*Sqrt[d*Csc[
a + b*x]]*Log[1 + Sqrt[2]*Sqrt[Tan[a + b*x]] + Tan[a + b*x]]*Sqrt[Tan[a + b
*x]])/(256*Sqrt[2]*b*c^2*d^4*Sqrt[c*Sec[a + b*x]])
```

Rubi [A] time = 0.389599, antiderivative size = 406, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 11, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.44, Rules used = {2627, 2628, 2629, 3476, 329, 211, 1165, 628, 1162, 617, 204}

$$-\frac{5\tan^{-1}\left(1-\sqrt{2}\sqrt{\tan(a+bx)}\right)\sqrt{\tan(a+bx)}\sqrt{d\csc(a+bx)}}{128\sqrt{2}bc^2d^4\sqrt{c\sec(a+bx)}}+\frac{5\tan^{-1}\left(\sqrt{2}\sqrt{\tan(a+bx)}+1\right)\sqrt{\tan(a+bx)}\sqrt{d\csc(a+bx)}}{128\sqrt{2}bc^2d^4\sqrt{c\sec(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d*Csc[a + b*x])^(7/2)*(c*Sec[a + b*x])^(5/2)),x]
```


```
[Out] -c/(6*b*d*(d*Csc[a + b*x])^(5/2)*(c*Sec[a + b*x])^(7/2)) - (5*c)/(48*b*d^3*
Sqrt[d*Csc[a + b*x]]*(c*Sec[a + b*x])^(7/2)) + 5/(192*b*c*d^3*Sqrt[d*Csc[a
+ b*x]]*(c*Sec[a + b*x])^(3/2)) - (5*ArcTan[1 - Sqrt[2]*Sqrt[Tan[a + b*x]]]
*Sqrt[d*Csc[a + b*x]]*Sqrt[Tan[a + b*x]])/(128*Sqrt[2]*b*c^2*d^4*Sqrt[c*Sec
[a + b*x]]) + (5*ArcTan[1 + Sqrt[2]*Sqrt[Tan[a + b*x]]]*Sqrt[d*Csc[a + b*x]
]*Sqrt[Tan[a + b*x]])/(128*Sqrt[2]*b*c^2*d^4*Sqrt[c*Sec[a + b*x]]) - (5*Sqr
t[d*Csc[a + b*x]]*Log[1 - Sqrt[2]*Sqrt[Tan[a + b*x]] + Tan[a + b*x]]*Sqrt[T
an[a + b*x]])/(256*Sqrt[2]*b*c^2*d^4*Sqrt[c*Sec[a + b*x]]) + (5*Sqrt[d*Csc[
a + b*x]]*Log[1 + Sqrt[2]*Sqrt[Tan[a + b*x]] + Tan[a + b*x]]*Sqrt[Tan[a + b
*x]])/(256*Sqrt[2]*b*c^2*d^4*Sqrt[c*Sec[a + b*x]])
```


Rule 2627

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Simp[(b*(a*Csc[e + f*x])^(m + 1)*(b*Sec[e + f*x])^(n - 1)
)/(a*f*(m + n)), x] + Dist[(m + 1)/(a^2*(m + n)), Int[(a*Csc[e + f*x])^(m +
 2)*(b*Sec[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -1] &
& NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2628

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> -Simp[(a*(a*Csc[e + f*x])^(m - 1)*(b*Sec[e + f*x])^(n + 1
))/(b*f*(m + n)), x] + Dist[(n + 1)/(b^2*(m + n)), Int[(a*Csc[e + f*x])^m*(
b*Sec[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && LtQ[n, -1]
&& NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2629

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Dist[((a*Csc[e + f*x])^m*(b*Sec[e + f*x])^n)/Tan[e + f*x]^
n, Int[Tan[e + f*x]^n, x], x] /; FreeQ[{a, b, e, f, m, n}, x] &&  !IntegerQ
[n] && EqQ[m + n, 0]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
```

`a, 0] || LtQ[b, 0])`

Rubi steps

$$\int \frac{1}{(d\csc(a+bx))^{7/2}(c\sec(a+bx))^{5/2}}\,dx = -\frac{c}{6bd(d\csc(a+bx))^{5/2}(c\sec(a+bx))^{7/2}} + \frac{5\int \frac{1}{(d\csc(a+bx))^{3/2}(c\sec(a+bx))^{5/2}}\,dx}{12d^2}$$

$$= -\frac{c}{6bd(d\csc(a+bx))^{5/2}(c\sec(a+bx))^{7/2}} - \frac{5c}{48bd^3\sqrt{d\csc(a+bx)}(c\sec(a+bx)}$$

$$= -\frac{c}{6bd(d\csc(a+bx))^{5/2}(c\sec(a+bx))^{7/2}} - \frac{5c}{48bd^3\sqrt{d\csc(a+bx)}(c\sec(a+bx)}$$

$$= -\frac{c}{6bd(d\csc(a+bx))^{5/2}(c\sec(a+bx))^{7/2}} - \frac{5c}{48bd^3\sqrt{d\csc(a+bx)}(c\sec(a+bx)}$$

$$= -\frac{c}{6bd(d\csc(a+bx))^{5/2}(c\sec(a+bx))^{7/2}} - \frac{5c}{48bd^3\sqrt{d\csc(a+bx)}(c\sec(a+bx)}$$

$$= -\frac{c}{6bd(d\csc(a+bx))^{5/2}(c\sec(a+bx))^{7/2}} - \frac{5c}{48bd^3\sqrt{d\csc(a+bx)}(c\sec(a+bx)}$$

$$= -\frac{c}{6bd(d\csc(a+bx))^{5/2}(c\sec(a+bx))^{7/2}} - \frac{5c}{48bd^3\sqrt{d\csc(a+bx)}(c\sec(a+bx)}$$

$$= -\frac{c}{6bd(d\csc(a+bx))^{5/2}(c\sec(a+bx))^{7/2}} - \frac{5c}{48bd^3\sqrt{d\csc(a+bx)}(c\sec(a+bx)}$$

$$= -\frac{c}{6bd(d\csc(a+bx))^{5/2}(c\sec(a+bx))^{7/2}} - \frac{5c}{48bd^3\sqrt{d\csc(a+bx)}(c\sec(a+bx)}$$

$$= -\frac{c}{6bd(d\csc(a+bx))^{5/2}(c\sec(a+bx))^{7/2}} - \frac{5c}{48bd^3\sqrt{d\csc(a+bx)}(c\sec(a+bx)}$$

Mathematica [C] time = 0.411069, size = 95, normalized size = 0.23

$$-\frac{\cot(a+bx)\sqrt{d\csc(a+bx)}\left(10\text{Hypergeometric2F1}\left(\frac{3}{4},1,\frac{7}{4},-\cot^2(a+bx)\right)+3\cos(2(a+bx))-9\cos(4(a+bx))+2}{384bc^2d^4\sqrt{c\sec(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d*Csc[a + b*x])^(7/2)*(c*Sec[a + b*x])^(5/2)),x]
```

```
[Out] -(Cot[a + b*x]*Sqrt[d*Csc[a + b*x]]*(4 + 3*Cos[2*(a + b*x)] - 9*Cos[4*(a +
b*x)] + 2*Cos[6*(a + b*x)] + 10*Hypergeometric2F1[3/4, 1, 7/4, -Cot[a + b*x
]^2]))/(384*b*c^2*d^4*Sqrt[c*Sec[a + b*x]])
```

Maple [C] time = 0.221, size = 722, normalized size = 1.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*csc(b*x+a))^(7/2)/(c*sec(b*x+a))^(5/2),x)
```

```
[Out] 1/768/b*2^(1/2)*(64*2^(1/2)*cos(b*x+a)^7-64*2^(1/2)*cos(b*x+a)^6-104*2^(1/2
)*cos(b*x+a)^5+15*I*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a)+sin(
b*x+a))/sin(b*x+a))^(1/2)*(-(-1+cos(b*x+a)-sin(b*x+a))/sin(b*x+a))^(1/2)*El
lipticPi((-(-1+cos(b*x+a)-sin(b*x+a))/sin(b*x+a))^(1/2),1/2+1/2*I,1/2*2^(1/
2))*sin(b*x+a)-15*I*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a)+sin(
b*x+a))/sin(b*x+a))^(1/2)*(-(-1+cos(b*x+a)-sin(b*x+a))/sin(b*x+a))^(1/2)*El
lipticPi((-(-1+cos(b*x+a)-sin(b*x+a))/sin(b*x+a))^(1/2),1/2-1/2*I,1/2*2^(1/
2))*sin(b*x+a)+104*2^(1/2)*cos(b*x+a)^4-15*sin(b*x+a)*(-(-1+cos(b*x+a)-sin(
b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-
1+cos(b*x+a))/sin(b*x+a))^(1/2)*EllipticPi((-(-1+cos(b*x+a)-sin(b*x+a))/sin
(b*x+a))^(1/2),1/2+1/2*I,1/2*2^(1/2))-15*sin(b*x+a)*EllipticPi((-(-1+cos(b*
x+a)-sin(b*x+a))/sin(b*x+a))^(1/2),1/2-1/2*I,1/2*2^(1/2))*(-(-1+cos(b*x+a)-
sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)
*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)+30*sin(b*x+a)*(-(-1+cos(b*x+a)-sin(b*x+
a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+co
s(b*x+a))/sin(b*x+a))^(1/2)*EllipticF((-(-1+cos(b*x+a)-sin(b*x+a))/sin(b*x+
a))^(1/2),1/2*2^(1/2))+10*cos(b*x+a)^3*2^(1/2)-10*cos(b*x+a)^2*2^(1/2))/(-1
+cos(b*x+a))/cos(b*x+a)^3/sin(b*x+a)^3/(d/sin(b*x+a))^(7/2)/(c/cos(b*x+a))^
(5/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(d\csc(bx+a))^{\frac{7}{2}}(c\sec(bx+a))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*csc(b*x+a))^(7/2)/(c*sec(b*x+a))^(5/2),x, algorithm="maxima"
)
```

```
[Out] integrate(1/((d*csc(b*x + a))^(7/2)*(c*sec(b*x + a))^(5/2)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*csc(b*x+a))^(7/2)/(c*sec(b*x+a))^(5/2),x, algorithm="fricas"
)
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*csc(b*x+a))**(7/2)/(c*sec(b*x+a))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(d\csc(bx+a))^{\frac{7}{2}}(c\sec(bx+a))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*csc(b*x+a))^(7/2)/(c*sec(b*x+a))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(1/((d*csc(b*x + a))^(7/2)*(c*sec(b*x + a))^(5/2)), x)
```

3.280 $\int \csc^n(e+fx)\sec^m(e+fx)\,dx$

Optimal. Leaf size=81

$$\frac{\cos^2(e+fx)^{\frac{m+1}{2}}\sec^{m+1}(e+fx)\csc^{n-1}(e+fx)\text{Hypergeometric2F1}\left(\frac{m+1}{2},\frac{1-n}{2},\frac{3-n}{2},\sin^2(e+fx)\right)}{f(1-n)}$$

```
[Out] ((Cos[e + f*x]^2)^((1 + m)/2)*Csc[e + f*x]^(-1 + n)*Hypergeometric2F1[(1 +
m)/2, (1 - n)/2, (3 - n)/2, Sin[e + f*x]^2]*Sec[e + f*x]^(1 + m))/(f*(1 - n
))
```

Rubi [A] time = 0.073849, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2631, 2577}

$$\frac{\cos^2(e+fx)^{\frac{m+1}{2}}\sec^{m+1}(e+fx)\csc^{n-1}(e+fx)\,{}_2F_1\left(\frac{m+1}{2},\frac{1-n}{2};\frac{3-n}{2};\sin^2(e+fx)\right)}{f(1-n)}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^n*Sec[e + f*x]^m,x]
```

```
[Out] ((Cos[e + f*x]^2)^((1 + m)/2)*Csc[e + f*x]^(-1 + n)*Hypergeometric2F1[(1 +
m)/2, (1 - n)/2, (3 - n)/2, Sin[e + f*x]^2]*Sec[e + f*x]^(1 + m))/(f*(1 - n
))
```

Rule 2631

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Dist[(a^2*(a*Csc[e + f*x])^(m - 1)*(b*Sec[e + f*x])^(n + 1
)*(a*Sin[e + f*x])^(m - 1)*(b*Cos[e + f*x])^(n + 1))/b^2, Int[1/((a*Sin[e +
 f*x])^m*(b*Cos[e + f*x])^n), x], x] /; FreeQ[{a, b, e, f, m, n}, x] &&  !S
implerQ[-m, -n]
```

Rule 2577

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Fra
cPart[(n - 1)/2])*(a*Sin[e + f*x])^(m + 1)*Hypergeometric2F1[(1 + m)/2, (1
- n)/2, (3 + m)/2, Sin[e + f*x]^2])/(a*f*(m + 1)*(Cos[e + f*x]^2)^FracPart[
(n - 1)/2]), x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rubi steps

$$\int \csc^n(e+fx)\sec^m(e+fx)\,dx = \left(\cos^{1+m}(e+fx)\csc^{-1+n}(e+fx)\sec^{1+m}(e+fx)\sin^{-1+n}(e+fx)\right)\int \cos^{-m}(e+fx)$$

$$= \frac{\cos^2(e+fx)^{\frac{1+m}{2}}\csc^{-1+n}(e+fx)\,{}_2F_1\left(\frac{1+m}{2},\frac{1-n}{2};\frac{3-n}{2};\sin^2(e+fx)\right)\sec^{1+m}(e+fx)}{f(1-n)}$$

Mathematica [C] time = 2.24891, size = 278, normalized size = 3.43

$$-\frac{(n-3)\sec^m(e+fx)\csc^{n-1}(e+}{f(n-1)\left((n-3)F_1\left(\frac{1}{2}-\frac{n}{2};m,-m-n+1;\frac{3}{2}-\frac{n}{2};\tan^2\left(\frac{1}{2}(e+fx)\right),-\tan^2\left(\frac{1}{2}(e+fx)\right)\right)-2\tan^2\left(\frac{1}{2}(e+fx)\right)\left((m+\right.\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Csc[e + f*x]^n*Sec[e + f*x]^m,x]
```

```
[Out] -(((-3 + n)*AppellF1[1/2 - n/2, m, 1 - m - n, 3/2 - n/2, Tan[(e + f*x)/2]^2
, -Tan[(e + f*x)/2]^2]*Csc[e + f*x]^(-1 + n)*Sec[e + f*x]^m)/(f*(-1 + n)*((
-3 + n)*AppellF1[1/2 - n/2, m, 1 - m - n, 3/2 - n/2, Tan[(e + f*x)/2]^2, -T
an[(e + f*x)/2]^2] - 2*((-1 + m + n)*AppellF1[3/2 - n/2, m, 2 - m - n, 5/2
- n/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + m*AppellF1[3/2 - n/2, 1 +
 m, 1 - m - n, 5/2 - n/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e
+ f*x)/2]^2)))
```

Maple [F] time = 0.565, size = 0, normalized size = 0.

$$\int \left(\csc \left(fx+e\right)\right)^{n} \left(\sec \left(fx+e\right)\right)^{m}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^n*sec(f*x+e)^m,x)
```

```
[Out] int(csc(f*x+e)^n*sec(f*x+e)^m,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \csc \left(fx+e\right)^{n} \sec \left(fx+e\right)^{m}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^n*sec(f*x+e)^m,x, algorithm="maxima")
```

```
[Out] integrate(csc(f*x + e)^n*sec(f*x + e)^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\csc \left(fx+e\right)^{n} \sec \left(fx+e\right)^{m}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^n*sec(f*x+e)^m,x, algorithm="fricas")
```

```
[Out] integral(csc(f*x + e)^n*sec(f*x + e)^m, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \csc^{n}\left(e+fx\right) \sec^{m}\left(e+fx\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**n*sec(f*x+e)**m,x)
```

```
[Out] Integral(csc(e + f*x)**n*sec(e + f*x)**m, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \csc\left(fx+e\right)^{n} \sec\left(fx+e\right)^{m} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^n*sec(f*x+e)^m,x, algorithm="giac")
```

```
[Out] integrate(csc(f*x + e)^n*sec(f*x + e)^m, x)
```

3.281 $\int \csc^n(e+fx)(a\sec(e+fx))^m\,dx$

Optimal. Leaf size=86

$$\frac{\cos^2(e+fx)^{\frac{m+1}{2}}\csc^{n-1}(e+fx)(a\sec(e+fx))^{m+1}\text{Hypergeometric2F1}\left(\frac{m+1}{2},\frac{1-n}{2},\frac{3-n}{2},\sin^2(e+fx)\right)}{af(1-n)}$$

```
[Out] ((Cos[e + f*x]^2)^((1 + m)/2)*Csc[e + f*x]^(-1 + n)*Hypergeometric2F1[(1 +
m)/2, (1 - n)/2, (3 - n)/2, Sin[e + f*x]^2]*(a*Sec[e + f*x])^(1 + m))/(a*f*
(1 - n))
```

Rubi [A] time = 0.0864896, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2631, 2577}

$$\frac{\cos^2(e+fx)^{\frac{m+1}{2}}\csc^{n-1}(e+fx)(a\sec(e+fx))^{m+1}\,{}_2F_1\left(\frac{m+1}{2},\frac{1-n}{2};\frac{3-n}{2};\sin^2(e+fx)\right)}{af(1-n)}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^n*(a*Sec[e + f*x])^m,x]
```

```
[Out] ((Cos[e + f*x]^2)^((1 + m)/2)*Csc[e + f*x]^(-1 + n)*Hypergeometric2F1[(1 +
m)/2, (1 - n)/2, (3 - n)/2, Sin[e + f*x]^2]*(a*Sec[e + f*x])^(1 + m))/(a*f*
(1 - n))
```

Rule 2631

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Dist[(a^2*(a*Csc[e + f*x])^(m - 1)*(b*Sec[e + f*x])^(n + 1
)*(a*Sin[e + f*x])^(m - 1)*(b*Cos[e + f*x])^(n + 1))/b^2, Int[1/((a*Sin[e +
 f*x])^m*(b*Cos[e + f*x])^n), x], x] /; FreeQ[{a, b, e, f, m, n}, x] &&  !S
implerQ[-m, -n]
```

Rule 2577

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Fra
cPart[(n - 1)/2])*(a*Sin[e + f*x])^(m + 1)*Hypergeometric2F1[(1 + m)/2, (1
- n)/2, (3 + m)/2, Sin[e + f*x]^2])/(a*f*(m + 1)*(Cos[e + f*x]^2)^FracPart[
(n - 1)/2]), x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rubi steps

$$\int \csc^n(e+fx)(a\sec(e+fx))^m\,dx = \frac{\left((a\cos(e+fx))^{1+m}\csc^{-1+n}(e+fx)(a\sec(e+fx))^{1+m}\sin^{-1+n}(e+fx)\right)\int(a\cos(e+}{a^2}$$

$$= \frac{\cos^2(e+fx)^{\frac{1+m}{2}}\csc^{-1+n}(e+fx)\,{}_2F_1\left(\frac{1+m}{2},\frac{1-n}{2};\frac{3-n}{2};\sin^2(e+fx)\right)(a\sec(e+fx))^{1+}}{af(1-n)}$$

Mathematica [C] time = 0.549284, size = 280, normalized size = 3.26

$$-\frac{(n-3)\csc^{n-1}(e+fx)(a\sec(e+fx))}{f(n-1)\left((n-3)F_1\left(\frac{1}{2}-\frac{n}{2};m,-m-n+1;\frac{3}{2}-\frac{n}{2};\tan^2\left(\frac{1}{2}(e+fx)\right),-\tan^2\left(\frac{1}{2}(e+fx)\right)\right)-2\tan^2\left(\frac{1}{2}(e+fx)\right)\left((m+n-\right.\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Csc[e + f*x]^n*(a*Sec[e + f*x])^m,x]
```

```
[Out] -(((-3 + n)*AppellF1[1/2 - n/2, m, 1 - m - n, 3/2 - n/2, Tan[(e + f*x)/2]^2
, -Tan[(e + f*x)/2]^2]*Csc[e + f*x]^(-1 + n)*(a*Sec[e + f*x])^m)/(f*(-1 + n
)*((-3 + n)*AppellF1[1/2 - n/2, m, 1 - m - n, 3/2 - n/2, Tan[(e + f*x)/2]^2
, -Tan[(e + f*x)/2]^2] - 2*((-1 + m + n)*AppellF1[3/2 - n/2, m, 2 - m - n,
5/2 - n/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + m*AppellF1[3/2 - n/2,
 1 + m, 1 - m - n, 5/2 - n/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan
[(e + f*x)/2]^2)))
```

Maple [F] time = 0.509, size = 0, normalized size = 0.

$$\int \left(\csc\left(fx+e\right)\right)^{n} \left(a\sec\left(fx+e\right)\right)^{m}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^n*(a*sec(f*x+e))^m,x)
```

```
[Out] int(csc(f*x+e)^n*(a*sec(f*x+e))^m,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\sec\left(fx+e\right)\right)^{m} \csc\left(fx+e\right)^{n}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^n*(a*sec(f*x+e))^m,x, algorithm="maxima")
```

```
[Out] integrate((a*sec(f*x + e))^m*csc(f*x + e)^n, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(a\sec\left(fx+e\right)\right)^{m} \csc\left(fx+e\right)^{n}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^n*(a*sec(f*x+e))^m,x, algorithm="fricas")
```

```
[Out] integral((a*sec(f*x + e))^m*csc(f*x + e)^n, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\sec\left(e+fx\right)\right)^{m} \csc^{n}\left(e+fx\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**n*(a*sec(f*x+e))**m,x)
```

```
[Out] Integral((a*sec(e + f*x))**m*csc(e + f*x)**n, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\sec\left(fx+e\right)\right)^{m}\csc\left(fx+e\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^n*(a*sec(f*x+e))^m,x, algorithm="giac")
```

```
[Out] integrate((a*sec(f*x + e))^m*csc(f*x + e)^n, x)
```

3.282 $\int (b\csc(e+fx))^n \sec^m(e+fx)\,dx$

Optimal. Leaf size=84

$$\frac{b\cos^2(e+fx)^{\frac{m+1}{2}}\sec^{m+1}(e+fx)(b\csc(e+fx))^{n-1}\text{Hypergeometric2F1}\left(\frac{m+1}{2},\frac{1-n}{2},\frac{3-n}{2},\sin^2(e+fx)\right)}{f(1-n)}$$

```
[Out] (b*(Cos[e + f*x]^2)^((1 + m)/2)*(b*Csc[e + f*x])^(-1 + n)*Hypergeometric2F1
[(1 + m)/2, (1 - n)/2, (3 - n)/2, Sin[e + f*x]^2]*Sec[e + f*x]^(1 + m))/(f*
(1 - n))
```

Rubi [A] time = 0.0868945, antiderivative size = 84, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2631, 2577}

$$\frac{b\cos^2(e+fx)^{\frac{m+1}{2}}\sec^{m+1}(e+fx)(b\csc(e+fx))^{n-1}\,{}_2F_1\left(\frac{m+1}{2},\frac{1-n}{2};\frac{3-n}{2};\sin^2(e+fx)\right)}{f(1-n)}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Csc[e + f*x])^n*Sec[e + f*x]^m,x]
```

```
[Out] (b*(Cos[e + f*x]^2)^((1 + m)/2)*(b*Csc[e + f*x])^(-1 + n)*Hypergeometric2F1
[(1 + m)/2, (1 - n)/2, (3 - n)/2, Sin[e + f*x]^2]*Sec[e + f*x]^(1 + m))/(f*
(1 - n))
```

Rule 2631

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Dist[(a^2*(a*Csc[e + f*x])^(m - 1)*(b*Sec[e + f*x])^(n + 1
)*(a*Sin[e + f*x])^(m - 1)*(b*Cos[e + f*x])^(n + 1))/b^2, Int[1/((a*Sin[e +
 f*x])^m*(b*Cos[e + f*x])^n), x], x] /; FreeQ[{a, b, e, f, m, n}, x] &&  !S
implerQ[-m, -n]
```

Rule 2577

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Fra
cPart[(n - 1)/2])*(a*Sin[e + f*x])^(m + 1)*Hypergeometric2F1[(1 + m)/2, (1
- n)/2, (3 + m)/2, Sin[e + f*x]^2])/(a*f*(m + 1)*(Cos[e + f*x]^2)^FracPart[
(n - 1)/2]), x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rubi steps

$$\int (b\csc(e+fx))^n\sec^m(e+fx)\,dx = \left(b^2\cos^{1+m}(e+fx)(b\csc(e+fx))^{-1+n}\sec^{1+m}(e+fx)(b\sin(e+fx))^{-1+n}\right)\int \cos$$

$$= \frac{b\cos^2(e+fx)^{\frac{1+m}{2}}(b\csc(e+fx))^{-1+n}\,{}_2F_1\left(\frac{1+m}{2},\frac{1-n}{2};\frac{3-n}{2};\sin^2(e+fx)\right)\sec^{1+m}(e}{f(1-n)}$$

Mathematica [C] time = 0.311531, size = 281, normalized size = 3.35

$$-\frac{b(n-3)\sec^m(e+fx)(b\csc(e+fx))^{n-}}{f(n-1)\left((n-3)F_1\left(\frac{1-n}{2};m,-m-n+1;\frac{3-n}{2};\tan^2\left(\frac{1}{2}(e+fx)\right),-\tan^2\left(\frac{1}{2}(e+fx)\right)\right)-2\tan^2\left(\frac{1}{2}(e+fx)\right)\left((m+n-1\right.\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(b*Csc[e + f*x])^n*Sec[e + f*x]^m,x]
```

```
[Out] -((b*(-3 + n)*AppellF1[(1 - n)/2, m, 1 - m - n, (3 - n)/2, Tan[(e + f*x)/2]
^2, -Tan[(e + f*x)/2]^2]*(b*Csc[e + f*x])^(-1 + n)*Sec[e + f*x]^m)/(f*(-1 +
 n)*((-3 + n)*AppellF1[(1 - n)/2, m, 1 - m - n, (3 - n)/2, Tan[(e + f*x)/2]
^2, -Tan[(e + f*x)/2]^2] - 2*((-1 + m + n)*AppellF1[(3 - n)/2, m, 2 - m - n
, (5 - n)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + m*AppellF1[(3 - n)/
2, 1 + m, 1 - m - n, (5 - n)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*T
an[(e + f*x)/2]^2)))
```

Maple [F] time = 0.529, size = 0, normalized size = 0.

$$\int \left(b\csc\left(fx+e\right)\right)^{n}\left(\sec\left(fx+e\right)\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*csc(f*x+e))^n*sec(f*x+e)^m,x)
```

```
[Out] int((b*csc(f*x+e))^n*sec(f*x+e)^m,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\csc\left(fx+e\right)\right)^{n}\sec\left(fx+e\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*csc(f*x+e))^n*sec(f*x+e)^m,x, algorithm="maxima")
```

```
[Out] integrate((b*csc(f*x + e))^n*sec(f*x + e)^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(b\csc\left(fx+e\right)\right)^{n}\sec\left(fx+e\right)^{m},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*csc(f*x+e))^n*sec(f*x+e)^m,x, algorithm="fricas")
```

```
[Out] integral((b*csc(f*x + e))^n*sec(f*x + e)^m, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\csc\left(e+fx\right)\right)^{n}\sec^{m}\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*csc(f*x+e))**n*sec(f*x+e)**m,x)
```

```
[Out] Integral((b*csc(e + f*x))**n*sec(e + f*x)**m, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\csc\left(fx+e\right)\right)^{n}\sec\left(fx+e\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*csc(f*x+e))^n*sec(f*x+e)^m,x, algorithm="giac")
```

```
[Out] integrate((b*csc(f*x + e))^n*sec(f*x + e)^m, x)
```

3.283 $\int (b\csc(e+fx))^n (a\sec(e+fx))^m\,dx$

Optimal. Leaf size=89

$$\frac{b\cos^2(e+fx)^{\frac{m+1}{2}}(a\sec(e+fx))^{m+1}(b\csc(e+fx))^{n-1}\text{Hypergeometric2F1}\left(\frac{m+1}{2},\frac{1-n}{2},\frac{3-n}{2},\sin^2(e+fx)\right)}{af(1-n)}$$

```
[Out] (b*(Cos[e + f*x]^2)^((1 + m)/2)*(b*Csc[e + f*x])^(-1 + n)*Hypergeometric2F1
[(1 + m)/2, (1 - n)/2, (3 - n)/2, Sin[e + f*x]^2]*(a*Sec[e + f*x])^(1 + m))
/(a*f*(1 - n))
```

Rubi [A] time = 0.0942339, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2631, 2577}

$$\frac{b\cos^2(e+fx)^{\frac{m+1}{2}}(a\sec(e+fx))^{m+1}(b\csc(e+fx))^{n-1}\,{}_2F_1\left(\frac{m+1}{2},\frac{1-n}{2};\frac{3-n}{2};\sin^2(e+fx)\right)}{af(1-n)}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Csc[e + f*x])^n*(a*Sec[e + f*x])^m,x]
```

```
[Out] (b*(Cos[e + f*x]^2)^((1 + m)/2)*(b*Csc[e + f*x])^(-1 + n)*Hypergeometric2F1
[(1 + m)/2, (1 - n)/2, (3 - n)/2, Sin[e + f*x]^2]*(a*Sec[e + f*x])^(1 + m))
/(a*f*(1 - n))
```

Rule 2631

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Dist[(a^2*(a*Csc[e + f*x])^(m - 1)*(b*Sec[e + f*x])^(n + 1
)*(a*Sin[e + f*x])^(m - 1)*(b*Cos[e + f*x])^(n + 1))/b^2, Int[1/((a*Sin[e +
 f*x])^m*(b*Cos[e + f*x])^n), x], x] /; FreeQ[{a, b, e, f, m, n}, x] &&  !S
implerQ[-m, -n]
```

Rule 2577

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Fra
cPart[(n - 1)/2])*(a*Sin[e + f*x])^(m + 1)*Hypergeometric2F1[(1 + m)/2, (1
- n)/2, (3 + m)/2, Sin[e + f*x]^2])/(a*f*(m + 1)*(Cos[e + f*x]^2)^FracPart[
(n - 1)/2]), x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rubi steps

$$\int (b\csc(e+fx))^n(a\sec(e+fx))^m\,dx = \frac{\left(b^2(a\cos(e+fx))^{1+m}(b\csc(e+fx))^{-1+n}(a\sec(e+fx))^{1+m}(b\sin(e+fx))^{-1+n}\right)\int}{a^2}$$

$$= \frac{b\cos^2(e+fx)^{\frac{1+m}{2}}(b\csc(e+fx))^{-1+n}\,{}_2F_1\left(\frac{1+m}{2},\frac{1-n}{2};\frac{3-n}{2};\sin^2(e+fx)\right)(a\sec(e+}{af(1-n)}$$

Mathematica [C] time = 0.187567, size = 283, normalized size = 3.18

$$-\frac{b(n-3)(a\sec(e+fx))^m(b\csc(e+fx))^{n-1}I}{f(n-1)\left((n-3)F_1\left(\frac{1-n}{2};m,-m-n+1;\frac{3-n}{2};\tan^2\left(\frac{1}{2}(e+fx)\right),-\tan^2\left(\frac{1}{2}(e+fx)\right)\right)-2\tan^2\left(\frac{1}{2}(e+fx)\right)\left((m+n-1)F\right.\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(b*Csc[e + f*x])^n*(a*Sec[e + f*x])^m,x]
```

```
[Out] -((b*(-3 + n)*AppellF1[(1 - n)/2, m, 1 - m - n, (3 - n)/2, Tan[(e + f*x)/2]
^2, -Tan[(e + f*x)/2]^2]*(b*Csc[e + f*x])^(-1 + n)*(a*Sec[e + f*x])^m)/(f*(
-1 + n)*((-3 + n)*AppellF1[(1 - n)/2, m, 1 - m - n, (3 - n)/2, Tan[(e + f*x
)/2]^2, -Tan[(e + f*x)/2]^2] - 2*((-1 + m + n)*AppellF1[(3 - n)/2, m, 2 - m
 - n, (5 - n)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + m*AppellF1[(3 -
 n)/2, 1 + m, 1 - m - n, (5 - n)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2
])*Tan[(e + f*x)/2]^2)))
```

Maple [F] time = 0.558, size = 0, normalized size = 0.

$$\int \left(b \csc \left(fx+e\right)\right)^{n} \left(a \sec \left(fx+e\right)\right)^{m}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*csc(f*x+e))^n*(a*sec(f*x+e))^m,x)
```

```
[Out] int((b*csc(f*x+e))^n*(a*sec(f*x+e))^m,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b \csc \left(fx+e\right)\right)^{n} \left(a \sec \left(fx+e\right)\right)^{m}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*csc(f*x+e))^n*(a*sec(f*x+e))^m,x, algorithm="maxima")
```

```
[Out] integrate((b*csc(f*x + e))^n*(a*sec(f*x + e))^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\left(b \csc \left(fx+e\right)\right)^{n} \left(a \sec \left(fx+e\right)\right)^{m}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*csc(f*x+e))^n*(a*sec(f*x+e))^m,x, algorithm="fricas")
```

```
[Out] integral((b*csc(f*x + e))^n*(a*sec(f*x + e))^m, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a \sec \left(e+fx\right)\right)^{m} \left(b \csc \left(e+fx\right)\right)^{n}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*csc(f*x+e))**n*(a*sec(f*x+e))**m,x)
```

```
[Out] Integral((a*sec(e + f*x))**m*(b*csc(e + f*x))**n, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\csc\left(fx+e\right)\right)^{n}\left(a\sec\left(fx+e\right)\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*csc(f*x+e))^n*(a*sec(f*x+e))^m,x, algorithm="giac")
```

```
[Out] integrate((b*csc(f*x + e))^n*(a*sec(f*x + e))^m, x)
```

3.284 $\int (b\csc(e+fx))^n \sec^5(e+fx)\,dx$

Optimal. Leaf size=48

$$\frac{(b\csc(e+fx))^{n+5}\text{Hypergeometric2F1}\left(3,\frac{n+5}{2},\frac{n+7}{2},\csc^2(e+fx)\right)}{b^5f(n+5)}$$

```
[Out] ((b*Csc[e + f*x])^(5 + n)*Hypergeometric2F1[3, (5 + n)/2, (7 + n)/2, Csc[e
+ f*x]^2])/(b^5*f*(5 + n))
```

Rubi [A] time = 0.0508996, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2621, 364}

$$\frac{(b\csc(e+fx))^{n+5}\,{}_2F_1\left(3,\frac{n+5}{2};\frac{n+7}{2};\csc^2(e+fx)\right)}{b^5f(n+5)}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Csc[e + f*x])^n*Sec[e + f*x]^5,x]
```

```
[Out] ((b*Csc[e + f*x])^(5 + n)*Hypergeometric2F1[3, (5 + n)/2, (7 + n)/2, Csc[e
+ f*x]^2])/(b^5*f*(5 + n))
```

Rule 2621

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> -Dist[(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] && !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int (b\csc(e+fx))^n\sec^5(e+fx)\,dx &= -\frac{\text{Subst}\left(\int \frac{x^{4+n}}{\left(-1+\frac{x^2}{b^2}\right)^3}\,dx, x, b\csc(e+fx)\right)}{b^5f}\\ &= \frac{(b\csc(e+fx))^{5+n}\,{}_2F_1\left(3,\frac{5+n}{2};\frac{7+n}{2};\csc^2(e+fx)\right)}{b^5f(5+n)}\end{aligned}$$

Mathematica [A] time = 0.0512588, size = 51, normalized size = 1.06

$$-\frac{b(b\csc(e+fx))^{n-1}\text{Hypergeometric2F1}\left(3,\frac{1-n}{2},\frac{3-n}{2},\sin^2(e+fx)\right)}{f(n-1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Csc[e + f*x])^n*Sec[e + f*x]^5,x]
```

```
[Out] -((b*(b*Csc[e + f*x])^(-1 + n)*Hypergeometric2F1[3, (1 - n)/2, (3 - n)/2, S
in[e + f*x]^2])/(f*(-1 + n)))
```

Maple [F] time = 0.338, size = 0, normalized size = 0.

$$\int \left(b\csc\left(fx+e\right)\right)^{n} \left(\sec\left(fx+e\right)\right)^{5}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*csc(f*x+e))^n*sec(f*x+e)^5,x)
```

```
[Out] int((b*csc(f*x+e))^n*sec(f*x+e)^5,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\csc\left(fx+e\right)\right)^{n} \sec\left(fx+e\right)^{5}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*csc(f*x+e))^n*sec(f*x+e)^5,x, algorithm="maxima")
```

```
[Out] integrate((b*csc(f*x + e))^n*sec(f*x + e)^5, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(b\csc\left(fx+e\right)\right)^{n} \sec\left(fx+e\right)^{5}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*csc(f*x+e))^n*sec(f*x+e)^5,x, algorithm="fricas")
```

```
[Out] integral((b*csc(f*x + e))^n*sec(f*x + e)^5, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*csc(f*x+e))**n*sec(f*x+e)**5,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\csc\left(fx+e\right)\right)^{n}\sec\left(fx+e\right)^{5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*csc(f*x+e))^n*sec(f*x+e)^5,x, algorithm="giac")
```

```
[Out] integrate((b*csc(f*x + e))^n*sec(f*x + e)^5, x)
```

3.285 $\int (b\csc(e+fx))^n \sec^3(e+fx)\,dx$

Optimal. Leaf size=49

$$-\frac{(b\csc(e+fx))^{n+3}\text{Hypergeometric2F1}\left(2,\frac{n+3}{2},\frac{n+5}{2},\csc^2(e+fx)\right)}{b^3f(n+3)}$$

```
[Out] -(((b*Csc[e + f*x])^(3 + n)*Hypergeometric2F1[2, (3 + n)/2, (5 + n)/2, Csc[
e + f*x]^2])/(b^3*f*(3 + n)))
```

Rubi [A] time = 0.0510699, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2621, 364}

$$-\frac{(b\csc(e+fx))^{n+3}\,{}_2F_1\left(2,\frac{n+3}{2};\frac{n+5}{2};\csc^2(e+fx)\right)}{b^3f(n+3)}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Csc[e + f*x])^n*Sec[e + f*x]^3,x]
```

```
[Out] -(((b*Csc[e + f*x])^(3 + n)*Hypergeometric2F1[2, (3 + n)/2, (5 + n)/2, Csc[
e + f*x]^2])/(b^3*f*(3 + n)))
```

Rule 2621

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> -Dist[(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int (b\csc(e+fx))^n\sec^3(e+fx)\,dx &= -\frac{\text{Subst}\left(\int \frac{x^{2+n}}{\left(-1+\frac{x^2}{b^2}\right)^2}\,dx, x, b\csc(e+fx)\right)}{b^3f}\\ &= -\frac{(b\csc(e+fx))^{3+n}\,{}_2F_1\left(2,\frac{3+n}{2};\frac{5+n}{2};\csc^2(e+fx)\right)}{b^3f(3+n)}\end{aligned}$$

Mathematica [A] time = 0.0430347, size = 51, normalized size = 1.04

$$-\frac{b(b\csc(e+fx))^{n-1}\text{Hypergeometric2F1}\left(2,\frac{1-n}{2},\frac{3-n}{2},\sin^2(e+fx)\right)}{f(n-1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Csc[e + f*x])^n*Sec[e + f*x]^3,x]
```

```
[Out] -((b*(b*Csc[e + f*x])^(-1 + n)*Hypergeometric2F1[2, (1 - n)/2, (3 - n)/2, S
in[e + f*x]^2])/(f*(-1 + n)))
```

Maple [F] time = 0.323, size = 0, normalized size = 0.

$$\int \left(b \csc \left(fx+e\right)\right)^{n} \left(\sec \left(fx+e\right)\right)^{3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*csc(f*x+e))^n*sec(f*x+e)^3,x)
```

```
[Out] int((b*csc(f*x+e))^n*sec(f*x+e)^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b \csc \left(fx+e\right)\right)^{n} \sec \left(fx+e\right)^{3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*csc(f*x+e))^n*sec(f*x+e)^3,x, algorithm="maxima")
```

```
[Out] integrate((b*csc(f*x + e))^n*sec(f*x + e)^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(b \csc \left(fx+e\right)\right)^{n} \sec \left(fx+e\right)^{3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*csc(f*x+e))^n*sec(f*x+e)^3,x, algorithm="fricas")
```

```
[Out] integral((b*csc(f*x + e))^n*sec(f*x + e)^3, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*csc(f*x+e))**n*sec(f*x+e)**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\csc\left(fx+e\right)\right)^{n}\sec\left(fx+e\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*csc(f*x+e))^n*sec(f*x+e)^3,x, algorithm="giac")
```

```
[Out] integrate((b*csc(f*x + e))^n*sec(f*x + e)^3, x)
```

3.286 $\int (b\csc(e+fx))^n \sec(e+fx)\,dx$

Optimal. Leaf size=48

$$\frac{(b\csc(e+fx))^{n+1}\text{Hypergeometric2F1}\left(1,\frac{n+1}{2},\frac{n+3}{2},\csc^2(e+fx)\right)}{bf(n+1)}$$

```
[Out] ((b*Csc[e + f*x])^(1 + n)*Hypergeometric2F1[1, (1 + n)/2, (3 + n)/2, Csc[e
+ f*x]^2])/(b*f*(1 + n))
```

Rubi [A] time = 0.0355939, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2621, 364}

$$\frac{(b\csc(e+fx))^{n+1}\,{}_2F_1\left(1,\frac{n+1}{2};\frac{n+3}{2};\csc^2(e+fx)\right)}{bf(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Csc[e + f*x])^n*Sec[e + f*x],x]
```

```
[Out] ((b*Csc[e + f*x])^(1 + n)*Hypergeometric2F1[1, (1 + n)/2, (3 + n)/2, Csc[e
+ f*x]^2])/(b*f*(1 + n))
```

Rule 2621

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> -Dist[(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int (b\csc(e+fx))^n \sec(e+fx)\,dx &= -\frac{\text{Subst}\left(\int \frac{x^n}{-1+\frac{x^2}{b^2}}\,dx,x,b\csc(e+fx)\right)}{bf}\\ &= \frac{(b\csc(e+fx))^{1+n}\,{}_2F_1\left(1,\frac{1+n}{2};\frac{3+n}{2};\csc^2(e+fx)\right)}{bf(1+n)}\end{aligned}$$

Mathematica [A] time = 0.0325293, size = 51, normalized size = 1.06

$$-\frac{b(b\csc(e+fx))^{n-1}\text{Hypergeometric2F1}\left(1,\frac{1-n}{2},\frac{3-n}{2},\sin^2(e+fx)\right)}{f(n-1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Csc[e + f*x])^n*Sec[e + f*x],x]
```

```
[Out] -((b*(b*Csc[e + f*x])^(-1 + n)*Hypergeometric2F1[1, (1 - n)/2, (3 - n)/2, S
in[e + f*x]^2])/(f*(-1 + n)))
```

Maple [F] time = 0.31, size = 0, normalized size = 0.

$$\int \left(b\csc\left(fx+e\right)\right)^{n} \sec\left(fx+e\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*csc(f*x+e))^n*sec(f*x+e),x)
```

```
[Out] int((b*csc(f*x+e))^n*sec(f*x+e),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\csc\left(fx+e\right)\right)^{n} \sec\left(fx+e\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*csc(f*x+e))^n*sec(f*x+e),x, algorithm="maxima")
```

```
[Out] integrate((b*csc(f*x + e))^n*sec(f*x + e), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b\csc\left(fx+e\right)\right)^{n} \sec\left(fx+e\right), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*csc(f*x+e))^n*sec(f*x+e),x, algorithm="fricas")
```

```
[Out] integral((b*csc(f*x + e))^n*sec(f*x + e), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\csc\left(e+fx\right)\right)^{n} \sec\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*csc(f*x+e))**n*sec(f*x+e),x)
```

```
[Out] Integral((b*csc(e + f*x))**n*sec(e + f*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b\csc(fx+e))^n \sec(fx+e)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*csc(f*x+e))^n*sec(f*x+e),x, algorithm="giac")
```

```
[Out] integrate((b*csc(f*x + e))^n*sec(f*x + e), x)
```

3.287 $\int \cos(e+fx)(b\csc(e+fx))^n\,dx$

Optimal. Leaf size=24

$$\frac{b(b\csc(e+fx))^{n-1}}{f(1-n)}$$

```
[Out] (b*(b*Csc[e + f*x])^(-1 + n))/(f*(1 - n))
```

Rubi [A] time = 0.0335722, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2621, 30}

$$\frac{b(b\csc(e+fx))^{n-1}}{f(1-n)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]*(b*Csc[e + f*x])^n,x]
```

```
[Out] (b*(b*Csc[e + f*x])^(-1 + n))/(f*(1 - n))
```

Rule 2621

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> -Dist[(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \cos(e+fx)(b\csc(e+fx))^n\,dx &= -\frac{b\,\text{Subst}\left(\int x^{-2+n}\,dx, x, b\csc(e+fx)\right)}{f}\\ &= \frac{b(b\csc(e+fx))^{-1+n}}{f(1-n)}\end{aligned}$$

Mathematica [A] time = 0.0249491, size = 23, normalized size = 0.96

$$-\frac{b(b\csc(e+fx))^{n-1}}{f(n-1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]*(b*Csc[e + f*x])^n,x]
```

```
[Out] -((b*(b*Csc[e + f*x])^(-1 + n))/(f*(-1 + n)))
```

Maple [B] time = 0.07, size = 66, normalized size = 2.8

$$-2\,{\frac{\tan\left(1/2\,fx+e/2\right)}{f\left(-1+n\right)\left(1+\left(\tan\left(1/2\,fx+e/2\right)\right)^{2}\right)}{{\rm e}^{n\ln\left(1/2\,{\frac{b\left(1+\left(\tan\left(1/2\,fx+e/2\right)\right)^{2}\right)}{\tan\left(1/2\,fx+e/2\right)}}\right)}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)*(b*csc(f*x+e))^n,x)
```

```
[Out] -2/f/(-1+n)*tan(1/2*f*x+1/2*e)*exp(n*ln(1/2*b*(1+tan(1/2*f*x+1/2*e)^2)/tan(
1/2*f*x+1/2*e)))/(1+tan(1/2*f*x+1/2*e)^2)
```

Maxima [A] time = 1.14941, size = 39, normalized size = 1.62

$$-\frac{b^{n}\sin\left(fx+e\right)^{-n}\sin\left(fx+e\right)}{f(n-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(b*csc(f*x+e))^n,x, algorithm="maxima")
```

```
[Out] -b^n*sin(f*x + e)^(-n)*sin(f*x + e)/(f*(n - 1))
```

Fricas [A] time = 1.69949, size = 59, normalized size = 2.46

$$-\frac{\left(\frac{b}{\sin\left(fx+e\right)}\right)^{n}\sin\left(fx+e\right)}{fn-f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(b*csc(f*x+e))^n,x, algorithm="fricas")
```

```
[Out] -(b/sin(f*x + e))^n*sin(f*x + e)/(f*n - f)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\left(b\csc\left(e+fx\right)\right)^{n}\cos\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(b*csc(f*x+e))**n,x)
```

```
[Out] Integral((b*csc(e + f*x))**n*cos(e + f*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\csc\left(fx+e\right)\right)^{n}\cos\left(fx+e\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(b*csc(f*x+e))^n,x, algorithm="giac")
```

```
[Out] integrate((b*csc(f*x + e))^n*cos(f*x + e), x)
```

3.288 $\int \cos^3(e+fx)(b\csc(e+fx))^n\,dx$

Optimal. Leaf size=52

$$\frac{b(b\csc(e+fx))^{n-1}}{f(1-n)}-\frac{b^3(b\csc(e+fx))^{n-3}}{f(3-n)}$$

```
[Out] -((b^3*(b*Csc[e + f*x])^(-3 + n))/(f*(3 - n))) + (b*(b*Csc[e + f*x])^(-1 +
n))/(f*(1 - n))
```

Rubi [A] time = 0.0516861, antiderivative size = 52, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2621, 14}

$$\frac{b(b\csc(e+fx))^{n-1}}{f(1-n)}-\frac{b^3(b\csc(e+fx))^{n-3}}{f(3-n)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^3*(b*Csc[e + f*x])^n,x]
```

```
[Out] -((b^3*(b*Csc[e + f*x])^(-3 + n))/(f*(3 - n))) + (b*(b*Csc[e + f*x])^(-1 +
n))/(f*(1 - n))
```

Rule 2621

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> -Dist[(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \cos^3(e+fx)(b\csc(e+fx))^n\,dx&=-\frac{b^3\,\text{Subst}\left(\int x^{-4+n}\left(-1+\frac{x^2}{b^2}\right)dx,x,b\csc(e+fx)\right)}{f}\\&=-\frac{b^3\,\text{Subst}\left(\int\left(-x^{-4+n}+\frac{x^{-2+n}}{b^2}\right)dx,x,b\csc(e+fx)\right)}{f}\\&=-\frac{b^3(b\csc(e+fx))^{-3+n}}{f(3-n)}+\frac{b(b\csc(e+fx))^{-1+n}}{f(1-n)}\end{aligned}$$

Mathematica [A] time = 0.126489, size = 45, normalized size = 0.87

$$-\frac{b((n-1)\cos(2(e+fx))+n-5)(b\csc(e+fx))^{n-1}}{2f(n-3)(n-1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^3*(b*Csc[e + f*x])^n,x]
```

```
[Out] -(b*(-5 + n + (-1 + n)*Cos[2*(e + f*x)])*(b*Csc[e + f*x])^(-1 + n))/(2*f*(-
3 + n)*(-1 + n))
```

Maple [F] time = 1.153, size = 0, normalized size = 0.

$$\int \left(\cos \left(fx+e\right)\right)^{3} \left(b \csc \left(fx+e\right)\right)^{n}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^3*(b*csc(f*x+e))^n,x)
```

```
[Out] int(cos(f*x+e)^3*(b*csc(f*x+e))^n,x)
```

Maxima [A] time = 1.1554, size = 78, normalized size = 1.5

$$\frac{\frac{b^{n} \sin\left(fx+e\right)^{-n} \sin\left(fx+e\right)^{3}}{n-3} - \frac{b^{n} \sin\left(fx+e\right)^{-n} \sin\left(fx+e\right)}{n-1}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^3*(b*csc(f*x+e))^n,x, algorithm="maxima")
```

```
[Out] (b^n*sin(f*x + e)^(-n)*sin(f*x + e)^3/(n - 3) - b^n*sin(f*x + e)^(-n)*sin(f
*x + e)/(n - 1))/f
```

Fricas [A] time = 1.7469, size = 115, normalized size = 2.21

$$-\frac{\left((n-1)\cos\left(fx+e\right)^{2}-2\right)\left(\frac{b}{\sin\left(fx+e\right)}\right)^{n}\sin\left(fx+e\right)}{fn^{2}-4\,fn+3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^3*(b*csc(f*x+e))^n,x, algorithm="fricas")
```

```
[Out] -((n - 1)*cos(f*x + e)^2 - 2)*(b/sin(f*x + e))^n*sin(f*x + e)/(f*n^2 - 4*f*
n + 3*f)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**3*(b*csc(f*x+e))**n,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\csc\left(fx+e\right)\right)^{n}\cos\left(fx+e\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^3*(b*csc(f*x+e))^n,x, algorithm="giac")
```

```
[Out] integrate((b*csc(f*x + e))^n*cos(f*x + e)^3, x)
```

3.289 $\int \cos^5(e+fx)(b\csc(e+fx))^n\,dx$

Optimal. Leaf size=78

$$\frac{b^5(b\csc(e+fx))^{n-5}}{f(5-n)}-\frac{2b^3(b\csc(e+fx))^{n-3}}{f(3-n)}+\frac{b(b\csc(e+fx))^{n-1}}{f(1-n)}$$

```
[Out] (b^5*(b*Csc[e + f*x])^(-5 + n))/(f*(5 - n)) - (2*b^3*(b*Csc[e + f*x])^(-3 +
 n))/(f*(3 - n)) + (b*(b*Csc[e + f*x])^(-1 + n))/(f*(1 - n))
```

Rubi [A] time = 0.0642744, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2621, 270}

$$\frac{b^5(b\csc(e+fx))^{n-5}}{f(5-n)}-\frac{2b^3(b\csc(e+fx))^{n-3}}{f(3-n)}+\frac{b(b\csc(e+fx))^{n-1}}{f(1-n)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^5*(b*Csc[e + f*x])^n,x]
```

```
[Out] (b^5*(b*Csc[e + f*x])^(-5 + n))/(f*(5 - n)) - (2*b^3*(b*Csc[e + f*x])^(-3 +
 n))/(f*(3 - n)) + (b*(b*Csc[e + f*x])^(-1 + n))/(f*(1 - n))
```

Rule 2621

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> -Dist[(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \cos^5(e+fx)(b\csc(e+fx))^n\,dx &= -\frac{b^5\,\text{Subst}\left(\int x^{-6+n}\left(-1+\frac{x^2}{b^2}\right)^2\,dx,x,b\csc(e+fx)\right)}{f}\\ &= -\frac{b^5\,\text{Subst}\left(\int \left(x^{-6+n}-\frac{2x^{-4+n}}{b^2}+\frac{x^{-2+n}}{b^4}\right)dx,x,b\csc(e+fx)\right)}{f}\\ &= \frac{b^5(b\csc(e+fx))^{-5+n}}{f(5-n)}-\frac{2b^3(b\csc(e+fx))^{-3+n}}{f(3-n)}+\frac{b(b\csc(e+fx))^{-1+n}}{f(1-n)}\end{aligned}$$

Mathematica [A] time = 0.553882, size = 81, normalized size = 1.04

$$-\frac{\sin^5(e+fx)\left(\left(n^2-8n+15\right)\csc^4(e+fx)-2\left(n^2-6n+5\right)\csc^2(e+fx)+n^2-4n+3\right)(b\csc(e+fx))^n}{f(n-5)(n-3)(n-1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^5*(b*Csc[e + f*x])^n,x]
```

```
[Out] -(((b*Csc[e + f*x])^n*(3 - 4*n + n^2 - 2*(5 - 6*n + n^2)*Csc[e + f*x]^2 + (
15 - 8*n + n^2)*Csc[e + f*x]^4)*Sin[e + f*x]^5)/(f*(-5 + n)*(-3 + n)*(-1 +
n)))
```

Maple [F] time = 1.199, size = 0, normalized size = 0.

$$\int \left(\cos \left(fx+e\right)\right)^{5} \left(b \csc \left(fx+e\right)\right)^{n} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^5*(b*csc(f*x+e))^n,x)
```

```
[Out] int(cos(f*x+e)^5*(b*csc(f*x+e))^n,x)
```

Maxima [A] time = 1.14297, size = 116, normalized size = 1.49

$$-\frac{\frac{b^{n} \sin\left(fx+e\right)^{-n} \sin\left(fx+e\right)^{5}}{n-5} - \frac{2\,b^{n} \sin\left(fx+e\right)^{-n} \sin\left(fx+e\right)^{3}}{n-3} + \frac{b^{n} \sin\left(fx+e\right)^{-n} \sin\left(fx+e\right)}{n-1}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^5*(b*csc(f*x+e))^n,x, algorithm="maxima")
```

```
[Out] -(b^n*sin(f*x + e)^(-n)*sin(f*x + e)^5/(n - 5) - 2*b^n*sin(f*x + e)^(-n)*si
n(f*x + e)^3/(n - 3) + b^n*sin(f*x + e)^(-n)*sin(f*x + e)/(n - 1))/f
```

Fricas [A] time = 1.76295, size = 178, normalized size = 2.28

$$-\frac{\left(\left(n^{2}-4\,n+3\right)\cos\left(fx+e\right)^{4}-4\,(n-1)\cos\left(fx+e\right)^{2}+8\right)\left(\frac{b}{\sin\left(fx+e\right)}\right)^{n}\sin\left(fx+e\right)}{fn^{3}-9\,fn^{2}+23\,fn-15\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^5*(b*csc(f*x+e))^n,x, algorithm="fricas")
```

```
[Out] -((n^2 - 4*n + 3)*cos(f*x + e)^4 - 4*(n - 1)*cos(f*x + e)^2 + 8)*(b/sin(f*x
 + e))^n*sin(f*x + e)/(f*n^3 - 9*f*n^2 + 23*f*n - 15*f)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**5*(b*csc(f*x+e))**n,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\csc\left(fx+e\right)\right)^{n}\cos\left(fx+e\right)^{5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^5*(b*csc(f*x+e))^n,x, algorithm="giac")
```

```
[Out] integrate((b*csc(f*x + e))^n*cos(f*x + e)^5, x)
```

3.290 $\int (b\csc(e+fx))^n \sec^6(e+fx)\,dx$

Optimal. Leaf size=72

$$\frac{b\sqrt{\cos^2(e+fx)}\sec(e+fx)(b\csc(e+fx))^{n-1}\text{Hypergeometric2F1}\left(\frac{7}{2},\frac{1-n}{2},\frac{3-n}{2},\sin^2(e+fx)\right)}{f(1-n)}$$

```
[Out] (b*Sqrt[Cos[e + f*x]^2]*(b*Csc[e + f*x])^(-1 + n)*Hypergeometric2F1[7/2, (1
 - n)/2, (3 - n)/2, Sin[e + f*x]^2]*Sec[e + f*x])/(f*(1 - n))
```

Rubi [A] time = 0.0798404, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2631, 2577}

$$\frac{b\sqrt{\cos^2(e+fx)}\sec(e+fx)(b\csc(e+fx))^{n-1}\,{}_2F_1\left(\frac{7}{2},\frac{1-n}{2};\frac{3-n}{2};\sin^2(e+fx)\right)}{f(1-n)}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Csc[e + f*x])^n*Sec[e + f*x]^6,x]
```

```
[Out] (b*Sqrt[Cos[e + f*x]^2]*(b*Csc[e + f*x])^(-1 + n)*Hypergeometric2F1[7/2, (1
 - n)/2, (3 - n)/2, Sin[e + f*x]^2]*Sec[e + f*x])/(f*(1 - n))
```

Rule 2631

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Dist[(a^2*(a*Csc[e + f*x])^(m - 1)*(b*Sec[e + f*x])^(n + 1
)*(a*Sin[e + f*x])^(m - 1)*(b*Cos[e + f*x])^(n + 1))/b^2, Int[1/((a*Sin[e +
 f*x])^m*(b*Cos[e + f*x])^n), x], x] /; FreeQ[{a, b, e, f, m, n}, x] &&  !S
implerQ[-m, -n]
```

Rule 2577

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Fra
cPart[(n - 1)/2])*(a*Sin[e + f*x])^(m + 1)*Hypergeometric2F1[(1 + m)/2, (1
- n)/2, (3 + m)/2, Sin[e + f*x]^2])/(a*f*(m + 1)*(Cos[e + f*x]^2)^FracPart[
(n - 1)/2]), x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rubi steps

$$\begin{aligned}\int (b\csc(e+fx))^n\sec^6(e+fx)\,dx &= \left(b^2(b\csc(e+fx))^{-1+n}(b\sin(e+fx))^{-1+n}\right)\int \sec^6(e+fx)(b\sin(e+fx))^{-n}\,dx\\ &= \frac{b\sqrt{\cos^2(e+fx)}(b\csc(e+fx))^{-1+n}\,{}_2F_1\left(\frac{7}{2},\frac{1-n}{2};\frac{3-n}{2};\sin^2(e+fx)\right)\sec(e+fx)}{f(1-n)}\end{aligned}$$

Mathematica [A] time = 0.576896, size = 77, normalized size = 1.07

$$\frac{\tan(e+fx)\sec^2(e+fx)^{-n/2}(b\csc(e+fx))^n\text{Hypergeometric2F1}\left(-\frac{n}{2}-2,\frac{1}{2}-\frac{n}{2},\frac{3}{2}-\frac{n}{2},-\tan^2(e+fx)\right)}{f(1-n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Csc[e + f*x])^n*Sec[e + f*x]^6,x]
```

```
[Out] ((b*Csc[e + f*x])^n*Hypergeometric2F1[-2 - n/2, 1/2 - n/2, 3/2 - n/2, -Tan[
e + f*x]^2]*Tan[e + f*x])/(f*(1 - n)*(Sec[e + f*x]^2)^(n/2))
```

Maple [F] time = 0.352, size = 0, normalized size = 0.

$$\int \left(b\csc\left(fx+e\right)\right)^{n} \left(\sec\left(fx+e\right)\right)^{6}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*csc(f*x+e))^n*sec(f*x+e)^6,x)
```

```
[Out] int((b*csc(f*x+e))^n*sec(f*x+e)^6,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\csc\left(fx+e\right)\right)^{n} \sec\left(fx+e\right)^{6}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*csc(f*x+e))^n*sec(f*x+e)^6,x, algorithm="maxima")
```

```
[Out] integrate((b*csc(f*x + e))^n*sec(f*x + e)^6, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(b\csc\left(fx+e\right)\right)^{n} \sec\left(fx+e\right)^{6}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*csc(f*x+e))^n*sec(f*x+e)^6,x, algorithm="fricas")
```

```
[Out] integral((b*csc(f*x + e))^n*sec(f*x + e)^6, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*csc(f*x+e))**n*sec(f*x+e)**6,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\csc\left(fx+e\right)\right)^{n}\sec\left(fx+e\right)^{6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*csc(f*x+e))^n*sec(f*x+e)^6,x, algorithm="giac")
```

```
[Out] integrate((b*csc(f*x + e))^n*sec(f*x + e)^6, x)
```

3.291 $\int (b\csc(e+fx))^n \sec^4(e+fx)\,dx$

Optimal. Leaf size=72

$$\frac{b\sqrt{\cos^2(e+fx)}\sec(e+fx)(b\csc(e+fx))^{n-1}\text{Hypergeometric2F1}\left(\frac{5}{2},\frac{1-n}{2},\frac{3-n}{2},\sin^2(e+fx)\right)}{f(1-n)}$$

```
[Out] (b*Sqrt[Cos[e + f*x]^2]*(b*Csc[e + f*x])^(-1 + n)*Hypergeometric2F1[5/2, (1
 - n)/2, (3 - n)/2, Sin[e + f*x]^2]*Sec[e + f*x])/(f*(1 - n))
```

Rubi [A] time = 0.0794841, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2631, 2577}

$$\frac{b\sqrt{\cos^2(e+fx)}\sec(e+fx)(b\csc(e+fx))^{n-1}\,{}_2F_1\left(\frac{5}{2},\frac{1-n}{2};\frac{3-n}{2};\sin^2(e+fx)\right)}{f(1-n)}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Csc[e + f*x])^n*Sec[e + f*x]^4,x]
```

```
[Out] (b*Sqrt[Cos[e + f*x]^2]*(b*Csc[e + f*x])^(-1 + n)*Hypergeometric2F1[5/2, (1
 - n)/2, (3 - n)/2, Sin[e + f*x]^2]*Sec[e + f*x])/(f*(1 - n))
```

Rule 2631

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Dist[(a^2*(a*Csc[e + f*x])^(m - 1)*(b*Sec[e + f*x])^(n + 1
)*(a*Sin[e + f*x])^(m - 1)*(b*Cos[e + f*x])^(n + 1))/b^2, Int[1/((a*Sin[e +
 f*x])^m*(b*Cos[e + f*x])^n), x], x] /; FreeQ[{a, b, e, f, m, n}, x] &&  !S
implerQ[-m, -n]
```

Rule 2577

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Fra
cPart[(n - 1)/2])*(a*Sin[e + f*x])^(m + 1)*Hypergeometric2F1[(1 + m)/2, (1
- n)/2, (3 + m)/2, Sin[e + f*x]^2])/(a*f*(m + 1)*(Cos[e + f*x]^2)^FracPart[
(n - 1)/2]), x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rubi steps

$$\begin{aligned}\int (b\csc(e+fx))^n\sec^4(e+fx)\,dx &= \left(b^2(b\csc(e+fx))^{-1+n}(b\sin(e+fx))^{-1+n}\right)\int \sec^4(e+fx)(b\sin(e+fx))^{-n}\,dx\\ &= \frac{b\sqrt{\cos^2(e+fx)}(b\csc(e+fx))^{-1+n}\,{}_2F_1\left(\frac{5}{2},\frac{1-n}{2};\frac{3-n}{2};\sin^2(e+fx)\right)\sec(e+fx)}{f(1-n)}\end{aligned}$$

Mathematica [A] time = 0.508711, size = 77, normalized size = 1.07

$$\frac{\tan(e+fx)\sec^2(e+fx)^{-n/2}(b\csc(e+fx))^n\text{Hypergeometric2F1}\left(-\frac{n}{2}-1,\frac{1}{2}-\frac{n}{2},\frac{3}{2}-\frac{n}{2},-\tan^2(e+fx)\right)}{f(1-n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Csc[e + f*x])^n*Sec[e + f*x]^4,x]
```

```
[Out] ((b*Csc[e + f*x])^n*Hypergeometric2F1[-1 - n/2, 1/2 - n/2, 3/2 - n/2, -Tan[
e + f*x]^2]*Tan[e + f*x])/(f*(1 - n)*(Sec[e + f*x]^2)^(n/2))
```

Maple [F] time = 0.351, size = 0, normalized size = 0.

$$\int \left(b \csc \left(f x+e\right)\right)^{n} \left(\sec \left(f x+e\right)\right)^{4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*csc(f*x+e))^n*sec(f*x+e)^4,x)
```

```
[Out] int((b*csc(f*x+e))^n*sec(f*x+e)^4,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b \csc \left(f x+e\right)\right)^{n} \sec \left(f x+e\right)^{4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*csc(f*x+e))^n*sec(f*x+e)^4,x, algorithm="maxima")
```

```
[Out] integrate((b*csc(f*x + e))^n*sec(f*x + e)^4, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(b \csc \left(f x+e\right)\right)^{n} \sec \left(f x+e\right)^{4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*csc(f*x+e))^n*sec(f*x+e)^4,x, algorithm="fricas")
```

```
[Out] integral((b*csc(f*x + e))^n*sec(f*x + e)^4, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*csc(f*x+e))**n*sec(f*x+e)**4,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\csc\left(fx+e\right)\right)^{n}\sec\left(fx+e\right)^{4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*csc(f*x+e))^n*sec(f*x+e)^4,x, algorithm="giac")
```

```
[Out] integrate((b*csc(f*x + e))^n*sec(f*x + e)^4, x)
```

3.292 $\int (b\csc(e+fx))^n \sec^2(e+fx)\,dx$

Optimal. Leaf size=72

$$\frac{b\sqrt{\cos^2(e+fx)}\sec(e+fx)(b\csc(e+fx))^{n-1}\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{1-n}{2},\frac{3-n}{2},\sin^2(e+fx)\right)}{f(1-n)}$$

```
[Out] (b*Sqrt[Cos[e + f*x]^2]*(b*Csc[e + f*x])^(-1 + n)*Hypergeometric2F1[3/2, (1
 - n)/2, (3 - n)/2, Sin[e + f*x]^2]*Sec[e + f*x])/(f*(1 - n))
```

Rubi [A] time = 0.0782317, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2631, 2577}

$$\frac{b\sqrt{\cos^2(e+fx)}\sec(e+fx)(b\csc(e+fx))^{n-1}\,{}_2F_1\left(\frac{3}{2},\frac{1-n}{2};\frac{3-n}{2};\sin^2(e+fx)\right)}{f(1-n)}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Csc[e + f*x])^n*Sec[e + f*x]^2,x]
```

```
[Out] (b*Sqrt[Cos[e + f*x]^2]*(b*Csc[e + f*x])^(-1 + n)*Hypergeometric2F1[3/2, (1
 - n)/2, (3 - n)/2, Sin[e + f*x]^2]*Sec[e + f*x])/(f*(1 - n))
```

Rule 2631

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Dist[(a^2*(a*Csc[e + f*x])^(m - 1)*(b*Sec[e + f*x])^(n + 1
)*(a*Sin[e + f*x])^(m - 1)*(b*Cos[e + f*x])^(n + 1))/b^2, Int[1/((a*Sin[e +
 f*x])^m*(b*Cos[e + f*x])^n), x], x] /; FreeQ[{a, b, e, f, m, n}, x] &&  !S
implerQ[-m, -n]
```

Rule 2577

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Fra
cPart[(n - 1)/2])*(a*Sin[e + f*x])^(m + 1)*Hypergeometric2F1[(1 + m)/2, (1
- n)/2, (3 + m)/2, Sin[e + f*x]^2])/(a*f*(m + 1)*(Cos[e + f*x]^2)^FracPart[
(n - 1)/2]), x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rubi steps

$$\begin{aligned}\int (b\csc(e+fx))^n\sec^2(e+fx)\,dx &= \left(b^2(b\csc(e+fx))^{-1+n}(b\sin(e+fx))^{-1+n}\right)\int \sec^2(e+fx)(b\sin(e+fx))^{-n}\,dx\\ &= \frac{b\sqrt{\cos^2(e+fx)}(b\csc(e+fx))^{-1+n}\,{}_2F_1\left(\frac{3}{2},\frac{1-n}{2};\frac{3-n}{2};\sin^2(e+fx)\right)\sec(e+fx)}{f(1-n)}\end{aligned}$$

Mathematica [A] time = 0.442976, size = 75, normalized size = 1.04

$$\frac{\tan(e+fx)\sec^2(e+fx)^{-n/2}(b\csc(e+fx))^n\text{Hypergeometric2F1}\left(\frac{1}{2}-\frac{n}{2},-\frac{n}{2},\frac{3}{2}-\frac{n}{2},-\tan^2(e+fx)\right)}{f(1-n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Csc[e + f*x])^n*Sec[e + f*x]^2,x]
```

```
[Out] ((b*Csc[e + f*x])^n*Hypergeometric2F1[1/2 - n/2, -n/2, 3/2 - n/2, -Tan[e +
f*x]^2]*Tan[e + f*x])/(f*(1 - n)*(Sec[e + f*x]^2)^(n/2))
```

Maple [F] time = 0.323, size = 0, normalized size = 0.

$$\int \left(b\csc\left(fx+e\right)\right)^{n}\left(\sec\left(fx+e\right)\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*csc(f*x+e))^n*sec(f*x+e)^2,x)
```

```
[Out] int((b*csc(f*x+e))^n*sec(f*x+e)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\csc\left(fx+e\right)\right)^{n}\sec\left(fx+e\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*csc(f*x+e))^n*sec(f*x+e)^2,x, algorithm="maxima")
```

```
[Out] integrate((b*csc(f*x + e))^n*sec(f*x + e)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(b\csc\left(fx+e\right)\right)^{n}\sec\left(fx+e\right)^{2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*csc(f*x+e))^n*sec(f*x+e)^2,x, algorithm="fricas")
```

```
[Out] integral((b*csc(f*x + e))^n*sec(f*x + e)^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\csc\left(e+fx\right)\right)^{n}\sec^{2}\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*csc(f*x+e))**n*sec(f*x+e)**2,x)
```

```
[Out] Integral((b*csc(e + f*x))**n*sec(e + f*x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\csc\left(fx+e\right)\right)^{n}\sec\left(fx+e\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*csc(f*x+e))^n*sec(f*x+e)^2,x, algorithm="giac")
```

```
[Out] integrate((b*csc(f*x + e))^n*sec(f*x + e)^2, x)
```

3.293 $\int (b\csc(e+fx))^n\,dx$

Optimal. Leaf size=72

$$\frac{b\cos(e+fx)(b\csc(e+fx))^{n-1}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1-n}{2},\frac{3-n}{2},\sin^2(e+fx)\right)}{f(1-n)\sqrt{\cos^2(e+fx)}}$$

```
[Out] (b*Cos[e + f*x]*(b*Csc[e + f*x])^(-1 + n)*Hypergeometric2F1[1/2, (1 - n)/2,
 (3 - n)/2, Sin[e + f*x]^2])/(f*(1 - n)*Sqrt[Cos[e + f*x]^2])
```

Rubi [A] time = 0.030244, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {3772, 2643}

$$\frac{b\cos(e+fx)(b\csc(e+fx))^{n-1}\,{}_2F_1\left(\frac{1}{2},\frac{1-n}{2};\frac{3-n}{2};\sin^2(e+fx)\right)}{f(1-n)\sqrt{\cos^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Csc[e + f*x])^n,x]
```

```
[Out] (b*Cos[e + f*x]*(b*Csc[e + f*x])^(-1 + n)*Hypergeometric2F1[1/2, (1 - n)/2,
 (3 - n)/2, Sin[e + f*x]^2])/(f*(1 - n)*Sqrt[Cos[e + f*x]^2])
```

Rule 3772

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] && !IntegerQ[n]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}\int (b\csc(e+fx))^n\,dx &= (b\csc(e+fx))^n\left(\frac{\sin(e+fx)}{b}\right)^n\int\left(\frac{\sin(e+fx)}{b}\right)^{-n}dx\\ &= \frac{\cos(e+fx)(b\csc(e+fx))^n\,{}_2F_1\left(\frac{1}{2},\frac{1-n}{2};\frac{3-n}{2};\sin^2(e+fx)\right)\sin(e+fx)}{f(1-n)\sqrt{\cos^2(e+fx)}}\end{aligned}$$

Mathematica [A] time = 0.10017, size = 65, normalized size = 0.9

$$-\frac{\sin(e+fx)\cos(e+fx)\sin^2(e+fx)^{\frac{n-1}{2}}(b\csc(e+fx))^n\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{n+1}{2},\frac{3}{2},\cos^2(e+fx)\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Csc[e + f*x])^n,x]
```

```
[Out] -((Cos[e + f*x]*(b*Csc[e + f*x])^n*Hypergeometric2F1[1/2, (1 + n)/2, 3/2, C
os[e + f*x]^2]*Sin[e + f*x]*(Sin[e + f*x]^2)^((-1 + n)/2))/f)
```

Maple [F] time = 0.471, size = 0, normalized size = 0.

$$\int \left(b \csc \left(f x + e\right)\right)^{n}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*csc(f*x+e))^n,x)
```

```
[Out] int((b*csc(f*x+e))^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b \csc \left(f x + e\right)\right)^{n}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*csc(f*x+e))^n,x, algorithm="maxima")
```

```
[Out] integrate((b*csc(f*x + e))^n, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(b \csc \left(f x + e\right)\right)^{n}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*csc(f*x+e))^n,x, algorithm="fricas")
```

```
[Out] integral((b*csc(f*x + e))^n, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b \csc \left(e + f x\right)\right)^{n}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*csc(f*x+e))**n,x)
```

```
[Out] Integral((b*csc(e + f*x))**n, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\csc\left(fx+e\right)\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*csc(f*x+e))^n,x, algorithm="giac")
```

```
[Out] integrate((b*csc(f*x + e))^n, x)
```

3.294 $\int \cos^2(e+fx)(b\csc(e+fx))^n\,dx$

Optimal. Leaf size=72

$$\frac{b\cos(e+fx)(b\csc(e+fx))^{n-1}\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1-n}{2},\frac{3-n}{2},\sin^2(e+fx)\right)}{f(1-n)\sqrt{\cos^2(e+fx)}}$$

```
[Out] (b*Cos[e + f*x]*(b*Csc[e + f*x])^(-1 + n)*Hypergeometric2F1[-1/2, (1 - n)/2
, (3 - n)/2, Sin[e + f*x]^2])/(f*(1 - n)*Sqrt[Cos[e + f*x]^2])
```

Rubi [A] time = 0.0796031, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2631, 2577}

$$\frac{b\cos(e+fx)(b\csc(e+fx))^{n-1}\,{}_2F_1\left(-\frac{1}{2},\frac{1-n}{2};\frac{3-n}{2};\sin^2(e+fx)\right)}{f(1-n)\sqrt{\cos^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^2*(b*Csc[e + f*x])^n,x]
```

```
[Out] (b*Cos[e + f*x]*(b*Csc[e + f*x])^(-1 + n)*Hypergeometric2F1[-1/2, (1 - n)/2
, (3 - n)/2, Sin[e + f*x]^2])/(f*(1 - n)*Sqrt[Cos[e + f*x]^2])
```

Rule 2631

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Dist[(a^2*(a*Csc[e + f*x])^(m - 1)*(b*Sec[e + f*x])^(n + 1
)*(a*Sin[e + f*x])^(m - 1)*(b*Cos[e + f*x])^(n + 1))/b^2, Int[1/((a*Sin[e +
 f*x])^m*(b*Cos[e + f*x])^n), x], x] /; FreeQ[{a, b, e, f, m, n}, x] &&  !S
implerQ[-m, -n]
```

Rule 2577

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Fra
cPart[(n - 1)/2])*(a*Sin[e + f*x])^(m + 1)*Hypergeometric2F1[(1 + m)/2, (1
- n)/2, (3 + m)/2, Sin[e + f*x]^2])/(a*f*(m + 1)*(Cos[e + f*x]^2)^FracPart[
(n - 1)/2]), x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rubi steps

$$\int \cos^2(e+fx)(b\csc(e+fx))^n\,dx = \left(b^2(b\csc(e+fx))^{-1+n}(b\sin(e+fx))^{-1+n}\right)\int \cos^2(e+fx)(b\sin(e+fx))^{-n}\,dx$$
$$= \frac{b\cos(e+fx)(b\csc(e+fx))^{-1+n}\,{}_2F_1\left(-\frac{1}{2},\frac{1-n}{2};\frac{3-n}{2};\sin^2(e+fx)\right)}{f(1-n)\sqrt{\cos^2(e+fx)}}$$

Mathematica [B] time = 0.456792, size = 165, normalized size = 2.29

$$-\frac{2\tan\left(\frac{1}{2}(e+fx)\right)\sec^2\left(\frac{1}{2}(e+fx)\right)^{-n}(b\csc(e+fx))^n\left(\text{Hypergeometric2F1}\left(1-n,\frac{1}{2}-\frac{n}{2},\frac{3}{2}-\frac{n}{2},-\tan^2\left(\frac{1}{2}(e+fx)\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^2*(b*Csc[e + f*x])^n,x]
```

```
[Out] (-2*(b*Csc[e + f*x])^n*(Hypergeometric2F1[1 - n, 1/2 - n/2, 3/2 - n/2, -Tan
[(e + f*x)/2]^2] - 4*Hypergeometric2F1[2 - n, 1/2 - n/2, 3/2 - n/2, -Tan[(e
 + f*x)/2]^2] + 4*Hypergeometric2F1[3 - n, 1/2 - n/2, 3/2 - n/2, -Tan[(e +
f*x)/2]^2])*Tan[(e + f*x)/2])/(f*(-1 + n)*(Sec[(e + f*x)/2]^2)^n)
```

Maple [F] time = 0.946, size = 0, normalized size = 0.

$$\int \left(\cos \left(fx+e\right)\right)^{2} \left(b \csc \left(fx+e\right)\right)^{n}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(b*csc(f*x+e))^n,x)
```

```
[Out] int(cos(f*x+e)^2*(b*csc(f*x+e))^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b \csc \left(fx+e\right)\right)^{n} \cos \left(fx+e\right)^{2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(b*csc(f*x+e))^n,x, algorithm="maxima")
```

```
[Out] integrate((b*csc(f*x + e))^n*cos(f*x + e)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b \csc \left(fx+e\right)\right)^{n} \cos \left(fx+e\right)^{2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(b*csc(f*x+e))^n,x, algorithm="fricas")
```

```
[Out] integral((b*csc(f*x + e))^n*cos(f*x + e)^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b \csc \left(e+fx\right)\right)^{n} \cos^{2}\left(e+fx\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(b*csc(f*x+e))**n,x)
```

```
[Out] Integral((b*csc(e + f*x))**n*cos(e + f*x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\csc\left(fx+e\right)\right)^{n}\cos\left(fx+e\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(b*csc(f*x+e))^n,x, algorithm="giac")
```

```
[Out] integrate((b*csc(f*x + e))^n*cos(f*x + e)^2, x)
```

3.295 $\int \cos^4(e+fx)(b\csc(e+fx))^n\,dx$

Optimal. Leaf size=72

$$\frac{b\cos(e+fx)(b\csc(e+fx))^{n-1}\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{1-n}{2},\frac{3-n}{2},\sin^2(e+fx)\right)}{f(1-n)\sqrt{\cos^2(e+fx)}}$$

```
[Out] (b*Cos[e + f*x]*(b*Csc[e + f*x])^(-1 + n)*Hypergeometric2F1[-3/2, (1 - n)/2
, (3 - n)/2, Sin[e + f*x]^2])/(f*(1 - n)*Sqrt[Cos[e + f*x]^2])
```

Rubi [A] time = 0.0763823, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2631, 2577}

$$\frac{b\cos(e+fx)(b\csc(e+fx))^{n-1}\,{}_2F_1\left(-\frac{3}{2},\frac{1-n}{2};\frac{3-n}{2};\sin^2(e+fx)\right)}{f(1-n)\sqrt{\cos^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^4*(b*Csc[e + f*x])^n,x]
```


```
[Out] (b*Cos[e + f*x]*(b*Csc[e + f*x])^(-1 + n)*Hypergeometric2F1[-3/2, (1 - n)/2
, (3 - n)/2, Sin[e + f*x]^2])/(f*(1 - n)*Sqrt[Cos[e + f*x]^2])
```


Rule 2631

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Dist[(a^2*(a*Csc[e + f*x])^(m - 1)*(b*Sec[e + f*x])^(n + 1
)*(a*Sin[e + f*x])^(m - 1)*(b*Cos[e + f*x])^(n + 1))/b^2, Int[1/((a*Sin[e +
 f*x])^m*(b*Cos[e + f*x])^n), x], x] /; FreeQ[{a, b, e, f, m, n}, x] &&  !S
implerQ[-m, -n]
```

Rule 2577

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Fra
cPart[(n - 1)/2])*(a*Sin[e + f*x])^(m + 1)*Hypergeometric2F1[(1 + m)/2, (1
- n)/2, (3 + m)/2, Sin[e + f*x]^2])/(a*f*(m + 1)*(Cos[e + f*x]^2)^FracPart[
(n - 1)/2]), x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rubi steps

$$\begin{aligned}\int \cos^4(e+fx)(b\csc(e+fx))^n\,dx &= \left(b^2(b\csc(e+fx))^{-1+n}(b\sin(e+fx))^{-1+n}\right)\int \cos^4(e+fx)(b\sin(e+fx))^{-n}\,dx\\ &= \frac{b\cos(e+fx)(b\csc(e+fx))^{-1+n}\,{}_2F_1\left(-\frac{3}{2},\frac{1-n}{2};\frac{3-n}{2};\sin^2(e+fx)\right)}{f(1-n)\sqrt{\cos^2(e+fx)}}\end{aligned}$$

Mathematica [B] time = 0.619548, size = 246, normalized size = 3.42

$$-\frac{2\tan\left(\frac{1}{2}(e+fx)\right)\sec^2\left(\frac{1}{2}(e+fx)\right)^{-n}(b\csc(e+fx))^n\left(\text{Hypergeometric2F1}\left(1-n,\frac{1}{2}-\frac{n}{2},\frac{3}{2}-\frac{n}{2},-\tan^2\left(\frac{1}{2}(e+fx)\right)\right)-\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^4*(b*Csc[e + f*x])^n,x]
```

```
[Out] (-2*(b*Csc[e + f*x])^n*(Hypergeometric2F1[1 - n, 1/2 - n/2, 3/2 - n/2, -Tan
[(e + f*x)/2]^2] - 8*(Hypergeometric2F1[2 - n, 1/2 - n/2, 3/2 - n/2, -Tan[(
e + f*x)/2]^2] - 3*Hypergeometric2F1[3 - n, 1/2 - n/2, 3/2 - n/2, -Tan[(e +
 f*x)/2]^2] + 4*Hypergeometric2F1[4 - n, 1/2 - n/2, 3/2 - n/2, -Tan[(e + f*
x)/2]^2] - 2*Hypergeometric2F1[5 - n, 1/2 - n/2, 3/2 - n/2, -Tan[(e + f*x)/
2]^2]))*Tan[(e + f*x)/2])/(f*(-1 + n)*(Sec[(e + f*x)/2]^2)^n)
```

Maple [F] time = 1.065, size = 0, normalized size = 0.

$$\int \left(\cos \left(fx+e\right)\right)^{4}\left(b\csc \left(fx+e\right)\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^4*(b*csc(f*x+e))^n,x)
```

```
[Out] int(cos(f*x+e)^4*(b*csc(f*x+e))^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\csc \left(fx+e\right)\right)^{n}\cos \left(fx+e\right)^{4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4*(b*csc(f*x+e))^n,x, algorithm="maxima")
```

```
[Out] integrate((b*csc(f*x + e))^n*cos(f*x + e)^4, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(b\csc \left(fx+e\right)\right)^{n}\cos \left(fx+e\right)^{4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4*(b*csc(f*x+e))^n,x, algorithm="fricas")
```

```
[Out] integral((b*csc(f*x + e))^n*cos(f*x + e)^4, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**4*(b*csc(f*x+e))**n,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\csc\left(fx+e\right)\right)^{n}\cos\left(fx+e\right)^{4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4*(b*csc(f*x+e))^n,x, algorithm="giac")
```

```
[Out] integrate((b*csc(f*x + e))^n*cos(f*x + e)^4, x)
```

3.296 $\int (b\csc(e+fx))^n (c\sec(e+fx))^{3/2}\,dx$

Optimal. Leaf size=81

$$\frac{b\cos^2(e+fx)^{5/4}(c\sec(e+fx))^{5/2}(b\csc(e+fx))^{n-1}\text{Hypergeometric2F1}\left(\frac{5}{4},\frac{1-n}{2},\frac{3-n}{2},\sin^2(e+fx)\right)}{cf(1-n)}$$

```
[Out] (b*(Cos[e + f*x]^2)^(5/4)*(b*Csc[e + f*x])^(-1 + n)*Hypergeometric2F1[5/4,
(1 - n)/2, (3 - n)/2, Sin[e + f*x]^2]*(c*Sec[e + f*x])^(5/2))/(c*f*(1 - n))
```

Rubi [A] time = 0.114924, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2631, 2577}

$$\frac{b\cos^2(e+fx)^{5/4}(c\sec(e+fx))^{5/2}(b\csc(e+fx))^{n-1}\,{}_2F_1\left(\frac{5}{4},\frac{1-n}{2};\frac{3-n}{2};\sin^2(e+fx)\right)}{cf(1-n)}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Csc[e + f*x])^n*(c*Sec[e + f*x])^(3/2),x]
```

```
[Out] (b*(Cos[e + f*x]^2)^(5/4)*(b*Csc[e + f*x])^(-1 + n)*Hypergeometric2F1[5/4,
(1 - n)/2, (3 - n)/2, Sin[e + f*x]^2]*(c*Sec[e + f*x])^(5/2))/(c*f*(1 - n))
```

Rule 2631

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Dist[(a^2*(a*Csc[e + f*x])^(m - 1)*(b*Sec[e + f*x])^(n + 1
)*(a*Sin[e + f*x])^(m - 1)*(b*Cos[e + f*x])^(n + 1))/b^2, Int[1/((a*Sin[e +
 f*x])^m*(b*Cos[e + f*x])^n), x], x] /; FreeQ[{a, b, e, f, m, n}, x] &&  !S
implerQ[-m, -n]
```

Rule 2577

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Fra
cPart[(n - 1)/2])*(a*Sin[e + f*x])^(m + 1)*Hypergeometric2F1[(1 + m)/2, (1
- n)/2, (3 + m)/2, Sin[e + f*x]^2])/(a*f*(m + 1)*(Cos[e + f*x]^2)^FracPart[
(n - 1)/2]), x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rubi steps

$$\int (b\csc(e+fx))^n(c\sec(e+fx))^{3/2}\,dx = \frac{\left(b^2(c\cos(e+fx))^{5/2}(b\csc(e+fx))^{-1+n}(c\sec(e+fx))^{5/2}(b\sin(e+fx))^{-1+n}\right)}{c^2}$$

$$= \frac{b\cos^2(e+fx)^{5/4}(b\csc(e+fx))^{-1+n}\,{}_2F_1\left(\frac{5}{4},\frac{1-n}{2};\frac{3-n}{2};\sin^2(e+fx)\right)(c\sec(e+}{cf(1-n)}$$

Mathematica [A] time = 1.82734, size = 92, normalized size = 1.14

$$\frac{2\cot(e+fx)(c\sec(e+fx))^{3/2}\left(-\tan^2(e+fx)\right)^{\frac{n+1}{2}}(b\csc(e+fx))^n\text{Hypergeometric2F1}\left(\frac{n+1}{2},\frac{1}{4}(2n+3),\frac{1}{4}(2n+7),\right.}{f(2n+3)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Csc[e + f*x])^n*(c*Sec[e + f*x])^(3/2),x]
```

```
[Out] (2*Cot[e + f*x]*(b*Csc[e + f*x])^n*Hypergeometric2F1[(1 + n)/2, (3 + 2*n)/4
, (7 + 2*n)/4, Sec[e + f*x]^2]*(c*Sec[e + f*x])^(3/2)*(-Tan[e + f*x]^2)^((1
 + n)/2))/(f*(3 + 2*n))
```

Maple [F] time = 0.159, size = 0, normalized size = 0.

$$\int \left(b \csc \left(f x+e\right)\right)^{n} \left(c \sec \left(f x+e\right)\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*csc(f*x+e))^n*(c*sec(f*x+e))^(3/2),x)
```

```
[Out] int((b*csc(f*x+e))^n*(c*sec(f*x+e))^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(c \sec \left(f x+e\right)\right)^{\frac{3}{2}} \left(b \csc \left(f x+e\right)\right)^{n}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*csc(f*x+e))^n*(c*sec(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((c*sec(f*x + e))^(3/2)*(b*csc(f*x + e))^n, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\sqrt{c \sec \left(f x+e\right)} \left(b \csc \left(f x+e\right)\right)^{n} c \sec \left(f x+e\right), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*csc(f*x+e))^n*(c*sec(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*sec(f*x + e))*(b*csc(f*x + e))^n*c*sec(f*x + e), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*csc(f*x+e))**n*(c*sec(f*x+e))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (c\sec(fx+e))^{\frac{3}{2}} (b\csc(fx+e))^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*csc(f*x+e))^n*(c*sec(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((c*sec(f*x + e))^(3/2)*(b*csc(f*x + e))^n, x)
```

3.297 $\int (b\csc(e+fx))^n \sqrt{c\sec(e+fx)}\,dx$

Optimal. Leaf size=81

$$\frac{b\cos^2(e+fx)^{3/4}(c\sec(e+fx))^{3/2}(b\csc(e+fx))^{n-1}\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{1-n}{2},\frac{3-n}{2},\sin^2(e+fx)\right)}{cf(1-n)}$$

```
[Out] (b*(Cos[e + f*x]^2)^(3/4)*(b*Csc[e + f*x])^(-1 + n)*Hypergeometric2F1[3/4,
(1 - n)/2, (3 - n)/2, Sin[e + f*x]^2]*(c*Sec[e + f*x])^(3/2))/(c*f*(1 - n))
```

Rubi [A] time = 0.0971722, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2631, 2577}

$$\frac{b\cos^2(e+fx)^{3/4}(c\sec(e+fx))^{3/2}(b\csc(e+fx))^{n-1}\,{}_2F_1\left(\frac{3}{4},\frac{1-n}{2};\frac{3-n}{2};\sin^2(e+fx)\right)}{cf(1-n)}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Csc[e + f*x])^n*Sqrt[c*Sec[e + f*x]],x]
```

```
[Out] (b*(Cos[e + f*x]^2)^(3/4)*(b*Csc[e + f*x])^(-1 + n)*Hypergeometric2F1[3/4,
(1 - n)/2, (3 - n)/2, Sin[e + f*x]^2]*(c*Sec[e + f*x])^(3/2))/(c*f*(1 - n))
```

Rule 2631

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Dist[(a^2*(a*Csc[e + f*x])^(m - 1)*(b*Sec[e + f*x])^(n + 1
)*(a*Sin[e + f*x])^(m - 1)*(b*Cos[e + f*x])^(n + 1))/b^2, Int[1/((a*Sin[e +
 f*x])^m*(b*Cos[e + f*x])^n), x], x] /; FreeQ[{a, b, e, f, m, n}, x] && !S
implerQ[-m, -n]
```

Rule 2577

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Fra
cPart[(n - 1)/2])*(a*Sin[e + f*x])^(m + 1)*Hypergeometric2F1[(1 + m)/2, (1
- n)/2, (3 + m)/2, Sin[e + f*x]^2])/(a*f*(m + 1)*(Cos[e + f*x]^2)^FracPart[
(n - 1)/2]), x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rubi steps

$$\int (b\csc(e+fx))^n\sqrt{c\sec(e+fx)}\,dx = \frac{\left(b^2(c\cos(e+fx))^{3/2}(b\csc(e+fx))^{-1+n}(c\sec(e+fx))^{3/2}(b\sin(e+fx))^{-1+n}\right)\int \frac{(b\sin\ldots}{\sqrt{c\ldots}}}{c^2}$$

$$= \frac{b\cos^2(e+fx)^{3/4}(b\csc(e+fx))^{-1+n}\,{}_2F_1\left(\frac{3}{4},\frac{1-n}{2};\frac{3-n}{2};\sin^2(e+fx)\right)(c\sec(e+fx))^{3\ldots}}{cf(1-n)}$$

Mathematica [A] time = 1.61025, size = 90, normalized size = 1.11

$$\frac{2\cot(e+fx)\sqrt{c\sec(e+fx)}\left(-\tan^2(e+fx)\right)^{\frac{n+1}{2}}(b\csc(e+fx))^n\text{Hypergeometric2F1}\left(\frac{n+1}{2},\frac{1}{4}(2n+1),\frac{1}{4}(2n+5),\sec^2(\ldots\right.}{2fn+f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Csc[e + f*x])^n*Sqrt[c*Sec[e + f*x]],x]
```

```
[Out] (2*Cot[e + f*x]*(b*Csc[e + f*x])^n*Hypergeometric2F1[(1 + n)/2, (1 + 2*n)/4
, (5 + 2*n)/4, Sec[e + f*x]^2]*Sqrt[c*Sec[e + f*x]]*(-Tan[e + f*x]^2)^((1 +
 n)/2))/(f + 2*f*n)
```

Maple [F] time = 0.152, size = 0, normalized size = 0.

$$\int \left(b \csc \left(fx + e\right)\right)^{n} \sqrt{c \sec \left(fx + e\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*csc(f*x+e))^n*(c*sec(f*x+e))^(1/2),x)
```

```
[Out] int((b*csc(f*x+e))^n*(c*sec(f*x+e))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{c \sec \left(fx + e\right)} \left(b \csc \left(fx + e\right)\right)^{n}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*csc(f*x+e))^n*(c*sec(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*sec(f*x + e))*(b*csc(f*x + e))^n, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{c \sec \left(fx + e\right)} \left(b \csc \left(fx + e\right)\right)^{n}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*csc(f*x+e))^n*(c*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*sec(f*x + e))*(b*csc(f*x + e))^n, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b \csc \left(e + fx\right)\right)^{n} \sqrt{c \sec \left(e + fx\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*csc(f*x+e))**n*(c*sec(f*x+e))**(1/2),x)
```

```
[Out] Integral((b*csc(e + f*x))**n*sqrt(c*sec(e + f*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{c \sec \left(fx+e\right)} \left(b \csc \left(fx+e\right)\right)^{n} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*csc(f*x+e))^n*(c*sec(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*sec(f*x + e))*(b*csc(f*x + e))^n, x)
```

3.298 $\int \frac{(b\csc(e+fx))^n}{\sqrt{c\sec(e+fx)}}\,dx$

Optimal. Leaf size=81

$$\frac{b\sqrt[4]{\cos^2(e+fx)}\sqrt{c\sec(e+fx)}(b\csc(e+fx))^{n-1}\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1-n}{2},\frac{3-n}{2},\sin^2(e+fx)\right)}{cf(1-n)}$$

```
[Out] (b*(Cos[e + f*x]^2)^(1/4)*(b*Csc[e + f*x])^(-1 + n)*Hypergeometric2F1[1/4,
(1 - n)/2, (3 - n)/2, Sin[e + f*x]^2]*Sqrt[c*Sec[e + f*x]])/(c*f*(1 - n))
```

Rubi [A] time = 0.0958881, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2631, 2577}

$$\frac{b\sqrt[4]{\cos^2(e+fx)}\sqrt{c\sec(e+fx)}(b\csc(e+fx))^{n-1}\,{}_2F_1\left(\frac{1}{4},\frac{1-n}{2};\frac{3-n}{2};\sin^2(e+fx)\right)}{cf(1-n)}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Csc[e + f*x])^n/Sqrt[c*Sec[e + f*x]],x]
```

```
[Out] (b*(Cos[e + f*x]^2)^(1/4)*(b*Csc[e + f*x])^(-1 + n)*Hypergeometric2F1[1/4,
(1 - n)/2, (3 - n)/2, Sin[e + f*x]^2]*Sqrt[c*Sec[e + f*x]])/(c*f*(1 - n))
```

Rule 2631

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Dist[(a^2*(a*Csc[e + f*x])^(m - 1)*(b*Sec[e + f*x])^(n + 1
)*(a*Sin[e + f*x])^(m - 1)*(b*Cos[e + f*x])^(n + 1))/b^2, Int[1/((a*Sin[e +
 f*x])^m*(b*Cos[e + f*x])^n), x], x] /; FreeQ[{a, b, e, f, m, n}, x] &&  !S
implerQ[-m, -n]
```

Rule 2577

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Fra
cPart[(n - 1)/2])*(a*Sin[e + f*x])^(m + 1)*Hypergeometric2F1[(1 + m)/2, (1
- n)/2, (3 + m)/2, Sin[e + f*x]^2])/(a*f*(m + 1)*(Cos[e + f*x]^2)^FracPart[
(n - 1)/2]), x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{(b\csc(e+fx))^n}{\sqrt{c\sec(e+fx)}}\,dx = \frac{\left(b^2\sqrt{c\cos(e+fx)}(b\csc(e+fx))^{-1+n}\sqrt{c\sec(e+fx)}(b\sin(e+fx))^{-1+n}\right)\int \sqrt{c\cos(e+fx)}(b\text{s}}{c^2}$$

$$= \frac{b\sqrt[4]{\cos^2(e+fx)}(b\csc(e+fx))^{-1+n}\,{}_2F_1\left(\frac{1}{4},\frac{1-n}{2};\frac{3-n}{2};\sin^2(e+fx)\right)\sqrt{c\sec(e+fx)}}{cf(1-n)}$$

Mathematica [C] time = 3.10075, size = 326, normalized size = 4.02

$$-\frac{4(n-3)\sin\left(\frac{1}{2}(e+fx)\right)\cos^3(}{f(n-1)\sqrt{c\sec(e+fx)}\left(2(3-2n)\sin^2\left(\frac{1}{2}(e+fx)\right)F_1\left(\frac{3}{2}-\frac{n}{2};-\frac{1}{2},\frac{5}{2}-n;\frac{5}{2}-\frac{n}{2};\tan^2\left(\frac{1}{2}(e+fx)\right),-\tan^2\left(\frac{1}{2}(e+fx)\right)\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(b*Csc[e + f*x])^n/Sqrt[c*Sec[e + f*x]],x]
```

```
[Out] (-4*(-3 + n)*AppellF1[1/2 - n/2, -1/2, 3/2 - n, 3/2 - n/2, Tan[(e + f*x)/2]
^2, -Tan[(e + f*x)/2]^2]*Cos[(e + f*x)/2]^3*(b*Csc[e + f*x])^n*Sin[(e + f*x
)/2])/(f*(-1 + n)*Sqrt[c*Sec[e + f*x]]*(2*(-3 + n)*AppellF1[1/2 - n/2, -1/2
, 3/2 - n, 3/2 - n/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Cos[(e + f*x
)/2]^2 - AppellF1[3/2 - n/2, 1/2, 3/2 - n, 5/2 - n/2, Tan[(e + f*x)/2]^2, -
Tan[(e + f*x)/2]^2]*(-1 + Cos[e + f*x]) + 2*(3 - 2*n)*AppellF1[3/2 - n/2, -
1/2, 5/2 - n, 5/2 - n/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sin[(e +
f*x)/2]^2))
```

Maple [F] time = 0.144, size = 0, normalized size = 0.

$$\int \left(b \csc \left(fx+e\right)\right)^{n} \frac{1}{\sqrt{c \sec \left(fx+e\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*csc(f*x+e))^n/(c*sec(f*x+e))^(1/2),x)
```

```
[Out] int((b*csc(f*x+e))^n/(c*sec(f*x+e))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b \csc \left(fx+e\right)\right)^{n}}{\sqrt{c \sec \left(fx+e\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*csc(f*x+e))^n/(c*sec(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b*csc(f*x + e))^n/sqrt(c*sec(f*x + e)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{c \sec \left(fx+e\right)} \left(b \csc \left(fx+e\right)\right)^{n}}{c \sec \left(fx+e\right)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*csc(f*x+e))^n/(c*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*sec(f*x + e))*(b*csc(f*x + e))^n/(c*sec(f*x + e)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b\csc\left(e+fx\right)\right)^{n}}{\sqrt{c\sec\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*csc(f*x+e))**n/(c*sec(f*x+e))**(1/2),x)
```

```
[Out] Integral((b*csc(e + f*x))**n/sqrt(c*sec(e + f*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b\csc\left(fx+e\right)\right)^{n}}{\sqrt{c\sec\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*csc(f*x+e))^n/(c*sec(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*csc(f*x + e))^n/sqrt(c*sec(f*x + e)), x)
```

3.299 $\int \frac{(b\csc(e+fx))^n}{(c\sec(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=81

$$\frac{b(b\csc(e+fx))^{n-1}\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1-n}{2},\frac{3-n}{2},\sin^2(e+fx)\right)}{cf(1-n)\sqrt[4]{\cos^2(e+fx)}\sqrt{c\sec(e+fx)}}$$

```
[Out] (b*(b*Csc[e + f*x])^(-1 + n)*Hypergeometric2F1[-1/4, (1 - n)/2, (3 - n)/2,
Sin[e + f*x]^2])/(c*f*(1 - n)*(Cos[e + f*x]^2)^(1/4)*Sqrt[c*Sec[e + f*x]])
```

Rubi [A] time = 0.107255, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2631, 2577}

$$\frac{b(b\csc(e+fx))^{n-1}{}_2F_1\left(-\frac{1}{4},\frac{1-n}{2};\frac{3-n}{2};\sin^2(e+fx)\right)}{cf(1-n)\sqrt[4]{\cos^2(e+fx)}\sqrt{c\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Csc[e + f*x])^n/(c*Sec[e + f*x])^(3/2),x]
```

```
[Out] (b*(b*Csc[e + f*x])^(-1 + n)*Hypergeometric2F1[-1/4, (1 - n)/2, (3 - n)/2,
Sin[e + f*x]^2])/(c*f*(1 - n)*(Cos[e + f*x]^2)^(1/4)*Sqrt[c*Sec[e + f*x]])
```

Rule 2631

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Dist[(a^2*(a*Csc[e + f*x])^(m - 1)*(b*Sec[e + f*x])^(n + 1
)*(a*Sin[e + f*x])^(m - 1)*(b*Cos[e + f*x])^(n + 1))/b^2, Int[1/((a*Sin[e +
 f*x])^m*(b*Cos[e + f*x])^n), x], x] /; FreeQ[{a, b, e, f, m, n}, x] &&  !S
implerQ[-m, -n]
```

Rule 2577

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Fra
cPart[(n - 1)/2])*(a*Sin[e + f*x])^(m + 1)*Hypergeometric2F1[(1 + m)/2, (1
- n)/2, (3 + m)/2, Sin[e + f*x]^2])/(a*f*(m + 1)*(Cos[e + f*x]^2)^FracPart[
(n - 1)/2]), x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{(b\csc(e+fx))^n}{(c\sec(e+fx))^{3/2}}\,dx &= \frac{\left(b^2(b\csc(e+fx))^{-1+n}(b\sin(e+fx))^{-1+n}\right)\int (c\cos(e+fx))^{3/2}(b\sin(e+fx))^{-n}\,dx}{c^2\sqrt{c\cos(e+fx)}\sqrt{c\sec(e+fx)}}\\ &= \frac{b(b\csc(e+fx))^{-1+n}{}_2F_1\left(-\frac{1}{4},\frac{1-n}{2};\frac{3-n}{2};\sin^2(e+fx)\right)}{cf(1-n)\sqrt[4]{\cos^2(e+fx)}\sqrt{c\sec(e+fx)}}\end{aligned}$$

Mathematica [A] time = 1.00577, size = 115, normalized size = 1.42

$$-\frac{2\cos(2(e+fx))\cot(e+fx)\sqrt{c\sec(e+fx)}\left(-\tan^2(e+fx)\right)^{\frac{n+1}{2}}(b\csc(e+fx))^n\text{Hypergeometric2F1}\left(\frac{n+1}{2},\frac{1}{4}(2n-3),\right.}{c^2f(2n-3)\left(\sec^2(e+fx)-2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Csc[e + f*x])^n/(c*Sec[e + f*x])^(3/2),x]
```

```
[Out] (-2*Cos[2*(e + f*x)]*Cot[e + f*x]*(b*Csc[e + f*x])^n*Hypergeometric2F1[(1 +
 n)/2, (-3 + 2*n)/4, (1 + 2*n)/4, Sec[e + f*x]^2]*Sqrt[c*Sec[e + f*x]]*(-Ta
n[e + f*x]^2)^((1 + n)/2))/(c^2*f*(-3 + 2*n)*(-2 + Sec[e + f*x]^2))
```

Maple [F] time = 0.136, size = 0, normalized size = 0.

$$\int \left(b \csc \left(fx+e\right)\right)^{n} \left(c \sec \left(fx+e\right)\right)^{-\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*csc(f*x+e))^n/(c*sec(f*x+e))^(3/2),x)
```

```
[Out] int((b*csc(f*x+e))^n/(c*sec(f*x+e))^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b \csc \left(fx+e\right)\right)^{n}}{\left(c \sec \left(fx+e\right)\right)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*csc(f*x+e))^n/(c*sec(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*csc(f*x + e))^n/(c*sec(f*x + e))^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{c \sec \left(fx+e\right)} \left(b \csc \left(fx+e\right)\right)^{n}}{c^{2} \sec \left(fx+e\right)^{2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*csc(f*x+e))^n/(c*sec(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*sec(f*x + e))*(b*csc(f*x + e))^n/(c^2*sec(f*x + e)^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b \csc \left(e+fx\right)\right)^{n}}{\left(c \sec \left(e+fx\right)\right)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*csc(f*x+e))**n/(c*sec(f*x+e))**(3/2),x)
```

```
[Out] Integral((b*csc(e + f*x))**n/(c*sec(e + f*x))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b\csc\left(fx+e\right)\right)^{n}}{\left(c\sec\left(fx+e\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*csc(f*x+e))^n/(c*sec(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*csc(f*x + e))^n/(c*sec(f*x + e))^(3/2), x)
```

Chapter 4

Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.0.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*          GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] :=
  If[ExpnType[result]<=ExpnType[optimal],
    If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
      If[LeafCount[result]<=2*LeafCount[optimal],
        "A",
      "B"],
    "C"],
  If[FreeQ[result,Integrate] && FreeQ[result,Int],
    "C",
  "F"]]


(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
```

```
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
        Exp,Log,
        Sin,Cos,Tan,Cot,Sec,Csc,
        ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
        Sinh,Cosh,Tanh,Coth,Sech,Csch,
        ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
        Erf, Erfc, Erfi,
        FresnelS, FresnelC,
        ExpIntegralE, ExpIntegralEi, LogIntegral,
        SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
        Gamma, LogGamma, PolyGamma,
        Zeta, PolyLog, ProductLog,
        EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]
```

```
AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.0.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems

GradeAntiderivative := proc(result,optimal)
local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal,
    debug:=false;

      leaf_count_result:=leafcount(result);
      #do NOT call ExpnType() if leaf size is too large. Recursion problem
      if leaf_count_result > 500000 then
          return "B";
      fi;

      leaf_count_optimal:=leafcount(optimal);

      ExpnType_result:=ExpnType(result);
      ExpnType_optimal:=ExpnType(optimal);

      if debug then
         print("ExpnType_result",ExpnType_result," ExpnType_optimal=",
    ExpnType_optimal);
      fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

  #This check below actually is not needed, since I only
  #call this grading only for passed integrals. i.e. I check
  #for "F" before calling this. But no harm of keeping it here.
  #just in case.


  if not type(result,freeof('int')) then
     return "F";
  end if;


  if ExpnType_result<=ExpnType_optimal then
      if debug then
           print("ExpnType_result<=ExpnType_optimal");
      fi;
      if is_contains_complex(result) then
         if is_contains_complex(optimal) then
            if debug then
```

```
               print("both result and optimal complex");
            fi;
            #both result and optimal complex
            if leaf_count_result<=2*leaf_count_optimal then
               return "A";
            else
               return "B";
            end if
         else #result contains complex but optimal is not
            if debug then
               print("result contains complex but optimal is not");
            fi;
            return "C";
         end if
      else # result do not contain complex
           # this assumes optimal do not as well
           if debug then
               print("result do not contain complex, this assumes optimal do not as well");
           fi;
           if leaf_count_result<=2*leaf_count_optimal then
              if debug then
                 print("leaf_count_result<=2*leaf_count_optimal");
              fi;
              return "A";
           else
              if debug then
                 print("leaf_count_result>2*leaf_count_optimal");
              fi;
              return "B";
           end if
      end if
  else #ExpnType(result) > ExpnType(optimal)
      if debug then
         print("ExpnType(result) > ExpnType(optimal)");
      fi;
      return "C";
  end if

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
```

```
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
```

```
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.0.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
           return True
```

```
        else:
           return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    leaf_count_result  = leaf_count(result)
```

```
    leaf_count_optimal = leaf_count(optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        return "F"

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
   well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```

4.0.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

def tree(expr):
    debug=False;
    if debug:
         print ("Enter tree(expr), expr=",expr)
         print ("expr.operator()=",expr.operator())
         print ("expr.operands()=",expr.operands())
         print ("map(tree, expr.operands()=",map(tree, expr.operands()))

    if expr.operator() is None:
        return expr
    else:
        return [expr.operator()]+list(map(tree, expr.operands()))

def leaf_count(anti):
    debug=False;

    if debug: print ("Enter leaf_count, anti=", anti, " len(anti)=", len(anti))

    if len(anti) == 0: #special check for optimal being 0 for some test cases.
       if debug: print ("len(anti) == 0")
       return 1
    else:
        if debug: print ("round(1.35*len(flatten(tree(anti))))=",round(1.35*len
   (flatten(tree(anti)))))
        return round(1.35*len(flatten(tree(anti)))) #fudge factor
                     #since this estimate of leaf count is bit lower than
```

```
                      #what it should be compared to Mathematica's

def is_sqrt(expr):
    debug=False;
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    debug = False

    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    debug = False

    if debug: print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e',
           'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath

```

```
def is_atom(expn):

    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-type-in-maple/
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()
        return False

    except AttributeError as error:
        return False


def expnType(expn):
    debug=False

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
            return expnType(expn.operands()[0])   #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinstance(expn,Mul)
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
```

```
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): # is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):
    debug = False;

    if debug: print ("Enter grade_antiderivative for sagemath")

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    if debug: print ("leaf_count_result=", leaf_count_result, " leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if debug: print ("expnType_result=", expnType_result, "expnType_optimal=", expnType_optimal)

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```